Spherical Collapse Model with Clustered Dark Energy

Chia-Chun Chang

Institute of Physics
National Chiao Tung University

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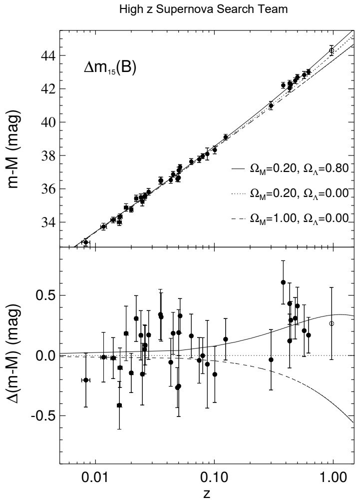
Outline

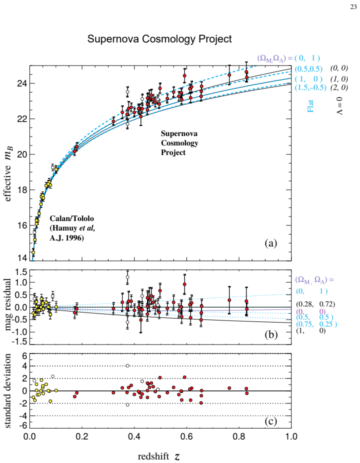
- Accelerating Expansion of the Universe
- Structure Formation
- Virialization
- Complete Journey of Matter Density Perturbation
- Structure Formation with Time-Varying Equation of State
- Conclusions

Accelerating Expansion of the Universe

Type Ia Supernova Data



Riess et al., 1998



Perlmutter et al., 1999

Accelerating Expansion of the Universe

According to the cosmological principle and the observation, the universe is homogeneous and isotropic on large scales (> 100 Mpc).

- FRW metric:

$$ds^2 = -dt^2 + \underline{a(t)}^2 \left(\frac{dr^2}{1 - \underline{k} r^2} + r^2 d\Omega^2 \right)$$

scale factor   curvature of the space

$\Rightarrow$ a spatially homogeneous and isotropic universe expanding as a function of time

Accelerating Expansion of the Universe

Friedmann Equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

if $P < -\frac{1}{3}\rho \Rightarrow \ddot{a} > 0$: acceleration (Dark Energy)

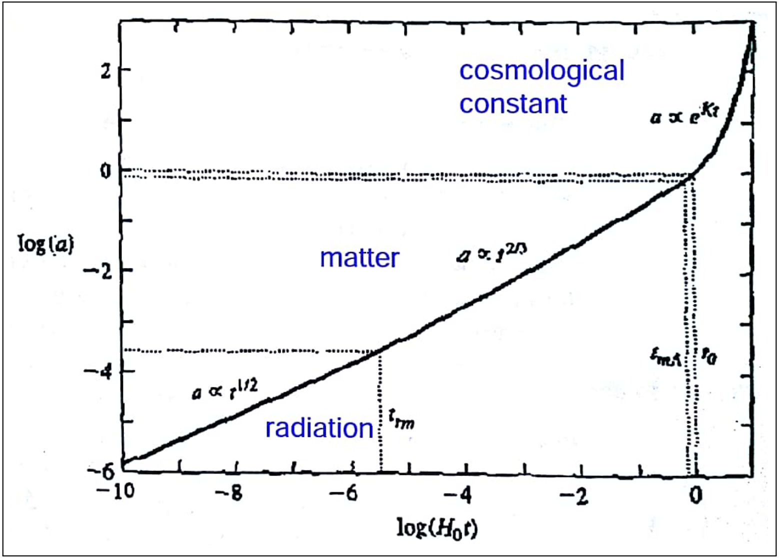
Fluid Equation

energy-momentum conservation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

for $P = w\rho \Rightarrow \rho(a) = \rho_0 a^{-3(1+w)}$

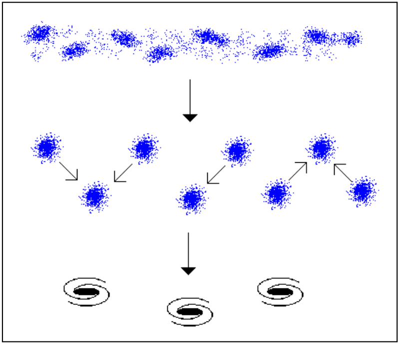
Accelerating Expansion of the Universe



(B. Ryden, Introduction to Cosmology (2002))

