Dynamical Instability of Holographic QCD at Finite Baryon Density

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Based on arXiv: 1004.0162
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Motivation

QCD phase diagram: **the first appearance** [Cabibbo and Parisi ‘75]

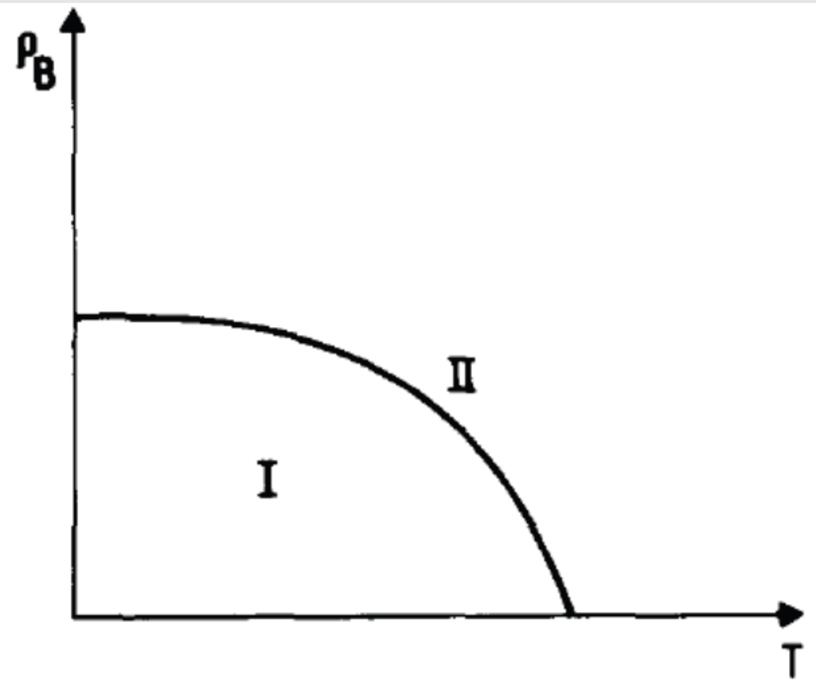
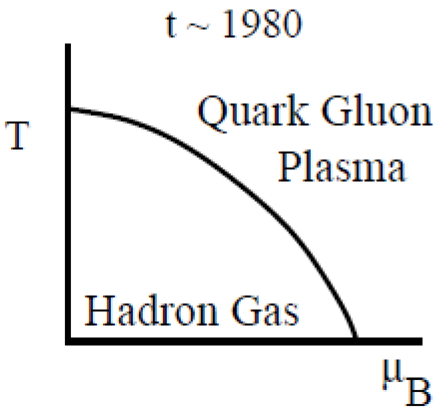


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

QCD phase diagram: the evolution

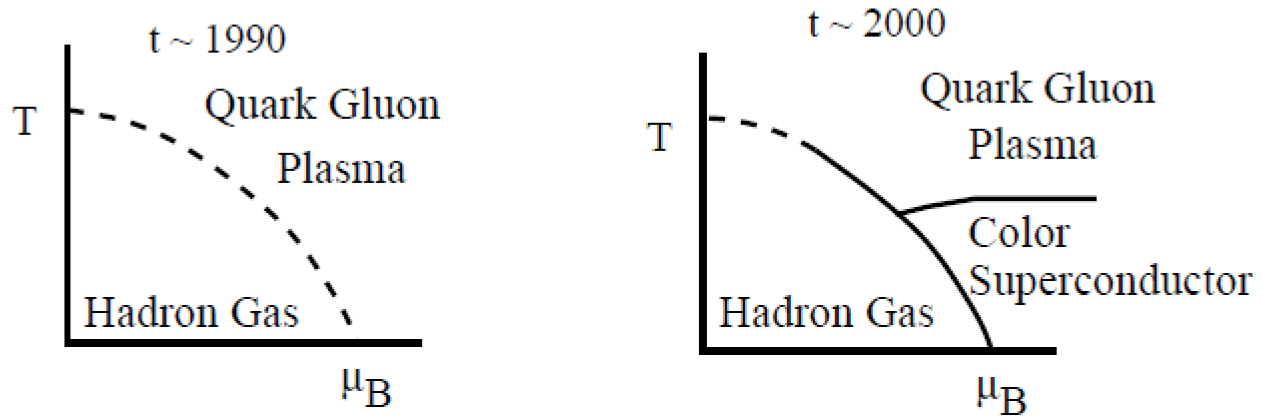
[McLerran, hep-ph/0202025]

The Evolving QCD Phase Transition



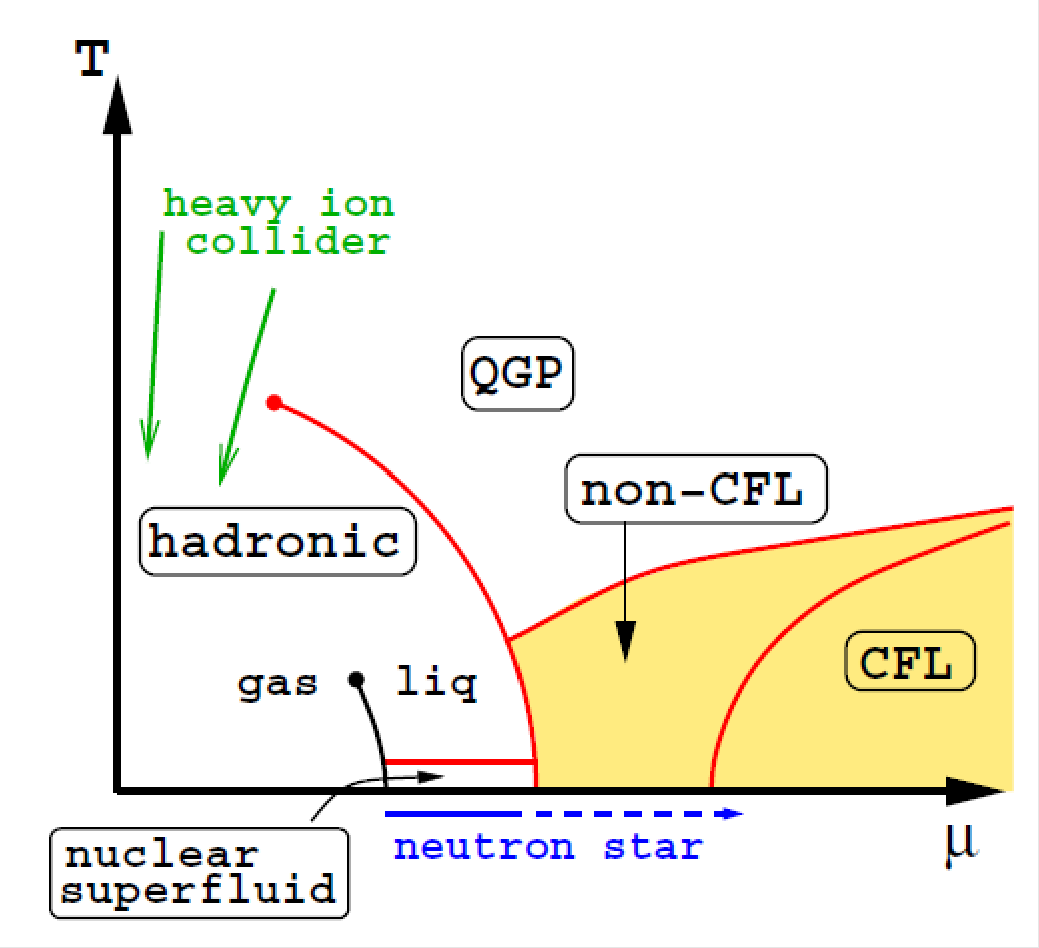
The ground state of cold, very dense QCD might be color superconductor!

