Issues in Dark Energy

& Modified Gravity

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Issues *related to* ***Dark Energy*** *&* ***Modified Gravity***

- *Dark Radiation* (maybe the only surprise in cosmology in recent years)

  from *dark energy*, *modified gravity*, or others?

- *Stability of the GR Limit in Modified Gravity*

– –

- *Unification of Scalar Fields*

- *Cosmological Constant Problem*

- *New Aspects of Gravity*

Dark Radiation

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Dark Radiation

(maybe the only surprise in cosmology in recent years)

Extra effective *relativistic* degrees of freedom, suggested by
- CMB observations, and consistent with
- light element observations compared with BBN prediction.

(conventional *fitting formula* for background expansion)

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\rho_{rad} + \rho_m + \rho_{DE}\right), \qquad \rho_{rad} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right]\rho_\gamma$$

- Energy contents of the universe in *Standard Cosmology*:
  baryons, electrons, photons, neutrinos, cold dark matter, dark energy.
  (photons: relativistic; neutrinos: relativistic (as least at early times))

- *Standard Model* of particle physics: 3 neutrino species → $N_{eff} = 3.046$
- Observations suggest: $N_{eff} = 4.34^{+0.86}_{-0.88}$ (68% CL) [WMAP7 & LSS]

⇒ *1 or 2 extra relativistic d.o.f. !!*

Observational & Experimental Constraints

- Number of light neutrino species: 2.984 ± 0.008 (decay width of Z boson [PDG2010])

- Number of effective relativistic particle species (d.o.f.):

Primordial ^{4}He abundance

- astro-ph/0408033 Cyburt: $N_{eff}(\text{BBN}) = 3.2 \pm 1.2$ (95% CL)
- 0803.3465 Simha & Steigman: $N_{eff}(\text{BBN}) = 2.4 \pm 0.4$ (68% CL)
- 1001.4440 Izotov & Thuan: $N_{eff}(\text{BBN}) = 3.68 +0.80 -0.70$ (2σ)
  Primordial ^{4}He abundance: $Y_p = 0.2565 \pm 0.0010(\text{stat}) \pm 0.0050(\text{syst})$
- 1001.5218 Aver, Olive & Skillman: $Y_p = 0.2561 \pm 0.0108$

CMB

- 1001.4538 Komatzu et al: $N_{eff}(\text{CMB}) = 4.34 +0.86 -0.88$ (68% CL)
  WMAP7 + BAO + H_0 (WMAP: 1st & 3rd peaks)
- 1009.0866 Dunkley et al: $N_{eff}(\text{CMB}) = 4.56 \pm 0.75$ (68% CL)
  ACT + BAO + H_0 (ACT: 3rd through 7th peaks)
- 1109.2767 Archidiacono, Calabrese, & Melchiorri: $N_{eff}(\text{CMB}) = 4.08 +0.71 -0.68$ (95% CL)
  WMAP7 + ACBAR + ACT + SPT + SDSS-DR7 + H_0
- Planck: $\delta N_{eff}(\text{CMB}) \approx 0.26$

Effects of additional *Relativistic d.o.f.*

BBN

- Make ν-*e*, ν-*p*-*n* decouple earlier (increase $T^{\nu}_{\rm de}$)
  $\Rightarrow$ increase n_n/n_p
  $\Rightarrow$ change abundance of light element (e.g. *more* ^{4}He)

***N*$_{\rm eff}$ ≥ 4 :** consistent with (but not required by) data

CMB

- Make *matter-radiation equality* later (z_{EQ} decrease)
  $\Rightarrow$ *enhance* early-time *integrated Sachs-Wolfe effect*
  favored by data
- If ***NOT*** *coupled* to the photon-baryon fluid (i.e. ***dark***)
  $\Rightarrow$ *free-streaming* out of grav. potential wells faster than c_s
  $\Rightarrow$ *phase shift* & *damping* of acoustic peaks
  favored by data
- If *coupled* to the photon-baryon fluid (i.e. not dark)
  $\Rightarrow$ largely change CMB spectrum (dangerous)
  NOT favored by data

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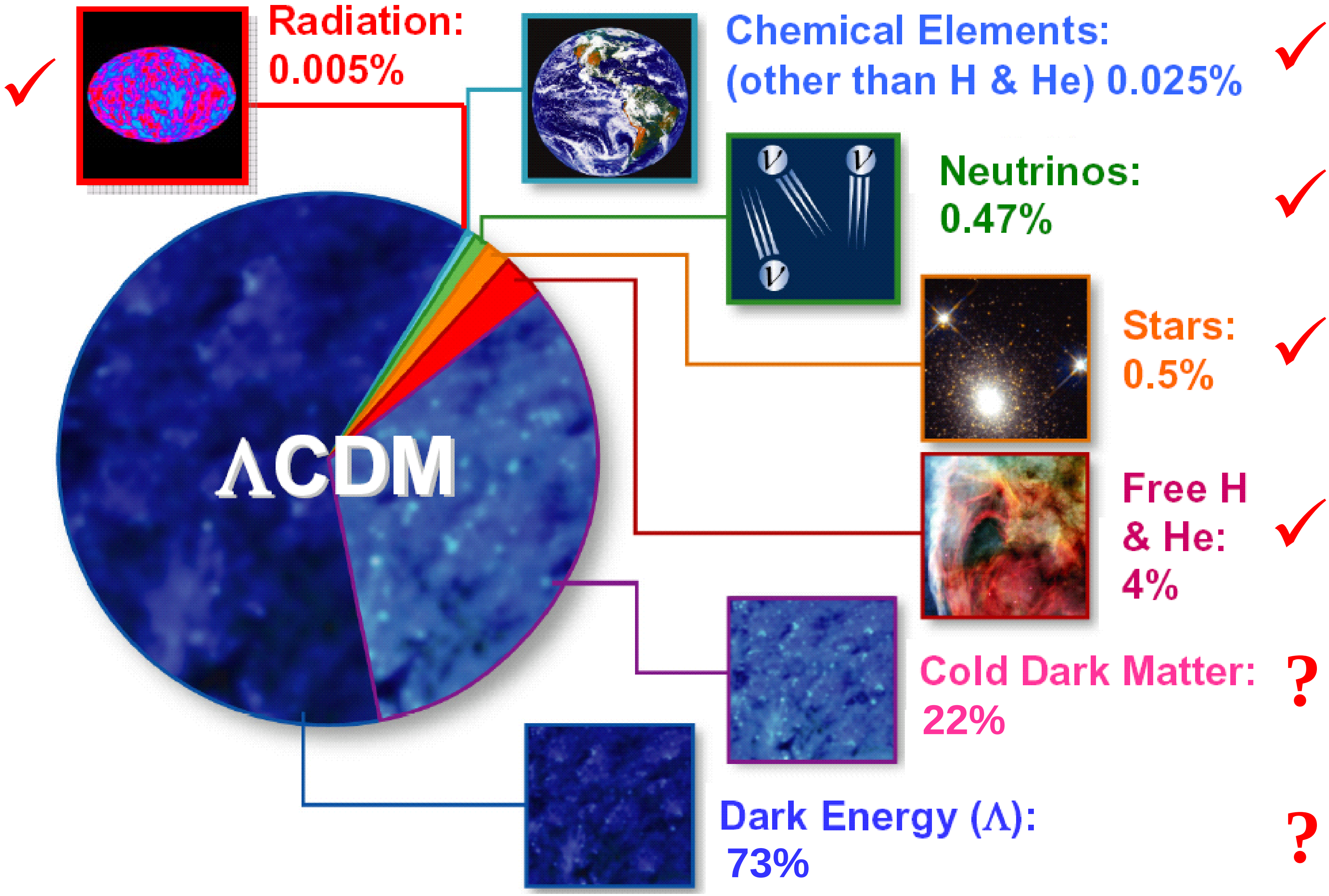
$$H^2 + k/a^2 = 8\pi G\left(\boxed{\rho_{rad} + \rho_b} + \boxed{\rho_{CDM} + \rho_{DE}} + \rho_{DR}\right)/3$$



