Poincaré-Snyder Relativity

and Quantum Physics

— Seminar at Chung Yuan Christian University
(May 2009)

OTTO C. W. KONG
— Nat'l Central U, Taiwan

Symmetry of " Space -Time-U..." : -

the New Arena for Fundamental Physics

Galilean Relativity

Einstein Relativity — special and general

(Special) Quantum Relativity

or Doubly (Triply) / Deformed Special Relativity

H.Synder / C.N.Yang (1947)
G.Amelino-Camelia (2001)
L.Smolin + (2002)

$\cdots\cdots$ General Quantum Relativity $\Longrightarrow$ Quantum Gravity +

“Quantum Space-Time/Gravity” :-
— My Basic Perspectives

- on foundations of the “Quantum”

  “hidden variable theory” beyond local versus nonlocal

  e.g. Adler’s (Matrix Model) Trace Mechanics

- more fundamental description of “Gravity”

  Non-Commutative Geometry *is to* Quantum Gravity

  as Non-Eucliean Geometry *is to* (Einstein) Gravity

- need *new conceptual thinking* about “Space-Time”

- an exploration into *new* ways to think about fundamental physics

  withhold phenomenological discussions

A New Relativity :-
— from Symmetry Stabilization

- mathematical — stabilization gives new symmetry
- philosophical — only stable symmetry can be *scientifically* correct
- physics — deformation parameters gives fundamental constants c, $\hbar$, and G

- **relativity symmetry tells about what is "space-time" or the basic arena for fundamental physics**

- **Relativity Symmetry $SO(2,4)$ for the Quantum World 'AdS$_5$' as a classical geometric view of the 4D noncommutative "Space-Time"**

A Stable Symmetry :-

R.V. Mendes (1994)
Chryssomalakos & Okon (2004)

- perturbations (deformations) of a symmetry algebra
- stable if deformed algebras isomorphic to original

— structure constants perturbation Vs experimental uncertainty

- Examples —

  1/ Galilean to Einstein Relativity ($1/c^2 \neq 0$)

  2/ classical to quantum mechanics ($\hbar \neq 0$)

  Poisson bracket $\Longrightarrow$ Moyal bracket

- deformation parameters $\longrightarrow$ fundamental constants

Space-Time + Quantum Symmetry

'Poincaré + Heisenberg' algebra

$\Longrightarrow$ SO(1,5) OR SO(2,4)

Quantum Relativity $SO(1,5)$ or $SO(2,4)$:-

$$[J_{AB}, J_{LN}] = i\,(\eta_{BL} J_{AN} - \eta_{AL} J_{BN} + \eta_{AN} J_{BL} - \eta_{BN} J_{AL})$$

with $\eta^{AB} = (1, -1, -1, -1, -1, \mp 1)$.

- linear realization on 6-geometry — $J_{\mathcal{AB}} = i\,(x_{\mathcal{A}} \partial_{\mathcal{B}} - x_{\mathcal{B}}\, \partial_{\mathcal{A}})$
- J_{AB} for $0 \to 3$ are the 10 generator for Lorentz symmetry $M_{\mu\nu}$
- NOT necessarily means extra space-time dimensions
- 3 deformations — $\boldsymbol{ISO(3)} \longrightarrow \boldsymbol{SO(1,3)} \hookrightarrow \boldsymbol{ISO(1,3)} \longrightarrow \boldsymbol{SO(1,4)} \hookrightarrow \boldsymbol{ISO(1,4)} \longrightarrow \boldsymbol{SO(1,5)}$ **or** $\boldsymbol{SO(2,4)}$
- 3 invariant quantities : $c \quad \kappa \quad \ell$

Kowalski-Glikman & Smolin(2004)
OK (2007)

- curved momentum space $\Longrightarrow$ non-commutative space-time
  $+$ curved space-time $\Longrightarrow$ non-commutative momentum space