Structure Formation

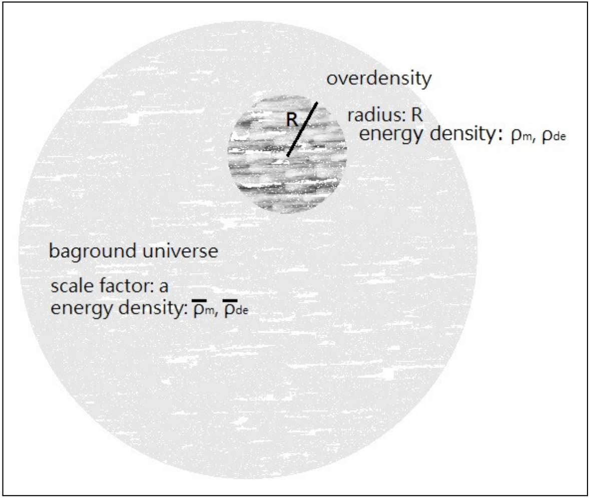
- Bottom-Up Structure Formation:
  The small galaxies form and attract each other by gravity and merge to form larger galaxies, and then cluster together to form clusters.

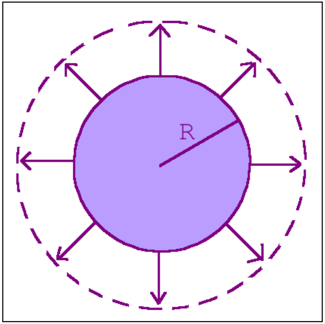


(https://www.astro.ufl.edu/~guzman/ast1002/class_notes/Ch15/Ch15b.html)

Gravity $\longleftrightarrow$ Universe Expansion

Structure Formation





- expand to $R_{\max}$
- turn around
- collapse to form structures

$\Rightarrow$ We use spherical collapse model to explore the nonlinear gravitational collapse in matter.

Structure Formation

Background Universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\bar{\rho}_{\rm m} + (1+3w)\bar{\rho}_{\rm de}\right]$$

$$\dot{\bar{\rho}}_{\rm m} + 3\left(\frac{\dot{a}}{a}\right)\bar{\rho}_{\rm m} = 0$$

$$\dot{\bar{\rho}}_{\rm de} + 3(1+w)\left(\frac{\dot{a}}{a}\right)\bar{\rho}_{\rm de} = 0$$

Spherical Overdensity

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left[\rho_{\rm m} + (1+3w)\rho_{\rm de}\right]$$

$$\dot{\rho}_{\rm m} + 3\left(\frac{\dot{R}}{R}\right)\rho_{\rm m} = 0$$

$$\dot{\rho}_{\rm de} + 3(1+w)\left(\frac{\dot{R}}{R}\right)\rho_{\rm de} = \alpha\Gamma,$$

where $\Gamma = 3(1+w)\left(\frac{\dot{R}}{R} - \frac{\dot{a}}{a}\right)\rho_{\rm de}$

with $0 \leq \alpha \leq 1$

Structure Formation

$$\dot{\rho}_{\rm de} + 3(1+w)\left(\frac{\dot{R}}{R}\right)\rho_{\rm de} = \alpha\Gamma,$$

where $\Gamma = 3(1+w)\left(\frac{\dot{R}}{R} - \frac{\dot{a}}{a}\right)\rho_{\rm de}$ with $0 \leq \alpha \leq 1$

$\alpha = 1$ (non-clustering DE)

$$\dot{\rho}_{\rm de} + 3(1+w)\left(\frac{\dot{a}}{a}\right)\rho_{\rm de} = 0 \Rightarrow \rho_{\rm de} = \bar{\rho}_{\rm de}$$

- DE is homogeneous.
- Energy does not conserve within the overdensity.

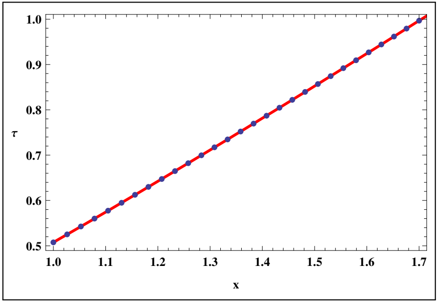
$\alpha = 0$ (clustering DE)

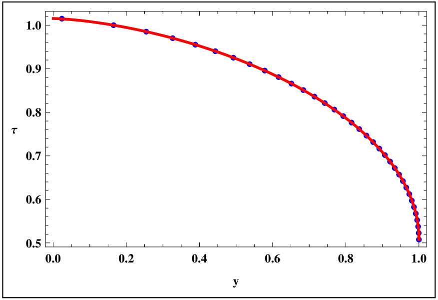
$$\dot{\rho}_{\rm de} + 3(1+w)\left(\frac{\dot{R}}{R}\right)\rho_{\rm de} = 0 \Rightarrow \rho_{\rm de} \neq \bar{\rho}_{\rm de}$$

- DE is inhomogeneous.
- The spherical overdensity is considered an isolated system satisfying the law of energy conservation.