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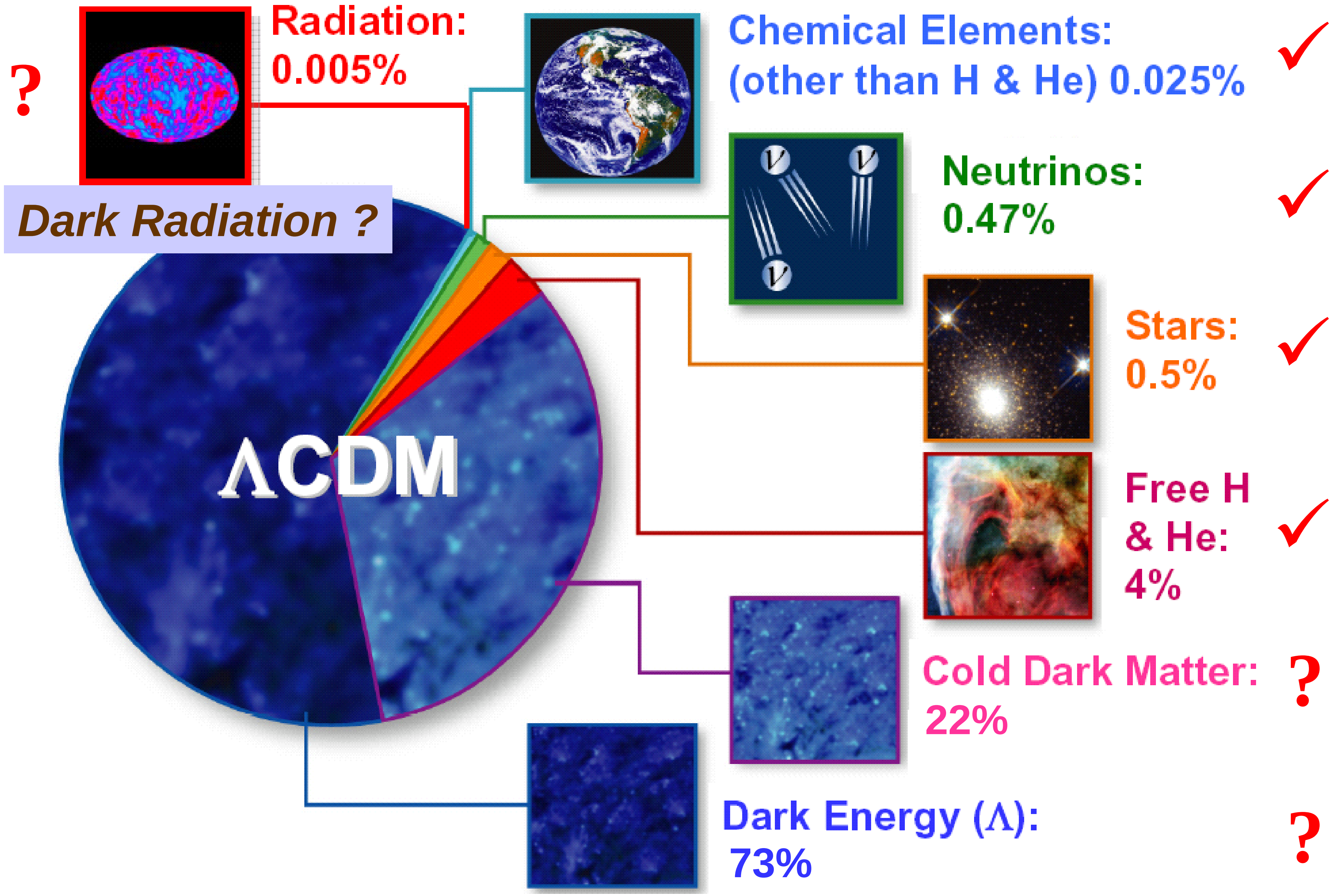
