Dynamical Gauge-Higgs Unificaion in Randall-Sundrum Spacetime

(based on HH, hep-Xh/0610XXX)

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Plan of this Talk

1. Introduction to Gauge-Higgs Unification

2. Hosotani Mechanism on RS

(a) SU(2) model
(b) SU(3) model (Application for Dynamical Gauge Higgs Unification on RS)

3. Summary/Discussion

1. Introduction

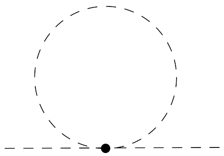
Gauge Hierarchy problem

- Difficulty in keeping Large Difference between M_{EW} (Electroweak Scale, Higgs mass scale $\sim O$(100GeV)) and $M_{GUT}, M_{pl} > 10^{15}$GeV (GUT/Planck Scales)

- Triplet Doublet Problem in GUT theories

- Quadratic divergence of Higgs mass correction

$$m_h^2 = m_{h,0}^2 + \lambda\Lambda^2 \quad (1)$$



$$\simeq \lambda\Lambda^2 \qquad \Lambda : \text{ cutoff scale} \quad (2)$$

This is finetuning if $m_{h,0}, \Lambda \sim M_{pl,GUT}$.

Solution to the Gauge Hierarchy Problem

In 4-dimensional world

- Supersymmetry
- Dynamical Symmetry Breaking(technicolor, top-condensation)
- pseudo Nambu-Goldstone boson (PNGB), Little Higgs

In higher-dimension

- Large Extra Dimension Arkani-Hamed etal 1998
- Warped Extra Dimension Randall,Sundrum 1999
- Gauge Higgs Unification N.S.Manton, D.B.Fairlie (1979) (next slide)

Gauge-Higgs Unification

Gauge-Higgs Unification – Basic Idea

- Consider Gauge Theory in Higher-Dimensional ($D > 4$) space-time
- regard extra component of gauge fields as higgs field in $4D$

$$(A^{\mu}, A^{y} = H) \tag{3}$$

- Higgs Potential can be induced dynamically
  "Dynamical Gauge-Higgs Unification"
  ("Hosotani mechanism" play a lore of generating effective potential, like "Coleman-Weinberg mechanism" in 4D Higgs theory)

Hosotani Mechanism

If the topology of the extra dimension allow the non-contractible loop along it,

- Wilson line $W = \oint dy g A_y$ becomes the non-integrable phase
- In non-abelian gauge theory, W may break the gauge symmetry
- In $5D$ theory, no potential term for A_y at tree level

Effective potential(flat extra dimension)

massless field (suppresion factor $\sim e^{-mR}$ appear for massive field)

$$F(\theta) \equiv \frac{3}{128\pi^7 R^5} \mathrm{Re}\,\mathrm{Li}_5(e^{i\theta}), \quad \mathrm{Re}\,\mathrm{Li}_5(e^{i\theta}) = \sum_{n=1}^{\infty} \frac{\cos n\theta}{n^5} \sim \cos\theta \tag{4}$$

$$V_{eff}^{fd} = f(5) \sum_{i=1}^{N_c} F(\theta_i), \quad (N_c \text{fermion}) \tag{5}$$

$$V_{eff}^{ad} = f(5) \sum_{i=1}^{N_c} F(\theta_i), \quad (\text{adjoint fermion}) \tag{6}$$

$$V_{eff}^{g+gh} = -(5-2) \sum_{i=1}^{N_c} F(\theta_i), \quad (\text{gauge} + \text{ghost}) \tag{7}$$

here $\langle A_y \rangle = \frac{1}{2\pi g R} \mathrm{diag}(\theta_1 \cdots \theta_{N_c})$

"Weirdness" of EWGHU

However, Naive models of GHU have several "weired" features.

1. "Yukawa" coupling

$$\int dy\,(\bar{\psi}_f g A_y \psi_f) \tag{8}$$

naively looks flavor-universal. It seems defficult to get mass hierarchy and mixing in this model

2. mass of fermions

$$m_f \sim \sqrt{\frac{\theta^2}{R^2} + m_{f,0}^2} \gtrsim \frac{\theta}{R} \tag{9}$$

θ : Wilson-line phase,

R : size of compactification

If mass of electron (or up,down quark) was generated by this mechanism, $R^{-1} \sim m_e$ (too large extra dimension), otherwise $|\theta| << 1$ (fine tuning)

3. Higgs mass is coming from second derivative of the effective potential ($\because$ no bare mass term for "Higgs" in DGHU)

$$m_H^2 = \left.\frac{\partial^2 V(\langle H \rangle)}{\partial \langle H \rangle^2}\right|_{\langle H \rangle = \langle H \rangle_{min}} \simeq \sqrt{\alpha_W} m_W / \theta_W \qquad (10)$$

($m_W = 80.4$GeV, $\alpha_W = g^2_{SU(2)4D}/4\pi$),
$\rightarrow$ $m_H \sim 10$GeV.

