Dark Energy Phenomenology

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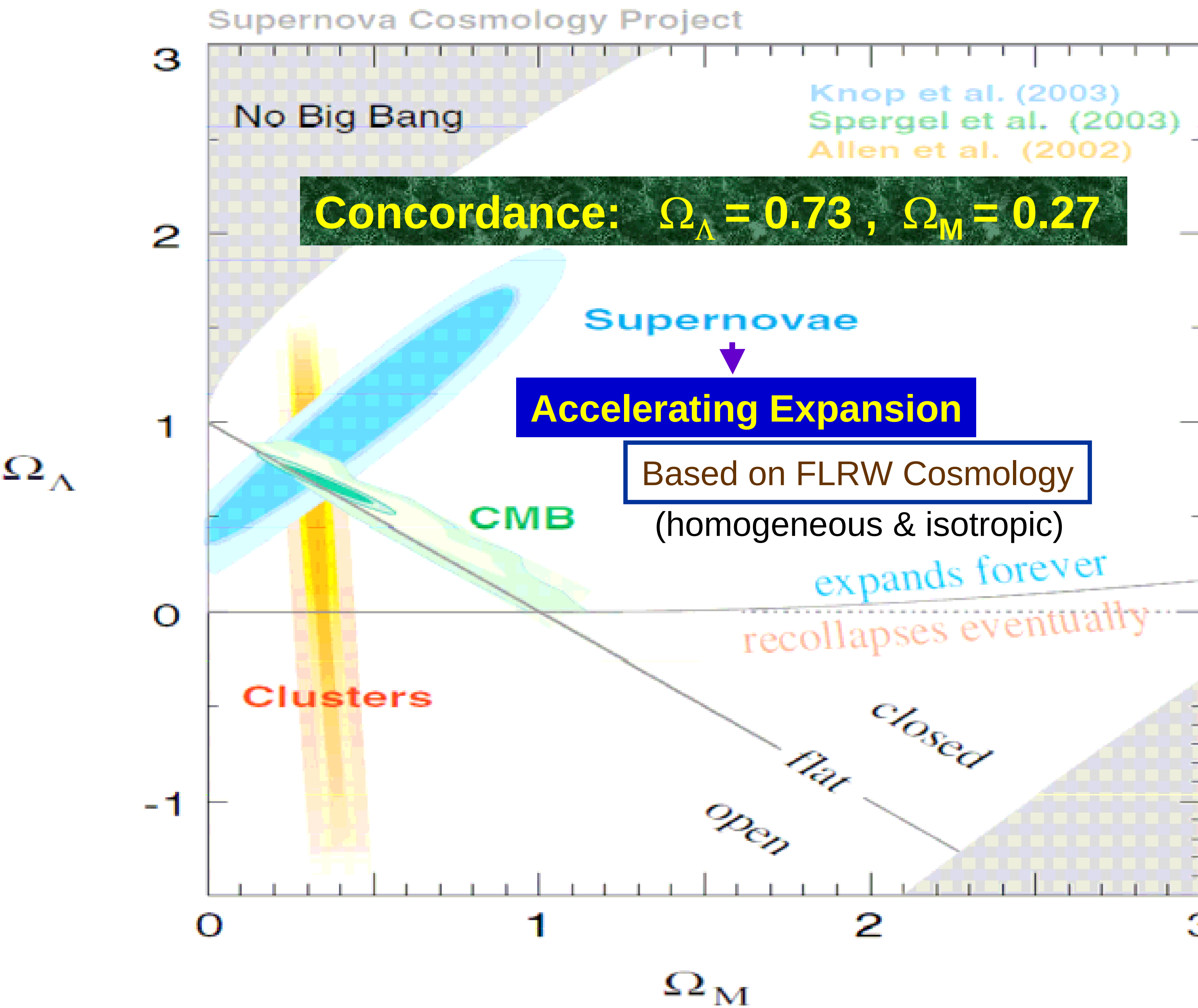
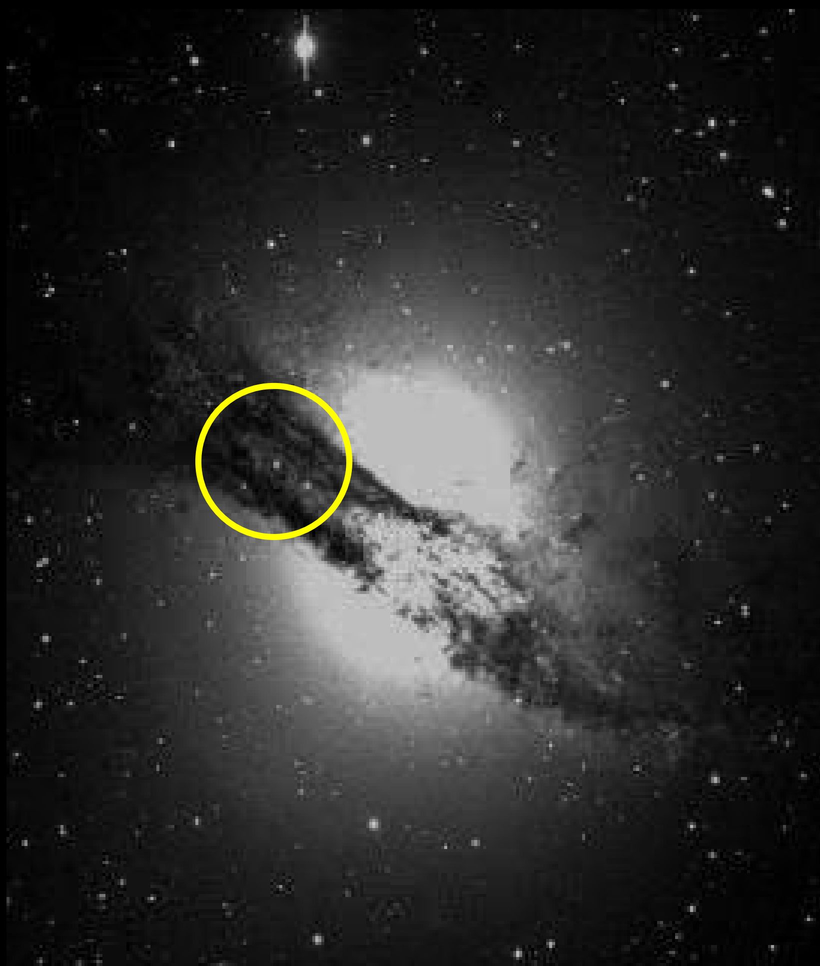
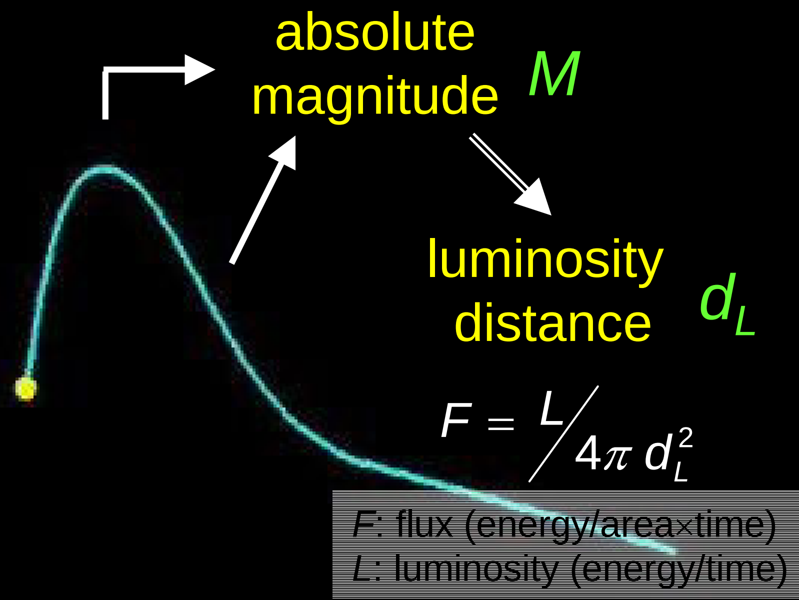
Introduction

(Basic Knowledge, SN, SNAP)


Supernova Cosmology Project
No Big Bang
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)
Concordance: $\Omega_\Lambda = 0.73$, $\Omega_M = 0.27$
Supernovae
Accelerating Expansion
Based on FLRW Cosmology
(homogeneous & isotropic)
CMB
expands forever
recollapses eventually
Clusters
closed
flat
open
Ω_Λ
Ω_M
3
2
1
0
-1
0
1
2
3





SN Ia Data: $d_L(z)$ [i.e, $d_{L,i}(z_i)$]

distance modulus $\mu = 5\, log_{10}(d_L / 10\text{pc})$

from Supernova Cosmology Project: http://www-supernova.lbl.gov/

Supernova (SN) : mapping out the evolution herstory history

Type Ia Supernova (SN Ia) : **(standard candle)**

– thermonulear explosion of carbon-oxide white dwarfs –

- Correlation between the peak luminosity and the decline rate

$\Rightarrow$ absolute magnitude M

$\Rightarrow$ luminosity distance d_L

$$F = L / 4\pi d_L^2$$

F: flux (energy/area$\times$time)
L: luminosity (energy/time)

(distance precision: σ_{mag} = 0.15 mag $\rightarrow$ $\delta d_L / d_L$ ~ 7%)

- Spectral information $\rightarrow$ redshift z

$$F \propto 100^{-m/5}$$
$$M = m(10\ \text{pc})$$

SN Ia Data: $d_L(z)$ [i.e, $d_{L,i}(z_i)$] [~ x(t) ~ position (time)]

