Disentanglement of Two Harmonic Oscillators in Relativistic Motion

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Outline

I. Introduction: Quantum Entanglement

II. The Model: Unruh-DeWitt Detector Theory

III. Results

IV. Summary

I. Introduction

- Quantum Entanglement

"When two systems, of which we know the states by their respective representatives, ... after a time of mutual influence the systems separate again, then they can no longer be described... by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical line of thought."

- Erwin Schrödinger (1936), in response to...

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

I. Introduction

Entangled/Separable States

- pure state

$$|\psi_{AB}\rangle = |\phi_A\rangle \otimes |\phi_B\rangle$$: separable, otherwise entangled.

Ex. 1. $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ is entangled.

2. $|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$ is entangled.

3. $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$ is separable.

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2. $|\uparrow\ \uparrow\rangle + |\downarrow\ \downarrow\rangle$ is entangled.

→ "non-local"

3. $|\uparrow\ \downarrow\rangle + |\downarrow\ \uparrow\rangle + |\uparrow\ \uparrow\rangle + |\downarrow\ \downarrow\rangle$ is separable.

$$= (|\uparrow\rangle + |\downarrow\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle)$$

Local operation on A

$|\uparrow\rangle \otimes (|\uparrow\rangle + |\downarrow\rangle)$ or $|\downarrow\rangle \otimes (|\uparrow\rangle + |\downarrow\rangle)$ quantum state of B not affected

Outcome of LO on B is independent of outcome of LO on A.

I. Introduction

Entangled/Separable States

- mixed states

$$\hat{\rho}(1,2) = \hat{\rho}(1) \otimes \hat{\rho}(2)$$: simply separable or uncorrelated

$$\hat{\rho}(1,2) = \sum_j k_j \, \hat{\rho}^{(j)}(1) \otimes \hat{\rho}^{(j)}(2), \qquad \sum_j k_j = 1, \quad k_j \geq 0$$: separable or

classically correlated [Werner 1989] , otherwise entangled.

Any classically correlated state can be modeled by a LOCAL hidden-variable theory (described by classical statistical mechanics) and hence satisfies all generalized Bell's inequalities.

I. Introduction

- Criteria of entanglement/separability for mixed states

Positive Partial Transpose (PPT) criterion [Peres 1996]

For a bi-partite (2-party) system ρ (Q_A, P_A; Q_B, P_B), performing a time-reversal transformation on one of the parties (Q_A, P_A; Q_B, P_B) $\longrightarrow$ (Q_A, P_A; Q_B, $-P_B$) (i.e. partial transpose ρ), if the resulting new density matrix ρ^{PT} is a "good" quantum state, then ρ is separable.

* Sufficient and necessary conditions only for

- 2-Level Atom x 2LA, 2LA x 3LA [Horodecki, Horodecki, Horodecki 1997]
- Gaussian states with continuous variables
  [Duan(段路明), Giedke, Cirac, Zoller 2000, Simon 2000]

- Measure of entanglement
  - 2 x 2 : Concurrence [Wooters 1998]
  - Gaussian state : (Logarithm) negativity [Vidal, Werner 2002]

I. Introduction

In short,

- Quantum entanglement plays a crucial role in EPR paradox, violation of Bell's inequality, quantum teleportation, etc.
- Entanglement is essentially non-local.
- Entanglement entirely departs from classical line of thought.
- Measure of entanglement is known only for limited cases.

  - We lack in experience and intuition on entanglement.

I. Introduction

- A surprise: "Sudden death" of entanglement [Yu(于挺), Eberly 2004]

Concurrence

$$C(\rho(t)) = \frac{2}{3}\max\{0, \gamma^2 f(t)\}$$

$C > 0$: entangled.

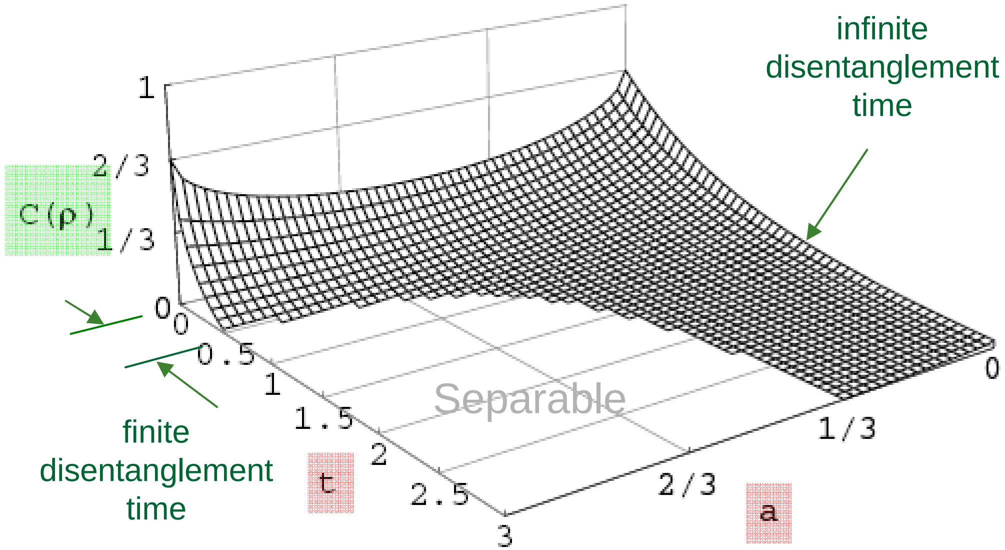
Initial state:

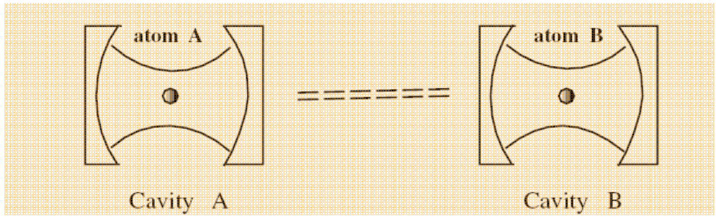
$$\rho_{\text{in}} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$





In Markovian regime

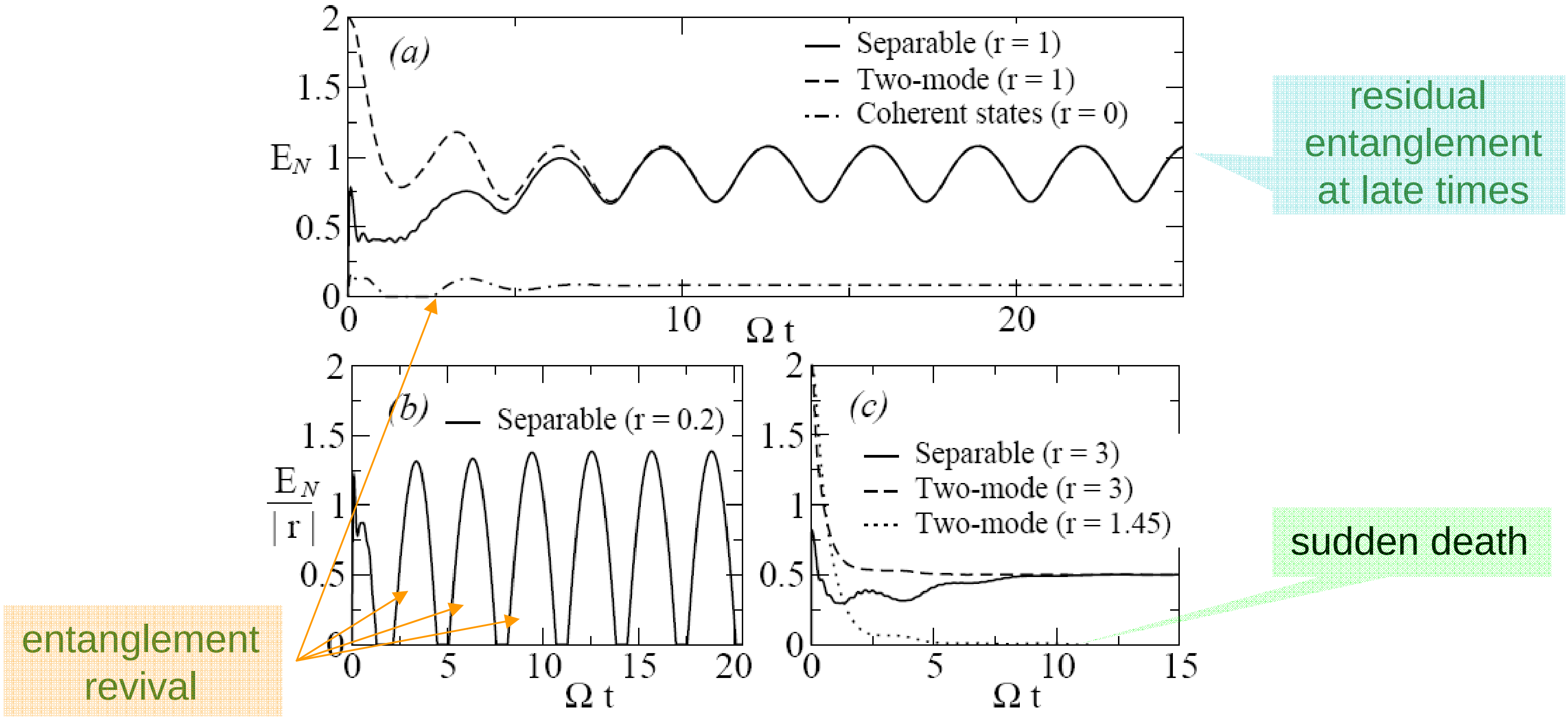
Two independent environments (cavities)