But as for the not-so-dense regime……???



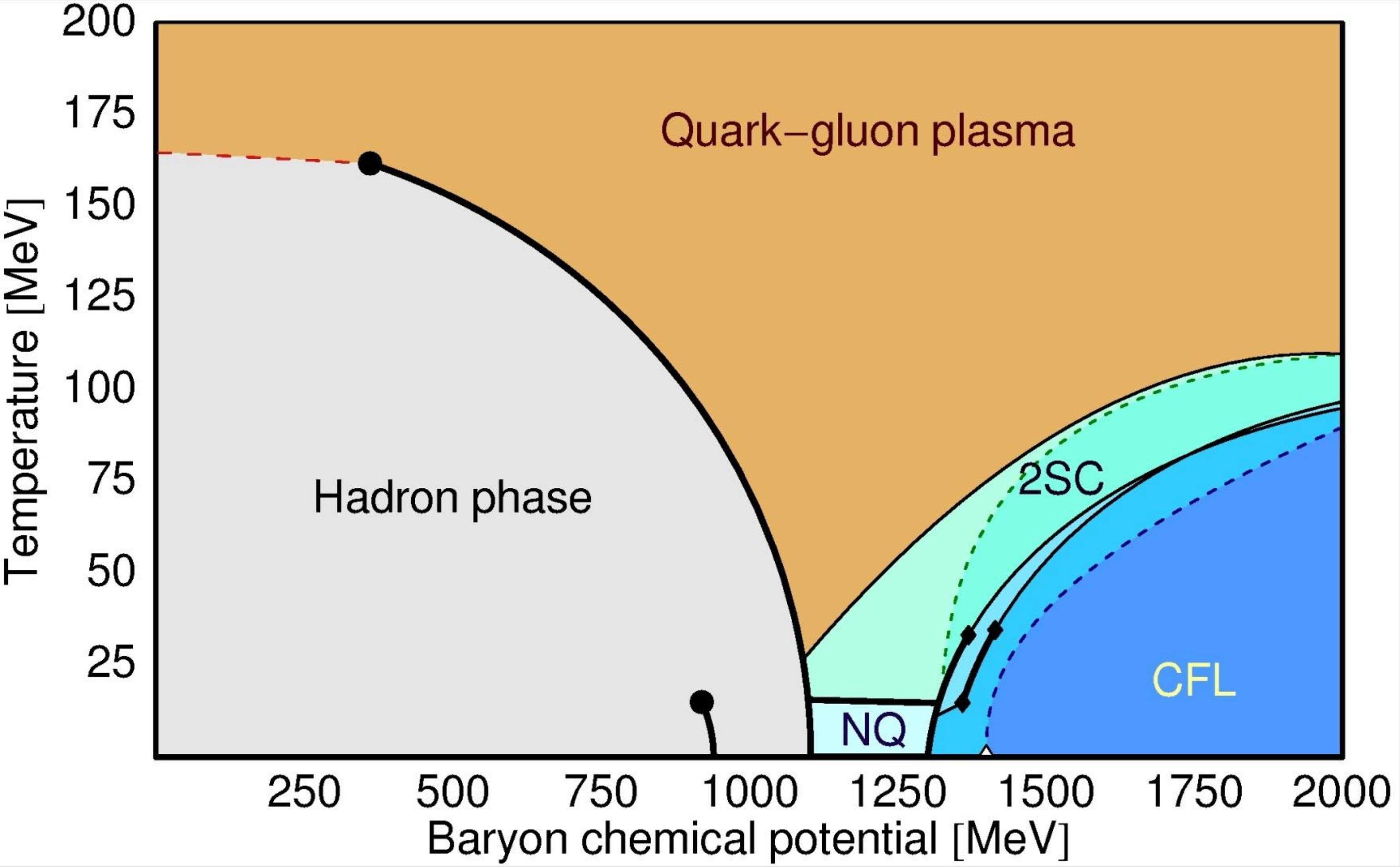
Real QCD phase diagram I

[Alford *et al.*, 0709.4635]



Real QCD phase diagram II

[Shovkovy *et al.*, hep-ph/0503184]



For large N_c QCD at low T and high density

At the limit $N_c \to \infty$ with fixed $\lambda = g^2 N_c$, the color superconductivity is suppressed due to color non-singlet quark "Cooper pair":

$$\langle \psi^T \psi \rangle \propto e^{-Const\frac{N_C}{\lambda}}$$

$\Rightarrow$ No color superconducting or CFL phase at high density!

What can we see at $N_c \to \infty$??

Chiral density wave (DGR Instability) [Deryagin, Grigoriev, and Rubakov `92]