$\mu(z)$

Distance Modulus $\mu \equiv m - M$

$$\mu \equiv m - M = 5\,log_{10}(d_L / \text{Mpc}) + 25$$

1998 Distance Modulus $\mu \equiv m - M$

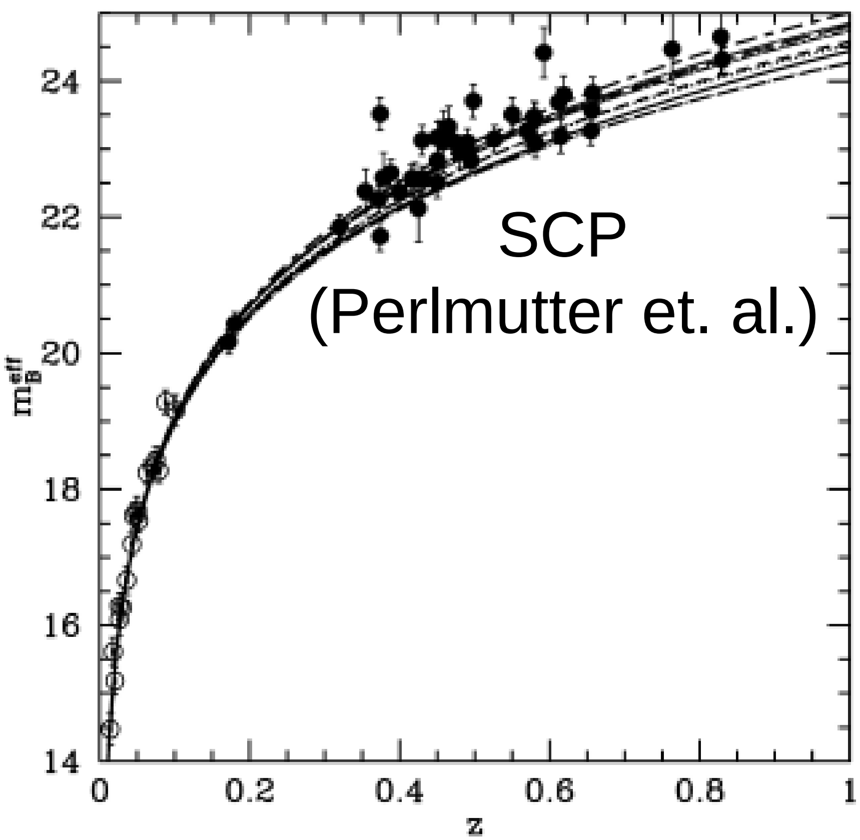
$$\mu \equiv m - M = 5\,log_{10}(d_L\,/\,Mpc) + 25$$

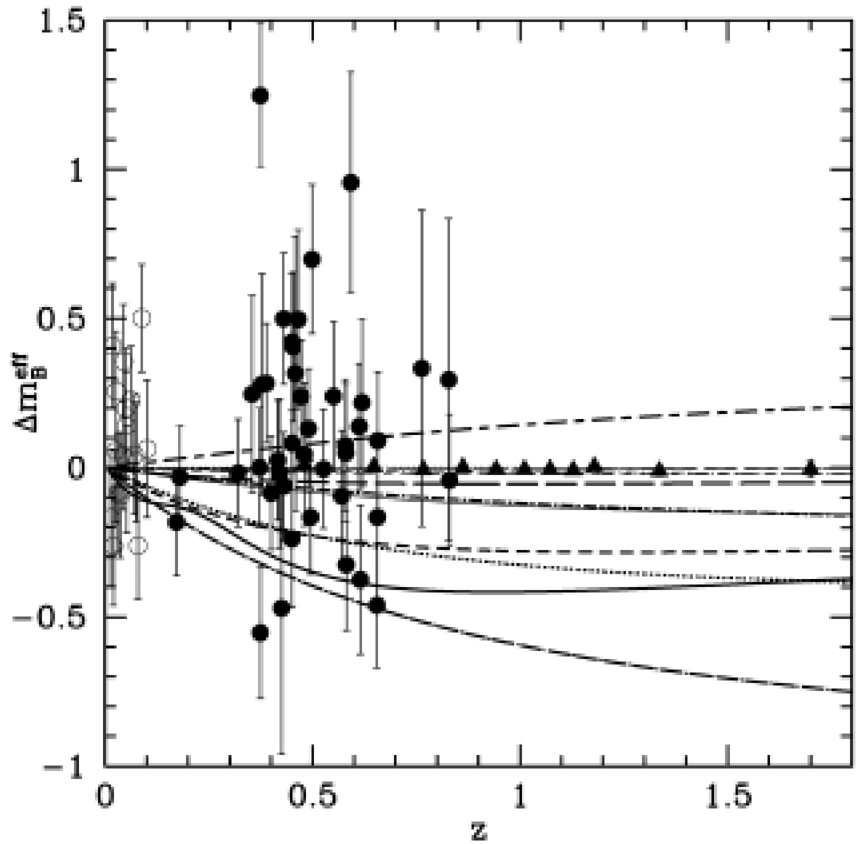
$$\Delta m(z) = m(z) - m_\Lambda(z)$$

$\uparrow$

fiducial model

$$\Omega_\Lambda = 0.7,\ \Omega_m = 0.3$$

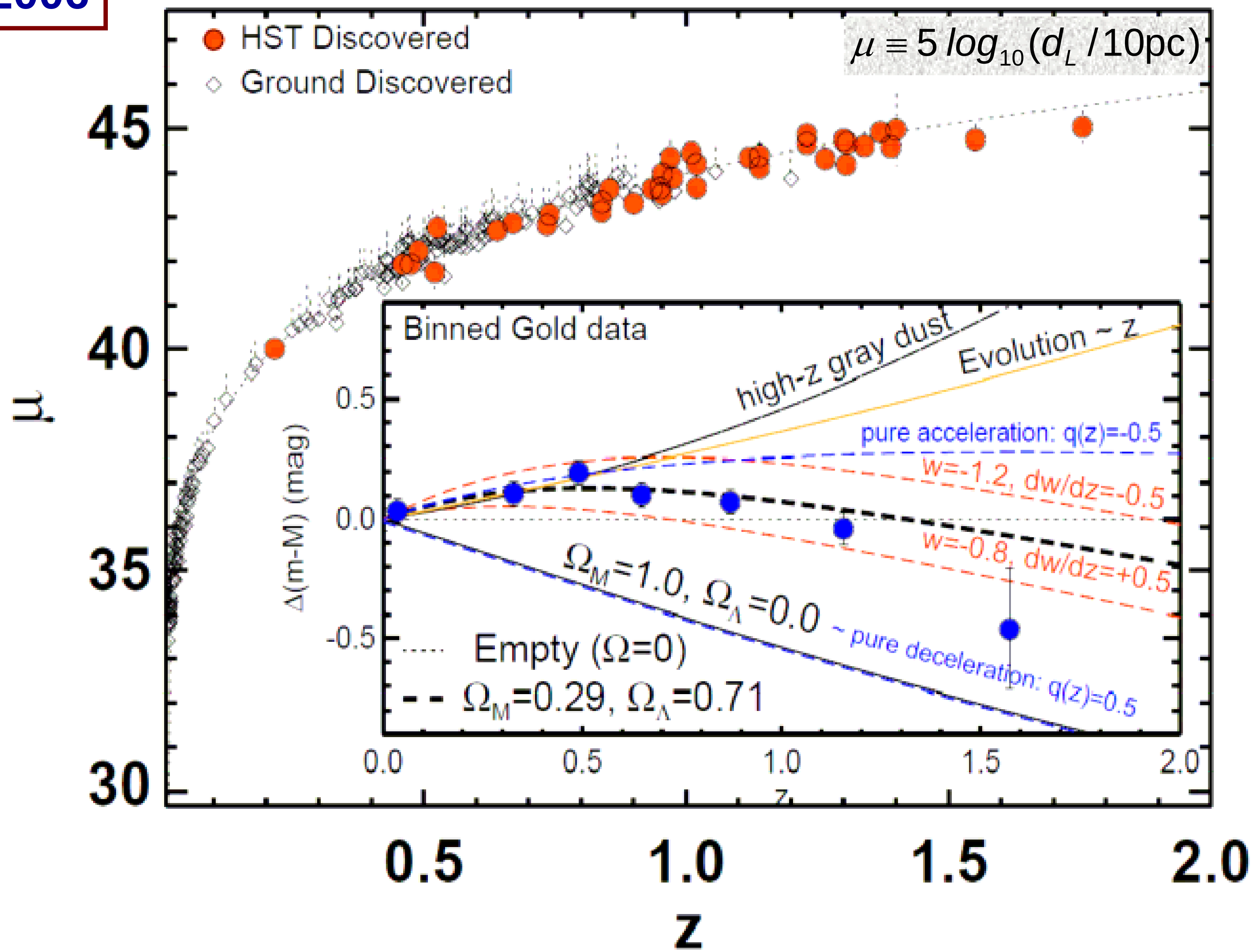




(can hardly distinguish different models)

2006

Riess et al., ApJ 659, 98 (2007) [astro-ph/0611572]