I. Introduction

- Residual entanglement, entanglement revival

Ex: 2 harmonic Oscillators located at a point in space in a quantum field

[Paz, Roncaglia 2008]



I. Introduction

- Entanglement generation (creation) [Braun 2002]

Ex: 2 qubits interacting separately in space with a spin chain

[Lai(賴承彥), Hung(洪若慈), Mou(牟中瑜), Chen(陳柏中) 2008]

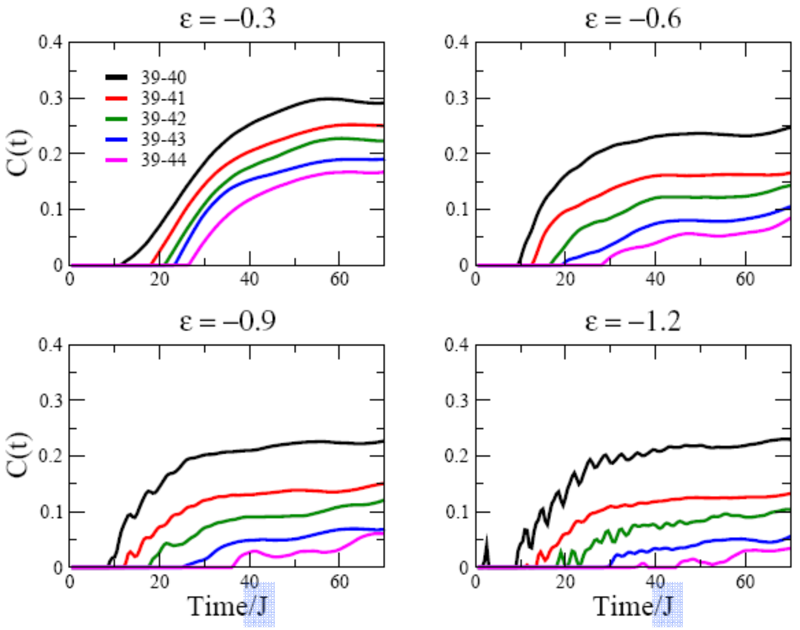


FIG. 7: Entanglement dynamics for an initially disentangled pair of qubits for the case of Heisenberg coupling. Here $\Delta = 0$ and $N = 80$. $(\epsilon_i^x = \epsilon_i^y = \epsilon_i^z \neq 0)$

"XY model"

$$\mathcal{H}_{bath} = J \sum \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right)$$

XXZ Heisenberg model

$$H_{int} = \sum_{i,\alpha} \epsilon_i^\alpha s_{A(B)}^\alpha S_i^\alpha$$

$$\epsilon_1 = \epsilon_2$$

Initial state (separable)

$$\frac{1}{\sqrt{4}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

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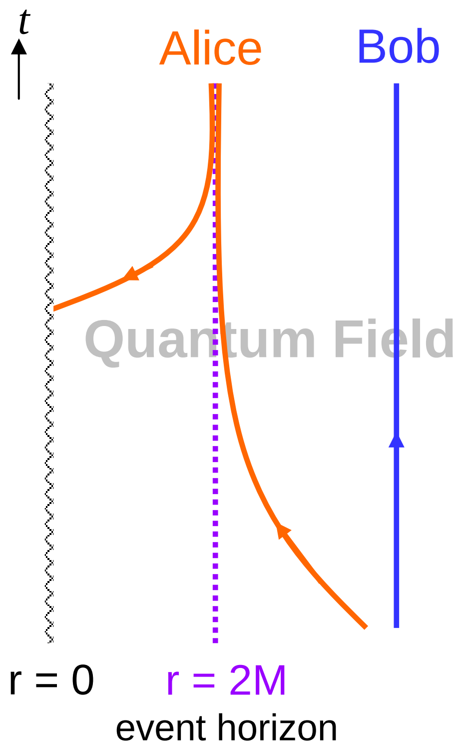
I. Introduction

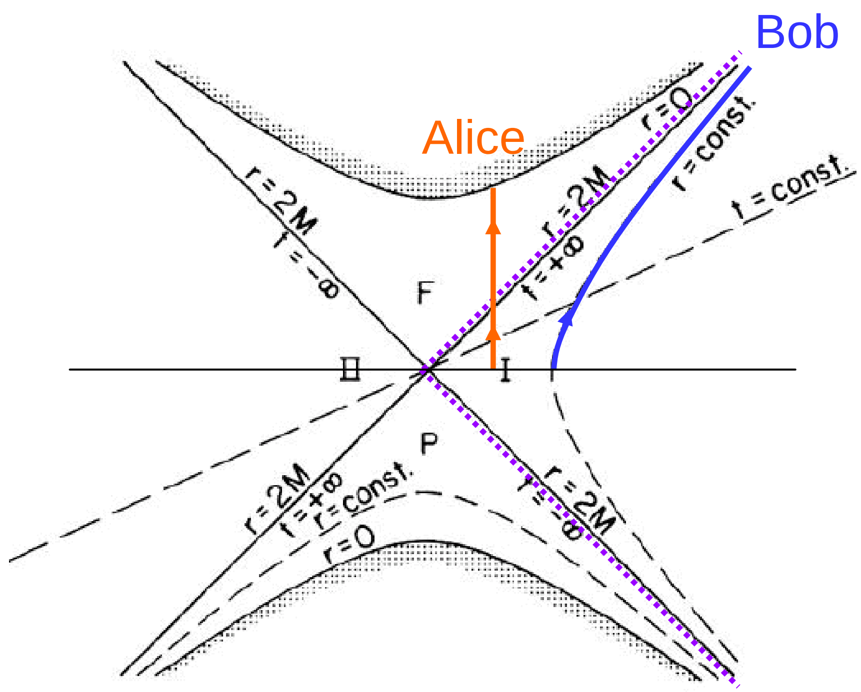
Q: Entanglement across the event horizon?

Schwarzschild black hole

Schwarzschild coordinate



Kruskal coordinate



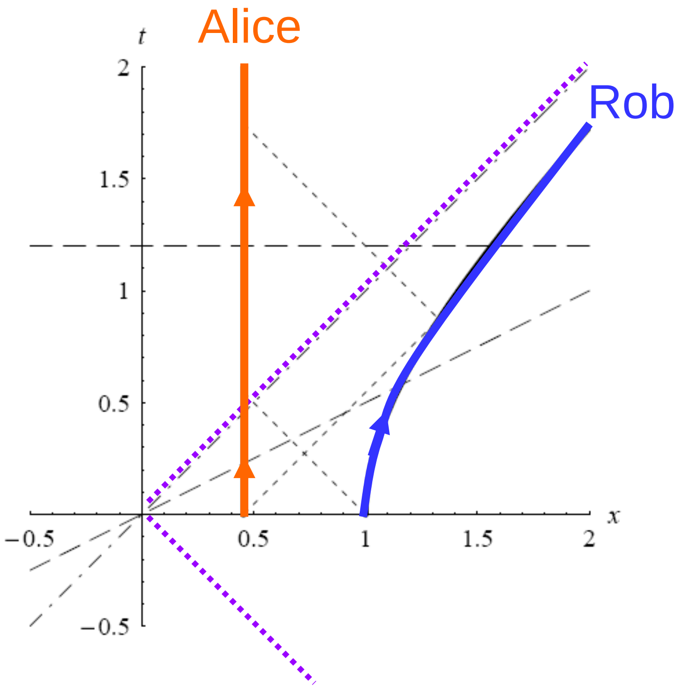
I. Introduction

Entanglement across the event horizon!

