Nonlocality Distillation

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Outline

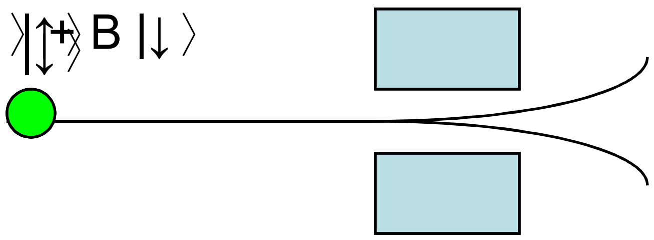
- Introduction
  - EPR-paradox
  - Locality
  - Hidden variable
- Nonlocality
  - Bell's inequality, CHSH inequality
  - Correlation
- Nonlocality distillation
  - FWW protocol
- Conclusion

Review of quantum mechanics

- 一個spin-1/2的粒子通過Z-方向磁場

Collapse

A |↑〉+ B |↓〉



|↑〉 $|A|^2$ A |↑〉 + B |↓〉 → |↑〉

|↓〉 $|B|^2$ A |↑〉 + B |↓〉 → | ↓〉

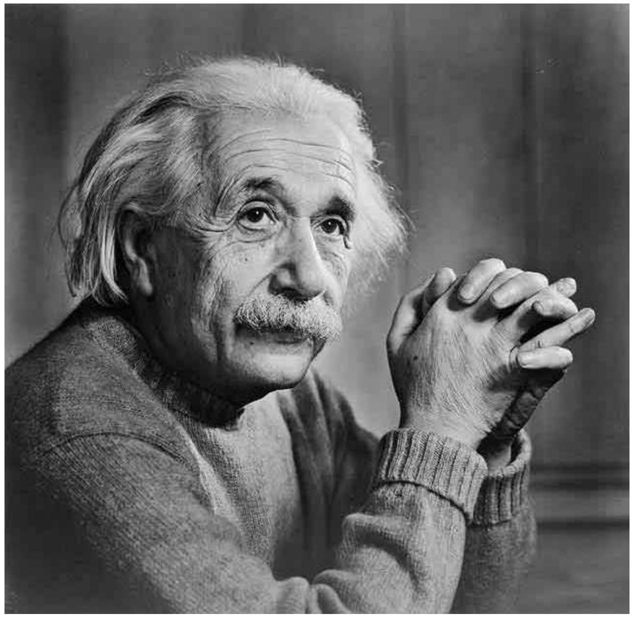
測量會造成collapse

測量**前**無法決定粒子會collapse到那個狀態

理論能夠預測的只有**機率**

上帝是不玩骰子的

"I, at any rate, am convinced that He [God] does not throw dice."



EPR Paradox

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Collins: English Dictionary Definition (Meaning) of

spook (INFORMAL)

N

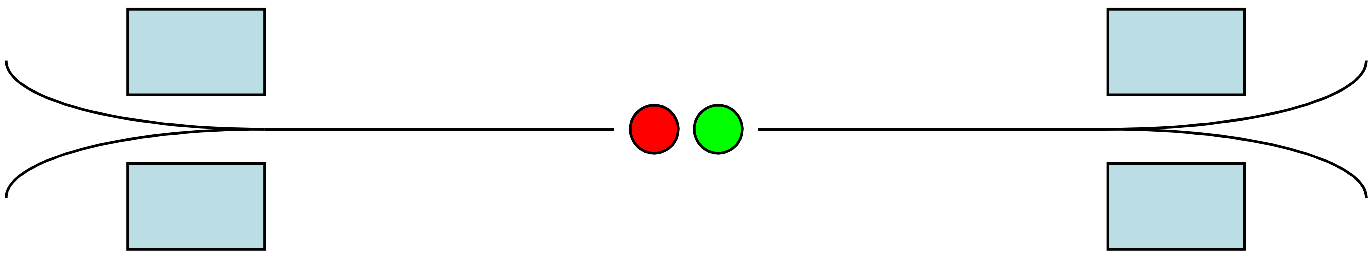
1. a ghost,

2. a strange and frightening person,

Spooky Action

measurement setups (magnetic direction ↑)

measurement results (spin up + spin down −)



$$\frac{1}{\sqrt{2}} |\uparrow \downarrow \rangle - \frac{1}{\sqrt{2}} |\downarrow \uparrow \rangle \twoheadrightarrow \| \uparrow\updownarrow\downarrow\updownarrow \rangle\!\rangle$$

physics should represent a reality in time and space, free from spooky actions at a distance