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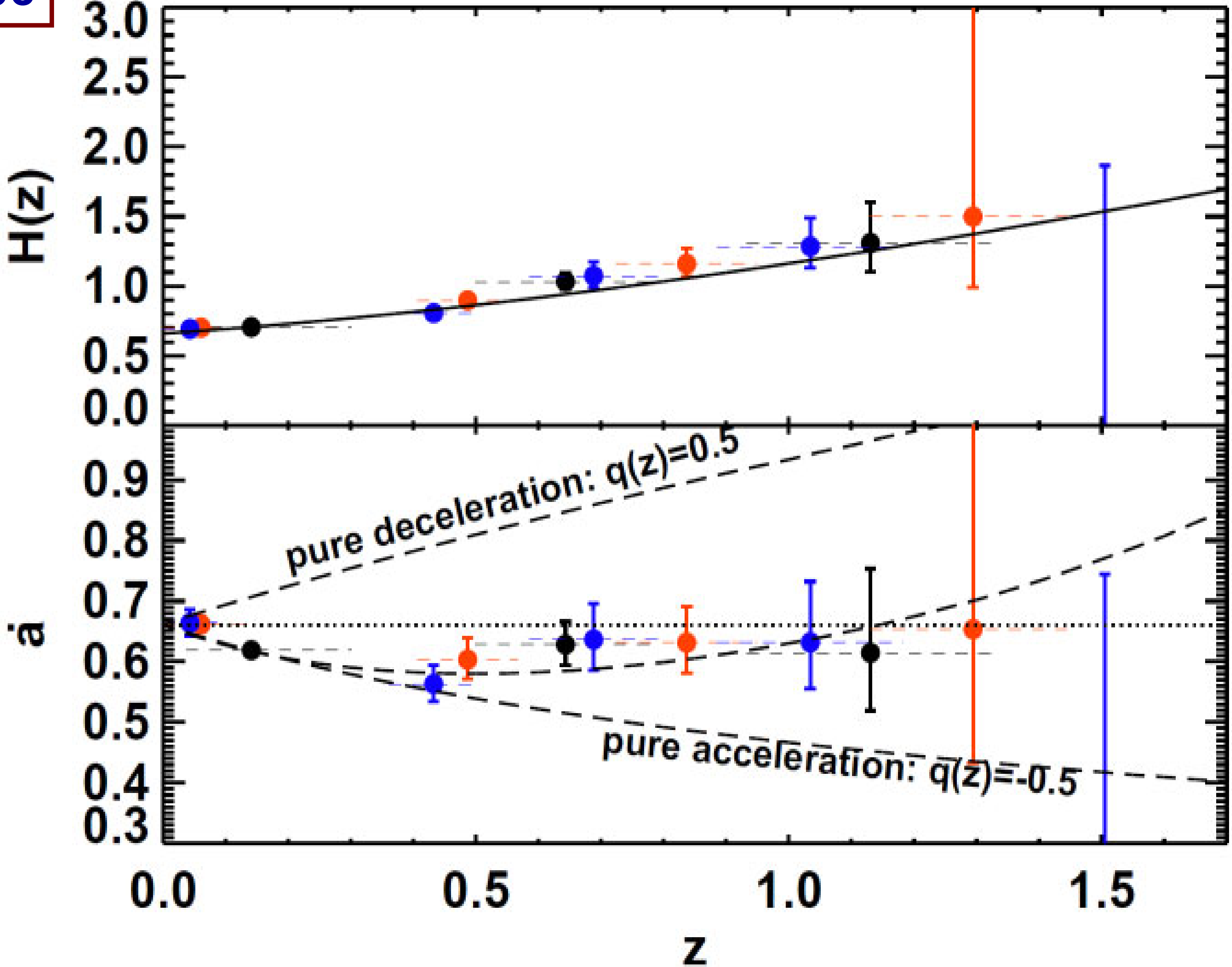


Supernova / Acceleration Probe (SNAP)

observe ~2000 SNe in 2 years

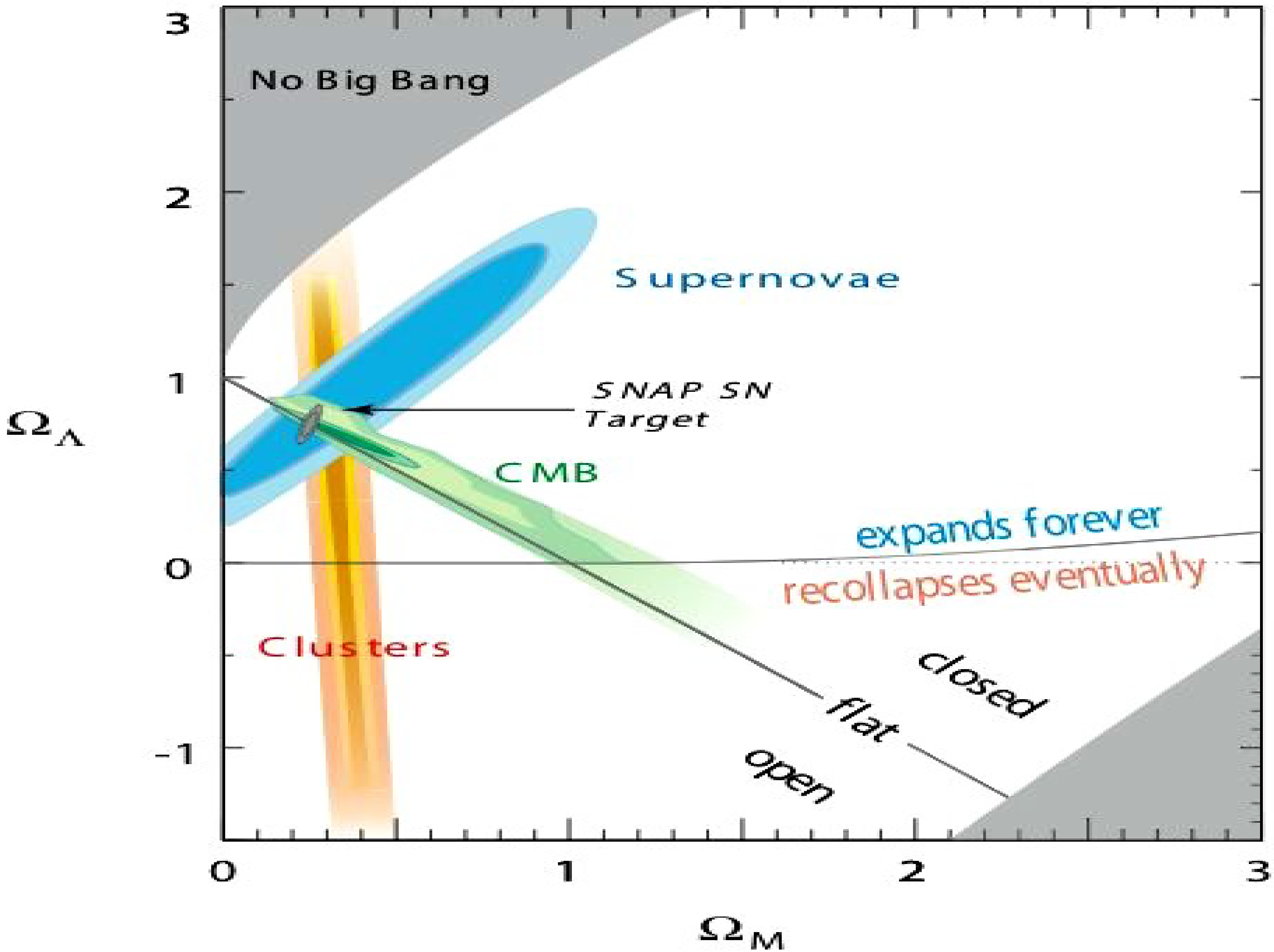
(2366 in our analysis)

statistical uncertainty $\sigma_{\mathrm{mag}} = 0.15$ mag $\rightarrow$ 7% uncertainty in d_L

σ_{sys} = 0.02 mag at z =1.5

σ_z = 0.002 mag (negligible)

z	0~0.2	0.2~1.2	1.2~1.4	1.4~1.7
# of SN	50	1800	50	15


No Big Bang
Supernovae
SNAP SN
Target
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Ω_Λ
Ω_M

Models: Dark Geometry vs. Dark Energy

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry ↑ Dark Geometry

Matter/Energy ↑ Dark Matter / Energy

- Modification of Gravity
- Extra Dimensions
- Averaging Einstein Equations for an inhomogeneous universe

(Non-FLRW)

- Λ (from vacuum energy)
- Quintessence/Phantom

(based on FLRW)

Supernova Data Analysis

— Parametrization / Fitting Formula —

Data vs. Models

Observations Data

mapping out
the evolution history
(e.g. SNe Ia , BAO)

Data Analysis

(e.g. χ^2 fitting)

Models Theories

$\mathbf{M_1}$
$\mathbf{M_2}$
$\mathbf{M_3}$

Quintessence
a dynamical scalar field

M_1 (tracker quint., exponential)

$$V(\phi) = V_A \exp\left[-(\phi - A_1) / M \right]$$

constraints on parameters : $\{V_A, A_1, M, \phi_0, \dot{\phi}_0\}$

M_2 (tracker quint., power-law)

$$V(\phi) = m^{4-n} (\phi - A_2)^n$$

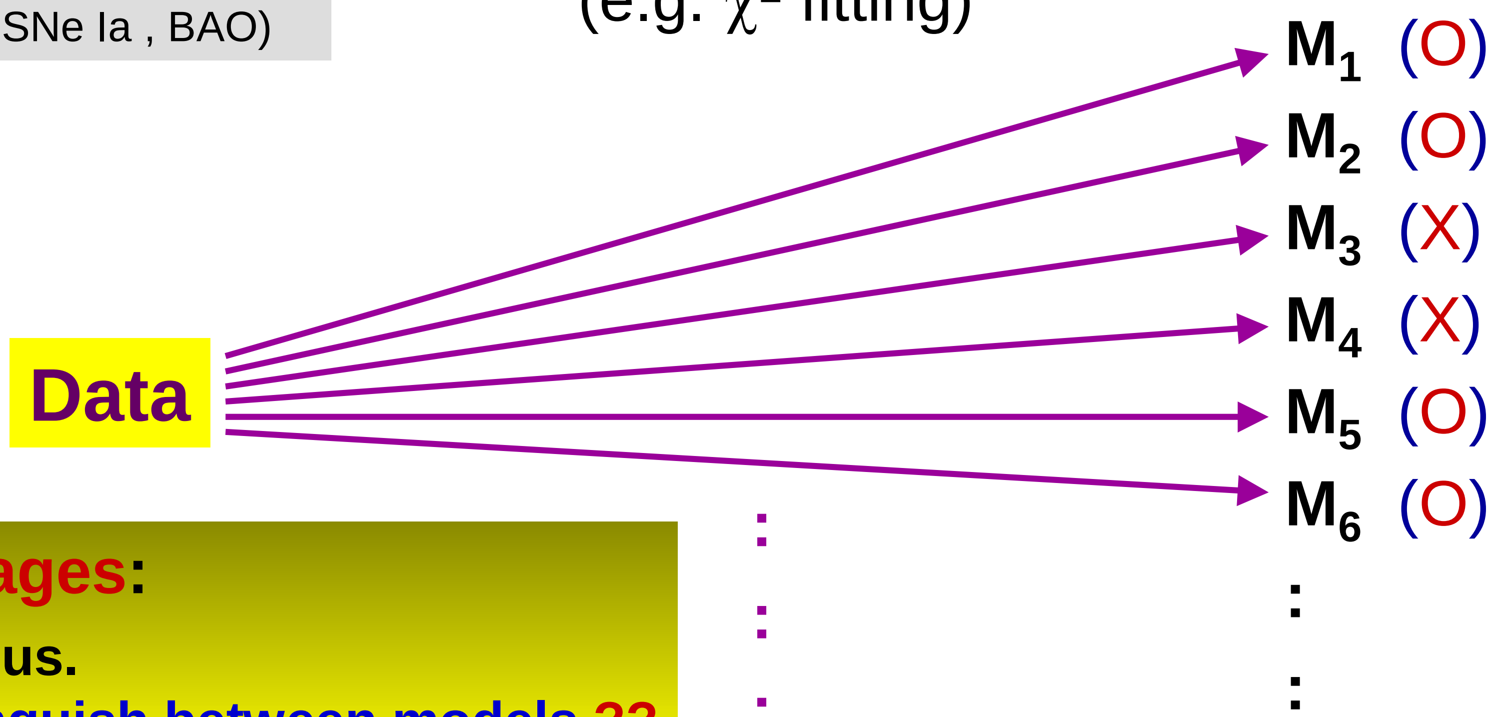
constraints on parameters : $\{n, A_2, m, \phi_0, \dot{\phi}_0\}$