Minkowski Space

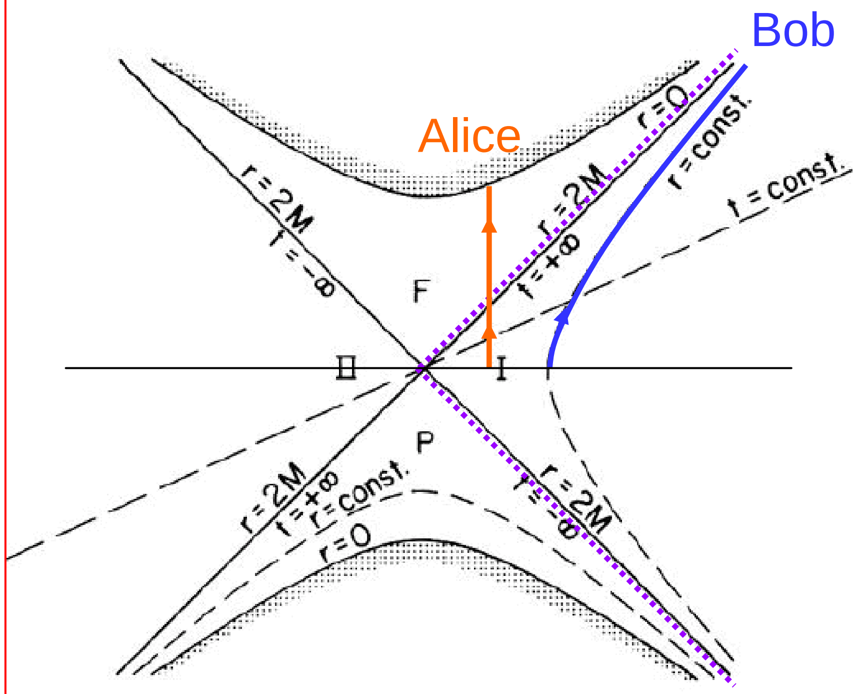
[Alsing and Milburn 2003]

[Fuentes-Schuller and Mann 2005]



Schwarzschild black hole

Kruskal coordinate



I. Introduction

Entanglement across the event horizon!

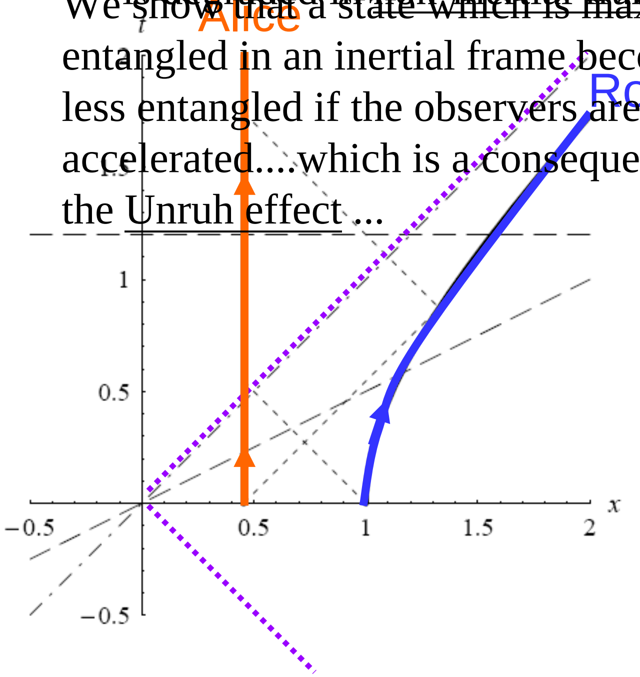
Minkowski Space

[Alsing and Milburn PRL 2003]:

...our result suggests that quantum entanglement is degraded in non-inertial frames.

[Fuentes-Schuller and Mann PRL 2005]:

We show that a state which is maximally entangled in an inertial frame becomes less entangled if the observers are relatively accelerated....which is a consequence of the Unruh effect ...



(See also: Comment by Schützhold and Unruh, [quant-ph/0506028].)

Unruh effect : a detector uniformly accelerated in Minkowski vacuum will experience a thermal bath at Unruh temperature $T = a/2\pi$.

II. The Model

II. The Model

- 2 identical Unruh-DeWitt detectors in (3+1)D Minkowski space

$$S = -\int d^4x \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi$$

- massless scalar field

$$+\int d\tau_A \frac{1}{2}\left[(\partial_{\tau_A} Q_A)^2 - \Omega_0^2 Q_A^2\right] + \int d\tau_B \frac{1}{2}\left[(\partial_{\tau_B} Q_B)^2 - \Omega_0^2 Q_B^2\right]$$

- internal: HO

$$+\lambda_0 \int d^4x \Phi(x) \left[\int d\tau_A Q_A(\tau_A)\delta^4\left(x^\mu - z_A^\mu(\tau_A)\right) + \int d\tau_B Q_B(\tau_B)\delta^4\left(x^\mu - z_B^\mu(\tau_B)\right)\right]$$

- bilinear interaction [DeWitt 1979]
  Detectors A, B are point-like objects.

cf. 2 HO Quantum Brownian Motion [Chou, Yu, Hu 2007; Paz, Roncaglia 2008]

$$H_{\rm tot} = H_{\rm sys} + H_{\rm bath} + H_{\rm int}$$

$$H_{\rm sys} = \frac{P_1^2}{2M} + \frac{1}{2}M\Omega^2 x_1^2 + \frac{P_2^2}{2M} + \frac{1}{2}M\Omega^2 x_2^2 + \kappa(x_1 - x_2)^k$$

$$H_{\rm bath} = \sum_{n=1}^{N_B}\left(\frac{p_n^2}{2m_n} + \frac{1}{2}m_n\omega_n^2 q_n^2\right) \qquad H_{\rm int} = (x_1 + x_2)\sum_{n=1}^{N_B} C_n q_n$$

~ 2 inertial HOs at the same space-point

($x_i \sim Q_i$, $q_n \sim \phi_k$, $C_n \sim -\lambda_0 e^{ikz}$ with $a = \kappa = 0$, $M = 1$.)

II. The Model

- **Motion of Detectors**

  Q_A : at rest, $(b>2a)$

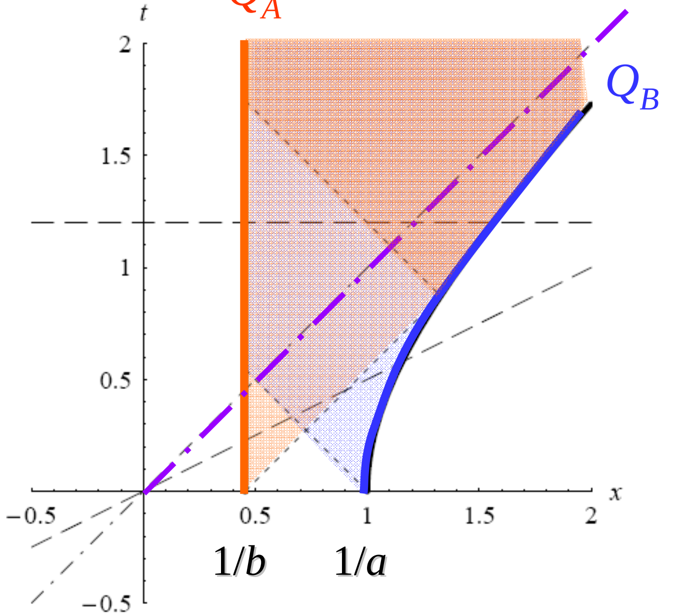
  $$z_A^\mu(t) = (t, 1/b, 0, 0)$$

  Q_B : uniformaly accelerated,

  $$z_B^\mu(\tau) = (a^{-1}\sinh a\tau, a^{-1}\cosh a\tau, 0, 0)$$

  a: proper acceleration

Event horizon for B can be clearly defined since B is ultra-localized.



- **Initial state at $t = \tau = 0$,**

$$|\,\psi(0)\,\rangle = |\,q_A, q_B\,\rangle \otimes |\,0_M\,\rangle$$

$|\,0_M\,\rangle$: Minkowski vacuum

$|\,q_A, q_B\,\rangle$ ~ two-mode squeezed state, represented by Wigner function,

$$W(Q_A,P_A,Q_B,P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$$

$|\,\psi(0)\,\rangle$ is a Gaussian state!

II. The Model

Dynamics of entanglement between A and B

Define $$\Sigma(t, \tau = a^{-1}\sinh^{-1} at) \equiv \det\left[\mathbf{V}^{PT} + i\frac{\hbar}{2}\mathbf{M}\right]$$

$$\mathbf{M} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

For Gaussian states, **Σ** < 0 ⟺ entangled, otherwise separable [Simon 2000]

$\mathbf{V}^{PT} = \mathbf{\Lambda V \Lambda}$: Partial Transposition of **V** , $\mathbf{\Lambda} = \mathrm{diag}(1, 1, 1, -1)$

$$V_{\mu\nu}(t, \tau) = \langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle \equiv \frac{1}{2}\langle\, (\mathcal{R}_\mu \mathcal{R}_\nu + \mathcal{R}_\nu \mathcal{R}_\mu)\,\rangle$$: 10 symmetric two-point functions (variances) of two detectors.

$$\mathcal{R}_\mu = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t))$$

Note: This criterion is testing the property of the PT Wigner functions, thus the reduced density matrix, of the detectors:

$$\rho^R(Q, Q'; \tau) = \int \mathcal{D}\Phi_k \psi_0[Q, \Phi_k; \tau]\psi_0^*[Q', \Phi_k; \tau]$$

So t and **τ** in **Σ** must be on the same time-slice as the field's.