Structure Formation

non-clustering DE ($\alpha = 1$), $\rho_{\rm de} = \bar{\rho}_{\rm de}$ (Lee and Ng, JCAP 10 (2010) 028)





- $w = -\frac{1}{3}$, $z_{\rm ta} = -0.8$, $\Omega_{\rm m,0} = 0.3$

$*d\tau \equiv H_{\rm ta}\sqrt{\Omega_{\rm m}(x_{\rm ta})}dt$, $x \equiv \frac{a}{a_{\rm ta}}$, $y \equiv \frac{R}{R_{\rm ta}}$

Structure Formation

clustering DE ($\alpha = 0$), $\rho_{\rm de} \neq \bar{\rho}_{\rm de}$

- Difficulties:
  $\rho_{\rm de}$ within the spherical overdensity is unknown.

$\Rightarrow$ model DE by various types of scalar fields

- Creminelli et al., JCAP 03 (2010) 027
- Nunes and Mota, MNRAS 368 (2006) 751
- Mota and Bruck, Astron. Astrophys 421 (2004) 71

$\Rightarrow$ Might it oversimplify the property of DE?
(e.g. early DE model)

- Huey et al., PRD 59 (1999) 063005
- Doran and Lilley, MNRAS 330 (2001) 965
- Bean et al., PRD 64 (2001) 103508
- Lee and Ng, PRD 67 (2003) 107302

Structure Formation

clustering DE ($\alpha = 0$), $\rho_{\rm de} \neq \bar{\rho}_{\rm de}$

$$\frac{dx}{d\tau} = \sqrt{x^{-1} + \frac{1}{Q_{\rm ta}} x^{-3w-1}}$$

$$\Rightarrow \quad \tau_{\rm ta} = \frac{2}{3} F\left[\frac{1}{2}, -\frac{1}{2w}, 1 - \frac{1}{2w}, -Q_{\rm ta}^{-1}\right]$$

$$\frac{d^2y}{d\tau^2} = -\frac{1}{2}\left[\zeta y^{-2} + (1+3w)\frac{1}{Q_{\rm cta}} y^{-3(1+w)+1}\right]$$

$$\zeta : \quad \rho_{\rm m} = ? \quad \Rightarrow y(\tau_{\rm c}) = y(2\tau_{\rm ta}) = 0$$

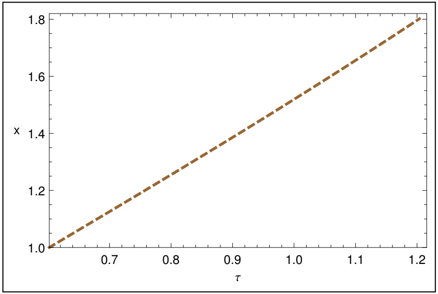
$$Q_{\rm cta} : \quad \rho_{\rm de} = ? \quad \Rightarrow \boxed{\textit{Assuming } \delta^{\rm NL}_{\rm de,ta} = r\delta^{\rm NL}_{\rm m,ta}}$$

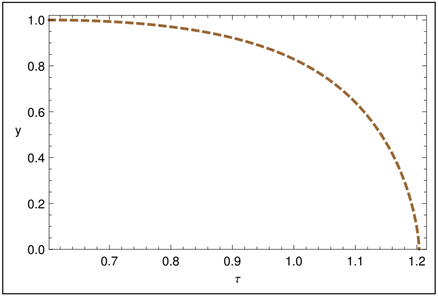
$$\Rightarrow Q_{\rm cta} = \frac{Q_{\rm ta}}{1 + r[\zeta(w, z_{\rm ta}, r) - 1]}$$

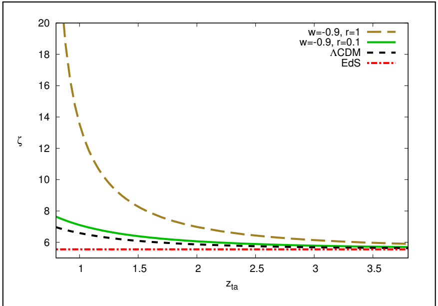
$^* d\tau \equiv H_{\rm ta}\sqrt{\Omega_{\rm m}(x_{\rm ta})}dt$, $x \equiv \frac{a}{a_{\rm ta}}$, $y \equiv \frac{R}{R_{\rm ta}}$, $\zeta \equiv \left.\frac{\rho_{\rm m}}{\bar{\rho}_{\rm m}}\right|_{z_{\rm ta}}$,
$Q_{\rm ta} \equiv \left.\frac{\bar{\rho}_{\rm m}}{\bar{\rho}_{\rm de}}\right|_{z_{\rm ta}} = \frac{\Omega^0_{\rm m}}{\Omega^0_{\rm de}}(1+z_{\rm ta})^{-3w}$, $Q_{\rm cta} \equiv \left.\frac{\bar{\rho}_{\rm m}}{\rho_{\rm de}}\right|_{z_{\rm ta}}$,
$\delta^{\rm NL}_{\rm m} \equiv \rho_{\rm m}/\bar{\rho}_{\rm m} - 1$, and $\delta^{\rm NL}_{\rm de} \equiv \rho_{\rm de}/\bar{\rho}_{\rm de} - 1$