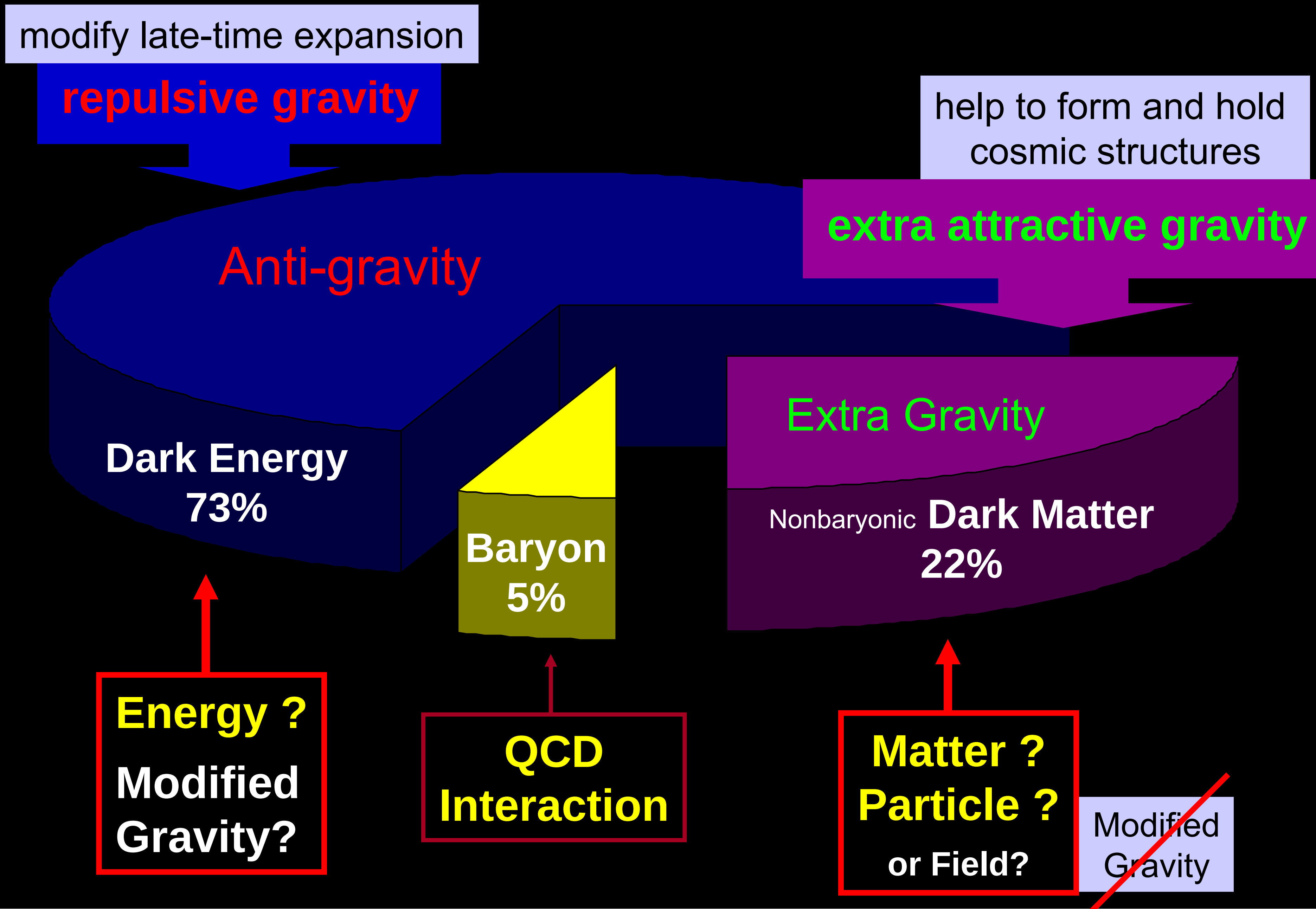
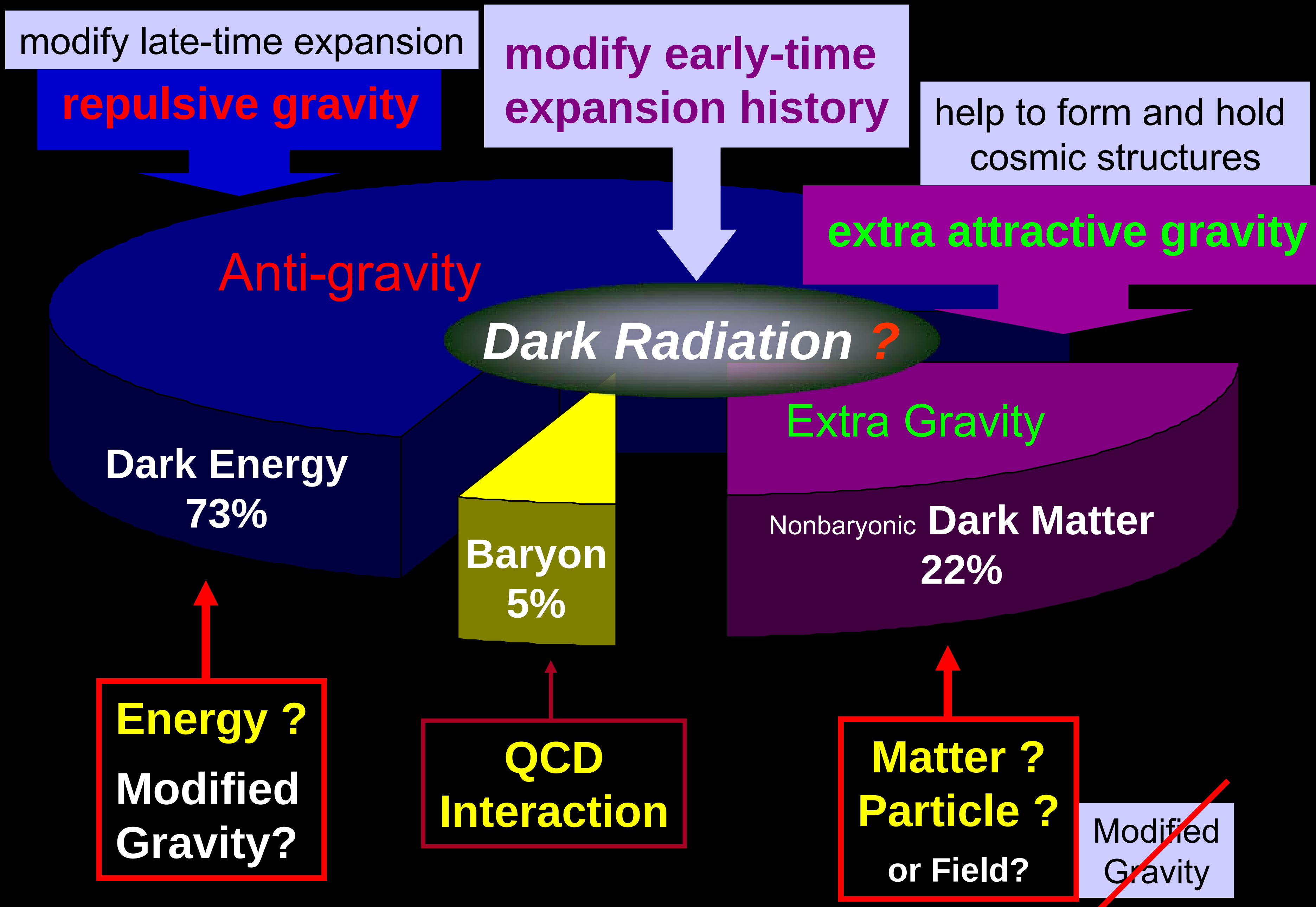