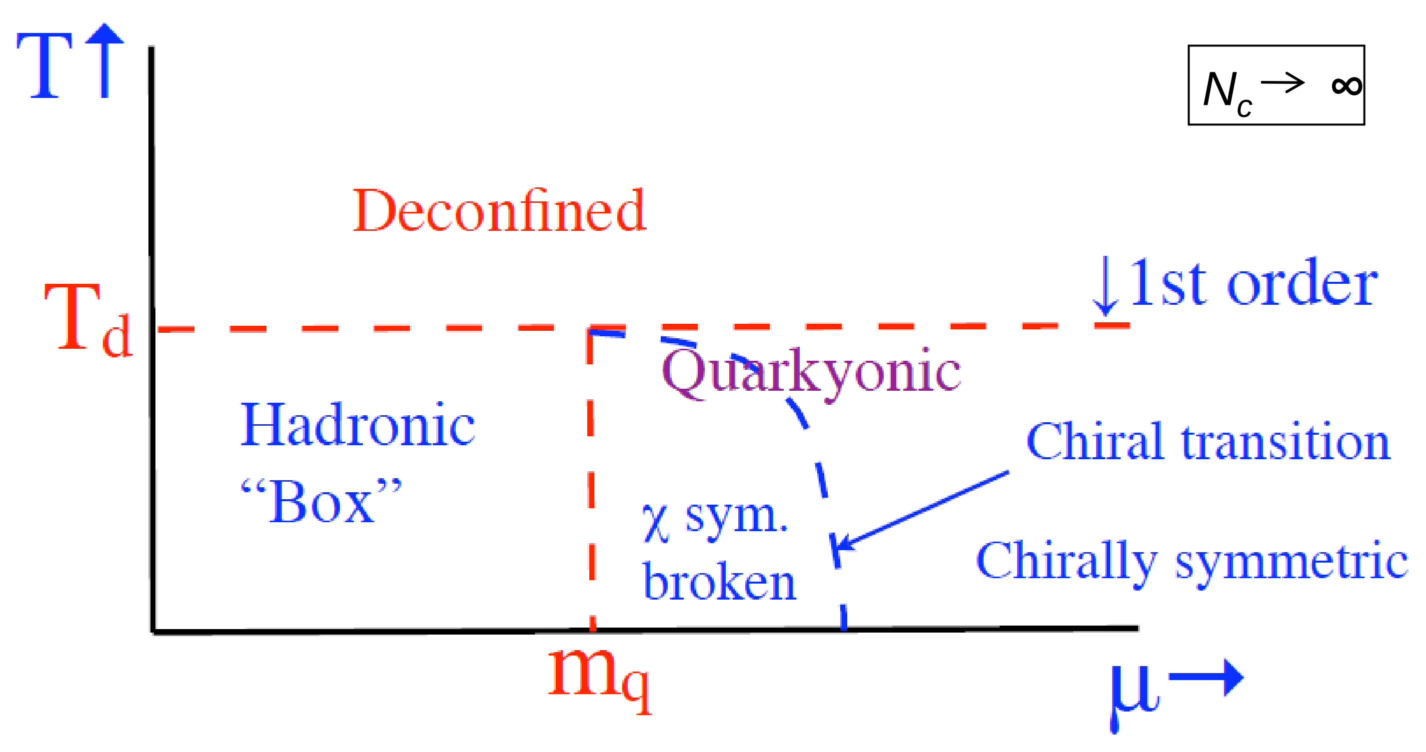
In the perturbative regime $g^2 N_c << 1$,

$$\langle \overline{\psi}(x)\psi(y) \rangle \propto e^{i\mathbf{p}\cdot(\mathbf{x}+\mathbf{y})} F(x-y) \quad \text{with} \quad F(0) \propto e^{-\frac{C}{\sqrt{\lambda}}}$$

- Color singlet condensate, dominating the $N_c \to \infty$ limit.
- spatially modulated chiral density wave
- Instability of the Fermi surface against formation of the chiral density wave

Or ……

Quarkyonic phase?? [McLerran & Pisarski, 0706.2191; Kojo *et al.*, 0912.3800]

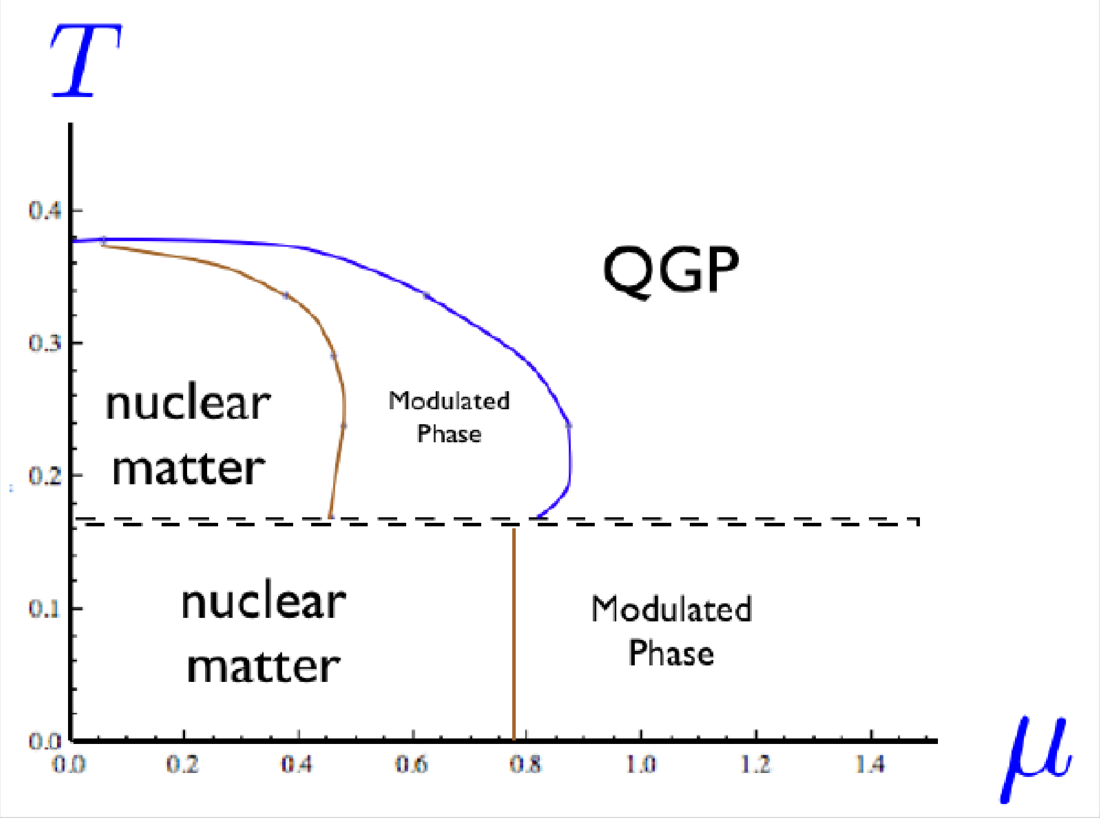


(Plot adapted from Pizarski's talk „*Phase Diagram of QCD at large Nc*" in 2007)

Question to answer in today‘s talk:

What is the phase at finite density in the picture of holographic QCD?

Is there any dynamical instability?