Reality : Many models survive

Observations Data ⟷ **Models Theories**

mapping out
the evolution history
(e.g. SNe Ia , BAO)

Data Analysis
(e.g. χ^2 fitting)



Shortages:

- **Tedious.**
- **Distinguish between models ??**
- **Reality: many models survive.** → **Dark Energy info NOT clear.**

<u>Reality</u> : Many models survive

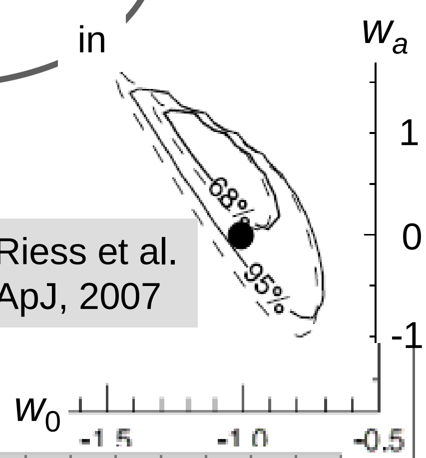
- Instead of comparing models and data (thereby ruling out models), Extract **information** about **dark energy** directly from data by invoking **model independent** parametrizations.

Model-independent Analysis

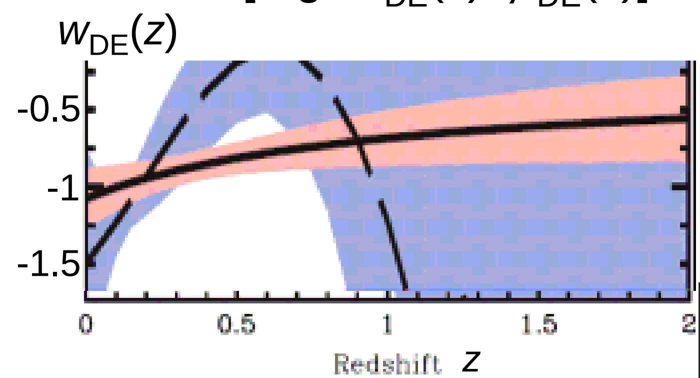
Data — analyzed by invoking → **Constraints on Parameters** → **Dark Energy Information**

[e.g. $w_{DE}(z)$, $\rho_{DE}(z)$]

Parametrization Fitting Formula ← in (Constraints on Parameters)



Riess et al.
ApJ, 2007

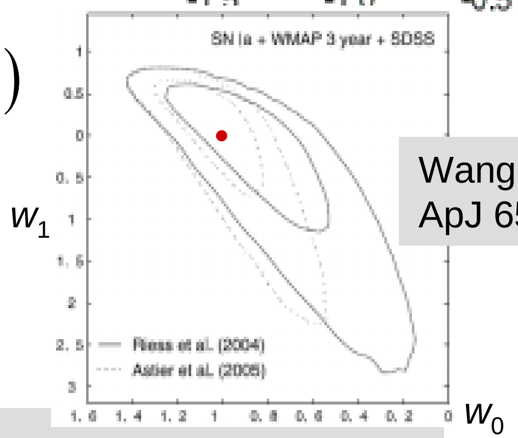


<u>Example</u>

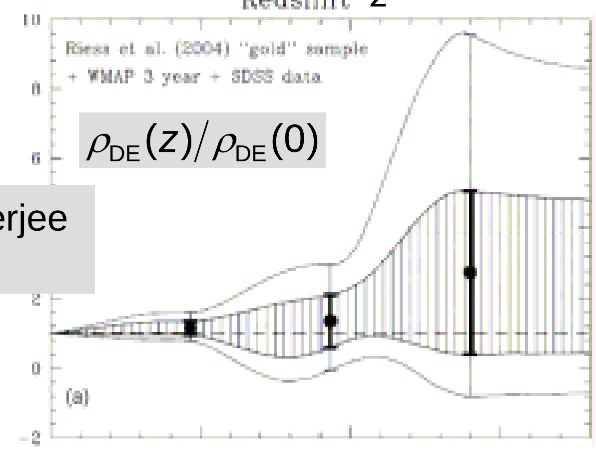
- Linder, PRL, 2003

$$w_{DE}(z) = w_0 + w_a(1-a)$$
$$= w_0 + w_a z/(1+z)$$

- $w_{DE}(z) = w_0 + w_1 z$



Wang & Mukherjee
ApJ 650, 2006



w : equation of state, an important quantity characterizing the nature of an energy content.
It corresponds to how fast the energy density changes along with the expansion.

Answering questions about Dark Energy

❖ Instead of comparing models and data (thereby ruling out models),
Extract **information** about **dark energy** directly from data
by invoking **model independent** parametrizations,
thereby answering essential questions about dark energy.

Issue

? **How to choose the parametrization ?**

- Linder, PRL, 2003 : $w_{\mathrm{DE}}(z) = w_0 + w_a(1-a) = w_0 + w_a z/(1+z)$ "standard parametrization" ?
- Taylor expansion : $w_{\mathrm{DE}}(z) = w_0 + w_1 z$

A New Parametrization

(proposed by Gu, Lo, and Yan)

New Parametrization

by Gu, Lo & Yan

SN Ia Data: $d_L(z)$ [i.e, $d_{L,i}(z_i)$ (and errors)]

For a supernova located at $r(t_{emit})$ when it emitted photons (at time t_{emit}) that we observe today,

$$1+z=\frac{a_0}{a(t_{emit})}, \qquad a_0 \equiv a(t_0), \qquad t_0 : \text{ present time}$$

$$a_0 r(z) = c\cdot\int_0^z \frac{dz'}{H(z')}, \qquad H \equiv \frac{\dot{a}}{a} = \frac{1}{a}\frac{da}{dt}$$

(H: Hubble expansion rate)

$$d_L(z) = a_0 \cdot r(z)\cdot(1+z) = c(1+z)\cdot\int_0^z \frac{dz'}{H(z')}$$

Assume flat universe.

$$H^2(z) = \frac{8\pi G}{3}\rho_{\text{total}}(z) = H_0^2\cdot\frac{\rho_{\text{total}}(z)}{\rho_c}, \quad \rho_c \equiv \frac{3H_0^2}{8\pi G}$$

$$\rho_{\text{total}}(z) = \rho_m(z)+\rho_r(z)+\rho_x(z)$$

$$= \rho_m(0)(1+z)^3 + \rho_r(0)(1+z)^4 + \rho_x(0)\exp\left\{3\int_0^z\left[1+w_x(z')\right]\frac{dz'}{1+z'}\right\}$$

"m": NR matter – DM & baryons
"r": radiation
"x": dark energy

New Parametrization

by Gu, Lo & Yan

Separate $\rho_x(z)$ into 2 parts : $\rho_x(z) = \rho_{w_0}(z) + \delta\rho_x(z)$

(i) $\rho_{w_0}(z) = \rho_x(0)(1+z)^{3(1+w_0)}$

i.e., the energy density of an energy source

with a constant equation of state $w_0 \equiv w_x(0)$ and $\rho_{w_0}(0) \equiv \rho_x(0)$

(ii) $\delta\rho_x(z)$: the correction/difference $\left[\rho_{w_0}(0) = \rho_x(0) \Rightarrow \delta\rho_x(0) = 0\right]$

$$\rho_{\text{total}}(z) = \rho_m(z) + \rho_r(z) + \rho_{w_0}(z) + \delta\rho_x(z)$$

$$\delta(z) \equiv \frac{\delta\rho_x(z)}{\rho_m(z) + \rho_r(z) + \rho_{w_0}(z)} \equiv \frac{\rho_x(z) - \rho_{w_0}(z)}{\rho_m(z) + \rho_r(z) + \rho_{w_0}(z)} \equiv \frac{\rho_x(z) - \rho_x(0)(1+z)^{3(1+w_0)}}{\rho_m(z) + \rho_r(z) + \rho_x(0)(1+z)^{3(1+w_0)}}$$

$$\rho_{\text{total}}(z) = \left[\rho_m(z) + \rho_r(z) + \rho_{w_0}(z)\right] \cdot \left[1 + \underline{\delta(z)}\right]$$

The density fraction δ**(z)** is the very quantity to parametrize.