modify late-time expansion
repulsive gravity
help to form and hold cosmic structures
extra attractive gravity
Anti-gravity
Extra Gravity
Dark Energy 73%
Baryon 5%
Nonbaryonic Dark Matter 22%
Energy ?
Modified Gravity?
QCD Interaction
Matter ?
Particle ?
or Field?
Modified Gravity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Dark Radiation

Dark Radiation:

Effective relativistic degrees of freedom, suggested by

- CMB observations, and consistent with
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– –

Candidates

- sterile neutrino
- *early dark energy* ($w_{early} \sim w_r = 1/3$, while $w_{late} < -1/3$)
- decaying particles
- nonstandard thermal history
- higher-dimensional brane-world scenario
- *modified gravity*, e.g., Horava gravity, scalar d.o.f. from MG

etc.

Stability in Modified Gravity

GR limit in MG: exist? stable?

- ***Einstein's GR***: $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$ ⟵ 2nd-order PDE of $g_{\mu\nu}$

- ***Modified Gravity***: involving higher-order derivatives of $g_{\mu\nu}$
  (e.g. $f(R)$ gravity)

- ***Cosmology***: require ***MG*** ~ ***GR*** at early times (CMB, BBN)
  ⇒ Consider MG models which ~ GR at *action level* at early times
  —(naively)→ **?** ***GR***: a good approximation for describing early-time cosmic evolution (background expansion) and cosmic structure formation (cosmological perturbations)

? Approximate a higher-order PDE with 2nd PDE

We found hints opposing this naive thinking.

for both background and perturbations

Details under investigations | To be shown

(Toy Example)

$$\varepsilon\ddot{x}(t)+b\dot{x}(t)+cx(t)=0 \xrightarrow[\textbf{?}]{\varepsilon\to 0} b\dot{x}(t)+cx(t)=0$$

$(\varepsilon > 0)$

Solution: $x = Ae^{-ct/b}e^{-(c^2/b^3)\varepsilon t} + Be^{-bt/\varepsilon}$ (ignore? Ignorant?) $\quad x = Ae^{-ct/b}$

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$b=1, c=1$: $x=Ae^{-t}e^{-\varepsilon t}+Be^{-t/\varepsilon}$ ok $\qquad x=Ae^{-t}$

$b=1, c=-1$: $x=Ae^{t}e^{-\varepsilon t}+Be^{-t/\varepsilon}$ ok $\qquad x=Ae^{t}$

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? Set B to zero by fine-tuning initial condition?

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Not negligible when $|\varepsilon t|\gtrsim 1$ (i.e. in the long run)

? Set B to zero by fine-tuning initial co

Warning !!

(Toy Example)

$$\varepsilon \ddot{x}(t) + b\dot{x}(t) + cx(t) = 0 \quad \xrightarrow[\textbf{?}]{\varepsilon \to 0} \quad b\dot{x}(t) + cx(t) = 0$$

$(\varepsilon > 0)$

Solution: $$x = Ae^{-ct/b} e^{-\left(c^2/b^3\right)\varepsilon t} + Be^{-bt/\varepsilon} \qquad x = Ae^{-ct/b}$$

(ignore? Ignorant?)

A better approximation, via iteration:

$$x(t) = x_0(t) + \varepsilon x_1(t) \quad \begin{cases} b\dot{x}_0(t) + cx_0(t) = 0 \quad \Rightarrow \quad x_0(t) = Ae^{-ct/b} \\ b\dot{x}_1(t) + cx_1(t) = -\varepsilon \ddot{x}_0(t) \qquad (\varepsilon^2 \ddot{x}_1 \text{ neglected.}) \end{cases}$$

$$\Rightarrow \quad x(t) = Ae^{-ct/b}\left[1 - \left(c^2/b^3\right)\varepsilon t\right] \quad \text{good when} \quad \left|\left(c^2/b^3\right)\varepsilon t\right| << 1$$

(Toy Example)

$$\varepsilon\ddot{x}(t)+b\dot{x}(t)+cx(t)=0 \xrightarrow[\textbf{?}]{\varepsilon\to 0} b\dot{x}(t)+cx(t)=0$$

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$$\Rightarrow \quad x(t)=Ae^{-ct/b}\left[1-(c^2/b^3)\varepsilon t\right]$$ good when $\underbrace{\left|(c^2/b^3)\varepsilon t\right| << 1}_{\text{(criterion)}}$

- *Benefits*: (1) better approximation to the 1st mode
  (2) provide a *criterion* for assessing the approximation
- *Drawbacks*: (1) still not good when $\left|(c^2/b^3)\varepsilon t\right| \geq 1$ (i.e. in the long run)
  (2) still ignore the 2nd mode

arXiv:1108.3038

Cosmological Perturbations in f(R) Gravity:

Early-time evolution

Motivation

Test *gravity theories cosmologically*, while background cosmic expansion cannot do the job alone.

Motivation

Explain *cosmic acceleration* (without dark energy).

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In most *MG* models, gravity modification is tiny.

GR ⟵?⟶ good approximation.

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Cosmological Perturbations in f(R) Gravity:

Early-time evolution

In most *MG* models, gravity modification is tiny.

GR ←?— good approximation.

Following the same principle presented by the toy example.

We found:
even a tiny deviation from GR may have non-negligible effect on the evolution of the cosmological perturbations, e.g. CMB photon density perturbation.

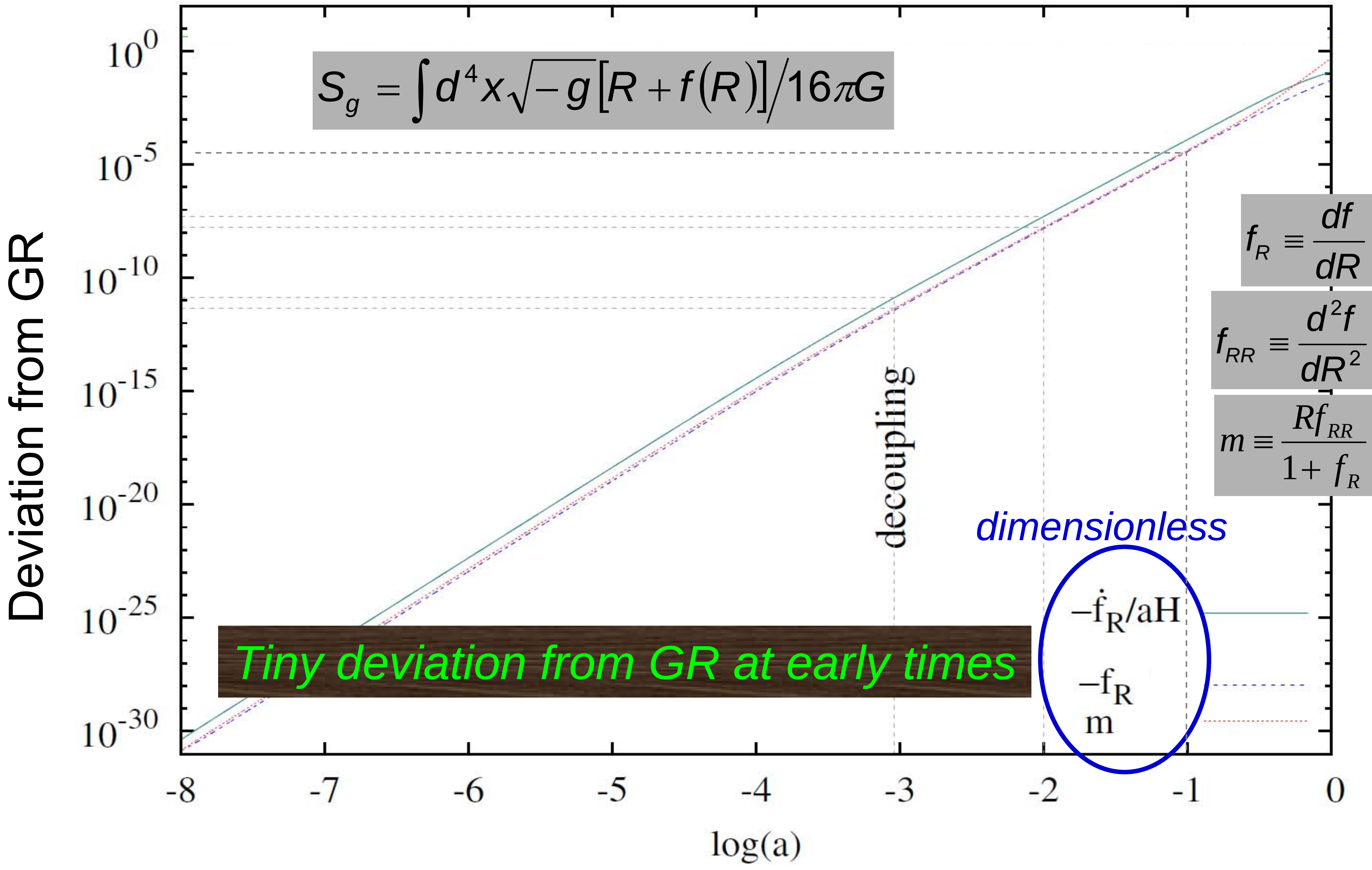
❖ "Early-Time Evolution of Cosmological Perturbations in $f(R)$ Gravity,"
Je-An Gu, Tse-Chun Wang, Yen-Ting Wu, Pisin Chen, and W-Y. Pauchy Hwang,
arXiv:1108.3038.