Holographic QCD

Gravity/Gauge Theory correspondence

- A map between gravity and gauge theory

Large N gauge theory In d-dim spacetime	$\Longleftrightarrow$	Classical gravitational theory In (d+1)-dim spacetime

- The best developed example

N=4 Super Yang-Mills	$\lambda = g_{YM}^2 N$	$\lambda / 4\pi N_C$	$\mathcal{O}$	Δ	$SO(2,4)\times SO(6)$
String theory in $AdS_5 \times S^5$	R^4 / α'^2	g_S	ϕ	m	$SO(2,4)\times SO(6)$

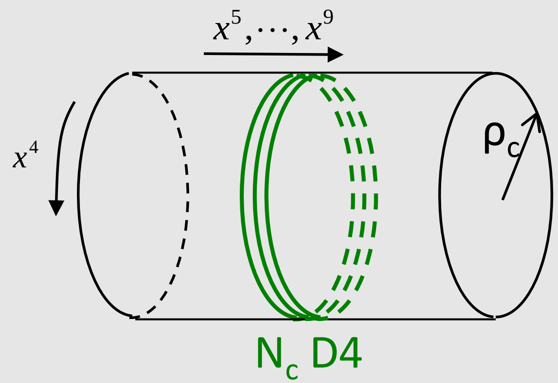
Prescription:

$$\langle e^{\int d^4x \phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{CFT} = \mathcal{Z}_{string}\left[\phi(\vec{x},z)\Big|_{z=0} = \phi_0(\vec{x})\right]$$

[Witten, hep-th/9803131]

Witten's Idea of Holographic Realization of 4d pure Yang-Mills

Starting from N=2 supersymmetric gauge theory on N_c D4-branes (5d) compactified on S^1 with radius ρ_c:



N_c D4: (x^0 x^1 x^2 x^3 x^4)
(Before taking the weak coupling limit)

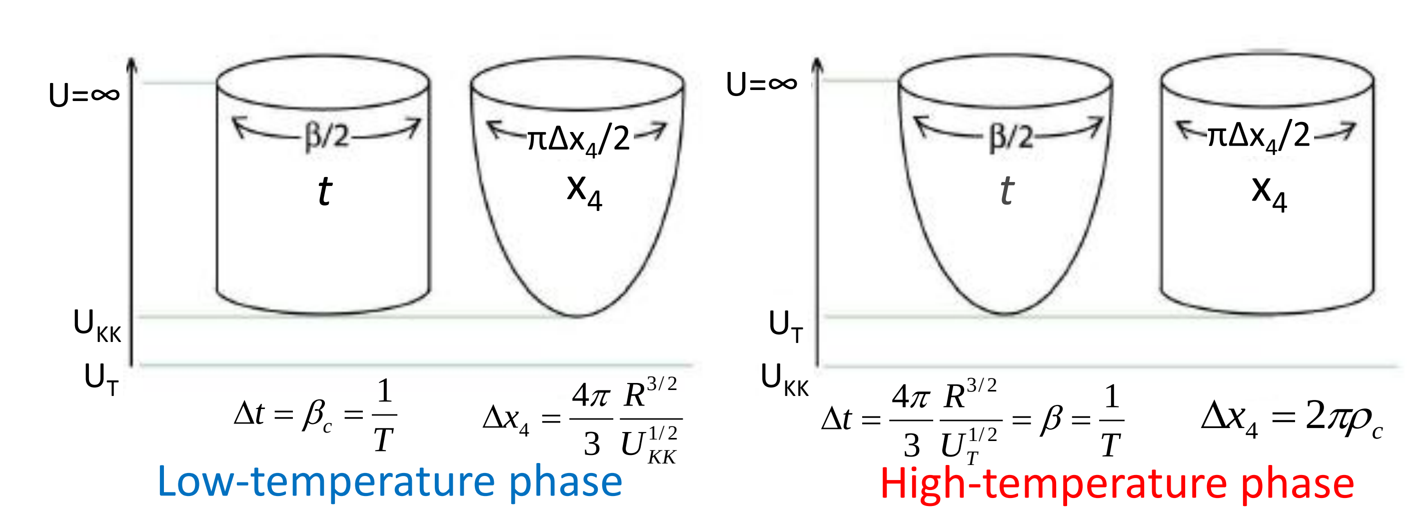
Taking antiperiodic boundary condition for the fermions to break SUSY:

- Fermions mass ~ $1/\rho_c$ (tree level)
- scalars mass ~ 1-loop effect

At scale << compactification radius ρ_c, the effective theory is 4d pure QCD.

[Aharony *et al.*, hep-th/0604161]

Dual geometry of thermal gauge theory (in the near horizon limit of the D4-branes)

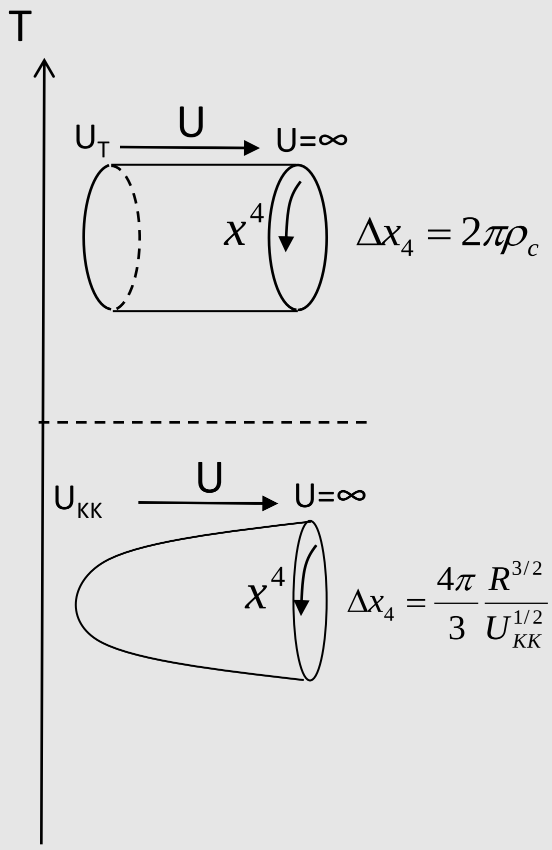


Low-temperature phase: $$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}}\left(dt^2 + dx_i^2 + f(U)dx_4^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \qquad f(U) = 1 - \frac{U_{KK}^3}{U^3}$$

High-temperature phase: $$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}}\left(f(U)dt^2 + dx_i^2 + dx_4^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \qquad f(U) = 1 - \frac{U_T^3}{U^3}$$

$$R^3 = \pi g_s N_c l_s^3,$$

Bulk thermal transition: (analogous to Hawking-Page transition)



High-temperature phase:

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}}\left(f(U)dt^2 + dx_i^2 + dx_4^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \qquad f(U) = 1 - \frac{U_T^3}{U^3}$$

$$\Delta t = \frac{4\pi}{3}\frac{R^{3/2}}{U_T^{1/2}} = \beta = \frac{1}{T}$$

Quark-anti-quark potential decays with distance:

$\Rightarrow$ deconfined phase

Low-temperature phase:

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}}\left(dt^2 + dx_i^2 + f(U)dx_4^2\right) + \left(\frac{R}{U}\right)^{\frac{3}{2}}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \qquad f(U) = 1 - \frac{U_{KK}^3}{U^3}$$

$$\Delta t = \beta_c = \frac{1}{T}$$

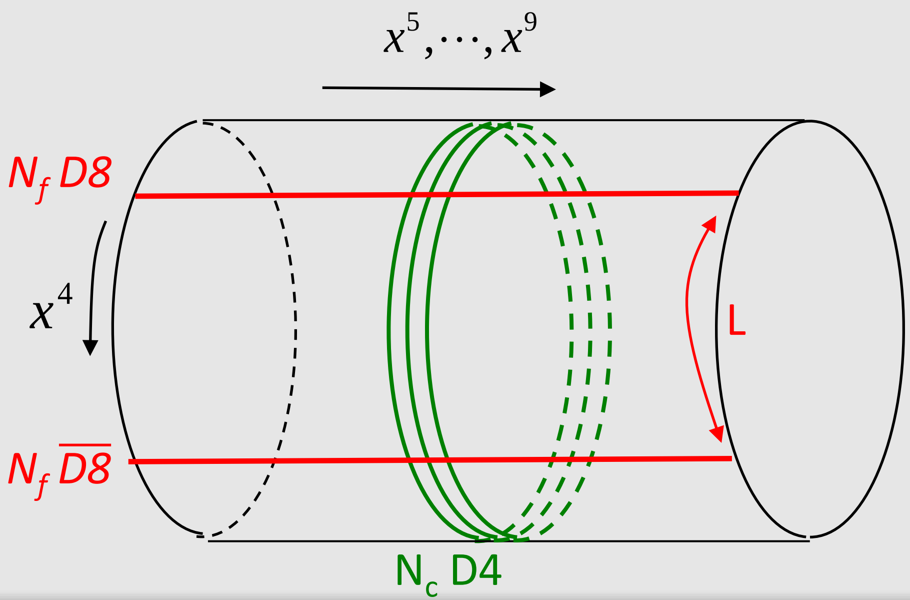
Quark-anti-quark potential grows with distance:

$\Rightarrow$ confined phase

Sakai-Sugimoto (SS) Holographic QCD Model:

adding quarks by introducing N_f D8 and N_f $\overline{\text{D8}}$-brane probes: ($N_f << N_c$)

N_c D4	0	1	2	3	(4)					
N_f D8/$\overline{\text{D8}}$	0	1	2	3		5	6	7	8	9



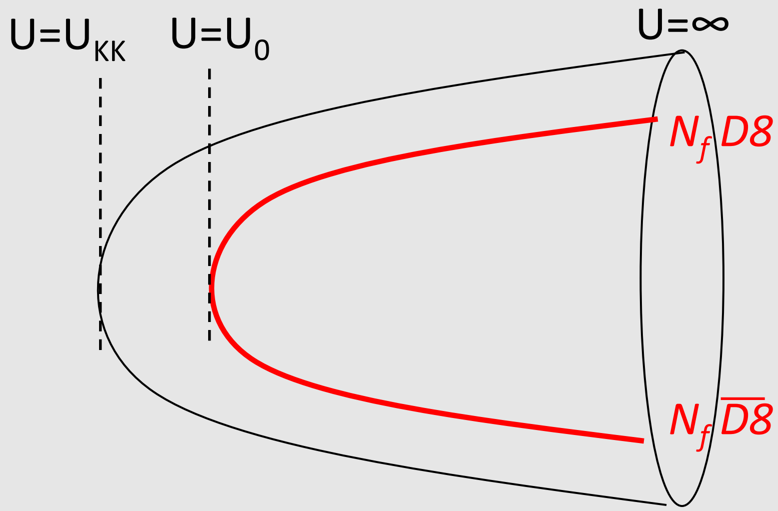
Picture before taking the weak coupling limit

4-8 strings: chiral quark
4-$\overline{8}$ strings: anti-chiral quark

Massless U(Nc) QCD in 4d with N_f flavors:

$$U(N_c)\times U(N_f)_L \times U(N_f)_R$$

- Supergravity dual (weak coupling limit) of confined phase



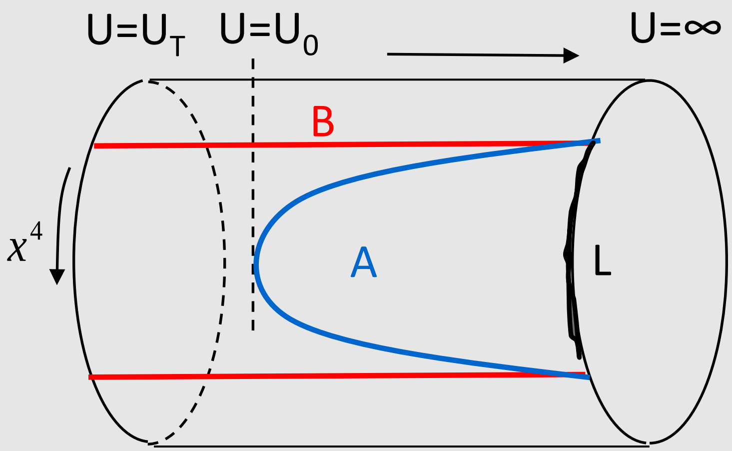
D8- and $\overline{\text{D8}}$-branes are embedded in the cigar-shaped background

D8- and $\overline{\text{D8}}$-branes are smoothly connected at U_0 $\Longrightarrow$ Chiral symmetry is broken

$$U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_{\text{diag}}$$

In which $U(1)_B$ corresponds to the conserved number of quarks

• Supergravity dual (weak coupling limit) of deconfined phase



There are two D8-$\overline{\text{D8}}$ pair configurations for certain T and L:

A: chiral symmetry broken

B: Chiral symmetry restored (QGP)

The thermodynamically favored configuration carry lower free energy.