Separable vs. Entangled

$$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \qquad + |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$$

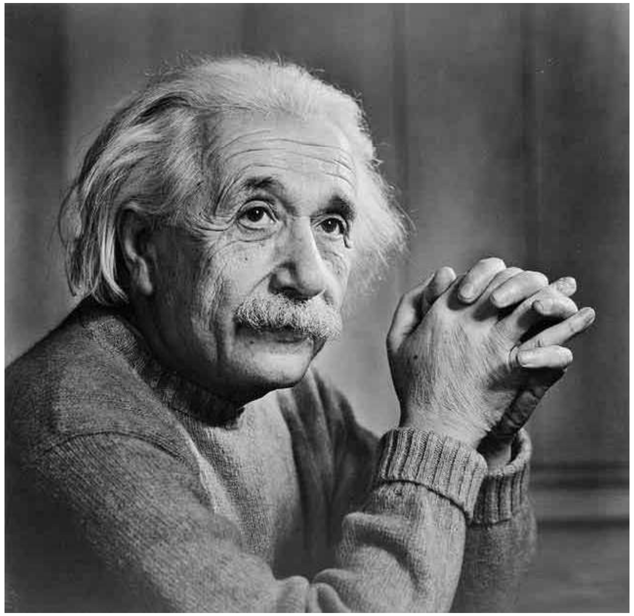
$$= (|\uparrow\rangle + |\downarrow\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle)$$

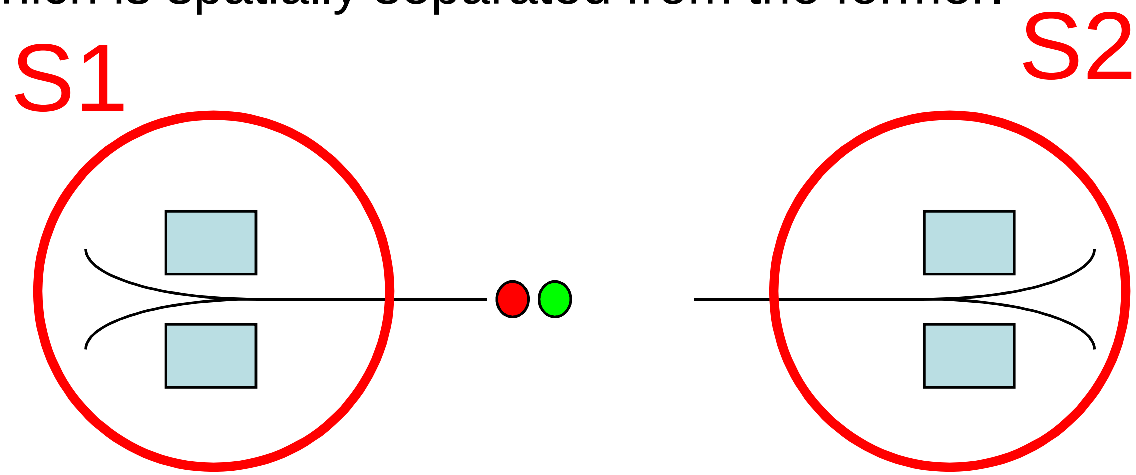
Separable state → $\boldsymbol{\psi = \psi_{S1} \otimes \psi_{S2}}$

Entangled state → ***Not*** separable state

Locality

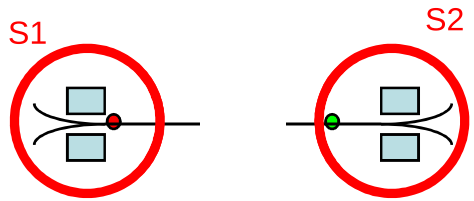
- The real factual situation of the system S2 is independent of what is done with the system S1, which is spatially separated from the former.





Hidden Variable

Local Hidden Variable (LHV)



	QM	LHV
粒子狀態	1. 測量前是疊加態 2. 測量後collapse到特定的eigenstate	測量前已經決定
機率行為	是大自然的特性	Hidden variable的資訊不足
區域性	Nonlocal	Local

PHYSICAL REVIEW VOLUME 85, NUMBER 2 JANUARY 15, 1952

A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I

DAVID BOHM*
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received July 5, 1951)

PHYSICAL REVIEW VOLUME 108, NUMBER 4 NOVEMBER 15, 1957

Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky

D. BOHM AND Y. AHARONOV
Technion, Haifa, Israel
(Received May 10, 1957)

REVIEWS OF
MODERN PHYSICS

VOLUME 33, NUMBER 1 JANUARY, 1961

Completeness of Quantum Mechanics and Charge-Conjugation Correlations of Theta Particles*