The Three Deformations :-

$\Delta x^i(t) = v^i \cdot t$	$\Delta x^\mu(\sigma) = p^\mu \cdot \sigma$	$\Delta x^A(\rho) = z^A \cdot \rho$
$\lvert v^i \rvert \le c$ $-\eta_{ij} v^i v^j = c^2\left(1 - \frac{1}{\gamma^2}\right)$	$\lvert p^\mu \rvert \le \kappa\, c$ $\eta_{\mu\nu} p^\mu p^\nu = \kappa^2 c^2\left(1 - \frac{1}{\Gamma^2}\right)$	$\lvert z^A \rvert \le i\ell$ $\eta_{AB}\, z^A z^B = -\ell^2\left(1 + \frac{1}{G^2}\right)$
$M_{0i} \equiv N_i \sim P_i$ $[N_i, N_j] \longrightarrow -i\, M_{ij}$	$J_{\mu 4} \equiv O_\mu \sim P_\mu$ $[O_\mu, O_\nu] \longrightarrow i\, M_{\mu\nu}$	$J_{A5} \equiv O'_A \sim P_A$ $[O'_A, O'_B] \longrightarrow i\, J_{AB}$
$\vec{u}^4 = \frac{\gamma}{c}(c, v^i)$ $\eta_{\mu\nu} u^\mu u^\nu = 1$ $\mathbb{R}^3 \to SO(1,3)/SO(3)$	$\vec{\pi}^5 = \frac{\Gamma}{\kappa\, c}(p^\mu, \kappa\, c)$ $\eta_{AB}\, \pi^A \pi^B = -1$ $M^4 \to SO(1,4)/SO(1,3)$	$\vec{X}^6 = \frac{G}{\ell}(z^A, \ell)$ $\eta_{\mathcal{MN}} X^{\mathcal{M}} X^{\mathcal{N}} = -1$ $M^5 \to SO(2,4)/SO(2,3)$

- $z^A = \frac{dx^\mu}{d\rho}$ is *chosen as* a “length”

$\Longrightarrow$ **Quantum World as ‘AdS$_5$’** (classical 6-geometry)

- **only $SO(2,4)$** no $\hookrightarrow ISO(2,4)$: no coordinate translation

The Invariant Planck Scale :-

- Galilean Relativity admits no invariant speed c
- Snyder suggested an invariant Planck scale modifies Relativity

- Lie algebra Vs quantum group as symmetry (1990–)
- linear realization (u-coordinates) Das & OK 2006 (Girelli & Livine)
- two deformations to quantum, $\hbar = \kappa\, c\, \ell$ OK 2008

- Galilean $\longrightarrow$ Einstein $\hookrightarrow$ Poincaré-Snyder $\longrightarrow$ Snyder $\Rightarrow$ Quantum
  $(x^i : t)$ (ct, x^i) $(ct, x^i : \sigma, \rho)$ $(ct, x^i, \kappa c\sigma : \rho)$ $(\ell\rho)$

Poincaré-Snyder/Snyder Relativity :-

Quantum	Snyder	Poincaré-Snyder	Einstein	Galilean
$SO(2,4)$	$ISO(1,4)$	$G(1,3)$	$ISO(1,3)$	$G(3)$
'AdS$_5$'	M^5	M^4 (with σ)	M^4	$I\!R^3$ (with t)
J_{ij}	J_{ij}	J_{ij}	J_{ij}	J_{ij}
J_{0i}	J_{0i}	J_{0i}	J_{0i}	K_i
J_{04}	J_{04}	K'_0	P_0	H
J_{i4}	J_{i4}	K'_i	P_i	P_i
J_{05}	P_0	P_0		
J_{i5}	P_i	P_i		
J_{45}	P_4	H'		

- Inönü-Wigner contractions :

$$SO(1,4) \longrightarrow ISO(1,3) \longrightarrow G(3)$$

$$SO(2,4) \longrightarrow ISO(1,4) \longrightarrow G(1,3)$$

Physics of the Peculiar $\boldsymbol{\sigma}$ Coordinate :-

$\star$ $p^\mu = \frac{dx^\mu}{d\sigma}$ Vs $m\,c\,u^\mu = \gamma\,m\,c\,\frac{dx^\mu}{dx^0}$ (as Einstein/classical limit)