[instead of $\rho_x(z)$ or $w_x(z)$]

New Parametrization

by Gu, Lo & Yan

General Features

$\delta\rho_x(0) = 0 \Rightarrow \delta(0) = 0$

$|\delta(z)| < 1$ in most cases

$|\delta(z)| << 1$ in earlier times

$|\delta(z)| \to 0$ as $z \to \infty$

$\Rightarrow$ Boundary conditions for $\delta(z)$: $\begin{cases} \delta(0) = 0 \\ \delta(\infty) = 0 \end{cases}$

$z = \tan(\theta/4)$ (change of variable)

$0 \leq z < \infty \quad \Leftrightarrow \quad 0 \leq \theta < 2\pi$

$\Rightarrow$ Boundary conditions for $\delta(\theta)$: $\begin{cases} \delta(0) = 0 \\ \delta(2\pi) = 0 \end{cases}$

Invoke Fourier series: $\delta(\theta) = \frac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta + \sum_{n=1}^{\infty} B_n \sin n\theta$

New Parametrization

by Gu, Lo & Yan

Invoke Fourier series: $\delta(\theta) = \frac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta + \sum_{n=1}^{\infty} B_n \sin n\theta$

Boundary Condition $\Rightarrow$ $\frac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n = 0$

$w_0 = w_x(0) \Rightarrow \sum_{n=1}^{\infty} nB_n = 0$

The lowest-order nontrivial parametrization:

$$\delta(\theta) = \frac{1}{2}A(1 - \cos\theta) = A\sin^2(\theta/2)$$

i.e., $\delta(z) = \frac{4Az^2}{(1+z^2)^2}$

$A > -1$ is required for guaranteeing $\rho_x(z) > 0$

New Parametrization

by Gu, Lo & Yan

$$\rho_{\text{total}}(z) \to H(z) \to d_L(z) \quad \text{[cf. Data]}$$

$$\rho_{\text{m}}(z) = \rho_{\text{m}}(0)(1+z)^3$$
$$\rho_{\text{r}}(z) = \rho_{\text{r}}(0)(1+z)^4$$
$$\rho_{w_0}(z) = \rho_{\text{x}}(0)(1+z)^{3(1+w_0)}$$
$$\delta(z) = 4Az^2/(1+z^2)^2$$

$$\begin{aligned}\rho_{\text{total}}(z) &= \rho_{\text{m}}(z) + \rho_{\text{r}}(z) + \rho_{\text{x}}(z) \\ &= \rho_{\text{m}}(z) + \rho_{\text{r}}(z) + \rho_{w_0}(z) + \delta\rho_{\text{x}}(z) \\ &= \left[\rho_{\text{m}}(z) + \rho_{\text{r}}(z) + \rho_{w_0}(z)\right]\cdot\left[1+\delta(z)\right]\end{aligned}$$

i.e.,

$$\Omega_i \equiv \rho_i(0)/\rho_{\text{c}}$$

$$\frac{\rho_{\text{total}}(z)}{\rho_{\text{c}}} = \left[1 + \frac{4Az^2}{(1+z^2)^2}\right]\cdot\left[\Omega_{\text{m}}(0)(1+z)^3 + \Omega_{\text{r}}(0)(1+z)^4 + \Omega_{\text{x}}(0)(1+z)^{3(1+w_0)}\right]$$

parameters : $\{\Omega_{\text{m}}, \Omega_{\text{r}}, \Omega_{\text{x}}, w_0, A\}$

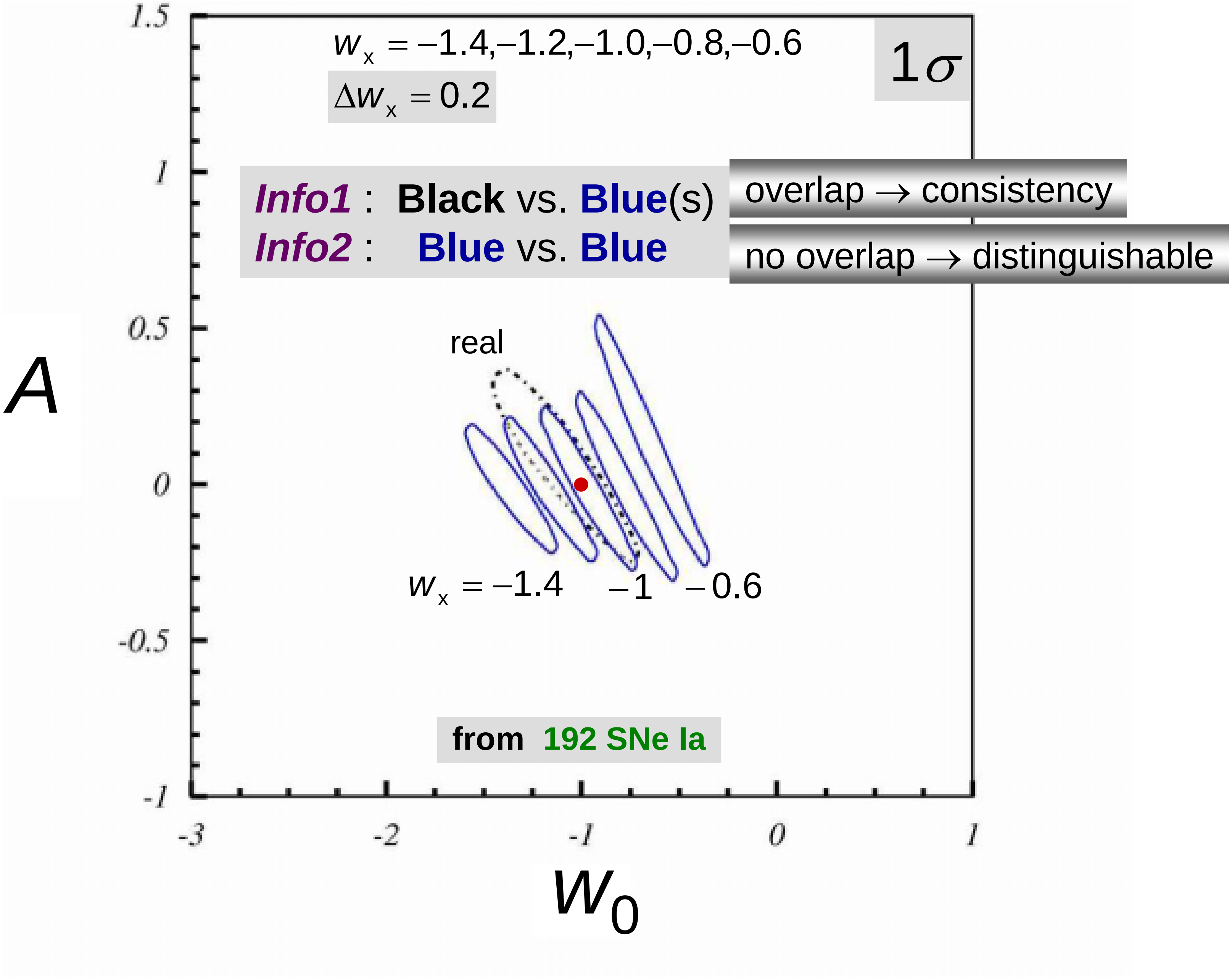
w_0 : present value of DE EoS
$A \propto$ "amplitude" of the correction $\delta\rho_{\text{x}}(z)$

In our analysis, we set $\Omega_{\text{m}} = 0.27,\ \ \Omega_{\text{r}} = 0,\ \ \Omega_{\text{x}} = 1 - \Omega_{\text{m}} = 0.73$