D. R. INGLIS
Argonne National Laboratory, Argonne, Illinois

Bell’s inequality

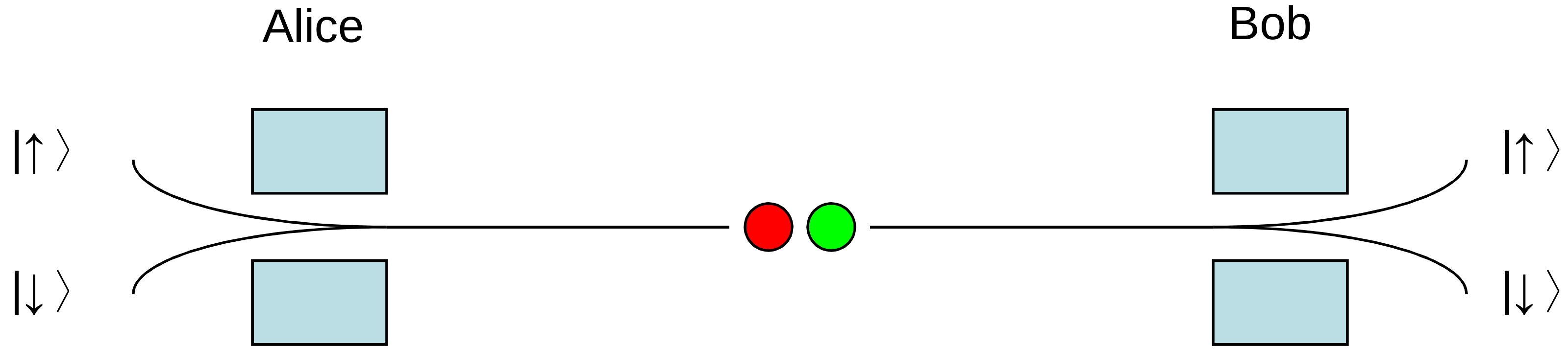
- Bell 在1961年提出第1個不等式，可以做實驗判斷 local hidden variable 是否存在？

- 之後有其他推廣的不等式，稱為Bell-type inequality

- 最常用的Bell-type inequality是 CHSH inequality，由 Clauser, Horne, Shimony and Holt在1969年提出

Correlation

$$E(AB)=\frac{1}{n}\sum_{1}^{n}[outcome(A)\times outcome(B)]$$



1. 將實驗測到的eigenvalue ±h/2 normalized
   將 spin up |↑〉的測量結果 h/2 設為 1
   將 spin up |↓〉的測量結果 -h/2 設為 -1