$\sigma \longrightarrow \frac{\tau}{m}$ for an Einstein particle

recall : "space-like" geometrical signature

- clearly neither space nor time

$\star$ Quantization with Poincaré-Snyder Relativity

— $\boldsymbol{ISO(1,3)}$ + commuting momentum boosts ($\Delta x = p^\mu \sigma$)

- covariant Schrödinger eqn. $\rightarrow$ metric fluctuation ($\sim \tau$ or τ/m)
- quantization as representation of $U(1)$ central extension ($\sim \tau$)
- stochastic quantization (extra fictitious imaginary time)

σ-Hamiltonian Dynamics :-

OK & Hung-yi Lee

- Lorentz invariant canonical mechanics straightforward
— similar in form to 'relativistic' evolution parameterized by τ
- ? σ as external evolution parameter
  — at least relational interpretation
- 'kinetic energy' term forced to be $\frac{p^\mu p_\mu}{2}$
  — same for all 'free particles'
  c.f. role of m from Poincaré down to Galilean
- 'potential' term $V(x^\mu)$ possible
  — 'bypass' no-interaction theorem of 'relativistic mechanics'
- *under investigation* $\cdots\cdots$

Quantization as $U(1)$ Central Extension :-

V. Aldaya & J.A. de Azcárraga '81

- symmetry realized projectively for Q.M.

— equivalent to unitary representation of $U(1)$ central extension

$\Longrightarrow$ (geometric) quantization based only on symmetry group

— successful for group with nontrivial extension [*e.g.* $G(3)$]

(cohomology analysis of mechanics)

- fail for Lorentz or Poincaré, symmetry with only trivial extension

- works for Poincaré-Snyder $G(1,3)$!

— with no quantum deformation yet $\rightarrow$ there is quantization

obtain covariant Schrödinger equation with σ evolution

- *again*, potential $V(x^\mu)$ possible $\cdots\cdots$

Extended $G(1,3)$ Action and Cocycle :-

- transformation $g = (b, A^\mu, p^\mu, \Lambda^\mu{}_\nu, e^{i\theta})$ in matrix form

$$g = \begin{pmatrix} \Lambda^\mu{}_\nu & p^\mu & A^\mu & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ -zp_\mu\Lambda^\mu{}_\nu & -\frac{z}{2}p^\mu p_\mu & \theta & 1 \end{pmatrix}$$

- classical : action on M_5 : $(x^\mu, \sigma, 1)^t$

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu + p^\mu\sigma + A^\mu$$
$$\sigma' = \sigma + b$$

- quantum : unitary action on section of $M_5 \times \mathbb{C}$

$$[U(b, A^\mu, p^\mu, \Lambda^\mu{}_\nu, e^{i\theta})\psi](x', t') = \exp\left[-iz\left(\Lambda_\mu{}^\nu p_\nu x^\mu + \frac{1}{2}p^\mu p_\mu \sigma + \frac{\theta}{z}\right)\right]\psi(x, t)$$

- nontrivial cocycle from cohomology

Geometric Quantization of $G(1,3)$:-

- a different cocycle (add coboundary)
- group product

$$A''^{\mu} = \Lambda'^{\mu}{}_{\nu} A^{\nu} + p'^{\mu} b + A'^{\mu} \;, \qquad b'' = b' + b \;,$$
$$p''^{\mu} = \Lambda'^{\mu}{}_{\nu} p^{\nu} + p'^{\mu} \;, \qquad \Lambda''^{\mu}{}_{\nu} = \Lambda'^{\mu}{}_{\rho} \Lambda^{\rho}{}_{\nu}$$

$$\theta'' = \theta' + \theta - z\left[A'_{\mu}\Lambda'^{\mu}{}_{\nu}p^{\nu} + b(p'^{\mu}\Lambda'_{\mu}{}^{\nu}p_{\nu} + \frac{1}{2}p'^{\mu}p'_{\mu})\right]$$

- (vertical) quantization 1-form ($\omega = d\Theta$)

$$\Theta = zA^{\nu}\Lambda_{\nu}{}^{\mu}dp_{\mu} + \tfrac{z}{2}p^{\mu}p_{\mu}db + \frac{\hbar d\zeta}{i\zeta}$$