Here, behavior of **Σ** ~ (logarithm) negativity (by Vidal & Werner, 2002).

II. The Model

Sketch of calculation

- <u>Evolution of operators $Q_A, P_A, Q_B, P_B, \boldsymbol{\Phi}, \boldsymbol{\Pi}$ in Heisenberg picture.</u>

$$\hat{Q}_i(\tau_i) = \sqrt{\frac{\hbar}{2\Omega_r}}\sum_j \left[q_i^{(j)}(\tau_i)\hat{a}_j + q_i^{(j)*}(\tau_i)\hat{a}_j^\dagger\right] + \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}\left[q_i^{(+)}(\tau_i,\mathbf{k})\hat{b}_\mathbf{k} + q_i^{(-)}(\tau_i,\mathbf{k})\hat{b}_\mathbf{k}^\dagger\right],$$

$$\hat{\Phi}(x) = \sqrt{\frac{\hbar}{2\Omega_r}}\sum_j \left[f^{(j)}(x)\hat{a}_j + f^{(j)*}(x)\hat{a}_j^\dagger\right] + \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}\left[f^{(+)}(x,\mathbf{k})\hat{b}_\mathbf{k} + f^{(-)}(x,\mathbf{k})\hat{b}_\mathbf{k}^\dagger\right],$$

with $i,j = A,B,\ \tau_A = t,\ \tau_B = \tau$.

Heisenberg equations imply

$$\left(\partial_{\tau_i}^2 + \Omega_0^2\right) q_i^{(j)}(\tau_i) = \lambda_0 f^{(j)}(z_i^\mu(\tau_i)),$$

~ damped HO

$$\left(\partial_t^2 - \nabla^2\right) f^{(j)}(x) = \lambda_0\left[\int_0^\infty dt\, q_A^{(j)}\delta^4(x - z_A(t)) + \int_0^\infty d\tau\, q_B^{(j)}\delta^4(x - z_B(\tau))\right],$$

$$\left(\partial_{\tau_i}^2 + \Omega_0^2\right) q_i^{(+)}(\tau_i,\mathbf{k}) = \lambda_0 f^{(+)}(z_i^\mu(\tau_i),\mathbf{k}),$$

~ damped HO driven by vacuum fluctuations

$$\left(\partial_t^2 - \nabla^2\right) f^{(+)}(x,\mathbf{k}) = \lambda_0\left[\int_0^\infty dt\, q_A^{(+)}(t,\mathbf{k})\delta^4(x - z_A(t)) + \int_0^\infty d\tau\, q_B^{(+)}(\tau,\mathbf{k})\delta^4(x - z_B(\tau))\right].$$

- solving EOM/FE for c-number functions with proper initial conditions.

II. The Model

Sketch of calculation

- Evolution of operators $Q_A, P_A, Q_B, P_B, \Phi, \Pi$ in Heisenberg picture.

$$\hat{Q}_i(\tau_i) = \sqrt{\frac{\hbar}{2\Omega_r}} \sum_j \left[q_i^{(j)}(\tau_i)\hat{a}_j + q_i^{(j)*}(\tau_i)\hat{a}_j^\dagger \right] + \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{\hbar}{2\omega}} \left[q_i^{(+)}(\tau_i, \mathbf{k})\hat{b}_\mathbf{k} + q_i^{(-)}(\tau_i, \mathbf{k})\hat{b}_\mathbf{k}^\dagger \right]$$

damped HO **damped driven HO**

- 10 symmetric two-point functions (variances)

$$V_{\mu\nu}(t,\tau) = \langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle \equiv \frac{1}{2} \langle\, (\mathcal{R}_\mu \mathcal{R}_\nu + \mathcal{R}_\nu \mathcal{R}_\mu)\, \rangle$$

where $\mathcal{R}_\mu = (Q_B(\tau), P_B(\tau), Q_A(t), P_A(t)),\ \mu, \nu = 1, 2, 3, 4.$

- Operators sandwiched by the initial state $|\,\psi(0)\,\rangle = |\, q_A, q_B \,\rangle \otimes |\, 0_M \,\rangle$

→ $\langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle = \langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle_{\rm v} + \langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle_{\rm a}$

where

$$\langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle_{\rm v} = \frac{1}{2} \langle\, 0_M | (\mathcal{R}_\mu \mathcal{R}_\nu + \mathcal{R}_\nu \mathcal{R}_\mu) | 0_M \,\rangle$$

$$\langle\, \mathcal{R}_\mu, \mathcal{R}_\nu \,\rangle_{\rm a} = \frac{1}{2} \langle\, q_A, q_B | (\mathcal{R}_\mu \mathcal{R}_\nu + \mathcal{R}_\nu \mathcal{R}_\mu) | q_A, q_B \,\rangle$$

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- <u>10 symmetric two-point functions (variances)</u>

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$$\langle \mathcal{R}_\mu, \mathcal{R}_\nu \rangle = \langle \mathcal{R}_\mu, \mathcal{R}_\nu \rangle_{\mathrm{v}} + \langle \mathcal{R}_\mu, \mathcal{R}_\nu \rangle_{\mathrm{a}}$$

introducing cut-offs

where

$$\langle \mathcal{R}_\mu, \mathcal{R}_\nu \rangle_{\mathrm{v}} = \frac{1}{2} \langle 0_M | (\mathcal{R}_\mu \mathcal{R}_\nu + \mathcal{R}_\nu \mathcal{R}_\mu) | 0_M \rangle = \mathrm{Re} \int \frac{\hbar d^3k}{(2\pi)^3 2\omega} r_\mu^{(+)}(t_\mu, \mathrm{k}) r_\nu^{(-)}(t_\nu, \mathrm{k})$$

$$\langle \mathcal{R}_\mu, \mathcal{R}_\nu \rangle_{\mathrm{a}} = \frac{1}{4} \Big\{ \hbar^2 \beta^{-2} \mathrm{Re}\left(r_\mu^{(A)} + r_\mu^{(B)}\right) \mathrm{Re}\left(r_\nu^{(A)} + r_\nu^{(B)}\right) + \alpha^2 \mathrm{Re}\left(r_\mu^{(A)} - r_\mu^{(B)}\right) \mathrm{Re}\left(r_\nu^{(A)} - r_\nu^{(B)}\right) + \Omega_r^{-2} \left[\beta^2 \mathrm{Im}\left(r_\mu^{(A)} + r_\mu^{(B)}\right) \mathrm{Im}\left(r_\nu^{(A)} + r_\nu^{(B)}\right) + \hbar^2 \alpha^{-2} \mathrm{Im}\left(r_\mu^{(A)} - r_\mu^{(B)}\right) \mathrm{Im}\left(r_\nu^{(A)} - r_\nu^{(B)}\right) \right] \Big\}$$

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- <u>Partial Transposition</u> $\mathbf{V}^{PT} = \mathbf{\Lambda V \Lambda}$ $\qquad \mathbf{\Lambda} = \mathrm{diag}(1,1,1,-1)$

<u>The quantity</u> $\Sigma(t, \tau = a^{-1}\sinh^{-1} at) \equiv \det\left[\mathbf{V}^{PT} + i\frac{\hbar}{2}\mathbf{M}\right]$ $\qquad \mathbf{M} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$

III. Results

III. Results

- Evolution of $\boldsymbol{\Sigma}$ in

  1. Ultraweak coupling limit, in view of A
  2. Weak coupling limit, both B and A are at rest
  3. high acceleration regime
  4. Ultraweak coupling limit, in view of B
  5. Non-Markovian regime

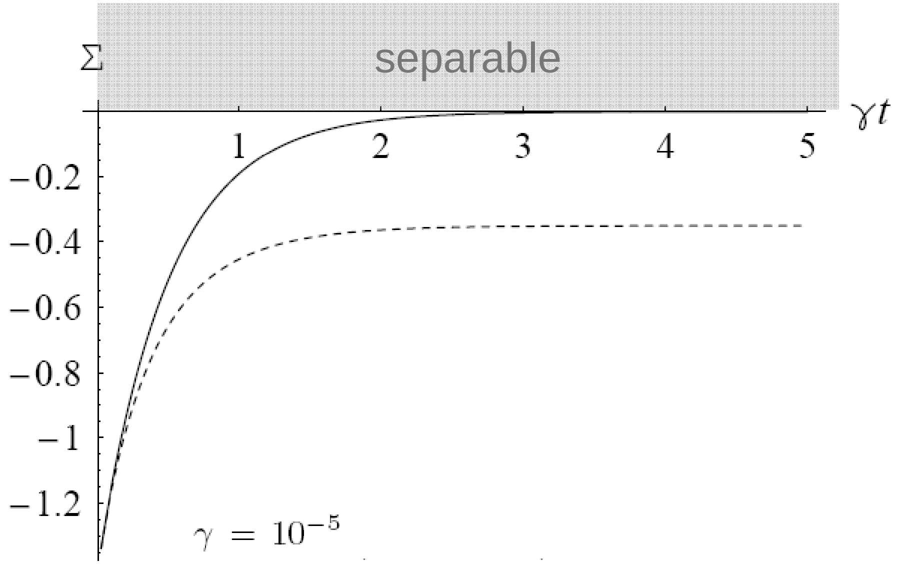
- Detector-detector entanglement vs. detector-field entanglement