2. 將每一次兩邊的測量結果相乘

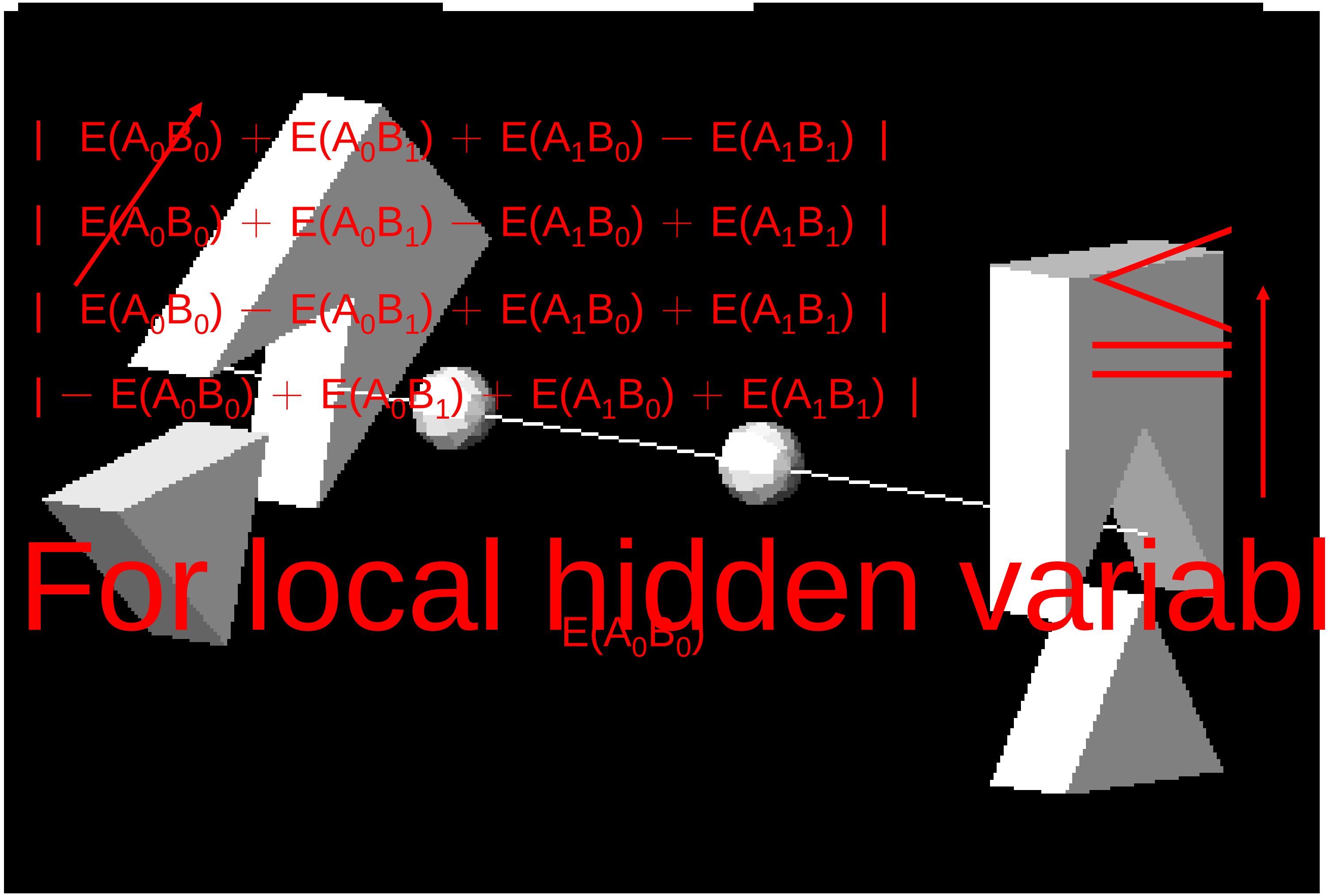
3. 求『測量結果相乘』的平均值

	Alice	Bob	相乘
第1次	1	-1	-1
第2次	1	1	1
⋮	⋮		⋮
第n次	-1	-1	1

$$\frac{-1+1+\cdots+1}{n}$$

CHSH inequality

1. 兩邊 各自選兩個測量方向　↗　↖　↑　→
2. 做4組實驗，得到４個correlations　A_0、A_1、B_0、B_1



$$| E(A_0B_0) + E(A_0B_1) + E(A_1B_0) - E(A_1B_1) |$$
$$| E(A_0B_0) + E(A_0B_1) - E(A_1B_0) + E(A_1B_1) |$$
$$| E(A_0B_0) - E(A_0B_1) + E(A_1B_0) + E(A_1B_1) |$$
$$| - E(A_0B_0) + E(A_0B_1) + E(A_1B_0) + E(A_1B_1) |$$
$$\leqq 2$$

$E(A_0B_1)$

$E(A_1B_1)$

$E(A_0B_0)$

For local hidden variable

CHSH nonlocality

$$\text{Nonlocality} =: \max \begin{cases} |\ E(A_0B_0) + E(A_0B_1) + E(A_1B_0) - E(A_1B_1)\ | \\ |\ E(A_0B_0) + E(A_0B_1) - E(A_1B_0) + E(A_1B_1)\ | \\ |\ E(A_0B_0) - E(A_0B_1) + E(A_1B_0) + E(A_1B_1)\ | \\ |-E(A_0B_0) + E(A_0B_1) + E(A_1B_0) + E(A_1B_1)\ | \end{cases}$$

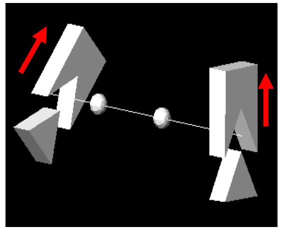
Nonlocality $\leqq 2$ The system is local