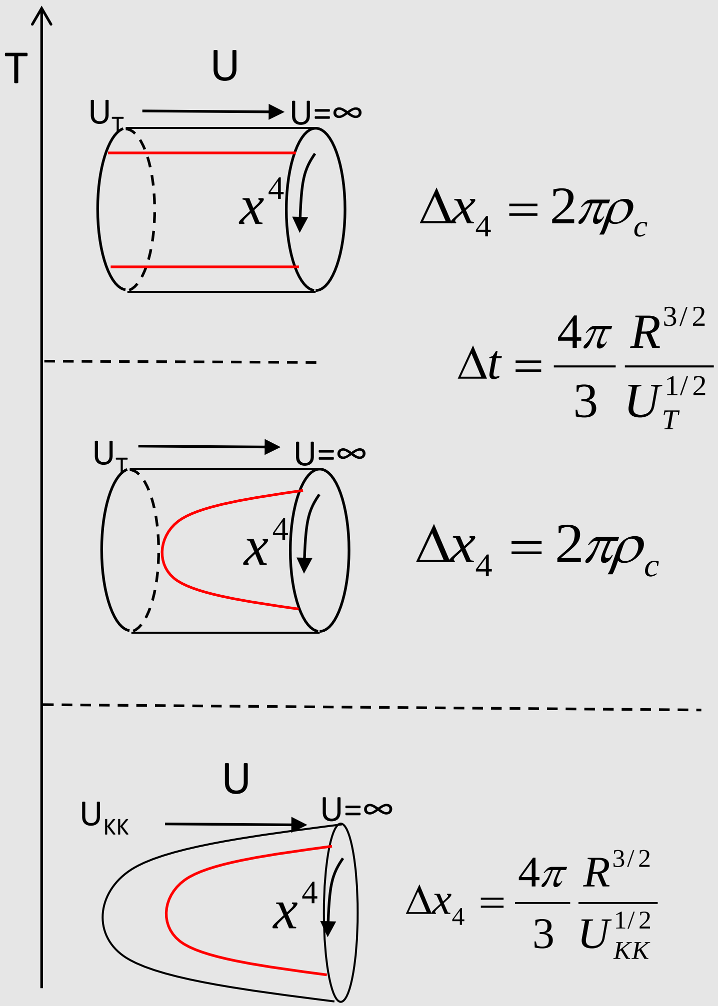
Phase transition:

As T increases, A $\longrightarrow$ B. In this case chiral symmetry restoration happens above deconfinement temperature

[Aharony *et al.*, hep-th/0604161]

As asymptotic separation L>0.97ρ_c, deconfinement coincides with χ-phase transition.

Bulk thermal transition in SS model:



$$\Delta x_4 = 2\pi\rho_c$$

Deconfined phase, restored chiral symmetry (QGP)

$$\Delta t = \frac{4\pi}{3}\frac{R^{3/2}}{U_T^{1/2}} = \beta = \frac{1}{T}$$

$$\Delta x_4 = 2\pi\rho_c$$

Deconfined phase, broken chiral symmetry

$$\Delta x_4 = \frac{4\pi}{3}\frac{R^{3/2}}{U_{KK}^{1/2}}$$

Confined phase, broken chiral symmetry

[Sakai & Sugimoto, hep-th/0412141]

[Hata-Sakai-Sugimoto-Yamato, hep-th/0701280]

Sakai-Sugimoto Model with Finite Baryon Density

$S_{D8} = S_{DBI} + S_{CS}$. The S_{DBI} part gives rise to 4d meson spectrum. The S_{CS} is given by

$$S_{CS} = T_8 \int_{D8} C_3 \wedge \mathrm{Tr}\exp(2\pi\alpha'\mathcal{F}) = T_8 \int_{D8} C_3 \wedge \mathrm{Tr}(2\pi\alpha'\mathcal{F})^3 = \frac{N_c}{24\pi^2}\int_{M_4\times R} \omega_5(\mathcal{A})$$

In general

$$\omega_5(\mathcal{A}) = tr(\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5)$$

If we expand the U(N_f) gauge field $\mathcal{A}$ into SU(N_f) part A and U(1) part Â:

$$\mathcal{A} = A^a T^a + \frac{1}{\sqrt{2N_f}}\hat{A}$$

For N_f=2, by solving the full YM+CS e.o.m of D8, one finds an instanton solution

Â = A_0, $A^a = (A^a)_{1,2,3,U}$ with $N_b = \frac{1}{32\pi^2}\int d^3x dU \mathrm{tr}(F\wedge F)$

such that $S_{CS} = \frac{N_c}{24\pi^2}\int_{M_4\times R} A_0 \mathrm{tr} F^2$, i.e U(1) gauge field is sourced by the instanton.

•Instanton as Baryon in SS model

The instanton sourcing U(1) electric gauge potential is regarded as a baryon by identifying it with D4-branes wrapping S^4.

Applying Witten's proposal in hep-th/9805112, the RR field $F_{(4)}$ charges up the D4-branes (wrapping S^4) under U(1) via

$$\frac{1}{2\pi}\int_{t\times S^4} A_0 \wedge F_{(4)} = N_c \int A_0$$

$\Rightarrow$ N_c Strings attaching the wrapped D4 and probe D8 (in SS model)

$\Rightarrow$ Baryonic vertex on D8

For this configuration to be stable, wrapped D4 must be attracted to D8

$\Rightarrow$ Instanton on the D8 worldvolume

[Domokos & Harvey, 0704.1604; Nakamura *et al.*, 0911.0679]

The CS term induces dynamic instability in holographic QCD, which we'll see later.

•The D4-instanton profile in our case:

The instanton locates at $U=U_c$ and smeared along (x_1,x_2,x_3):

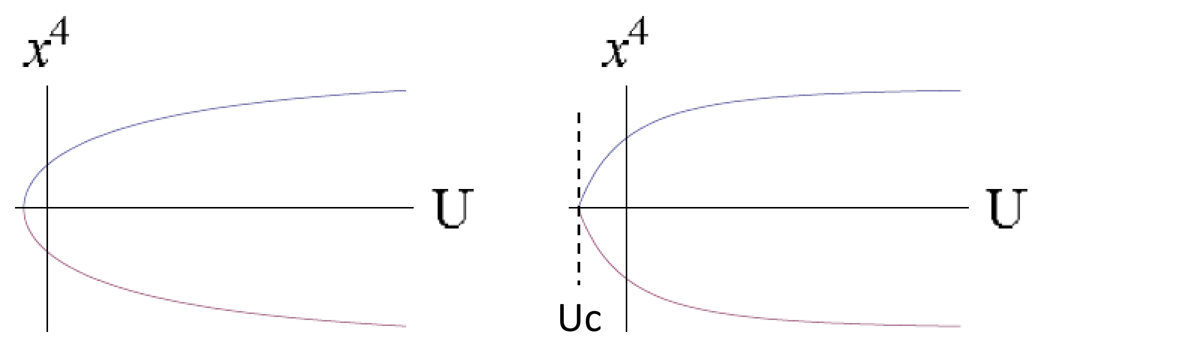
$$\frac{1}{8\pi^2}\mathrm{tr}F^2 = n_4\delta(U-U_c)d^3x dU$$