$\rightarrow$ *"Unbearable Lightness of Higgs"*

4. Hosotani mechanism in flat space is very different from 4D Higgs (Coleman-Weinberg) mechanism:

contribution to the symmetry breaking

	4D Higgs(CW)	HM - flat
fermions with small mass	decouple (small Yukawa)	large effect ($e^{-mR} \sim 1$)
fermions with large mass	large effect (large Yukawa)	decouple ($e^{-mR} \ll 1$)
1loop $V_{\rm eff}$	include Λ	finite

5. The Weinberg angle is determined by the gauge group G

6. For most of the group G, $\sin^2\theta_W$ is much different from experimental balue 0.23.

G	$\sin^2\theta_W$
$SU(3)$	$3/4$
$SO(5)$	$1/2$
G_2	$1/4$

Within the non-warped extra dimension,

1. tuning small θ
   (Haba-Takenaga-Yamashita 2005) (Sakamoto Takenaga 2006)

2. geometrical hierarchy
   (Murayama etal 2002, Ibanez PLB181-269(1986))

3. introducing extra $U(1)$ to tune $\sin^2\theta_W$.

We hope some of these problems will be cured
when the Hosotani mechanism be considered
in Randall-Sundrum Warped Space... (next chapter)

Recent Topics of Electroweak Gauge-Higgs Unification

- Proof of Higher-loop finiteness
  - Two-loop (Maru et.al,)
  - Any higher-loop (Hosotani,2006)
  - in deconstruction theory (Arkani-Hamed et.al, Phys.Lett. (2002)))
- Mass correction for simply connected (S^2) case
  - scalar QED on $M^d \times S^2$ (HH-Inami-Lim($d = 0$), Maru-Yamashita(general d))
- EWGHU on Warped spacetime (Next Chapter)

Gauge-Higgs Unification on RS spacetime

Outline of EWGHU on RS

- consider the Gauge theory $G \supset SU(2) \times U(1)$ on RS
- Gauge and Fermionic fields are living both on branes and in the bulk
- Hierarchy between EWPT and the Fundamental scale are obtained by RS mechanism
- EWPT is caused by the Hosotani mechanism (dynamical gauge higgs unification) – Higgs mass is stabilized by 5D gauge symmetry

DGHU on RS – Advantages

1. Orbifold (S^1/Z_2) topology ... Z_2 projection yieldls chiral fermion, and $SU(2)$ fundamental Higgs from adjoint gauge field

2. Fermion mass hierarchy is obtained order-one tuning of bulk mass parameters

3. “Higgs mass” is enhanced by warp index “kR”

4. “Higgs” is naturally localized on TeV brane

Recent Works for GHU on Warped Spacetime

- dual picture and phenomenology (Contino and Nomura 2003, Contino)
  utilizing AdS/CFT correspondence to consider composite Higgs model

- Hosotani mechanism
  - fermion part without bulk mass (Toms,)
  - gauge boson loop (Weiler-Oda, 2004)
  - Higgs mass and various phenomenology
    * $SU(3)$ model, W-boson coupling (Hosotan etal,05)
    * $SO(5) \times U(1)$, WWZ,WWH,ZZH-couplings (Sakamura etal, 06)

2. Hosotani Mechanism on RS

Preliminary

Details on this part will appear on HEP preprint server (`arXiv.org`) soonly...

3. Summary and Discussion

Summary

- calculate the effective potential for fermion with bulk mass
- apply to the $SU(3)$ model again. Numerically analyzed the dynamical gauge-Higgs unificaion in the model

$SU(3)$ model	flat	warped $(kR = 12.0)$
Z-boson mass	$2m_W = 160$GeV	$0 \sim 75$GeV
	(fixed)	depends on θ
fermion mass tuning	(hierarchical)	$\mathcal{O}(1)$ tuning
Higgs mass	~ 10GeV	$70 \sim 250$GeV

Hosotani mechanism in warped space has good compatibility with 4D Higgs (Coleman-Weinberg) mechanism

contribution to the symmetry breaking

	4D Higgs(CW)	HM - flat	HM - warped
fermions with small 4D-mass	decouple (small Yukawa)	large effect ($e^{-mR} \sim 1$)	decouple (small overlap)
fermions with large 4D-mass	large effect (large Yukawa)	decouple ($e^{-mR} \ll 1$)	large effect (large overlap)
1-loop $V_{\rm eff}$	include Λ	finite	finite

To be done...

1. How to get realistic $\sin^2\theta_W$?
2. How to give masses to down-type quarks ?
3. How to forbid massless fermions ("bino","higgsino") ?
4. This $SU(3)$ model doesn't have custodial symmetry..
5. Something happneing on m_Z/m_W side. How about g_2/g_1 side?