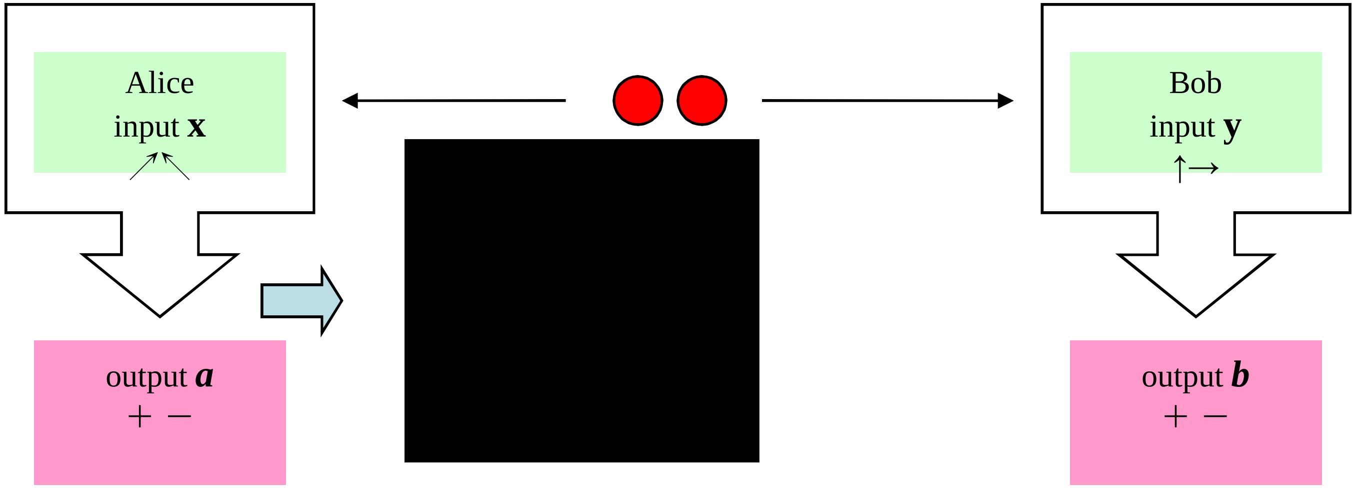
For Quantum system, Nonlocality $\leqq 2\sqrt{2}$

Bipartite binary input-output system

Input : measurement setups (magnetic direction ↗↖↑→)

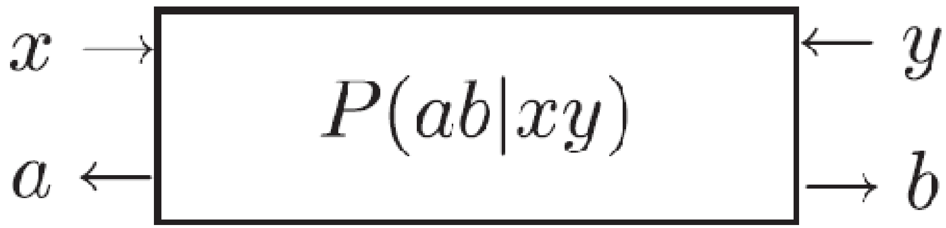
Output : measurement results (spin up ＋ spin down －)





Quantum Physics : the output (measurement result) is probabilistic.

Probability Box



$$\begin{pmatrix} P(00|00) & P(01|00) & P(10|00) & P(11|00) \\ P(00|10) & P(01|10) & P(10|10) & P(11|10) \\ P(00|01) & P(01|01) & P(10|01) & P(11|01) \\ P(00|11) & P(01|11) & P(10|11) & P(11|11) \end{pmatrix}$$

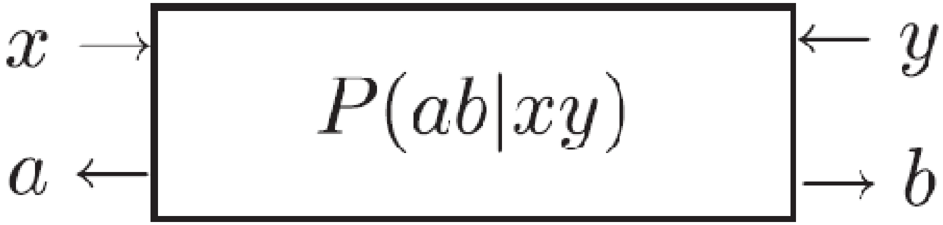
Non-Signaling

$$\sum_b P(ab|xy) = \sum_b P(ab|xy') \equiv P(a|x) \quad \forall\ a, x, y, y',$$

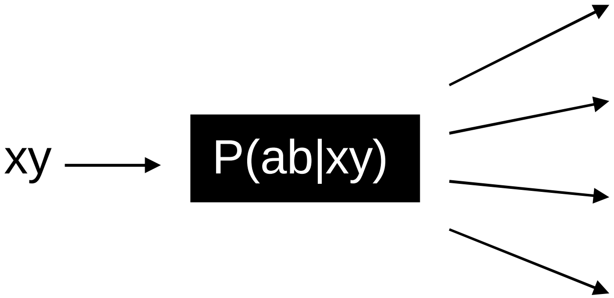
$$\sum_a P(ab|xy) = \sum_a P(ab|x'y) \equiv P(b|y) \quad \forall\ b, x, x', y.$$

