# Towards the Theory of

# *Quantum Relativity*

**OTTO C. W. KONG**
— Nat'l Central U, Taiwan

人之能知天也，
能知人之天也。

~王夫之

# Symmetry of " Space -Time-U..." : -

## *the New Arena for Fundamental Physics*

**Galilean Relativity**

**Einstein Relativity** — special and general

**(Special) Quantum Relativity**

*or* Doubly (Triply) / Deformed Special Relativity

H.Synder / C.N.Yang (1947)
G.Amelino-Camelia (2001)
L.Smolin + (2002)

$\cdots\cdots$ General Quantum Relativity $\Longrightarrow$ Quantum Gravity +

**My Basic Perspectives** :-
— on issues related to Quantum Space-Time

- need new understanding of "Space-Time" (relativity principle)
- within Space-Time Geometry

  Non-Commutative Geometry *is to* Quantum Gravity
  *as* Non-Euclean Geometry *is to* (Einstein) Gravity

- Quantum Field Theory, on (Minkowski/Einstein) Space-Time, perspective would be valid way beyond the experimentally probed energy/length scale — *a faith* I don't share *(Penrose)*

— Linear (Vs Nonlinear) Realization

- an exploration into **new** ways to think about fundamental physics
- withhold phenomenological discussions

## A Stable Symmetry :-

R.V. Mendes (1994)
Chryssomalakos & Okon (2004)

- perturbations (deformations) of a symmetry algebra
- stable if deformed algebras isomorphic to original

— structure constants perturbation Vs experimental uncertainty

- Examples —

  1/ Galilean to Einstein Relativity ($1/c^2 \neq 0$)

  2/ classical to quantum mechanics ($\hbar \neq 0$)

  Poisson bracket $\Longrightarrow$ Moyal bracket

- deformation parameters $\longrightarrow$ fundamental constants

**Space-Time + Quantum Symmetry**

‘Poincaré + Heisenberg’ algebra

$\Longrightarrow$ SO(1,5) OR SO(2,4)

## WI Contraction :-

$$\boldsymbol{SO(1,3)} \quad \longrightarrow \quad \boldsymbol{ISO(3)}$$

$$[N_i, N_j] = -i\,\frac{1}{c^2}\,M_{ij}$$

$\frac{1}{c^2} \rightarrow 0$ boosts commute (i.e. Galilean)

- $\boldsymbol{SO(1,3)}$ as the stablization :

$$[M_{ij}, L_{0k}] = i\,(\eta_{jk}L_{0i} - \eta_{ik}L_{0j}) \qquad\qquad [L_{0i}, L_{0j}] = -i\,M_{ij}$$

take $N_i \rightarrow \frac{1}{c}\,L_{0i}$ : isomorphic

? value of $c$ : $3 \times 10^8$ Vs 1 or $3 \times 10^{-7}$ ( km ps$^{-1}$ )

## Quantum Relativity $SO(1,5)$ or $SO(2,4)$ :-

$$[J_{AB}, J_{LN}] = i\,(\eta_{BL} J_{AN} - \eta_{AL} J_{BN} + \eta_{AN} J_{BL} - \eta_{BN} J_{AL})$$

with $\eta^{AB} = (1, -1, -1, -1, -1, \mp 1)$.

- linear realization on 6-geometry — $J_{\mathcal{AB}} = i\,(x_{\mathcal{A}} \partial_{\mathcal{B}} - x_{\mathcal{B}} \partial_{\mathcal{A}})$
- $J_{AB}$ for $0 \to 3$ are the 10 generator for Lorentz symmetry $M_{\mu\nu}$
- NOT necessarily means extra space-time dimensions
- 3 deformations — $\boldsymbol{ISO(3)} \longrightarrow \boldsymbol{SO(1,3)} \hookrightarrow \boldsymbol{ISO(1,3)}$ $\longrightarrow \boldsymbol{SO(1,4)} \hookrightarrow \boldsymbol{ISO(1,4)} \longrightarrow \boldsymbol{SO(1,5)}$ **or** $\boldsymbol{SO(2,4)}$
- 3 invariant quantities : $c \quad \kappa \quad \ell$

Kowalski-Glikman & Smolin(2004)
OK (2007)

- curved momentum space $\Longrightarrow$ non-commutative space-time
  $+$ curved space-time $\Longrightarrow$ non-commutative momentum space