Structure Formation

clustering DE ($\alpha = 0$), $\rho_{\rm de} \neq \bar{\rho}_{\rm de}$







- the upper: $r = 1$, $w = -0.8$, $z_{\rm ta} = 0.8$
- $\Omega_{\rm m,0} = 0.274$

* $r = \delta^{\rm NL}_{\rm de,ta}/\delta^{\rm NL}_{\rm m,ta}$

* $\zeta \equiv \left.\frac{\rho_{\rm m}}{\bar{\rho}_{\rm m}}\right|_{z_{\rm ta}}$

Virialization

(Maor and Lahav, Astropart. Phys. 07 (2005) 003)

virial theorem and energy conservation: $\left[U + \frac{R}{2}\frac{\partial U}{\partial R}\right]_{\rm vir} = U_{\rm ta}$

$$U = \frac{1}{2}\int_0^R \rho_{\rm m}\Phi_{\rm m} dV + \frac{1}{2}\int_0^R \rho_{\rm de}\Phi_{\rm m} dV + \frac{1}{2}\int_0^R \rho_{\rm m}\Phi_{\rm de} dV + \frac{1}{2}\int_0^R \rho_{\rm de}\Phi_{\rm de} dV$$

- whole system virializing

$$[1 + (2+3w)q + (1+3w)q^2]y_{\rm vir} - \frac{1}{2}(2+3w)(1-3w)q y_{\rm vir}^{-3w} - \frac{1}{2}(1+3w)(1-6w)q^2 y_{\rm vir}^{-6w} = \frac{1}{2}$$

- only matter virializing

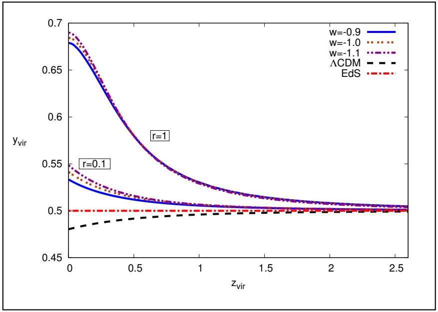
$$(1+q)y_{\rm vir} - \frac{q}{2}(1-3w)y_{\rm vir}^{-3w} = \frac{1}{2}$$

- EdS universe

$$y_{\rm vir} = \frac{1}{2}$$

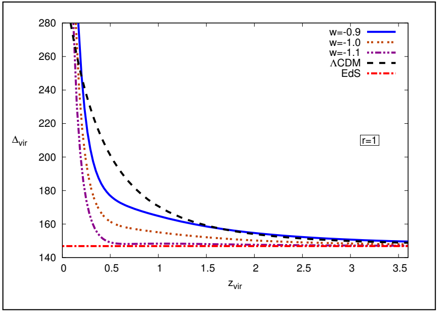
* $q \equiv \left.\frac{\rho_{\rm de}}{\rho_{\rm m}}\right|_{z_{\rm ta}} = \left(\frac{\delta^{\rm NL}_{\rm de,ta}+1}{\delta^{\rm NL}_{\rm m,ta}+1}\right)\frac{1-\Omega_{\rm m,0}}{\Omega_{\rm m,0}}(1+z_{\rm ta})^{3w} = \left(\frac{r(\zeta-1)+1}{\zeta}\right)\frac{1-\Omega_{\rm m,0}}{\Omega_{\rm m,0}}(1+z_{\rm ta})^{3w}$

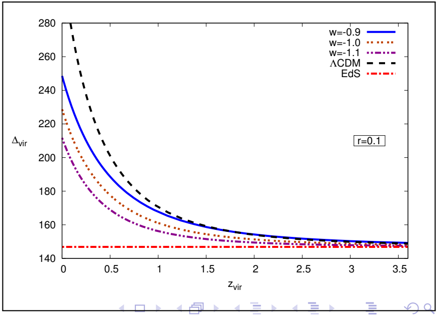
Virialization



- $\Omega_{m,0} = 0.274$

* $y_{vir} \equiv \dfrac{R_{vir}}{R_{ta}}$
* $\Delta_{vir} \equiv \dfrac{\rho_m}{\bar{\rho}_m}\Big|_{z_{vir}} = \zeta\left(\dfrac{x_{vir}}{y_{vir}}\right)^3$
* $r = \delta^{NL}_{de,ta}/\delta^{NL}_{m,ta}$