- Entanglement vs initial separation

III. Results

- Ultraweak coupling, in view of A $(\gamma\Lambda_1 \ll a,\ \Omega)$ $\gamma \equiv \lambda_0^2/8\pi$

Λ_1 ~ proper time resolution of detectors A and B



$\gamma = 10^{-5}$
$a = 0.1,\ b = 0.201,\ \alpha = 1.1,\ \beta = 4.5,$
$\Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3$ and $\hbar = 1$

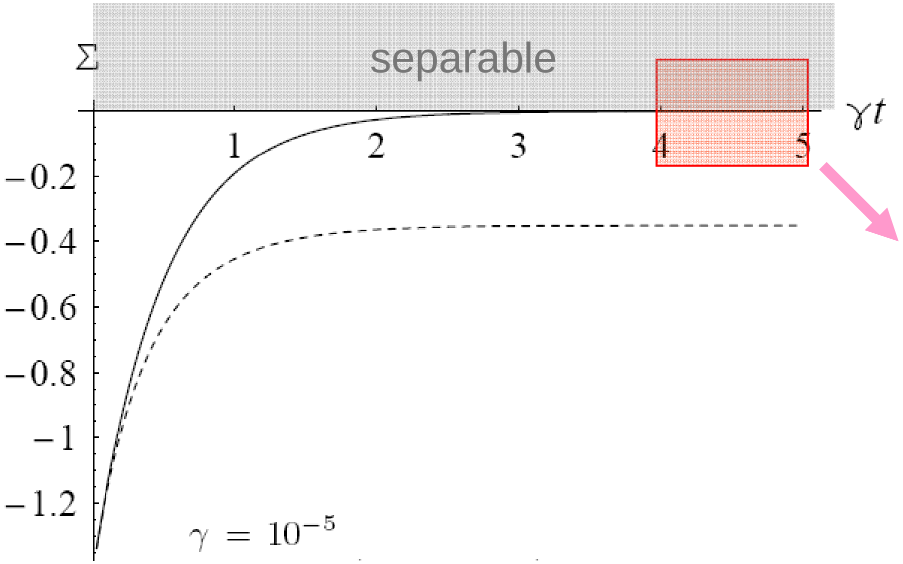
$$\Sigma \approx -\frac{\hbar^2}{16\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 e^{-2\gamma t}$$

Initially $W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2}\exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A+Q_B)^2 + \frac{1}{\alpha^2}(Q_A-Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A-P_B)^2 + \frac{1}{\beta^2}(P_A+P_B)^2\right]$

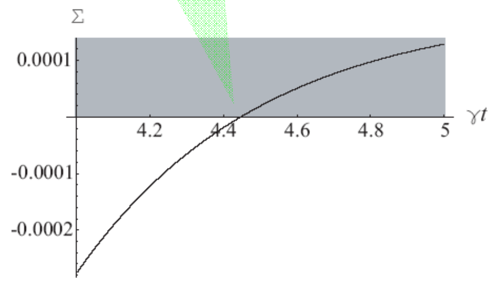
III. Results

- Ultraweak coupling, in view of A $(\gamma\Lambda_1 \ll a,\ \Omega)$

$\gamma \equiv \lambda_0^2/8\pi$



$t_{dE} \approx \frac{1}{2\gamma} \ln \frac{\pi\Omega}{\gamma\Lambda_1}$



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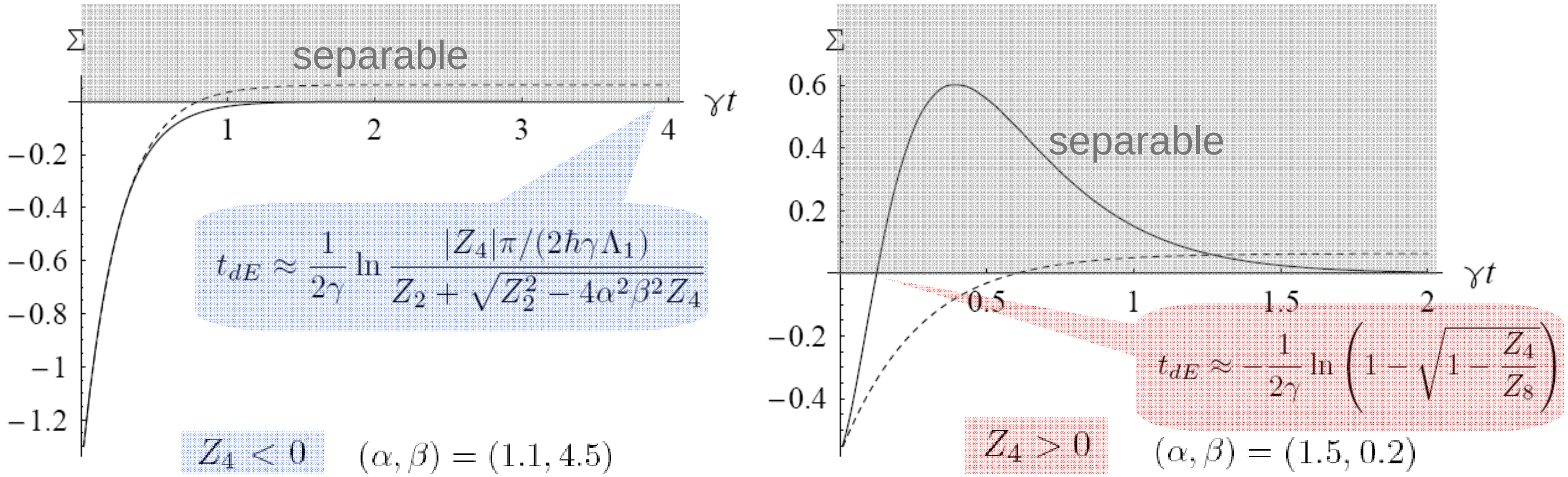
$a = 0.1,\ b = 0.201,\ \alpha = 1.1,\ \beta = 4.5,$
$\Lambda_0 = \Lambda_1 = 100,\ \Omega = 2.3 \text{ and } \hbar = 1$

$$\Sigma \approx -\frac{\hbar^2}{16\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 e^{-2\gamma t} + \frac{\hbar^2\gamma\Lambda_1}{16\pi\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2$$

Initially $W(Q_A, P_A, Q_B, P_B) = \frac{1}{\pi^2\hbar^2} \exp -\frac{1}{2}\left[\frac{\beta^2}{\hbar^2}(Q_A + Q_B)^2 + \frac{1}{\alpha^2}(Q_A - Q_B)^2 + \frac{\alpha^2}{\hbar^2}(P_A - P_B)^2 + \frac{1}{\beta^2}(P_A + P_B)^2\right]$

III. Results

- Ultraweak coupling, both at rest $\qquad \Omega \gg \gamma\Lambda_1 \gg a \to 0$



$\gamma = 10^{-5} \quad \Lambda_0 = \Lambda_1 = 100, \ \Omega = 2.3 \text{ and } \hbar = 1$

$$\Sigma \approx \frac{\hbar^2 e^{-4\gamma t}}{16\alpha^2\beta^2\Omega^2}\left[Z_8\left(e^{-4\gamma t} - 2e^{-2\gamma t}\right) + Z_4\right] + \frac{\hbar^3\gamma\Lambda_1}{4\pi\alpha^2\beta^2\Omega^2} Z_2 e^{-2\gamma t} + \frac{\hbar^4}{\pi^2\Omega^2}\gamma^2\Lambda_1^2$$

$$Z_8 \equiv \left(\hbar - \alpha^2\Omega\right)^2 \left(\beta^2 - \hbar\Omega\right)^2,$$

$$Z_4 \equiv \hbar^2\left(\beta^4 + \alpha^4\Omega^4 + 6\alpha^2\beta^2\Omega^2\right) - 2\hbar\Omega\left(\beta^2 + \alpha^2\Omega^2\right)\left(\hbar^2 + \alpha^2\beta^2\right).$$

$$Z_2 \equiv \alpha^2\left(\beta^2 - \hbar\Omega\right)^2 + \beta^2\left(\alpha^2\Omega - \hbar\right)^2$$

I. Introduction

- A surprise: "Sudden death" of entanglement [Yu(于挺), Eberly 2004]

"Concurrence"

$$C(\rho(t)) = \frac{2}{3}\max\{0, \gamma^2 f(t)\}$$

> 0 : entangled.