$\Rightarrow$ 2 parameters: w_0, A

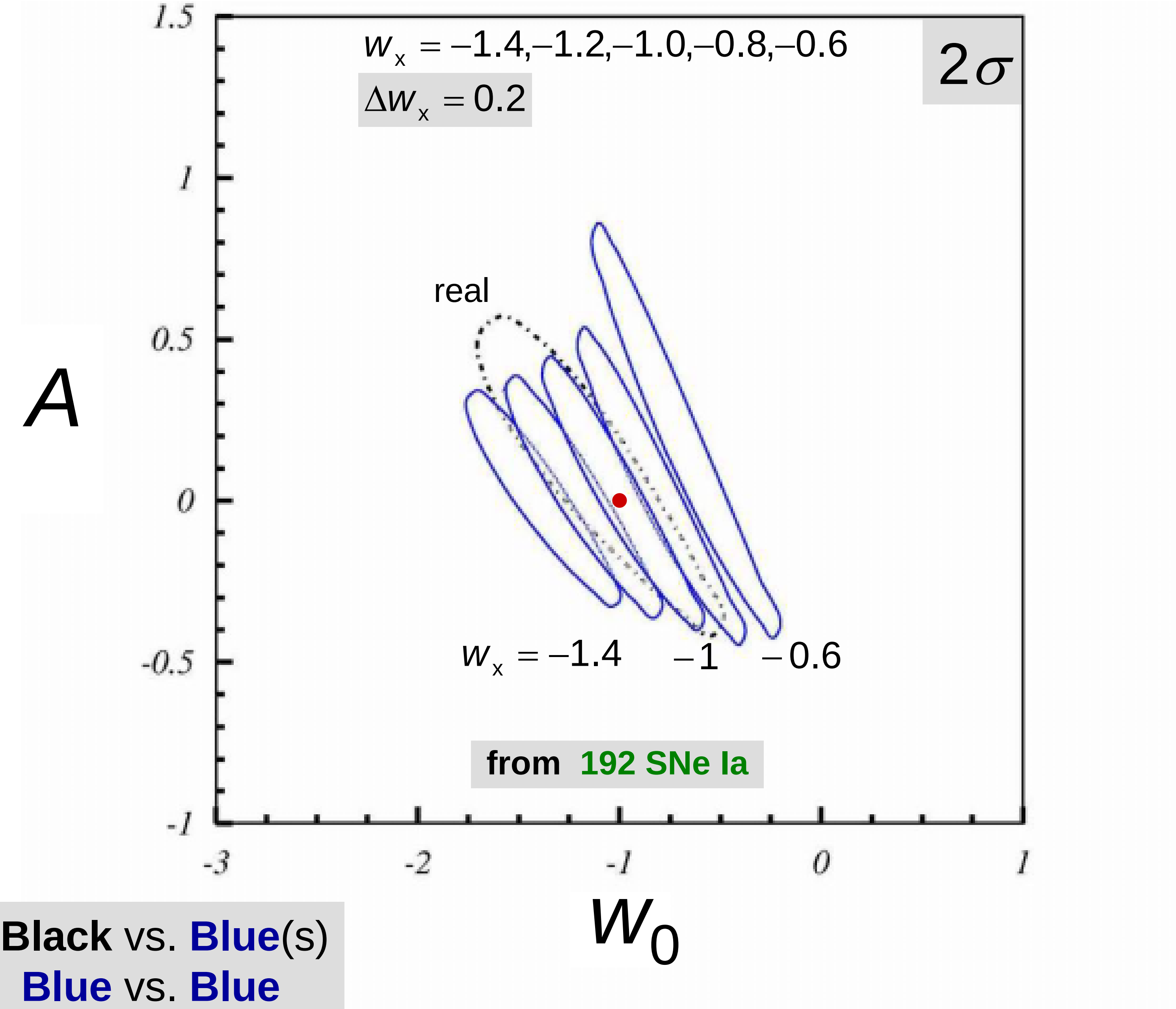
Test and Results

Gu, Lo & Yan



$w_x = -1.4, -1.2, -1.0, -0.8, -0.6$
$\Delta w_x = 0.2$
1σ
Info1 : **Black** vs. **Blue**(s)
Info2 : **Blue** vs. **Blue**
overlap → consistency
no overlap → distinguishable
real
$w_x = -1.4$
-1
-0.6
from 192 SNe Ia
1.5
1
0.5
0
-0.5
-1
-3
-2
-1
0
1
A
w_0

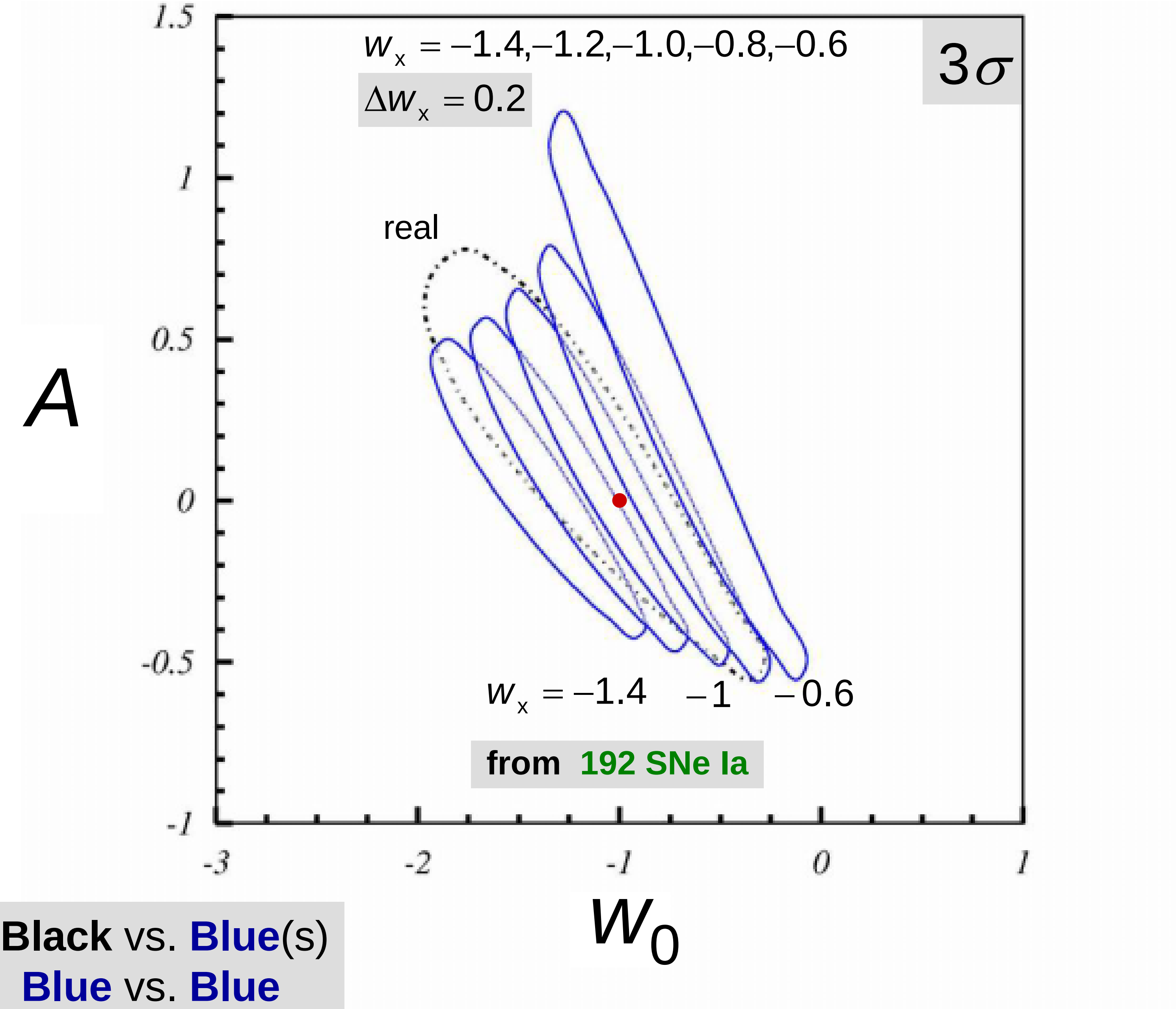
Gu, Lo & Yan



Info1 : **Black** vs. **Blue**(s)