$$\begin{pmatrix} P(00|00) & P(01|00) & P(10|00) & P(11|00) \\ P(00|10) & P(01|10) & P(10|10) & P(11|10) \\ P(00|01) & P(01|01) & P(10|01) & P(11|01) \\ P(00|11) & P(01|11) & P(10|11) & P(11|11) \end{pmatrix}$$

Correlation function



Correlation E(xy)= P(00|xy)-P(01|xy)-P(10|xy)+P(11|xy)

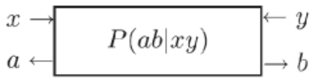


00	$++$	1	P(00|xy)
01	$+-$	-1	P(01|xy)
10	$-+$	-1	P(10|xy)
11	$--$	1	P(11|xy)

Given the input xy, the expectation value of output product is denoted by E(xy)

PR-box

Popescu, S. and Rohrlich, D. *Found. Phys.* **24**, 379 (1994)



$$\begin{pmatrix} P(00|00) & P(01|00) & P(10|00) & P(11|00) \\ P(00|10) & P(01|10) & P(10|10) & P(11|10) \\ P(00|01) & P(01|01) & P(10|01) & P(11|01) \\ P(00|11) & P(01|11) & P(10|11) & P(11|11) \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

Nonlocality = 4

Correlation E(xy)= P(00|xy)-P(01|xy)-P(10|xy)+P(11|xy)

E(00)= 1, E(01)= 1, E(10)= 1, E(11)= -1

Communication Complexity

Alice

Alice only knows ***X***
Bob only knows ***Y***
How many bits do they need to communicate to know the function F(***X***,***Y***)

Bob

Alice	Bob
x_1	y_1
x_2	y_2
x_3	y_3
:	:
:	:
:	:
x_n	y_n

$$F(X,Y) = (x_1 \oplus y_1) \oplus (x_2 \oplus y_2) \oplus \cdots (x_n \oplus y_n)$$

$$F(X,Y) = x_1 y_1 \oplus x_2 y_2 \oplus \cdots x_n y_n$$

$$= x_1 \oplus x_2 \oplus \cdots x_n \oplus y_1 \oplus y_2 \oplus \cdots \oplus y_n$$