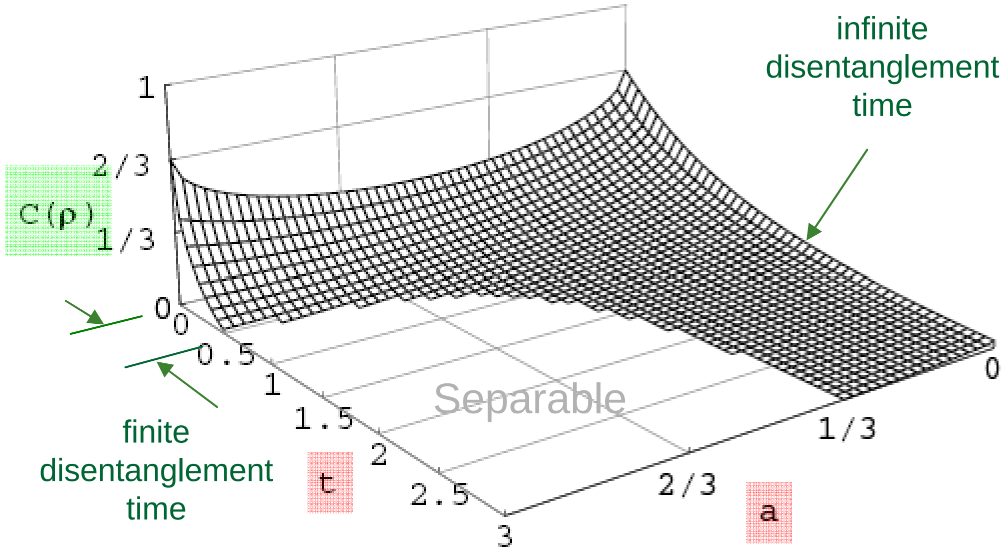
Initial state:

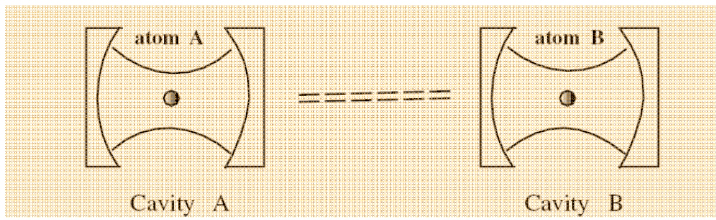
$$\rho_{\text{in}} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}$$

$$f(t) = 1 - \sqrt{a(1 - a + 2\omega^2 + \omega^4 a)}$$

$$\omega = \sqrt{1 - \exp[-\Gamma t]}$$

$$\gamma = \exp[-\Gamma t/2]$$



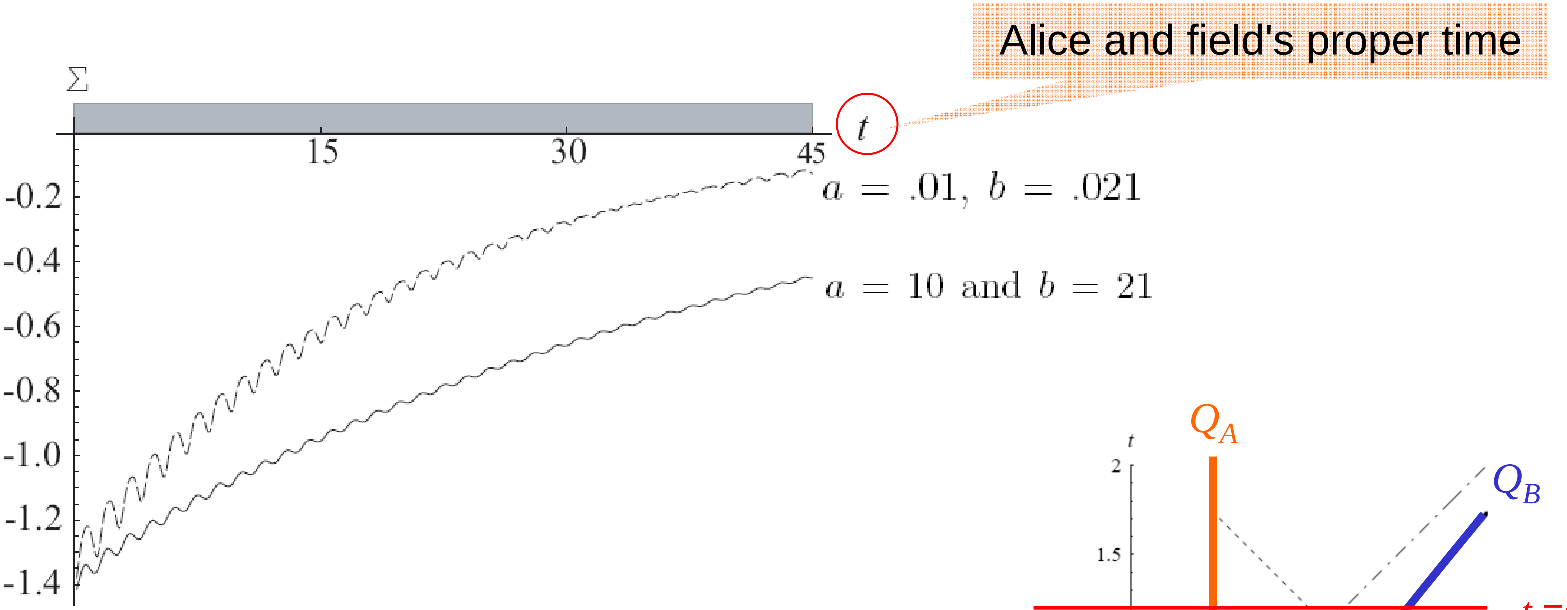


In Markovian regime

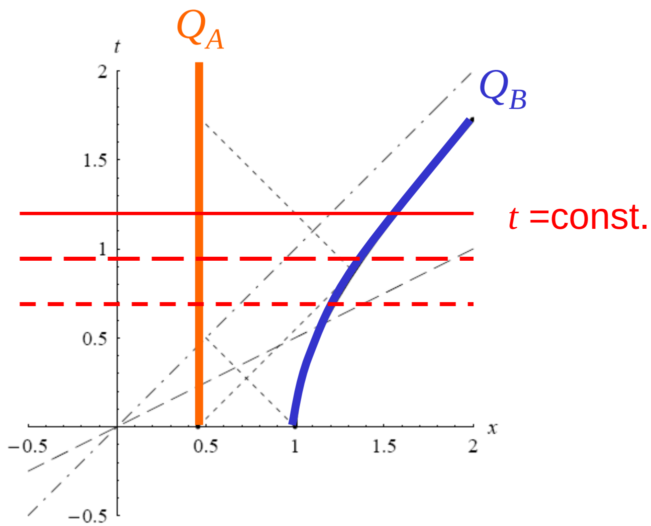
Two independent environments (cavities)

III. Results

- high acceleration (Unruh temperature) regime



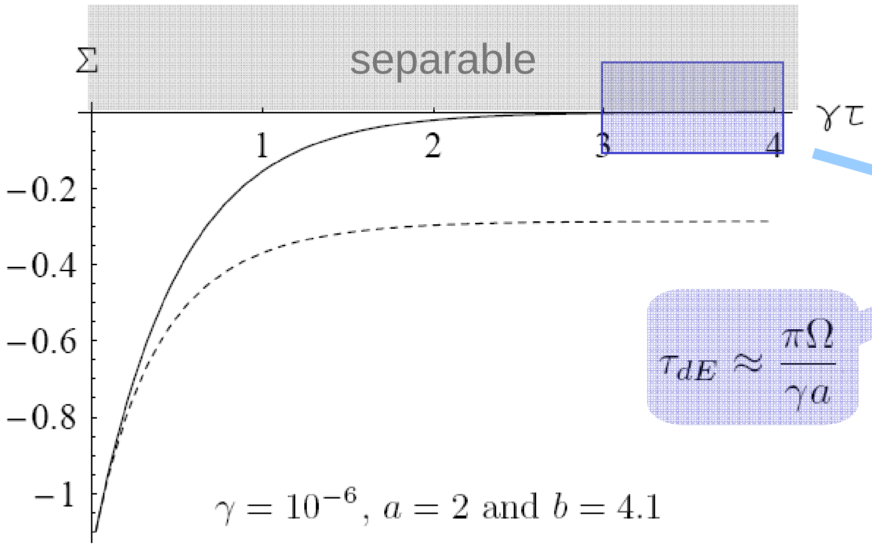
Slower growing rate due to
larger time-dilation of B in view of A