# The Three Deformations :-

$SO(1,4)$ :- $|z^A| \leq i\,\ell$
$\eta_{AB}\, z^A z^B = -\ell^2 \left(1 + \frac{1}{G^2}\right)$

| | | |
|---|---|---|
| $\Delta x^i(t) = v^i \cdot t$ | $\Delta x^\mu(\sigma) = p^\mu \cdot \sigma$ | $\Delta x^A(\rho) = z^A \cdot \rho$ |
| $\lvert v^i \rvert \leq c$ <br> $-\eta_{ij} v^i v^j = c^2\left(1 - \frac{1}{\gamma^2}\right)$ | $\lvert p^\mu \rvert \leq \kappa\, c$ <br> $\eta_{\mu\nu} p^\mu p^\nu = \kappa^2 c^2\left(1 - \frac{1}{\Gamma^2}\right)$ | $\lvert z^A \rvert \leq \ell$ <br> $\eta_{AB}\, z^A z^B = \ell^2\left(1 - \frac{1}{G^2}\right)$ |
| $M_{0i} \equiv N_i \sim P_i$ <br> $[N_i, N_j] \longrightarrow -i\, M_{ij}$ | $J_{\mu 4} \equiv O_\mu \sim P_\mu$ <br> $[O_\mu, O_\nu] \longrightarrow i\, M_{\mu\nu}$ | $J_{A5} \equiv O'_A \sim P_A$ <br> $[O'_A, O'_B] \longrightarrow i\, J_{AB}$ |
| $\vec{u}^4 = \frac{\gamma}{c}(c, v^i)$ <br> $\eta_{\mu\nu} u^\mu u^\nu = 1$ <br> $\mathbb{R}^3 \rightarrow SO(1,3)/SO(3)$ | $\vec{\pi}^5 = \frac{\Gamma}{\kappa\, c}(p^\mu, \kappa\, c)$ <br> $\eta_{AB}\pi^A \pi^B = -1$ <br> $M^4 \rightarrow SO(1,4)/SO(1,3)$ | $\vec{X}^6 = \frac{G}{\ell}(z^A, \ell)$ <br> $\eta_{\mathcal{MN}} X^{\mathcal{M}} X^{\mathcal{N}} = -1$ <br> $M^5 \rightarrow SO(1,5)/SO(1,4)$ |

- 4-momentum is *defined by* $p^\mu = \frac{dx^\mu}{d\sigma}$
- $z^A = \frac{dx^\mu}{d\rho}$ is *chosen as* a length
- without the $p^\mu$ deformation, $X^{\mathcal{M}}$ are just alternative coordinates of a de-Sitter space-time
- no $\hookrightarrow ISO(1,5)/ISO(2,4)$ : no naive *translation* symmetries

## Momentum Boosts ( with $\hat{X}_\mu$ ) :-

$$J_{\mu 4} = i\,(x_\mu \partial_4 - x_4\, \partial_\mu)$$

— rotations among $x_\mu$'s and $x_4 = -\kappa\, c\, \sigma$

A.Das & O.K.

★ **$\sigma$ is peculiar** — (*doubt* space-time interpretation !)

- $\boldsymbol{ISO(1,3)} \quad \longrightarrow \quad \boldsymbol{SO(1,4)}$ since Snyder

— $\kappa$ as Planck mass

- preferred frame for a quantum state : $\vec{\pi}^{\,5} = (0,0,0,0,1)$

  the reference frame does not see its own 4-momentum/mass

- boost characterized by $p^\mu$, hence mass dependent

— quantum frames

- $p^\mu = \frac{dx^\mu}{d\sigma}$ Vs $m\,c\,u^\mu = \gamma\,m\,c\,\frac{dx^\mu}{dx^0}$ (as Einstein/classical limit )

  $\sigma \longrightarrow \frac{\tau}{m}$ for an Einstein particle

  recall : "space-like" geometrical signature

- general transformation — on-shell condition not preserved

  classical to quantum frame, uncertainty : observer to observed

- $\sigma$-coordinate Vs literature

— Vs proper time / covariant Schrödinger QM

— system with indefinite mass

— QM on $M^4$ stochastic metric fluctuation

⋆ momentum boosts needed for group quantization (relativistic)

## Quantum Frame of Reference :-

Aharonov & Kaufherr, PR 30, 368
C. Rovelli, Class.Quantum.Grav. 8, 317

• context: nonrelativistic QM / gravitation

• characterized by *mass* (together with $v + \cdots$)

• interaction of frame (observer/device) with observed
— observation has nontrivial effect on observer

• uncertainty principle on frame (observer/device)
$\longrightarrow$ uncertainty on observed

• grav. — local gauge inv. observables obtained after dynamic properties of frame (observer/device) taken into consideration

⋆ *Is there such a thing as a practical quantum measurement ?*

## Translational Boosts ( with $P_a^{(\mathcal{L})}$ — $\hat{P}_\mu$ ) :-

$$J_{A5} = i\,(x_A \partial_5 - x_5\, \partial_A)$$

— rotations among $x_A$'s and $x_5 = -\ell\,\rho$

A.Das & O.K.
O.K.

⋆ $\rho$ may characterize the scale

• boost characterized by $z^A$, a location vector

sort of a translation

• preferred frame for a state : $\vec{X}^6 = (0,0,0,0,0,1)$

the reference frame does not see its own location

— $(0,0,0,0,0,1)$ characterizes the coordinate origin $z^A = 0$

⋆ **a reference frame does not observe itself** (no $v^i$, $p^\mu$, $z^A = 0$)

• Beltrami description *preserves* physics of 5-momentum constraint

- $\boldsymbol{ISO(1,4)} \quad \longrightarrow \quad \boldsymbol{SO(1,5)}$ A.Das & O.K.