Complete Journey of Matter Density Perturbation

- continuity eq.:

$$\dot{\rho}_{\rm x} + \overrightarrow{\nabla} \cdot [(\rho_{\rm x} + P_{\rm x})\vec{v}] = 0$$

- Euler eq.:

$$\dot{\vec{v}} + (\vec{v} \cdot \overrightarrow{\nabla})\vec{v} = -\overrightarrow{\nabla}\Phi$$

- adding small perturbations:

$$\rho_{\rm x} = \bar{\rho}_{\rm x}(1 + \delta_{\rm x}), \quad \overrightarrow{v} = H\overrightarrow{x} + \overrightarrow{u}$$

- Poisson eq.:

$$\overrightarrow{\nabla}^2 \delta\Phi = 4\pi G \left(\delta\rho_{\rm m} + \delta\rho_{\rm de} + 3\delta P_{\rm de}\right) a^2$$

Complete Journey of Matter Density Perturbation

δ in Linear Regime

(* $\tilde{x} \equiv a/a_0$, $\eta \equiv \sqrt{\Omega_{\mathrm{m},0}} H_0 t$)

- background universe

$$\frac{d\tilde{x}}{d\eta} = \frac{1}{\sqrt{\tilde{x}\Omega_{\mathrm{m}}(\tilde{x})}} \qquad \text{with } \Omega_{\mathrm{m}}(\tilde{x}) = \left(1 + \frac{1-\Omega_{\mathrm{m}}^0}{\Omega_{\mathrm{m}}^0}\tilde{x}^{-3w}\right)^{-1}$$

matter dominated universe: $\boxed{\tilde{x}_i = (3\eta_i/2)^{2/3}}$

- density perturbation

$$\frac{d^2\delta_{\mathrm{m}}}{d\eta^2} + \frac{2}{\tilde{x}}\frac{d\tilde{x}}{d\eta}\frac{d\delta_{\mathrm{m}}}{d\eta} = \frac{3}{2\tilde{x}^3}\left[\delta_{\mathrm{m}} + \frac{1-\Omega_{\mathrm{m}}^0}{\Omega_{\mathrm{m}}^0}(1+3w)\tilde{x}^{-3w}\delta_{\mathrm{de}}\right]$$

$$\frac{d\delta_{\mathrm{de}}}{d\eta} = (1+w)\frac{d\delta_{\mathrm{m}}}{d\eta} \Rightarrow \delta_{\mathrm{de}} = (1+w)\delta_{\mathrm{m}}$$

matter dominated universe:

$\delta_{\mathrm{m}} \propto a \Rightarrow \boxed{d\delta_{\mathrm{m}}/d\eta|_i = 2\delta_{\mathrm{m},i}/(3\eta_i)}$

Complete Journey of Matter Density Perturbation

δ in Non-Linear Regime (Creminelli et al., JCAP 03 (2010) 027)

- background universe $\frac{d\tilde{x}}{d\eta} = \frac{1}{\sqrt{\tilde{x}\Omega_{\rm m}(\tilde{x})}}$
- spherical overdense region

$$\frac{d^2\tilde{y}}{d\eta^2} + \frac{1}{2}\left[\frac{1+\delta_{{\rm m},i}}{\tilde{x}_i^3}\frac{1}{\tilde{y}^2} + (1+3w)(1+\delta_{\rm de}^{\rm NL})\frac{1-\Omega_{\rm m}^0}{\Omega_{\rm m}^0}\frac{\tilde{y}}{\tilde{x}^{3(1+w)}}\right] = 0$$

$\boxed{\tilde{y}(\eta_i) = \tilde{y}_i = 1}$ for $\tilde{y} \equiv \frac{R}{R_i}$

- density contrast

$$\frac{d\delta_{\rm de}^{\rm NL}}{d\eta} + 3(1+w)(1+\delta_{\rm de}^{\rm NL})\left(\frac{1}{\tilde{y}}\frac{d\tilde{y}}{d\eta} - \frac{1}{\tilde{x}}\frac{d\tilde{x}}{d\eta}\right) = 0$$

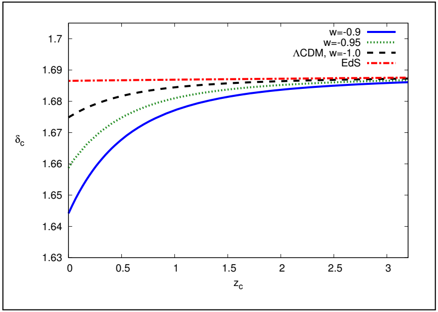
linear in early time: $\boxed{\delta_{{\rm de},i}^{\rm NL} \approx \delta_{{\rm de},i} = (1+w)\delta_{{\rm m},i}}$