1-bit

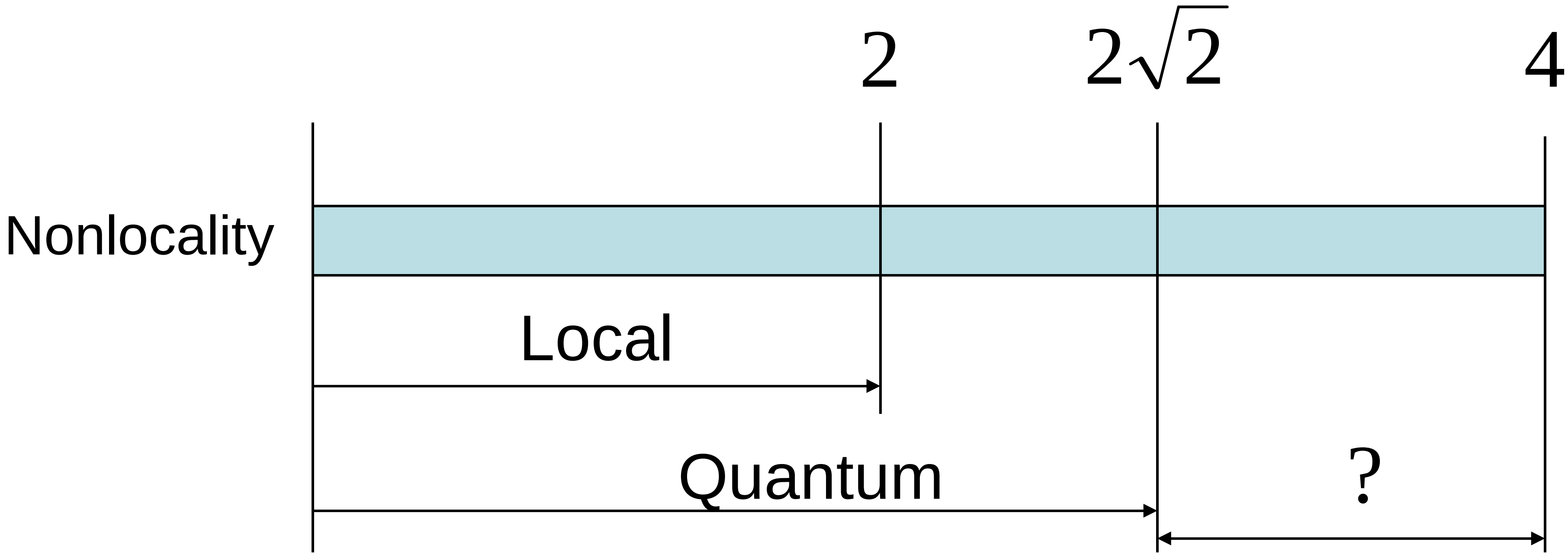
n-bit

But, if Alice and Bob have PR box, 1-bit is ok.

van Dam, W. Preprint at <http://arxiv.org/quant-ph/0501159> (2005).

Why nature is not more nonlocal

Popescu, Nature Physics **2**, 507 (2006)



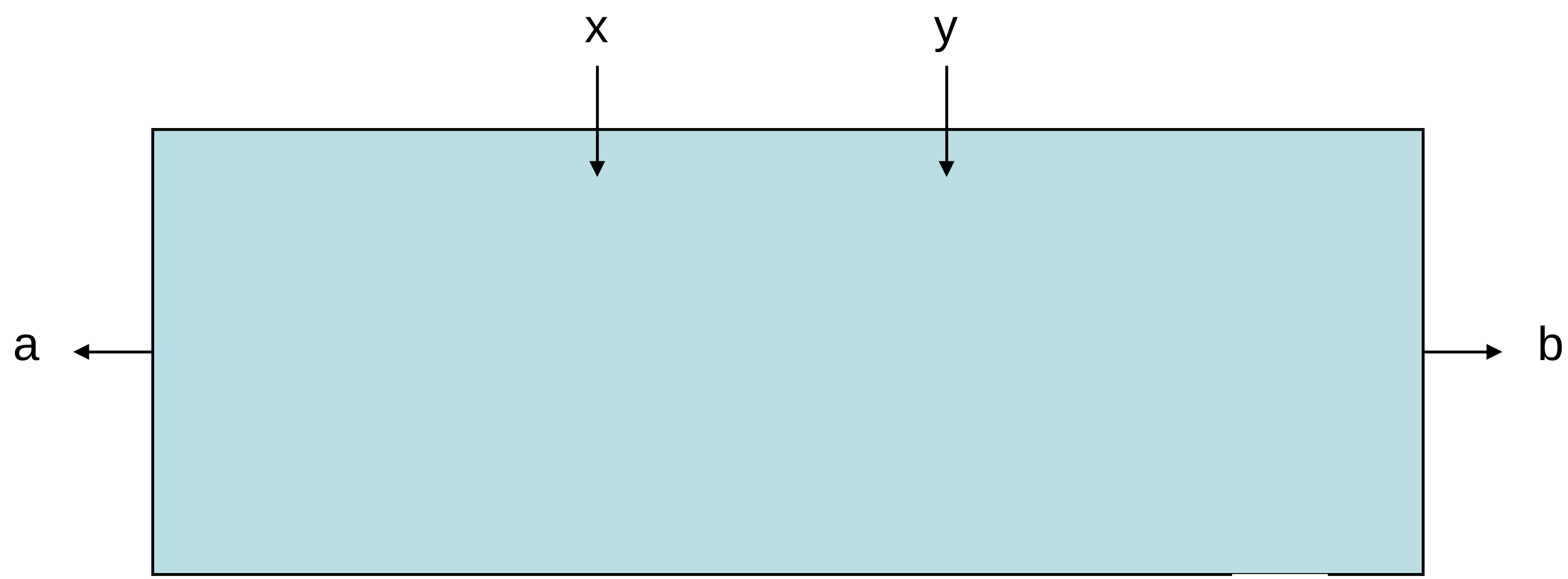
Nonlocality Distillation

$$P_{\varepsilon}^{PR} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1-\varepsilon}{2} & \frac{\varepsilon}{2} & \frac{\varepsilon}{2} & \frac{1-\varepsilon}{2} \end{pmatrix} = \varepsilon \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} + (1-\varepsilon) \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Can nonlocality be amplified by local operation ?