Info2 : **Blue** vs. **Blue**

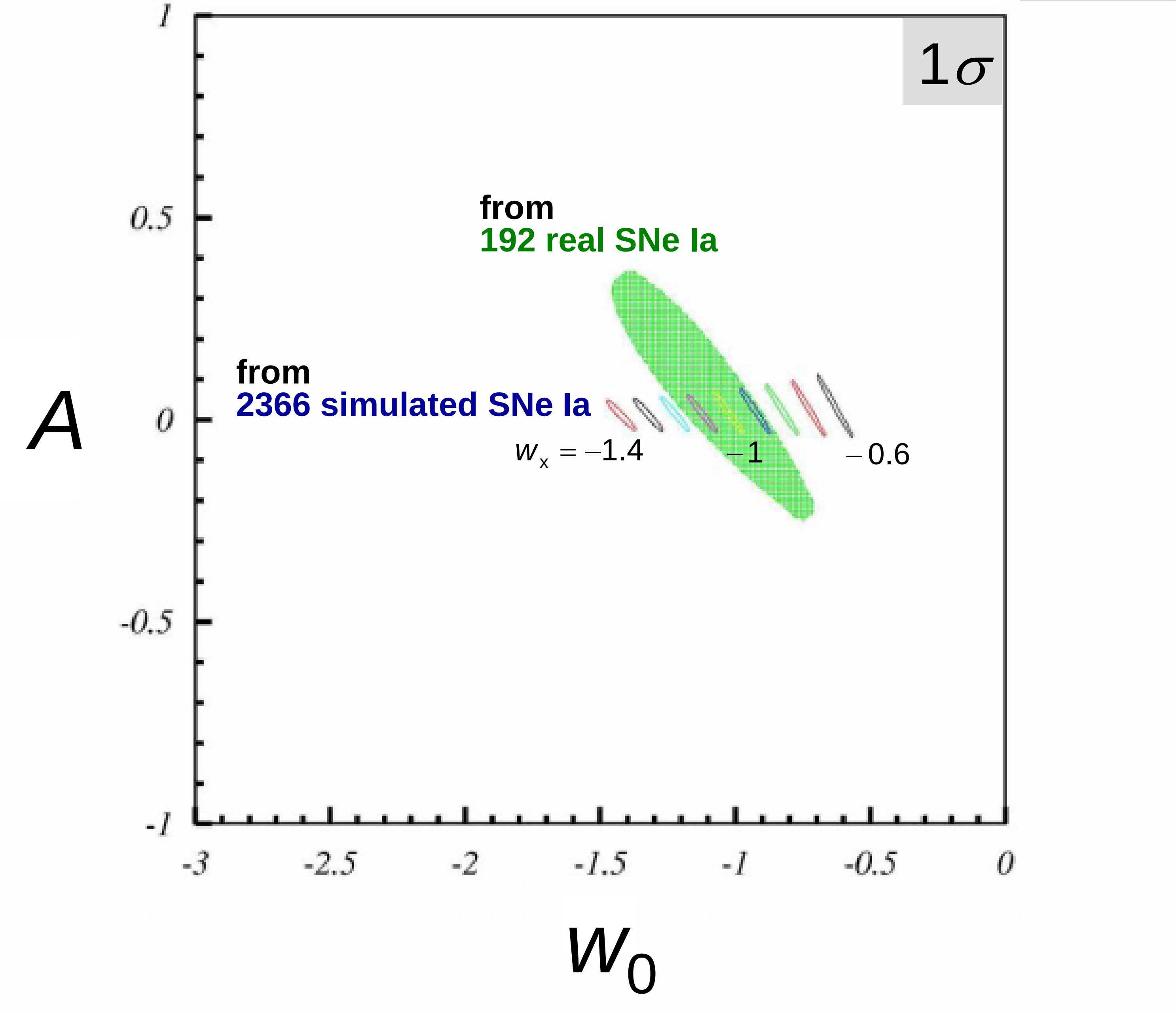
Gu, Lo & Yan



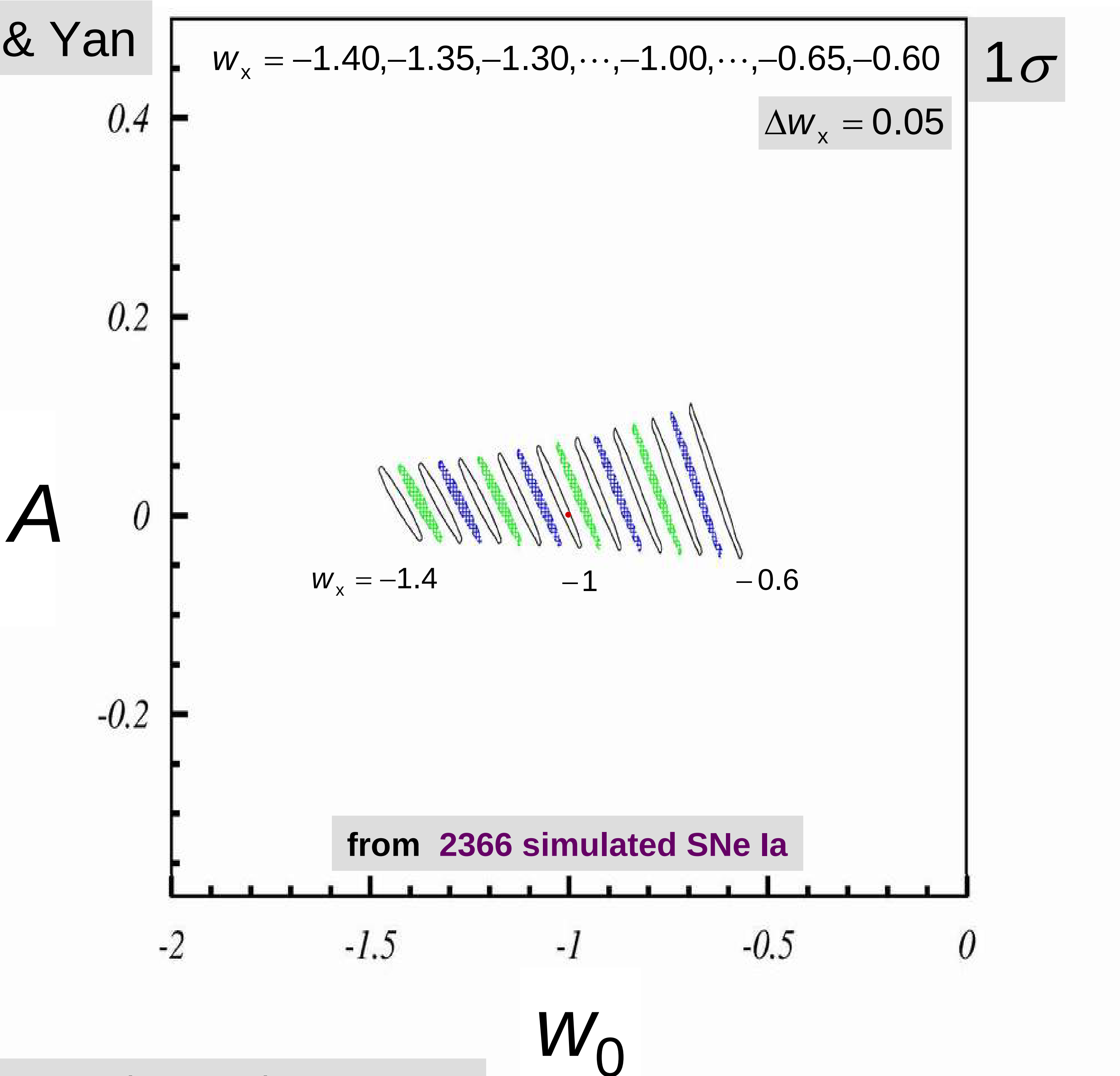
Info1 : **Black** vs. **Blue**(s)
Info2 : **Blue** vs. **Blue**

Gu, Lo & Yan

$w_x = -1.4, -1.3, -1.2, -1.1, -1.0, -0.9, -0.8, -0.7, -0.6$ $\Delta w_x = 0.1$

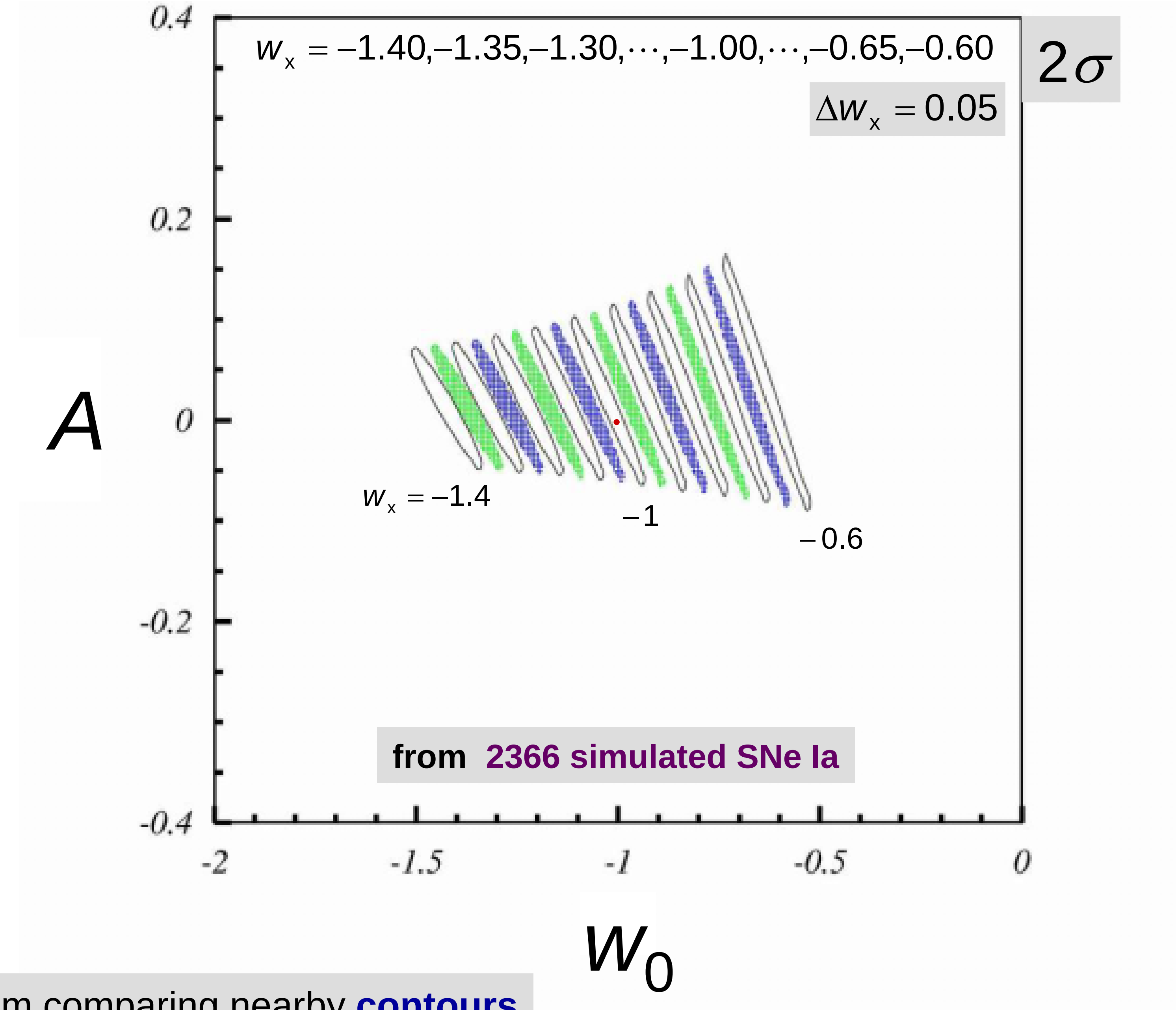


Gu, Lo & Yan



Info from comparing nearby **contours**

Gu, Lo & Yan



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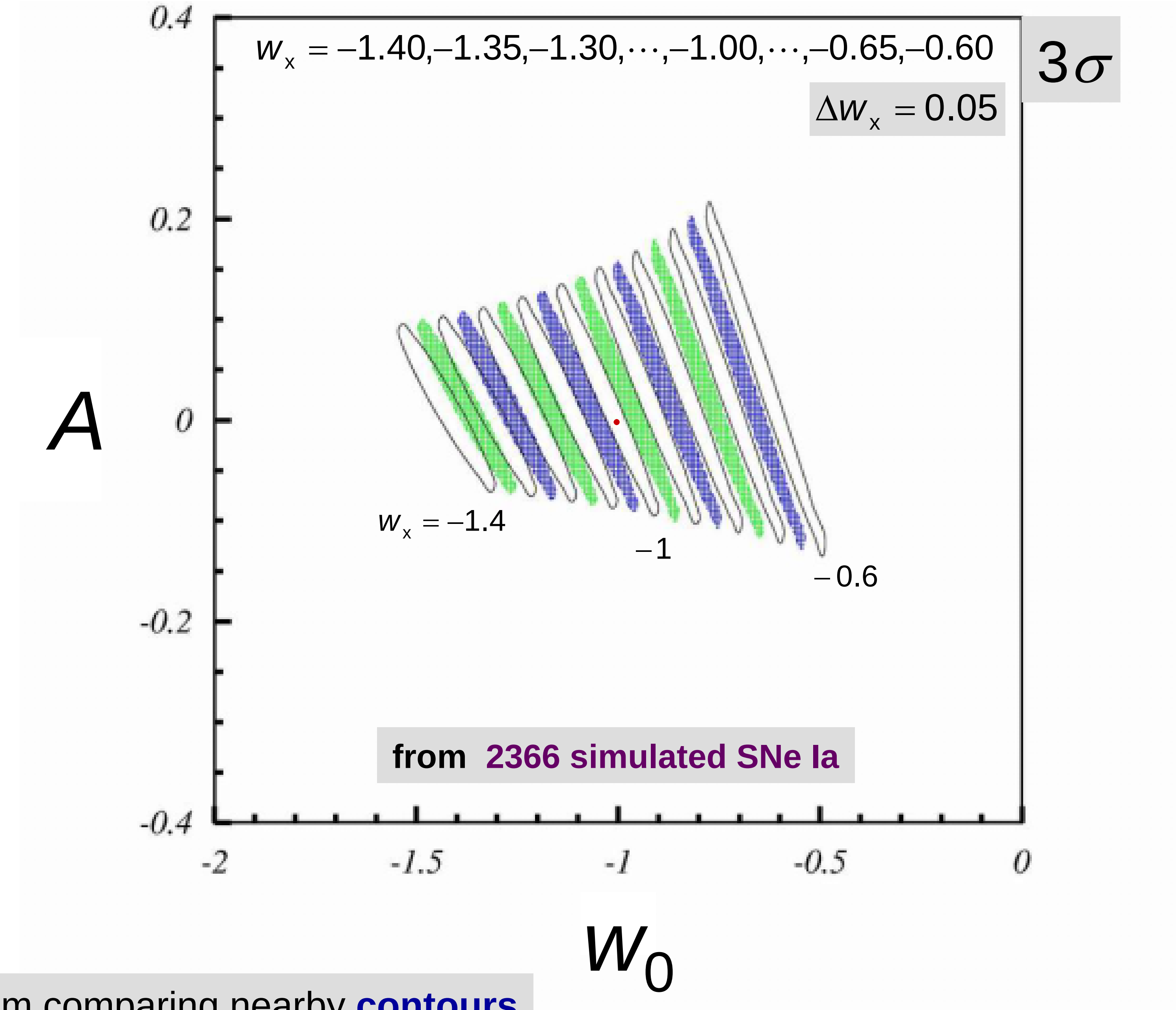