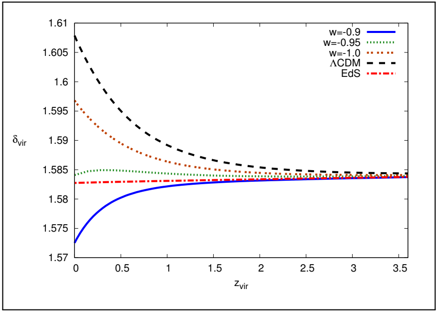
$$\frac{d\delta_{\rm m}^{\rm NL}}{d\eta} + 3(1+\delta_{\rm m}^{\rm NL})\left(\frac{1}{\tilde{y}}\frac{d\tilde{y}}{d\eta} - \frac{1}{\tilde{x}}\frac{d\tilde{x}}{d\eta}\right) = 0$$

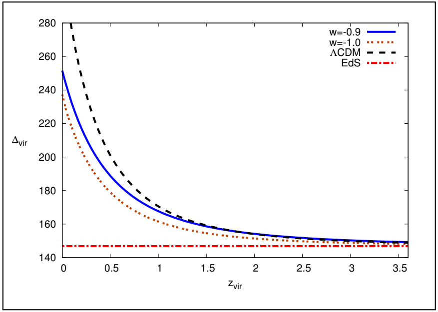
linear in early time:

$\boxed{d\tilde{y}/d\eta|_i \approx 2(1-\delta_{{\rm m},i}/3)/(3\eta_i)}$ $\quad (\delta_{\rm m} \propto \tilde{x} \propto \eta^{2/3})$

Complete Journey of Matter Density Perturbation







- $\Omega_{\mathrm{m},0} = 0.274$

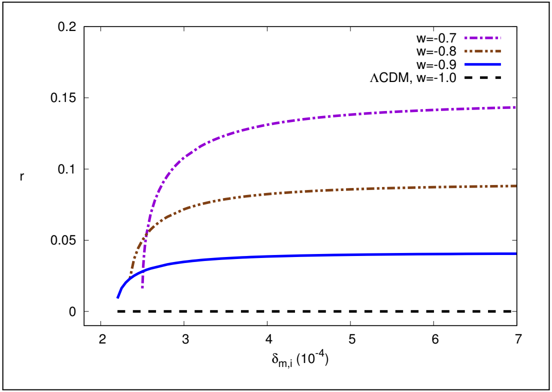
* $\delta_{\mathrm{c}} = \delta_{\mathrm{m}}(\eta_{\mathrm{c}})$

* $\delta_{\mathrm{vir}} = \delta_{\mathrm{m}}(\eta_{\mathrm{vir}})$

* $\Delta_{\mathrm{vir}} = \left(1 + \delta^{\mathrm{NL}}_{\mathrm{m,ta}}\right) \left(\frac{a_{\mathrm{vir}}/a_{\mathrm{ta}}}{R_{\mathrm{vir}}/R_{\mathrm{ta}}}\right)^3$

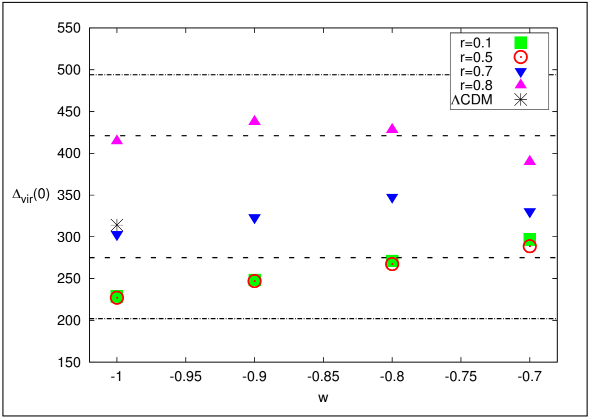
Complete Journey of Matter Density Perturbation

	I	II
$\rho_{\rm m}$	introduce $\zeta \equiv \left.\frac{\rho_{\rm m}}{\bar{\rho}_{\rm m}}\right\vert_{z_{\rm ta}}$ satisfying $y(\tau_{\rm c}) = y(2\tau_{\rm ta}) = 0$	give a large enough $\delta_{{\rm m},i}$
$\rho_{\rm de}$	introduce r with $\delta^{\rm NL}_{\rm de,ta} = r\delta^{\rm NL}_{\rm m,ta}$	$\delta^{\rm NL}_{{\rm de},i} \approx \delta_{{\rm de},i} = (1+w)\delta_{{\rm m},i}$



Complete Journey of Matter Density Perturbation

(Crook et al., ApJ 655 (2007) 790)