$$S_{CS} = \frac{N_c}{8\pi^2}\int A_0 \wedge \mathrm{Tr}(F^2) = N n_b A_0(U_c)$$

Where n_4 is the instanton number(D4-charges) density, n_b the baryon number density

$$N := \frac{T_8 R^5 N_f V_3 \beta \Omega_4}{g_s}$$

$$n_b := \frac{N_c N_f n_4 V_3 \beta}{2\pi\alpha' R^2 N}$$



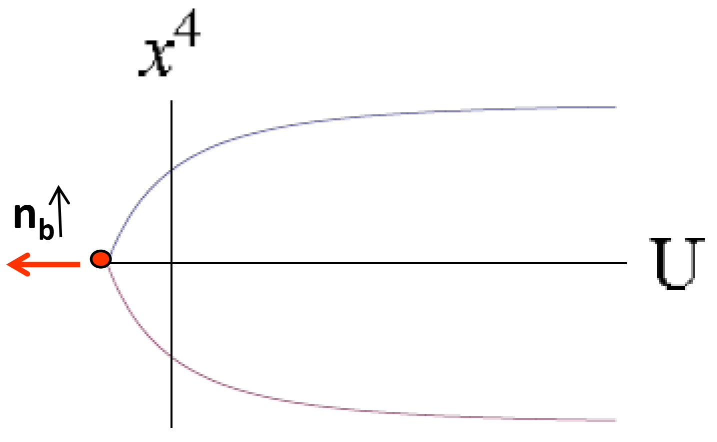
Smooth $D8-\overline{D8}$ for $n_b = 0$ $D8-\overline{D8}$ with a cusp for nonzero n_b

The D8-$\overline{\text{D8}}$ develops a cusp due to pulling of the instanton. The cusp angle

$$\cos\theta_c = \frac{n_b^2}{n_b^2 + U_c^5}$$

by the force balance condition.

• Effect of $\mathbf{n_b}$ on chiral symmetry restoration in the deconfined geometry



$D8 - \overline{D8}$ with a cusp for nonzero n_b

As baryon number density n_b increases (below a critical value), the tension of D4 increase, and the tip of D8-$\overline{\text{D8}}$ is pulled towards the horizon. However if n_b is too large, the joint D8-$\overline{\text{D8}}$ solution doesn't exist in HT phase $\Rightarrow$ QGP

Dynamical Instability

The dual geometries in SS model

- Low temperature phase

$$ds^2_{D8} = \left(\frac{U}{R}\right)^{\frac{3}{2}}(\eta_{\mu\nu}dx^\mu dx^\nu) + \left(\frac{R}{U}\right)^{\frac{3}{2}}\Big(\Big(\left(\frac{U}{R}\right)^3 h(U)x_4'^2 + \frac{1}{h(U)}\Big)dU^2 + U^2 d\Omega_4^2\Big)$$

$$F_4 = \frac{(2\pi)^3 \ell_s^3 N_c}{\Omega_4}\epsilon_4,\ e^\phi = g_s\left(\frac{U}{R}\right)^{\frac{3}{4}},\ h(U) = 1 - \frac{U_{KK}^3}{U^3},\ x_4 \sim x_4 + \frac{4\pi R^{3/2}}{3U_{KK}^{1/2}}$$

- High-temperature phase

$$ds^2 = (\frac{U}{R})^{3/2}[-h(U)dt^2 + dx_i^2] + ((\frac{U}{R})^{3/2}x_4'^2 + (\frac{R}{U})^{3/2}\frac{1}{h(U)})dU^2 + (\frac{R}{U})^{3/2}U^2 d\Omega_4^2$$

$$h(U) = 1 - \frac{U_T^3}{U^3}$$

- The total action of D8 is $S_{D8} = S_{DBI} + S_{CS}$

$$S_{D8} = -T\int d^9x\, e^{-\phi}\sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})} + S_{CS}$$

$$S_{CS} = \frac{N_c}{24\pi^2}\int_{M_4\times R_+} A_0 TrF^2 = Nn_b\int dU\delta(U-U_c)A_0$$

- The e.o.m

Low temperature phase

$$x_4' = \pm U^{-\frac{3}{2}} \frac{\sqrt{H_0 \sin^2\theta_c}}{h\sqrt{H - H_0 \sin^2\theta_c}}, \qquad E = -n_b U^{\frac{3}{2}} \frac{1}{\sqrt{H - H_0 \sin^2\theta_c}}$$

$$E = -\partial_U A_0 = -A_0'$$

where $H(U) = U^8 h(1 + \frac{n_b^2}{U^5}),\ H_0 = H(U_c)$ $\quad \cos^2\theta_c = \frac{n_b^2}{n_b^2 + U_c^5}$

$$L = \int_{U_c}^{\infty} dU x_4'$$

High-temperature phase

$$x_4' = \pm\, U^{-3/2} \sqrt{\frac{H_0 \sin^2\theta_c}{h(H - H_0 \sin^2\theta_c)}}, \qquad E = -n_b U^{3/2} \sqrt{\frac{h}{(H - H_0 \sin^2\theta_c)}}$$

The differential eqn x_4' can be solved (numerically) by the fixed-length condition

$$L = \int_{U_c}^{\infty} dU x_4'$$

Introducing the chemical potential

According to AdS/CFT prescription, the coupling occurs on the boundary

$$\int d^4 x A_\mu(x, U=\infty) j^\mu = \int d^4 x A_\mu(x, U=\infty) \bar{\psi}\gamma^\mu \psi$$

- For the <u>0-th component</u>:

$$\int d^4 x A_0(x, U=\infty) j^0 = \int d^4 x A_0(x, U=\infty) \rho$$

ρ: baryon number density

The chemical potential is identified as $A_0(x, U \to \infty)$ after solving A_0 in the e.o.m.

In our case, $\mu = N A_0(\infty)$, where the bulk U(1) gauge field is sourced by the instanton.

•For the *j*-th component (*j*=1,2,3):

$$\int d^4x A_i(x, U=\infty)\, j^i = \int d^4x A_i(x, U=\infty)\, \overline{\psi}\gamma^i\psi$$

A_i *(x, U→ ∞)* serves as the source to the current operator j^i on the boundary.