$$S_g = \int d^4x \sqrt{-g}\left[R + f(R)\right] / 16\pi G$$

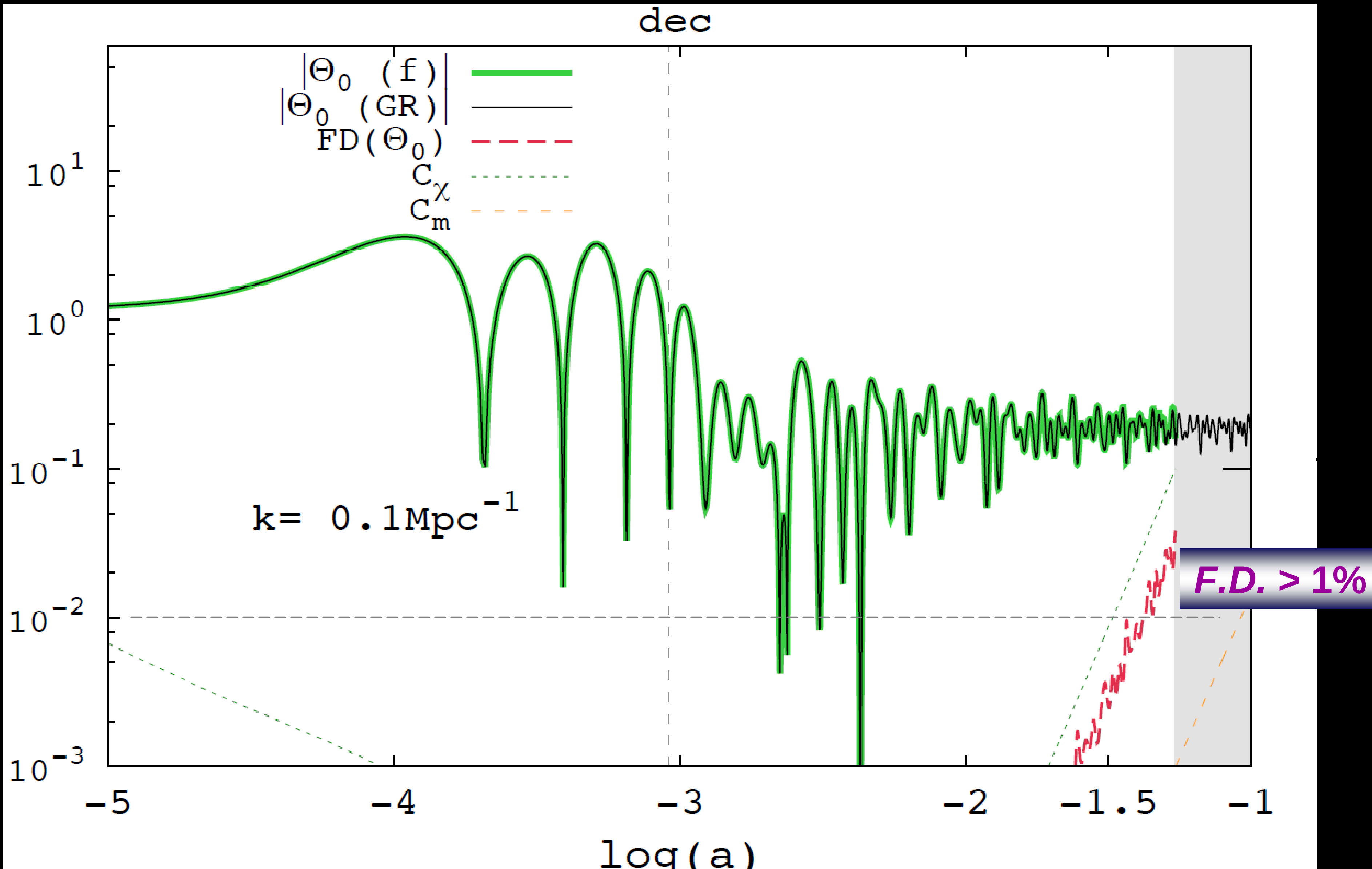
$$f_R \equiv \frac{df}{dR}$$

$$f_{RR} \equiv \frac{d^2 f}{dR^2}$$

$$m \equiv \frac{R f_{RR}}{1 + f_R}$$



❖ "Early-Time Evolution of Cosmological Perturbations in *f* (*R*) Gravity,"
Je-An Gu, Tse-Chun Wang, Yen-Ting Wu, Pisin Chen, and W-Y. Pauchy Hwang,
arXiv:1108.3038. *(Numerical results obtained by **modified CMBFAST.**)*