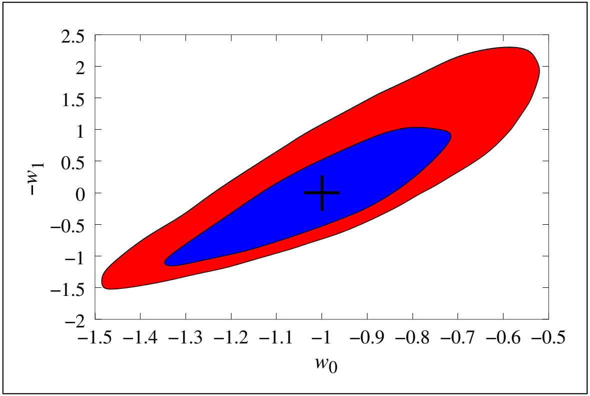
$$\Delta_{\rm vir} = \frac{3M_{\rm vir}}{4\pi\rho_{\rm c,0}\Omega_{\rm m,0}(1+z)^3 R_{\rm vir}^3}$$

- It is likely that $0.5 < r \leq 0.8$ for models containing clustered DE with $w \leq -0.9$ at a confidence level of 68%.

Structure Formation with Time-Varying w

CPL (Chevallier-Polarski-Linder) model:

$$w(a) = w_0 + w_1(1 - a)$$



(U. Seljak et al., PRD 71, 103515 (2005))

Structure Formation with Time-Varying w

CPL (Chevallier-Polarski-Linder) model:

$$
\begin{aligned}
w(a) &= w_0 + w_1(1-a) \\
&= [w_0 + w_1(1-a_{\rm ta})] + w_1 a_{\rm ta}\left(1 - \frac{a}{a_{\rm ta}}\right) \\
&= w_{\rm ta} + w_{1,\rm ta}\left(1 - \frac{a}{a_{\rm ta}}\right)
\end{aligned}
$$

$$
\begin{aligned}
w_b\left(\frac{a}{a_{\rm ta}}\right) &= w_{\rm ta} + w_{1,\rm ta}\left(1 - \frac{a}{a_{\rm ta}}\right) \\
w_c\left(\frac{R}{R_{\rm ta}}\right) &= w_{\rm ta} + w_{1,\rm ta}\left(1 - \frac{R}{R_{\rm ta}}\right)
\end{aligned}
$$

In this way, $w_c = w_b = w_{\rm ta}$ at $t = t_{\rm ta}$.

$$
\begin{aligned}
w_{\rm ta} &= w_0 + w_1(1-a_{\rm ta}) \\
&= w_0 + \frac{w_1 z_{\rm ta}}{1+z_{\rm ta}} \\
w_{1,\rm ta} &= w_1 a_{\rm ta} = \frac{w_1}{1+z_{\rm ta}}
\end{aligned}
$$

Structure Formation with Time-Varying w

Background Universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\bar{\rho}_{\rm m} + (1+3w_b)\bar{\rho}_{\rm de}\right]$$

$$\dot{\bar{\rho}}_{\rm m} + 3\left(\frac{\dot{a}}{a}\right)\bar{\rho}_{\rm m} = 0$$

$$\dot{\bar{\rho}}_{\rm de} + 3(1+w_b)\left(\frac{\dot{a}}{a}\right)\bar{\rho}_{\rm de} = 0$$

Spherical Overdensity

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left[\rho_{\rm m} + (1+3w_c)\rho_{\rm de}\right]$$

$$\dot{\rho}_{\rm m} + 3\left(\frac{\dot{R}}{R}\right)\rho_{\rm m} = 0$$

$$\dot{\rho}_{\rm de} + 3(1+w_c)\left(\frac{\dot{R}}{R}\right)\rho_{\rm de} = 0$$

$$\begin{aligned}
\Rightarrow \frac{\bar{\rho}_{\rm de}}{\bar{\rho}_{\rm de,ta}} &= \exp\left[3\int_{a/a_{\rm ta}}^{1}\frac{1+w_b(u)}{u}du\right] = f\left(\frac{a}{a_{\rm ta}}\right) \\
&= x^{-3(1+w_0+w_1)}e^{-3w_1(1-x)/(1+z_{\rm ta})} \\
\Rightarrow \frac{\rho_{\rm de}}{\rho_{\rm de,ta}} &= \exp\left[3\int_{R/R_{\rm ta}}^{1}\frac{1+w_c(u)}{u}du\right] = f\left(\frac{R}{R_{\rm ta}}\right) \\
&= y^{-3(1+w_0+w_1)}e^{-3w_1(1-y)/(1+z_{\rm ta})}
\end{aligned}$$

Structure Formation with Time-Varying w

$$\frac{dx}{d\tau} = \sqrt{x^{-1} + \frac{1}{Q_{\rm ta}} f\left(\frac{a}{a_{\rm ta}}\right) x^2}$$

$$= \sqrt{x^{-1} + \frac{1}{Q_{\rm ta}} x^{-1-3(w_0+w_1)} e^{-3w_1(1-x)/(1+z_{\rm ta})}}$$

$$\Rightarrow \tau_{\rm ta} = \int_0^1 \frac{dx'}{\sqrt{x'^{-1} + \frac{1}{Q_{\rm ta}} x'^{-1-3(w_0+w_1)} e^{-3w_1(1-x')/(1+z_{\rm ta})}}}$$