Dictionary of gauge/gravity correspondence

Nonnormalizable mode

normalizable mode

(U→∞)

bulk field $\phi(x,U) \approx AU^{-a} + BU^{b} + \cdots$

boundary $\int AO$ (source term) $\langle O \rangle$ (exp. value of operator $\mathcal{O}$)

The normalizable mode of A_i correspond to $\langle j^i \rangle$

Looking for Dynamical Instability

Now turn on the bulk perturbations:

U(1) gauge field: $\delta A = \{a_0,\ a_i, a_U\}$

D8-brane embedding funtion $\delta\, x_4 = y$

Expand the DBI+CS action upto quadratic order

$$L[A, x_4; a_\mu, a_U, y] = L_0 + L_1 + L_2 + \cdots$$

Vanish due to eom (L_0)

Contain 1st order pert. but vanish on shell (L_1)

gives eom for perturbations a_0, a_i, a_U, y (6 coupled diff eqns) (L_2)

Dynamical Instability from Chern-Simons coupling

Dynamical instabilities are found in the 3 equations of motions for a_i:
(master equations, derived from L_2)

$-\omega^2$ $-k^2$ **From Chern-Simons term**

$$\left(\frac{U^5\Delta_q}{L_q}f_k'\right)' + \frac{L_q}{U^3}\left(-\frac{Q}{\Delta_q}\partial_t^2 f_k + \partial_j^2 f_k\right) + 2\kappa\epsilon_{ijk}E\partial_j f_i = 0$$

$$f_i := \frac{1}{2}\epsilon_{ijk}f_{jk}$$

$$q = LT \quad L_{LT} := U^4\sqrt{\frac{U^5}{H - U_c^8 h_0}} \quad \Delta_{LT} := 1 \quad \delta_{LT} := h$$

$$q = HT \quad L_{HT} := U^4\sqrt{\frac{U^5 h}{H - U_c^8 h_0}} \quad \Delta_{HT} := h \quad \delta_{HT} := 1$$

$$\kappa := \frac{n_b}{4\pi^2 n_4} = 288\pi^2\frac{1}{\lambda^2}\frac{U_{KK}}{R}$$

$$Q := 1 + \frac{n_b^2}{U^5},\ h_0 := h(U_c)$$

$$\phi(t,x,U) = e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\, g(U)$$

Heuristically, take the ansatz for the perturbed fields

$$\phi(t,x,U) = e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}\, g(U)$$

The e.o.m's then take the form of 2nd order ordinary linear (eigenvalue) diff eqns.

$$A_{\mathbf{k}}(U)\, g''(U) + B_{\mathbf{k}}(U)\, g'(U) + C_{\mathbf{k}}(U)\, g(U) = \omega^2 g(U)$$

$$\Rightarrow \quad g(U) \approx m + \nu U^{-\alpha} + \cdots$$

Such that

$$\phi(t,x,U) \approx A + BU^{-\alpha} + \cdots \quad (U \to \infty)$$

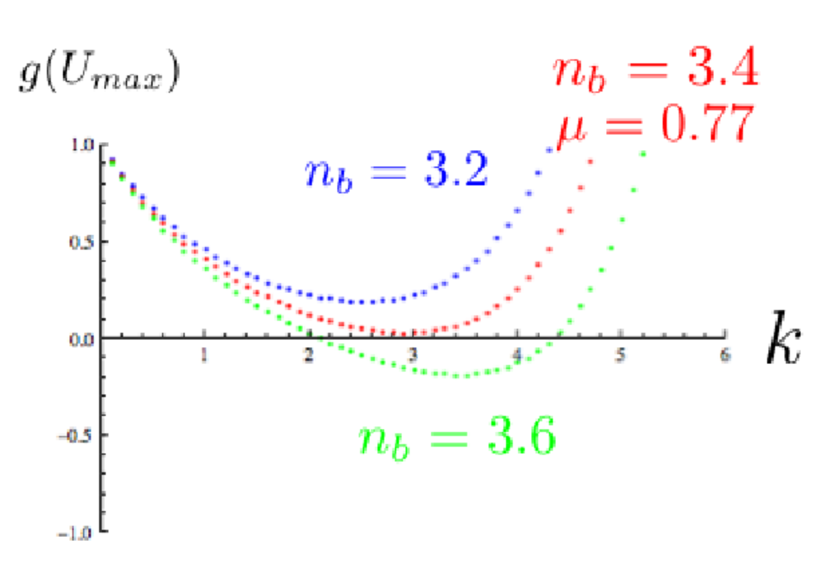
$$A \leftrightarrow \int AO \qquad B \leftrightarrow \langle O \rangle$$

We are looking for normalizable (*m*=0) growing mode such that $B \cong e^{\omega_I t}$

$\Rightarrow$ spontaneous symmetry breaking with order parameter $\langle O \rangle$, without the source term for $\mathcal{O}$

Numerical method: shooting

We look for the unstable mode by starting with the marginal case $\omega^2=0$ (onset of instability), and tune k for a certain value of n_b to "shoot" for the normalizable mode (m=0) for both HT and LT phases.



$$g(U) \approx m + \nu U^{-\alpha} + \cdots$$

Boundary conditions

At $U \to \infty$, m=0 (normalizable mode)

At $U=U_c$, a_i, a_U : Dirichlet or Neumann

y : Dirichlet (fixing the position of the tip)

a_0 : Neumann (fixing the electric source)

For normalizble unstable modes of a_i:

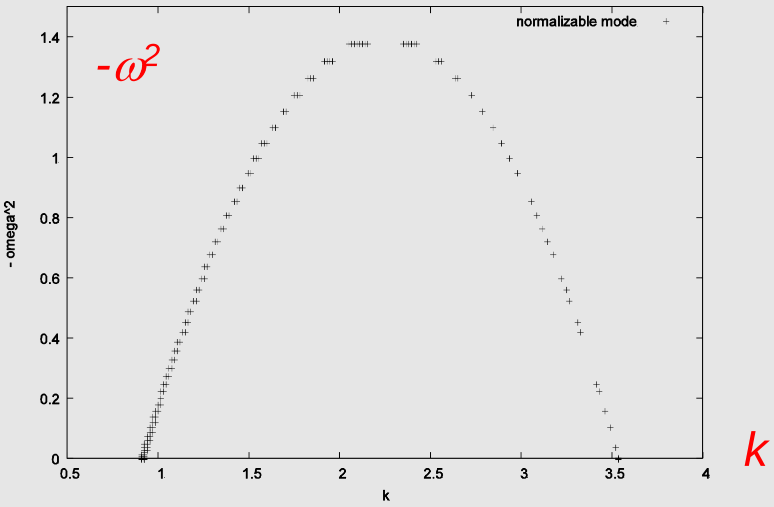
$$a_i \propto e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}} \nu U^{-\alpha}$$

$$\Updownarrow$$

$$\left\langle J^i \right\rangle \propto e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$$ unstable for nonzero ***k*** **:** **Spatially modulated**

Numerical Results

For k within certain range, there is unstable mode, with $\omega^2=0$ being the onset of instability. Thus bulk normalizable growing modes are found in a_i.



Conclusion: Holographic QCD Diagram