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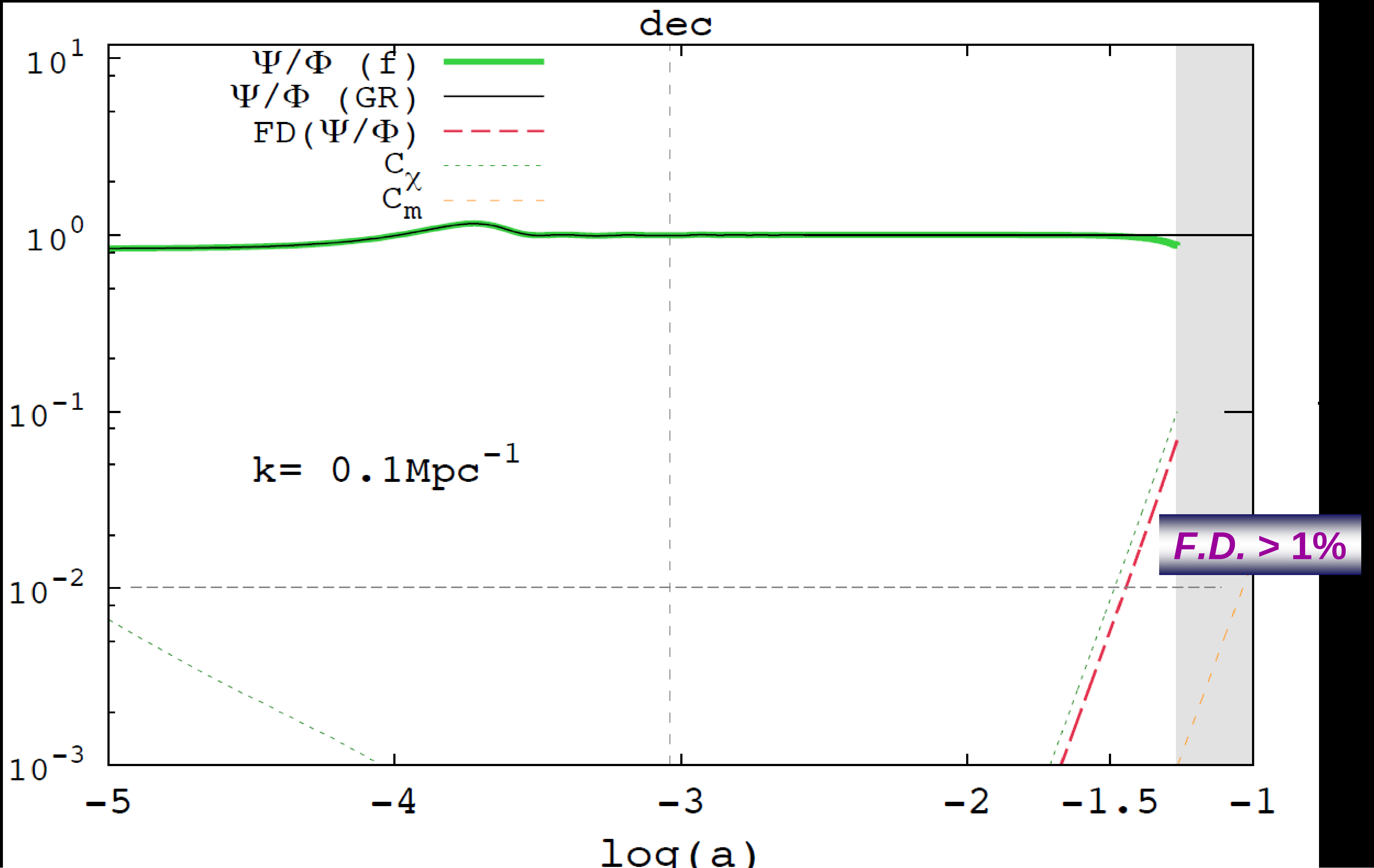

Unification !?

Scalar Fields

Inflaton, Higgs, Quintessence, Scalar DM, …

(Non is detected yet …)

Scalar Fields: "grand unification"

Unification of Inflaton, Higgs, Scalar DM, Quintessence (DE)

	Function	mass/energy scale	cross section	length scale & time period of its function
Quint. DE	anti-gravity	V'': 10^{-33}eV $\rho_0^{1/4}$: 10^{-3}eV ϕ_0: M_{pl}: 10^{29}eV	weak (dark)	
Inflaton	anti-gravity	GUT ?	strong? (for reheating)	
Higgs	give mass	TeV	strong (give ∨ mass) moderate	
Scalar DM	extra-gravity	>~ TeV	weak (dark)	

Scalar Fields: "grand unification"

Unification of Inflaton, Higgs, Scalar DM, Quintessence (DE)

	Function	length scale & time period of its function
Quint. DE	anti-gravity	length: only cosmological scale? (how about cluster & galaxy scales?) time: only present time? (how about the time when structures formed?)
Inflaton	anti-gravity	length: all scales? time: during inflation
Higgs	give mass	length: microscopic (< 1 fm ?) time: after EW SSB
Scalar DM	extra-gravity	length: from cluster to galaxy scales time: all time? (at least from $< t_{EQ}$ to now)

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Inflaton	anti-gravity	GUT ?	strong? (for reheating)	during inflation, all scales (?)
Higgs	give mass	TeV	strong (give ∨ moderate mass)	after EW SSB, microscopic
Scalar DM	extra-gravity	>~ TeV	weak (dark)	from $<t_{EQ}$ to now, from cluster to galaxy scales

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Chameleon may shed light !?

Cosmological Constant Problem

- ***Story* I** : Λ contribution from quantum fluctuations of vacuum
- ***Story* II**: Λ change led by SSB phase transition

Cosmological Constant Problem

Against :
- huge contributions from quantized fields
- huge change during SSB phase transition(s)

Pre-Dark-Energy

How to make Λ vanish ?

Post-Dark-Energy

- $\rho_{\text{dark energy}} \cong 3 \times 10^{-11} \text{eV}^{-4}$

How to generate such a small Λ ?

(Quantum) Vacuum Energy

QFT A quantized field can be regarded as

an ensemble of oscillators

each with frequency $\omega(k)$ (in k-space)

→ 'zero-point energy'

$$E_0 = \sum_k \frac{1}{2} \hbar \, \omega(k)$$

Vacuum Energy & Cosmological Constant

$$\rho_{vac} \sim \Lambda^4_{cutoff} \quad \text{vs.} \quad \rho_0 \sim (10^{-3}\,\text{eV})^4$$

Even *quantum fluctuations* at *atomic scales* can ruin our universe !!

(or micron scale ~ eV^{-1})

needless to say the case where $\Lambda_{cutoff} \sim M_{Pl}\,,\ M_{SUSY}\,,\ M_{EW}$

$\Rightarrow$ ***Dark Energy Crisis !!***

Crisis: We cannot exist if |vacuum energy| too large !!

To create a livable universe for human beings,

(naively)

the ***Creator*** needs to know in detail

the contributions to *vacuum energy* from *every quantized field*,

and then

make them *cancelled by each other*

or

cancelled by the bare cosmological constant

(nearly perfectly).

Λ Problem !! Fine Tuning Problem !!