- left- (right-) invariant vector fields and forms $\cdots\cdots$

• • • • • •

• (Hamiltonian) differential system $[\zeta = \exp(\frac{i}{\hbar}\theta)$ locally$]$

$$\mathbf{X}^L_b = \frac{\partial}{\partial b} + p^\mu \frac{\partial}{\partial A^\mu} - \frac{z}{2} p^\mu p_\mu \, \frac{i\zeta}{\hbar} \frac{\partial}{\partial \zeta}$$

• equations of motion : $\frac{d\zeta}{d\sigma} = -\frac{z}{2} p^\mu p_\mu \frac{i\zeta}{\hbar}$,

$$\frac{db}{d\sigma} = 1 \; , \qquad \frac{dA^\mu}{d\sigma} = p^\mu \; , \qquad \frac{dp^\mu}{d\sigma} = 0 \; , \qquad \frac{d\Lambda^\mu{}_\nu}{d\sigma} = 0$$

$$\Longrightarrow \qquad b = \sigma \; , \qquad A^\mu = p^\mu \sigma + K'^\nu \Lambda_\nu{}^\mu \; , \qquad p^\mu = P^\mu \; ,$$

$$\zeta = \zeta_o \exp(-\frac{iz}{2\hbar} p^\mu p_\mu \sigma) \; .$$

• taking $z = 1$, $H' = \frac{1}{2} p^\mu p_\mu$, and $K'^\mu = A^\nu \Lambda_\nu{}^\mu$

$$\omega = d\Theta = dK'^\mu \wedge dp_\mu + dH' \wedge db$$

• • • • • •

- σ-evolution $\quad [K'^{\mu} \longrightarrow x^{\mu} \;. \quad b \longrightarrow \sigma]$
- prequantum operator $\quad [i_{X_f}\omega = -df]$

$$\hat{f}\psi \equiv -i\hbar\tilde{X}f \cdot \psi = -i\hbar X_f \cdot \psi + [f - \Theta(X_f)]\psi$$

$$\Longrightarrow \quad \hat{K'}^{\mu} = -i\hbar\frac{\partial}{\partial P_{\mu}} \;, \quad \hat{P}_{\mu} = i\hbar\frac{\partial}{\partial K'^{\mu}} + P_{\mu} \;, \quad \hat{\sigma} = i\hbar\frac{\partial}{\partial H'} + \sigma \;, \quad \hat{H}' = i\hbar\frac{\partial}{\partial \sigma} \;.$$

- $\{\mathbf{X}^L_b, \mathbf{X}^L_{A^{\mu}}, \mathbf{X}^L_{\Lambda^{\mu}_{\ \nu}}\}$ span full polarization subalgebra ($\rightarrow$ reduction)
- wavefunction

$$\psi = \exp[-i\hbar(p_{\mu}x^{\mu} - \frac{1}{2}p^{\mu}p_{\mu}\sigma)]\;\phi(p^{\mu})$$

satisfies covariant Schrödinger equation (EL-eq. of $\mathbf{X}^L_b$)

$$i\hbar\partial_{\sigma}\psi - \frac{\hbar^2}{2}\partial_{\mu}\partial^{\mu}\psi = 0$$

Concluding Remarks :-

- for the Quantum Relativity Theory

the challenge is 'Dynamics'

— not dynamics as it used to be : Lagrangian ? Hamiltonian ?
lack of good physics handles

- Poincaré-Snyder dynamics as one of the first step

— probe physics of σ-coordinate

- experimentally accessible predictions may come from

— PS potential dynamics, classical and quantum
— PS (quantum) electrodynamics

$\cdots\cdots$ *concluding remarks*

• Poincaré-Snyder Relativity is *Not intrinsically Quantum*

— no deformation with $\hbar$-related parameter

— in fact, admits quantization (= nontrivial central extension)

$\star$ not Einstein Relativity

• (free particle) Poincaré-Snyder system successfully described

• may be Einstein relativistic physics is an incomplete description

—fixing m constant and $\sigma = \tau/m$

— putting $V(x^\mu)$ equals to zero (no-interaction)

$\cdots\cdots\cdots$

THANK YOU !