— $\ell$ an IF cutoff (Vs $\kappa$) $\leftrightarrow$ cosmological constant

Kowalski-Glikman & Smolin(2004)

— $dS_5$ as "quantum" world

- $\boldsymbol{ISO(1,4)} \quad \longrightarrow \quad \boldsymbol{SO(2,4)}$ O.K.

— $\ell_P$ as Planck length (UV cutoff); $\kappa\, c\, \ell_P = \hbar$

$$( \ \kappa^2 = c\,\hbar/G \quad , \quad \ell_P^2 = \hbar\, G/c^3 \ )$$

— $AdS_5$ as quantum world

- no $\boldsymbol{ISO(1,5)}$ or $\boldsymbol{ISO(2,4)}$ — no coordinate translations

- **quantum general relativity — the role of $G$ ?**

## Non-commutative (Space-Time) Geometry :-

*cf.* Kowalski-Glikman & Smolin
Chryssomalakos & Okon

- (four) space-time position operators $\hat{X}_\mu = -\frac{1}{\kappa c}\, i\,(x_\mu \partial_4 - x_4\, \partial_\mu)$

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i}{\kappa^2 c^2} M_{\mu\nu}$$

- (four) energy-momentum operators $\hat{P}_\mu = \frac{1}{\ell}\, i\,(x_\mu \partial_5 - x_5\, \partial_\mu)$

$$[\hat{P}_\mu, \hat{P}_\nu] = \frac{i}{\ell^2} M_{\mu\nu}$$

$$[\hat{X}_\mu, \hat{P}_\nu] = -i\,\eta_{\mu\nu} \hat{F}\ , \qquad [\hat{X}_\mu, \hat{F}] = +\frac{i}{\kappa^2 c^2} \hat{P}_\mu\ , \qquad [\hat{P}_\mu, \hat{F}] = -\frac{i}{\ell^2} \hat{X}_\mu$$

- $i\,\partial_\mu \ll \kappa c$ and $i\,\partial_4 = p_4 = -\kappa c$ : $\quad \hat{X}_\mu \longrightarrow x_\mu$
- $x_\mu \ll \ell$ and $x_5 = -\ell\ (\rho = 1)$ : $\quad \hat{P}_\mu \longrightarrow i\,\partial_\mu = p_\mu$

## $SO(2,4)$ Quantum Relativity :-

— AdS$_5$ as quantum world

- matching to 4D conformal symmetry :-

$$K_\mu \Rightarrow \sqrt{2} J_{\mu -} \qquad P_\mu \Rightarrow \sqrt{2} J_{\mu +} \qquad D \Rightarrow -J_{45}$$

$$J_{\mu\pm} \equiv i\hbar\,(x_\mu \partial_\pm - x_\pm \partial_\mu) = \frac{1}{\sqrt{2}}(J_{\mu 5} \pm J_{\mu 4})$$

- coordinate transformation — ( gives the right matching )

$$(x^\mu, x^4, x^5) \longrightarrow (y^\mu, \tfrac{1}{2}\eta_{\mu\nu} y^\mu y^\nu + \tfrac{1}{2}, \tfrac{1}{2}\eta_{\mu\nu} y^\mu y^\nu - \tfrac{1}{2})$$

- However : $\Rightarrow \qquad x_+ = x^- = \frac{1}{\sqrt{2}}(x^5 - x^4) = -\frac{1}{\sqrt{2}}$

$$x_- = x^+ = \frac{1}{\sqrt{2}}\, y^2 \qquad \partial_+ = 0 \qquad \partial_5 = -\partial_4 = \frac{1}{\sqrt{2}}\,\partial_- = x^\nu \partial_\nu$$

★ $\eta_{\mathcal{MN}} x^{\mathcal{M}} x^{\mathcal{N}} = \eta_{\mu\nu} x^\mu x^n u + 2\, x_+\, x_- = 0$ $(\neq -1)$ for conformal universe

★ $x^5$, $x^4$ (or $x_- = x^+$) translations related to scaling ?

## Remarks :-

- *(bottom line)* an interesting, radical but sensible, approach
- primitive stage — difficult to make (and identify) progress
- need creative but careful thinking about 'physics'
  *beyond* usual framework
- job for Einstein — hope we can make minor steps

" The chief cause of my failure was my clinging to the idea that the variable $t$ only can be considered as the true time and my local $t'$ must $\cdots\cdots$ In Einstein's theory, $\cdots\cdots$ $t'$ plays the same part as $t$ $\cdots\cdots$."

*The Theory of Electrons* (1916 ed.) — Lorentz ( *from* A.I. Miller)

**Space-Time-U ...**

***as the New Arena for Fundamental Physics***

# ***THANK YOU !***