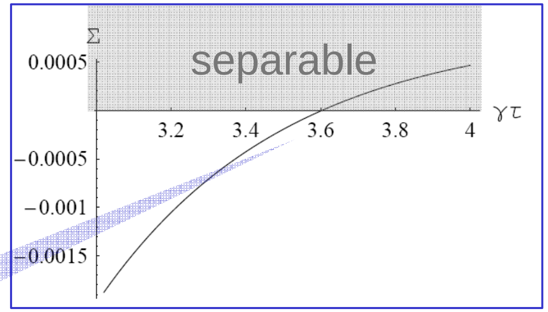
III. Results

- Ultraweak coupling limit, in view of B

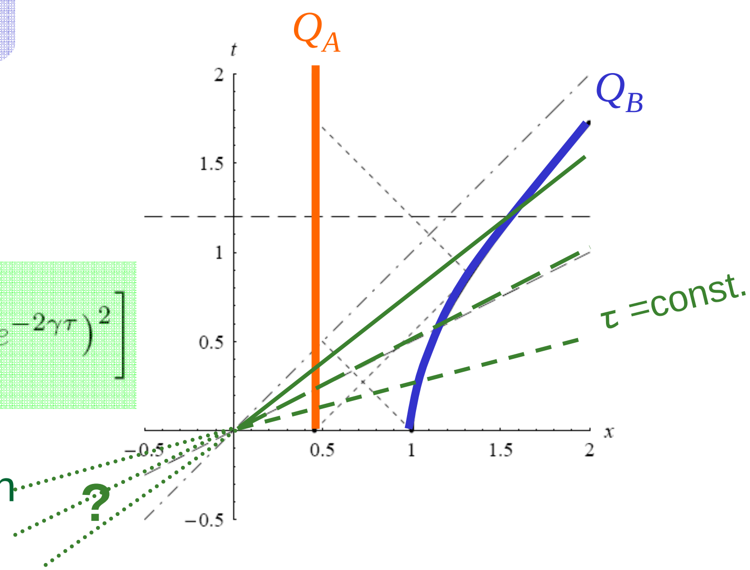
Entanglement is degraded by Unruh effect ?! (AM03)



$\gamma = 10^{-6}$, $a = 2$ and $b = 4.1$



$$\tau_{dE} \approx \frac{\pi\Omega}{\gamma a}$$

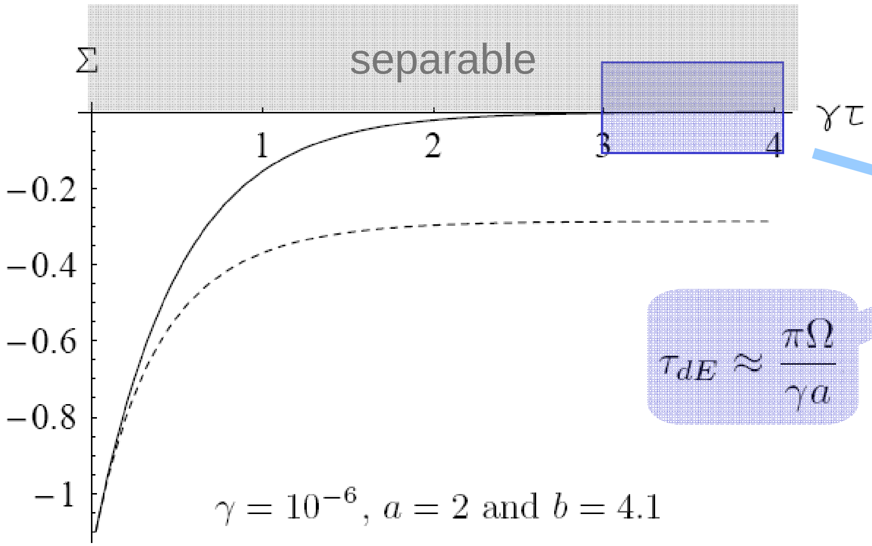


$$\Sigma \approx \frac{\hbar^2}{64\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2\left[\left(1 - e^{-2\gamma\tau}\right)^2 \coth^2\frac{\pi\Omega}{a} - \left(1 + e^{-2\gamma\tau}\right)^2\right]$$

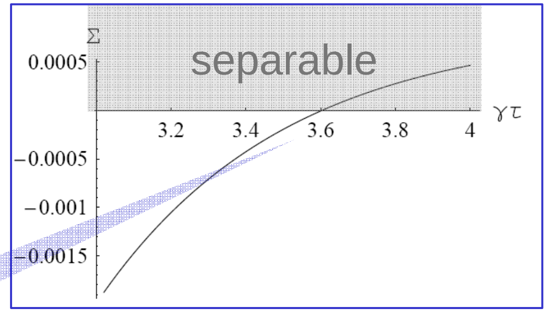
Unruh effect : a detector uniformly accelerated in Minkowski vacuum will experience a thermal bath at Unruh temperature $T = a/2\pi$.

III. Results

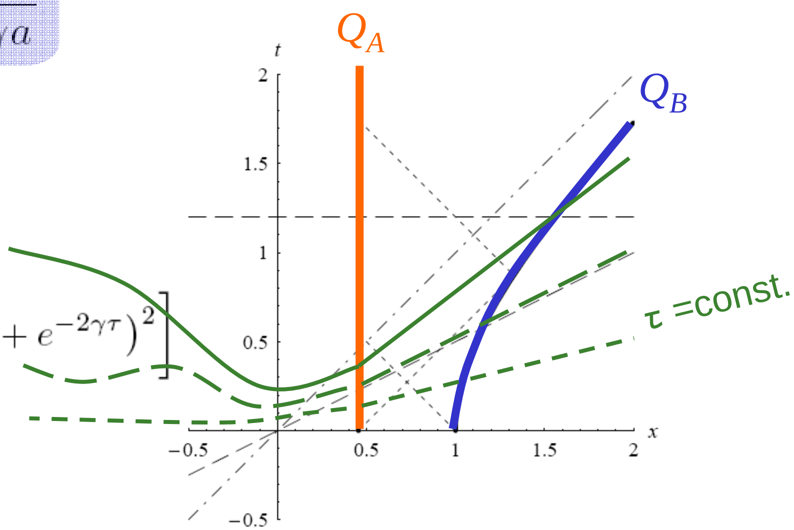
- Ultraweak coupling limit, in view of B



$\gamma = 10^{-6}$, $a = 2$ and $b = 4.1$



$$\tau_{dE} \approx \frac{\pi\Omega}{\gamma a}$$

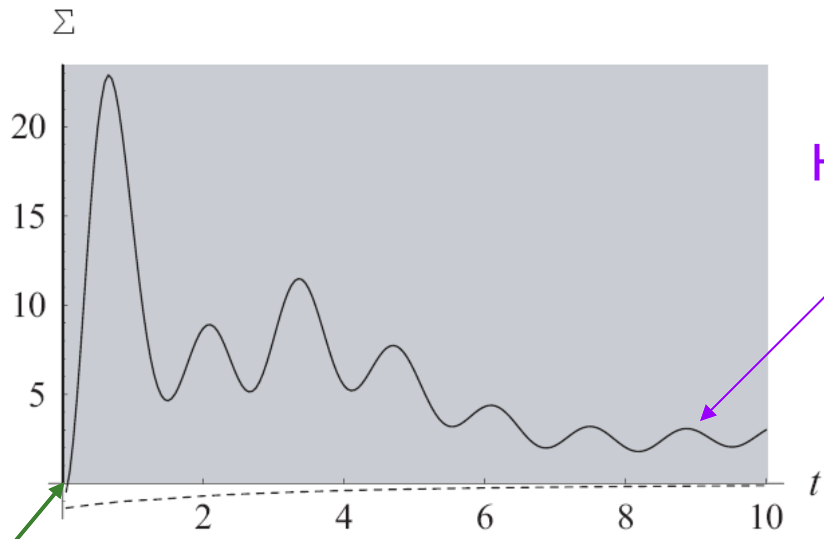


$$\Sigma \approx \frac{\hbar^2}{64\alpha^2\beta^2}\left(\hbar^2 - \alpha^2\beta^2\right)^2 \left[\left(1 - e^{-2\gamma\tau}\right)^2 \coth^2\frac{\pi\Omega}{a} - \left(1 + e^{-2\gamma\tau}\right)^2\right]$$

No guarantee on the Gaussianity of the field state in arbitrary time slicing.

III. Results

- Non-Markovian regime



$\gamma = 0.1,\ \Lambda_0 = \Lambda_1 = 50,\ a = 1,\ b = 2.01,$

Higher order corrections from mutual influences < 2% in this plot.

Quantum entanglement is destroyed right after the coupling is switched on.