Story II

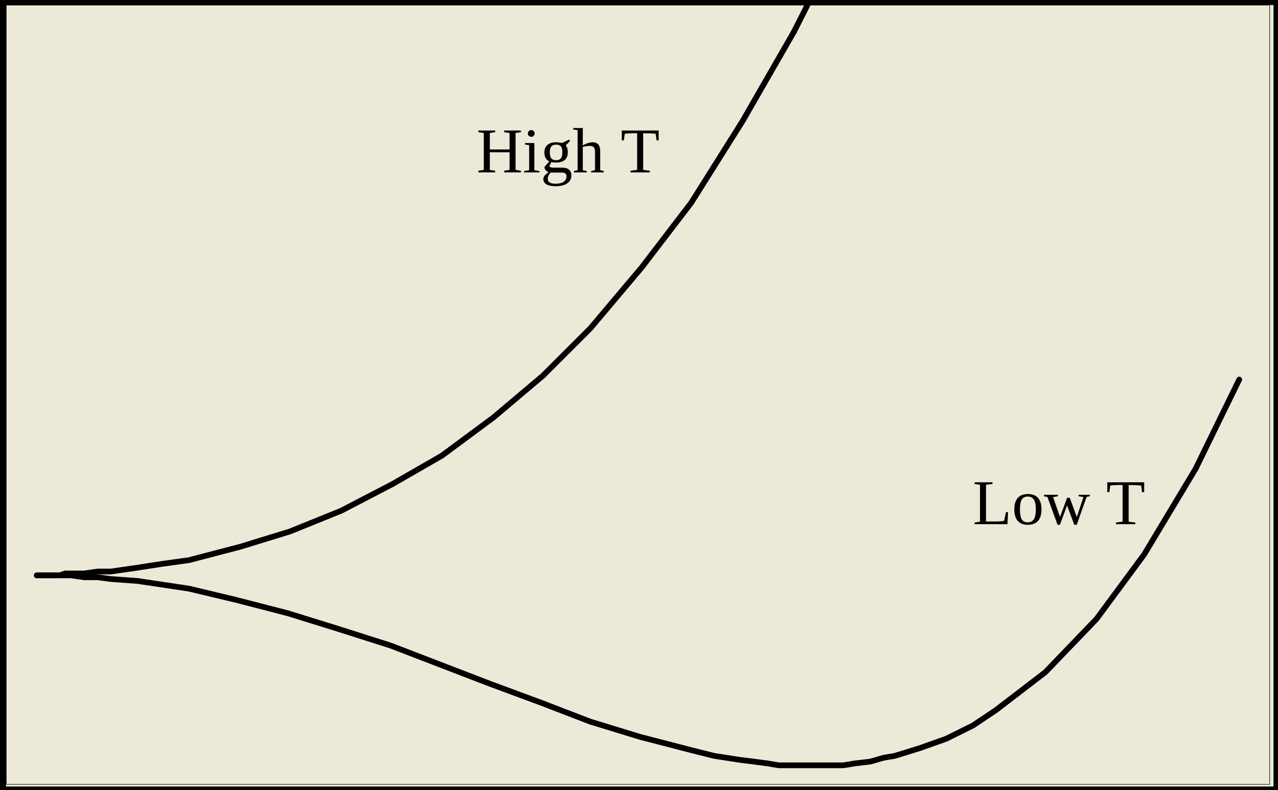
SSB Phase Transition → Λ Change

(spontaneous symmetry breaking)

$$L = \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi - V_T(\phi)$$

(T: temperature)

$V_T(\phi)$



ϕ

"Latent heat"

vacuum energy change

$\Delta\rho_{vac} \sim M_{SSB}^4$, e.g. $M_{EW} \sim 300$ GeV, vs. $\rho_0^{1/4} \sim 10^{-3}$ eV

SSB Phase Transition → Λ Change

Even if one obtains a small Λ before the SSB phase transition with a delicate device,

during the phase transition

the Λ energy density will drop for a certain amount on the scale of the phase transition,

(e.g. ~300GeV for the electroweak phase transition),

thereby *ruining the earlier* (nearly) *perfect cancellation*.

To create a livable universe for human beings,

[i.e., to have nearly perfect cancellation after the phase transition(s)]

the ***Creator*** needs to

foresee all possible SSB phase transitions

and

know the *very details of the Λ change* during each of them,

as detailed as 10^{-3}eV at least,

and then

make the earlier cancellation imperfect,

with the *energy deficit* on the scale **M_{SSB}**

with the precision 10^{-3}eV or better.

$$\rho_{vac}\,(\text{before SSB}) \sim M_{SSB}^4 + \left(10^{-3}\,\text{eV}\right)^4$$

Λ Problem !!   Fine Tuning Problem !!

Cosmological Constant Problem

$\Lambda_{\text{vacuum}}^{(q,l,g,\ldots)} + \Lambda_{\text{creator}}$ (?) how large? how precise?

$-\Delta\Lambda_{\text{Inf}}$ *Inflation*

$-\Delta\Lambda_{\text{PT1}}$ *SSB Phase Transition*

$-\Delta\Lambda_{\text{PTi}}$

(maybe also RG flow of Λ)

$\Lambda_{\text{now}} =$
$\Lambda_{\text{vacuum}}^{(q,l,g,\ldots)} + \Lambda_{\text{creator}} - \Delta\Lambda_{\text{Inf}} - \Delta\Lambda_{\text{PT1}} - \Delta\Lambda_{\text{PT2}} - \ldots - \Delta\Lambda_{\text{PTn}}$

$$\Lambda_{\text{creator}} = \left[-\Lambda_{\text{vacuum}}^{(q,l,g,\ldots)} + \Delta\Lambda_{\text{Inf}} + \Delta\Lambda_{\text{PT1}} + \Delta\Lambda_{\text{PT2}} + \ldots + \Delta\Lambda_{\text{PTn}} \right] - \Lambda_{\text{now}}$$

M_{Pl}	M_{GUT}?	*EW*	
10^{19} GeV	10^{16} GeV	300 GeV	10^{-3} eV

$\rho \sim M^4$

Cosmological Constant Problem

$$\Lambda_{\text{creator}} = \left[-\Lambda_{\text{vacuum}}^{(q,l,g,\dots)} + \Delta\Lambda_{\text{Inf}} + \Delta\Lambda_{\text{PT1}} + \Delta\Lambda_{\text{PT2}} + \dots + \Delta\Lambda_{\text{PTn}} \right] - \Lambda_{\text{now}}$$

M_{Pl} 10^{19} GeV	M_{GUT}? 10^{16} GeV	EW 300 GeV	10^{-3} eV

$\rho \sim M^4$

Unit: 10^{-11} eV^4

$-$ M_{Pl} 10^{19} GeV: 10^{112} eV^4

###########
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$+$ M_{GUT}? 10^{16} GeV: 10^{100} eV^4

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$+$ EW 300 GeV: 10^{46} eV^4

#######
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$-$ Λ_{now} 10^{-3} eV: 3×10^{-11} eV^4

Cosmological Constant Problem

stemming from

Tension *between*

Gravity *&* ***Quantum*** **!?**

- A (final) fundamental framework reconciling *gravity* with *quantum* should give a satisfactory solution to the *CC problem*.

New Aspects of Gravity

New Aspects of Gravity

References

T. Padmanabhan (IUCAA, Pune)
arXiv:0911.5004 "Thermodynamical Aspects of Gravity: New insights"
arXiv:1012.4476 "Lessons from Classical Gravity about the Quantum Structure of Spacetime"

L.D. Faddeev (Steklov Math. Inst., St. Petersburg)
arXiv:0906.4639 "New action for the Hilbert-Einstein equations"
arXiv:0911.0282 "New variables for the Einstein theory of gravitation"

Gravity vs. Quantum

Geometrical formulation

curvature $R_{\mu\nu}$ — **Geometry** ← Matter tells spacetime to curve. — **Matter Energy** — stress energy : $T_{\mu\nu}$ tensor

metric $g_{\mu\nu}$ — **Curved Space-Time** → Spacetime makes matter move. — **Particle Motion** — Eqn. of motion geodesic eqn.