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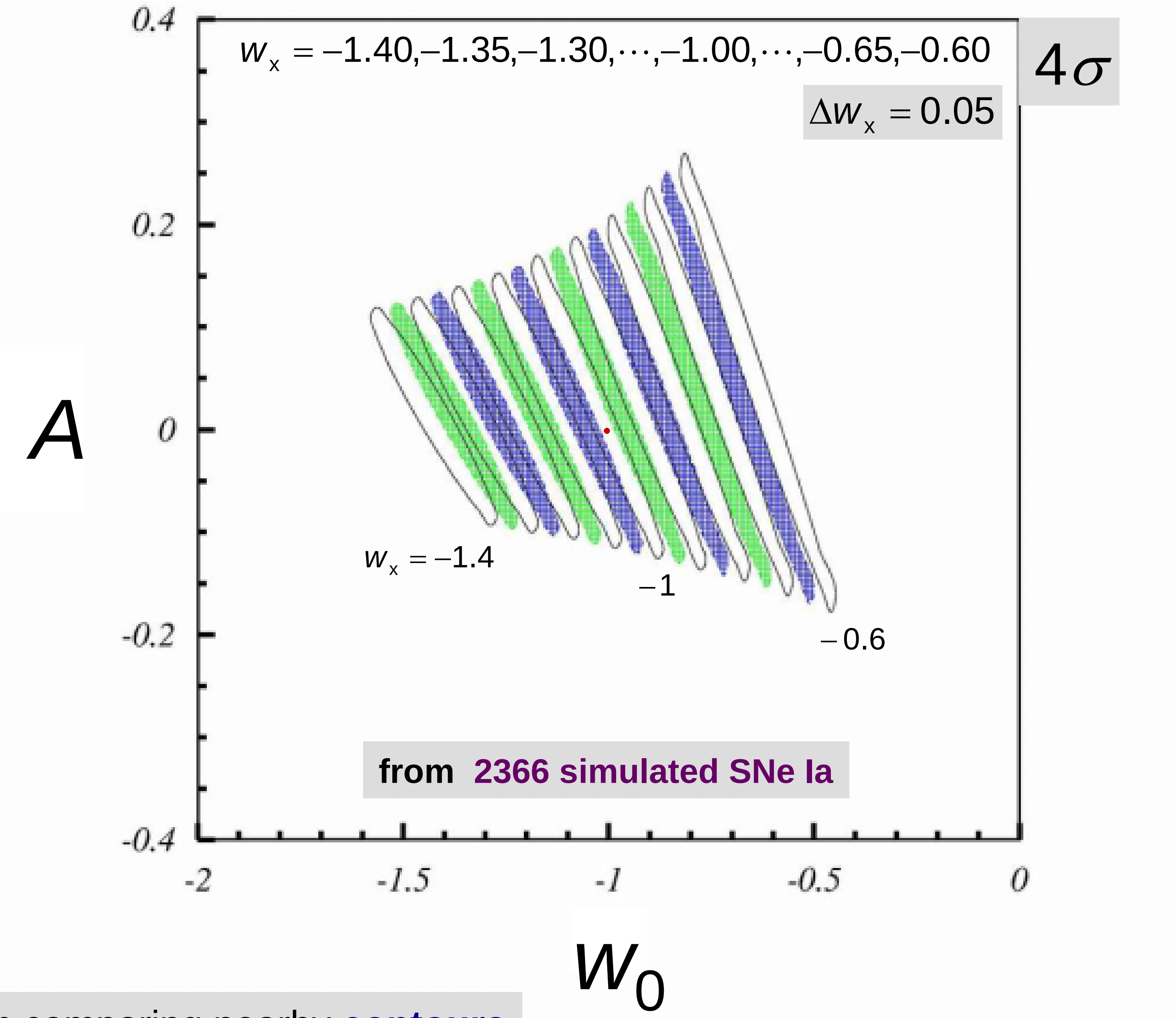

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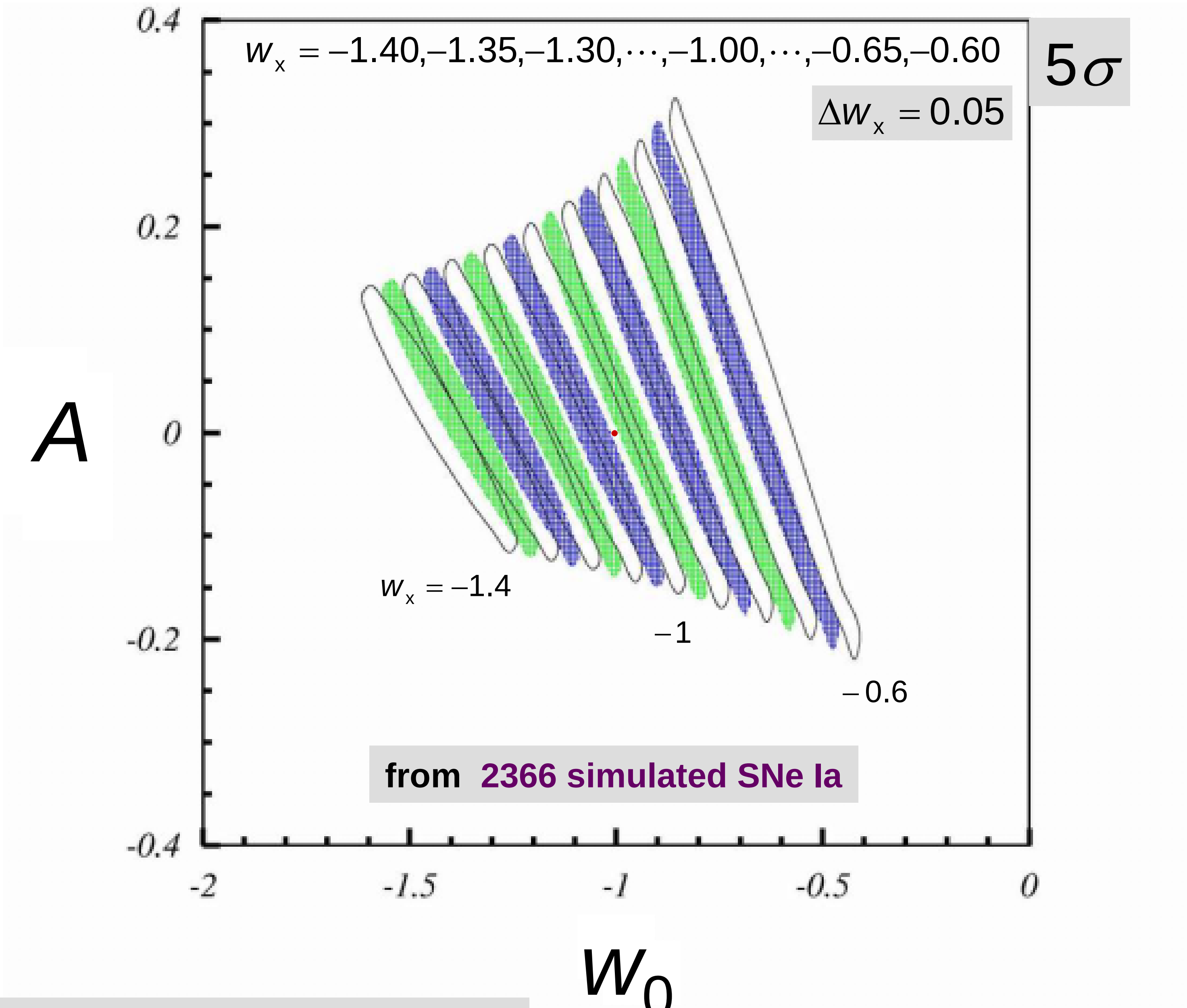

Info from comparing nearby **contours**

Summary

Summary

- A parametrization (by utilizing Fourier series) is proposed.
- This parametrization is particularly reasonable and controllable.
- It is systematical to include more terms and more parameters.

Via this parametrization:

- The current SN Ia data, including 192 SNe, can distinguish between constant-w_x models with $\Delta w_x = 0.2$ at 1σ confidence level.
- The future SN Ia data, if including 2366 SNe, can distinguish between constant-w_x models with $\Delta w_x = 0.05$ at 3σ confidence level.