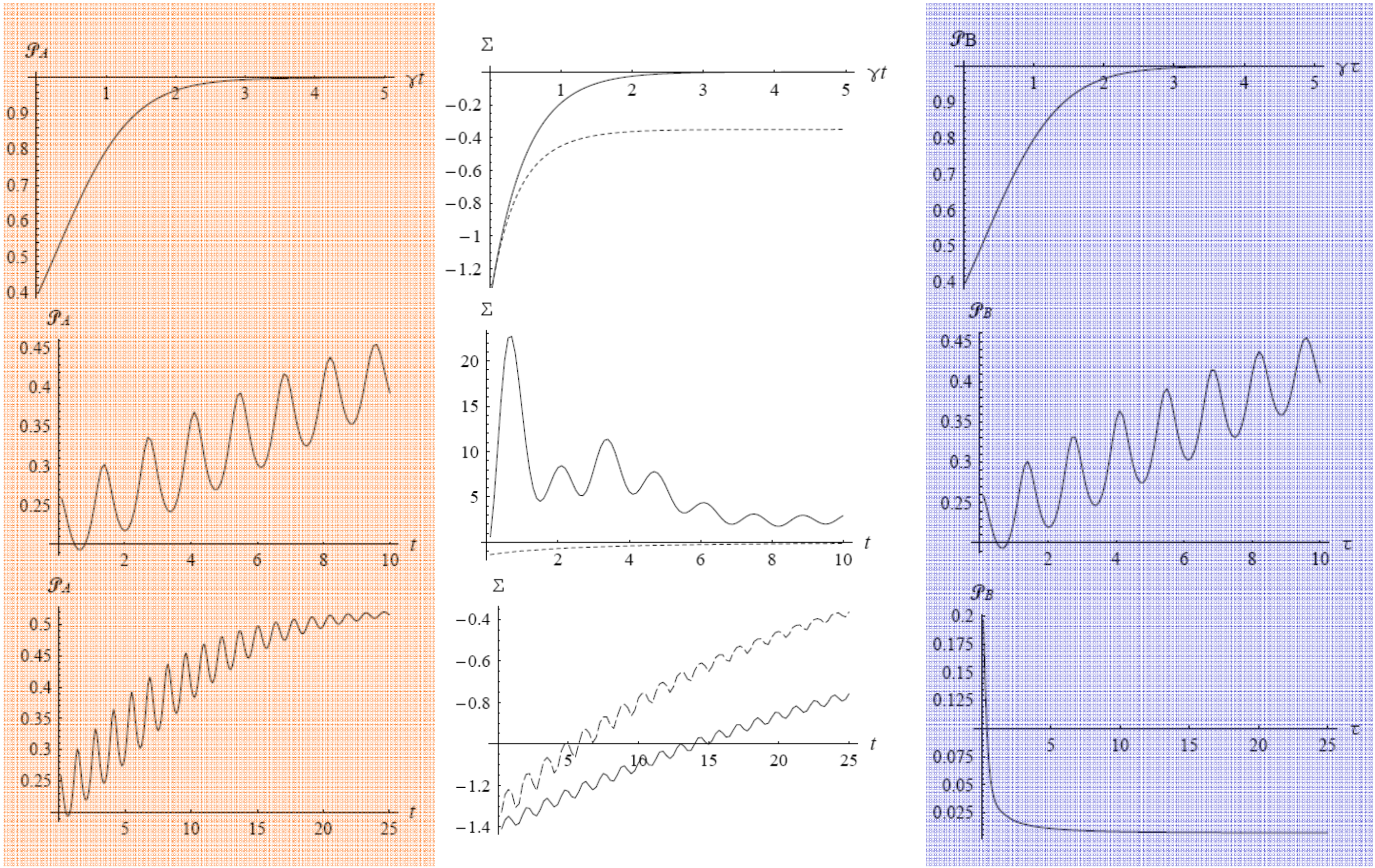
III. Results

- Detector-Detector Entanglement vs.
  Detector-Field Entanglement (Purity of Detector)

Ultraweak coupling

Non-Markovian

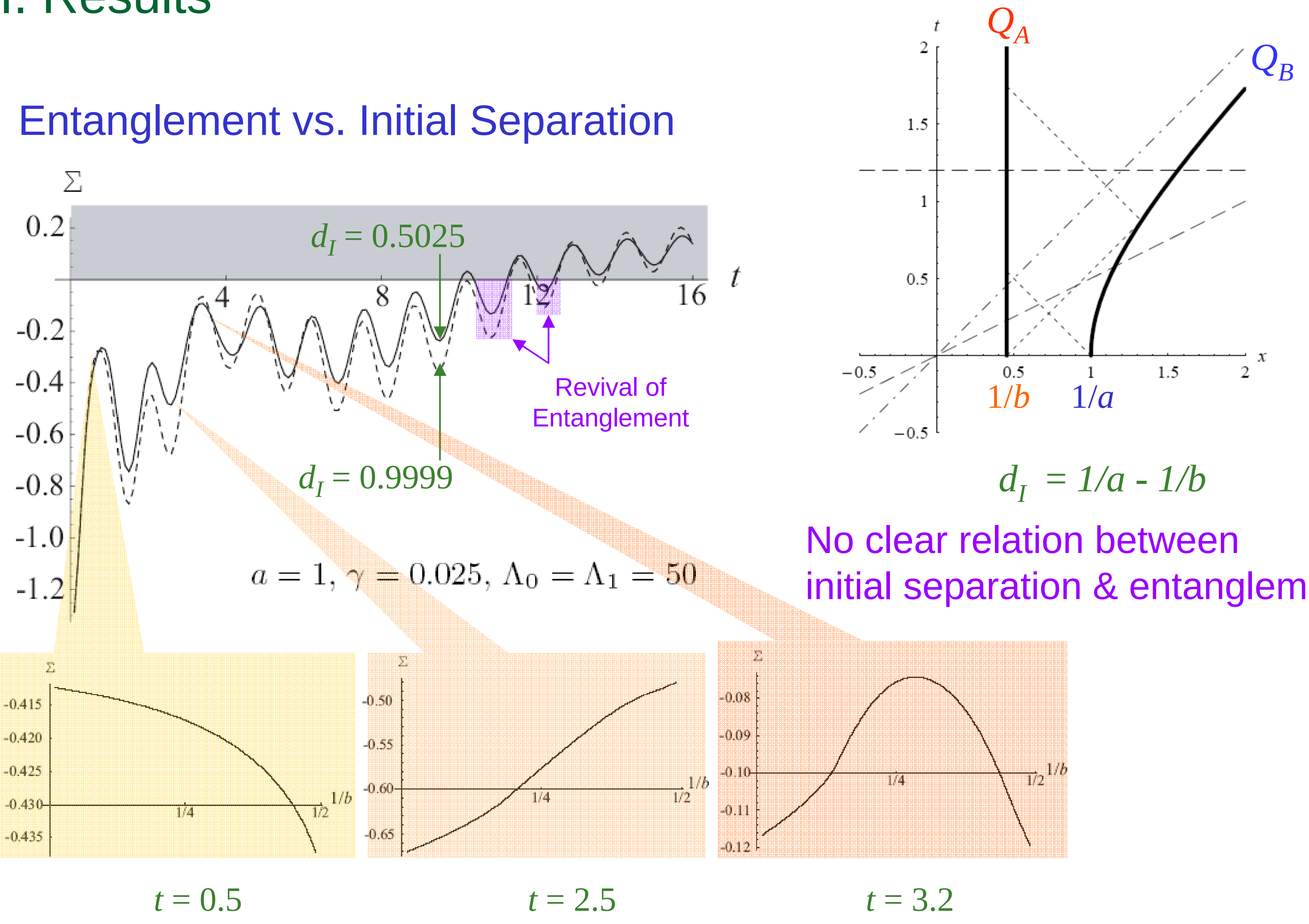
Ultrahigh acceleration



A ∞ (Field + B) | (A ∞ B) / Field | B ∞ (Field + A)

III. Results

- Entanglement vs. Initial Separation



d_I = 1/a - 1/b

No clear relation between initial separation & entanglement

IV. Summary

IV. Summary

- Interaction with the environment (QF) does induce disentanglement.
  - Disentanglement time of A and B in **all** cases we studied is **finite**;
    No residual A∞B (entanglement) at late times.
  - No long-time ($> O(1/\Omega)$) A∞B generated.

How generic are above features?

- We are considering an linearly coupling atom-field system in (3+1)D free space with no direct interaction between two spatially well-separated (and running away) atoms.

- Each reduced density matrix is associated with a time-slicing scheme, entanglement measures could be scheme dependent.
  - Quantum field offers a natural choice of coordinate ("new aether" by DeWitt).
  - In Rindler time, the greater a, the shorter disEnt time. (Unruh effect?!)
    However, the iff criterion (~ sgn Σ) could not be valid.
- No clear relation between initial separation and entanglement of two UD detectors (with small mutual influences.)