Hamiltonian formulation

- e.g. ADM (perturbatively, not renormalizable)
- utilized for quantizing gravity

- Self-consistent Quantum Gravity (QG):
  - Loop QG (calculations very difficult, no evidence)
  - String theory (many theories, no evidence)

So far, not satisfactory, still a puzzle, an open question.

❖ ***Conjectures*** about Einstein's GR (for the solution about QG):
- GR: effective theory, need a fundamental theory.
- $g_{\mu\nu}$: no good, need to choose other fundamental variables. [Faddeev]
- GR: macroscopic description, need microscopic description. [Padmanabhan]

L. D. Faddeev: new variable, new action

- arXiv:0906.4639 “New action for the Hilbert-Einstein equations”
- arXiv:0911.0282 “New variables for the Einstein theory of gravitation”

1970s

[past] ***Embedding approach***: 4D space-time M^4 as a hypersurface in 10D space R^{10}

- f^A, $A = 1,\ldots,10$: coordinates of R^{10}
- x^μ, $\mu = 1,\ldots,4$: coordinates of M^4
- on M^4, $f^A = f^A(x)$: functions of x^μ
- induced metric on M^4: $g_{\mu\nu}(x) = \partial_\mu f^A(x)\partial_\nu f^A(x) \equiv f_\mu^A f_\nu^A, \qquad f_\mu^A \equiv \partial_\mu f^A(x)$
- choose $f^A(x)$ as the dynamical variables

$$\Rightarrow \delta\int\sqrt{g}R d^4x = 2\int\sqrt{g}\partial_\mu\left(G^{\mu\nu}f_\nu^A\right)\delta f^A d^4x$$

← extra derivative

(drawback?)

Faddeev: choose $\partial_\mu f^A(x)$ as the dynamical variables $\Rightarrow$ Einstein equations

L. D. Faddeev: new variable, new action

- arXiv:0906.4639 “New action for the Hilbert-Einstein equations”
- arXiv:0911.0282 “New variables for the Einstein theory of gravitation”

1970s

[past] ***Embedding approach***: 4D space-time M^4 as a hypersurface in 10D space R^{10}

- f^A, A = 1,…,10: coordinates of R^{10}
- x^μ, μ = 1,…,4: coordinates of M^4
- on M^4, $f^A = f^A(x)$: functions of x^μ
- induced metric on M^4: $g_{\mu\nu}(x) = \partial_\mu f^A(x) \partial_\nu f^A(x) \equiv f_\mu^A f_\nu^A, \qquad f_\mu^A \equiv \partial_\mu f^A(x)$
- choose $f^A(x)$ as the dynamical variables

$$\Rightarrow \delta \int \sqrt{g} R d^4 x = 2 \int \sqrt{g} \partial_\mu \left(G^{\mu\nu} f_\nu^A \right) \delta f^A d^4 x$$

← extra derivative

(drawback?)

$$\Rightarrow \partial_\mu \left[\left(G^{\mu\nu} - \kappa T^{\mu\nu} \right) f_\nu^A \right] = 0$$

[Gu conjecture]

Einstein equations as a solution.

~ Cook’s gravity ?
Solve CC problem?

Faddeev: choose $\partial_\mu f^A(x)$ as the dynamical variables $\Rightarrow$ Einstein equations

T. Padmanabhan: *new aspects of gravity*

- arXiv:0911.5004 "Thermodynamical Aspects of Gravity: New insights"
- arXiv:1012.4476 "Lessons from Classical Gravity about the Quantum Structure of Spacetime"

- Einstein's GR: macroscopic description of space-time
- Need microscopic description:
  - "atoms" of space-time and the physical law of "atoms"

❖ Hints of QG: (something involving both quantum and gravity)

1970s

- Black Hole (BH) thermodynamics (Bekenstein, Hawking, …):
  temperature T and entropy S on BH horizon: $T = \kappa / 2\pi$, $S = A / 4$

  (one motivation: solving information loss problem)

  κ: surface gravity, A : area

- Unruh temperature/radiation for an accelerating observer:
  with constant acceleration κ, temperature $T = \kappa / 2\pi$

- Boltzmann: $T,S \rightarrow$ microstructures

- Padmanabhan: $\rightarrow$ microstructures of space-time

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❖ Hints of QG: (something involving both quantum and gravity)

200X

"Entropic Force":

require maximal free energy (e.g. entropy) on the horizon

$\longrightarrow$ gravitational field equations (Einstein eqns.)

- (original) for only BH horizon

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❖ Hints of QG: (something involving both quantum and gravity)

200X

“Emergent gravity” :

require maximal free energy (e.g. entropy) on the horizon

$\longrightarrow$ gravitational field equations (Einstein eqns.)

- (original) for only BH horizon
- (extended) for all local Rindler frames (accelerating frames)

$\Rightarrow$ gravitational field equations as a derived constraint

(instead of dynamical equations)

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❖ Analogy

[microscopic] *Statistical Mechanics* → "atoms" of Space-Time ? & physical laws of "atoms" ?

↓ maximizing free energy ↓ maximizing free energy

[macroscopic] *Thermodynamics* → *Gravitational field equations*

e.g. ideal gas: $PV = nRT$

e.g. Einstein: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

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❖ Analogy

[microscopic] *Statistical Mechanics* — “atoms” of Space-Time ? & physical laws of “atoms” ?

maximizing free energy — even without full knowledge of atoms — maximizing free energy

[macroscopic] *Thermodynamics* — *Gravitational field equations*

e.g. ideal gas: $PV = nRT$ — e.g. Einstein: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

- Thus, for QG, should quantize “atoms” of space-time instead of quantizing $g_{\mu\nu}$ (a macroscopic quantity)

Summary

Issues *related to* ***Dark Energy*** *&* ***Modified Gravity***

- *Dark Radiation*

  from dark energy / modified gravity ?

  (worth further investigations)

- *Stability in Modified Gravity* (whose action ~ GR at early times)

  GR: good approximation at early times ?

  and in the solar system ?

  (require careful examination)

- *Unification of scalar fields?* (Inflaton, Higgs, DE, DM)

  Mission difficult, but not impossible. ***Chameleon*** may shed light.

Issues related to *Dark Energy* & *Modified Gravity*

- *Cosmological constant problem*, *gravity vs. quantum* ?

  A (final) fundamental framework encompassing gravity & quantum should give a satisfactory solution to the CC problem.

- [*new aspects*] (Jacobson, Padmanabhan, …)

  Statistical derivation of gravitational field equations

  —?→ GR is a macroscopic description of space-time.

  In this regard,

  - Need to explore the "atoms" of space-time.
  - To quantize gravity,

    one should quantize the physics of space-time atoms, instead of quantizing GR ($g_{\mu\nu}$).

Thank you.