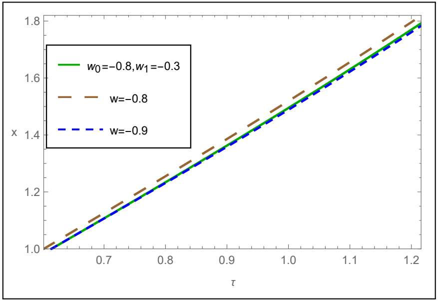
$$\frac{d^2y}{d\tau^2} = -\frac{1}{2}\left[\zeta y^{-2} + \left(\frac{1+3w_c}{Q_{\rm cta}}\right) f\left(\frac{R}{R_{\rm ta}}\right) y\right]$$

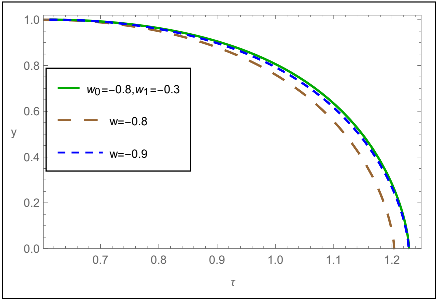
$$= -\frac{1}{2}\left\{\zeta y^{-2} + \frac{1}{Q_{\rm cta}}\left[1 + 3w_0 + 3w_1\left(1 - \frac{y}{1+z_{\rm ta}}\right)\right] y^{-2-3(w_0+w_1)} e^{-3w_1(1-y)/(1+z_{\rm ta})}\right\}$$

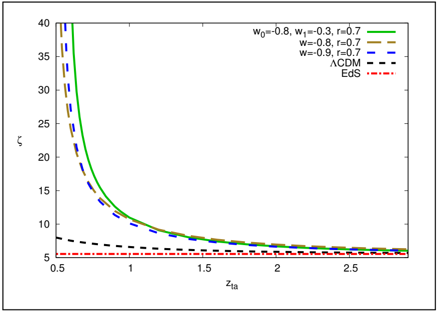
$$\star Q_{\rm ta} \equiv \left.\frac{\bar\rho_{\rm m}}{\bar\rho_{\rm de}}\right|_{z_{\rm ta}} = \frac{\Omega_{\rm m,0}}{\Omega_{\rm de,0}}(1+z_{\rm ta})^{-3(w_0+w_1)} e^{3w_1 z_{\rm ta}/(1+z_{\rm ta})}$$

$$\star Q_{\rm cta} \equiv \left.\frac{\bar\rho_{\rm m}}{\rho_{\rm de}}\right|_{z_{\rm ta}} = \frac{Q_{\rm ta}}{1 + r[\zeta(w_0, w_1, z_{\rm ta}, r) - 1]}$$

Structure Formation with Time-Varying w







- $\Omega_{m,0} = 0.274$
- $r = 0.7$

* $x \equiv \frac{a}{a_{ta}}$

* $y \equiv \frac{R}{R_{ta}}$

* $r = \delta^{NL}_{de,ta} / \delta^{NL}_{m,ta}$

* $\zeta \equiv \frac{\rho_m}{\bar{\rho}_m}\Big|_{z_{ta}}$

Conclusions

- We presuppose $\delta^{\rm NL}_{\rm de,ta} = r\delta^{\rm NL}_{\rm m,ta}$ to remove the difficulty with clustered DE in the SCM.
- We treat the overdense region as an isolate system and determin $y_{\rm vir}(= R_{\rm vir}/R_{\rm ta})$ and the nonlinear overdensity $\Delta_{\rm vir}$. Also we distinguish ΛCDM from the clustering model of DE with $w = -1$.
- $\Delta_{\rm vir}$ in ΛCDM reaches the maximum value while for other clustered DE models the amplitude of $\Delta_{\rm vir}$ is proportional to w.
- We track down the complete evolution of the clustered matter and DE by combining SCM with the linear evolution of density perturbation at early times, equivalent to introducing the new parameter r.
- We have found that the criterion $\delta_{{\rm m},i} \geq 2.2 \times 10^{-4} \sim 2.5 \times 10^{-4}$ depending on w has to be fulfilled to form a cosmic structure.
- The ratio r is less dependent on the initial matter density contrast for cases with $\delta_{{\rm m},i} \geq 5 \times 10^{-4}$.
- We use observational data of galaxy clusters to constrain the parameter r.
- Our method can be extended to DE with time-varying w following from the CPL model.