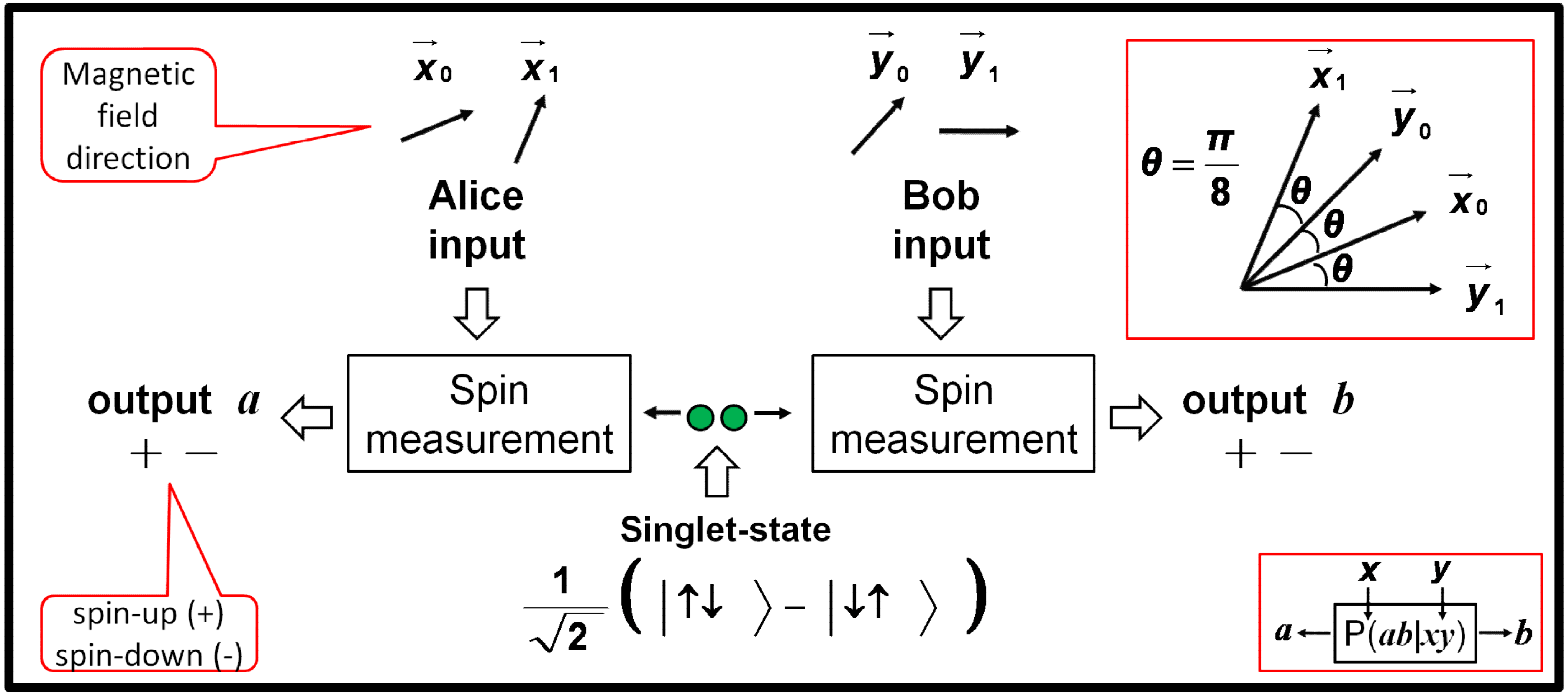
Forster, Winkler & Wolf, PRL **102**, 120401 (2009)

FWW Protocol



$$\begin{pmatrix} P'(00|xy) \\ P'(01|xy) \\ P'(10|xy) \\ P'(11|xy) \end{pmatrix} = \begin{pmatrix} P(00|xy) & P(01|xy) & P(10|xy) & P(11|xy) \\ P(01|xy) & P(00|xy) & P(11|xy) & P(10|xy) \\ P(10|xy) & P(11|xy) & P(00|xy) & P(01|xy) \\ P(11|xy) & P(10|xy) & P(01|xy) & P(00|xy) \end{pmatrix} \begin{pmatrix} P(00|xy) \\ P(01|xy) \\ P(10|xy) \\ P(11|xy) \end{pmatrix}$$

$$E(xy) \rightarrow E(xy)^2$$



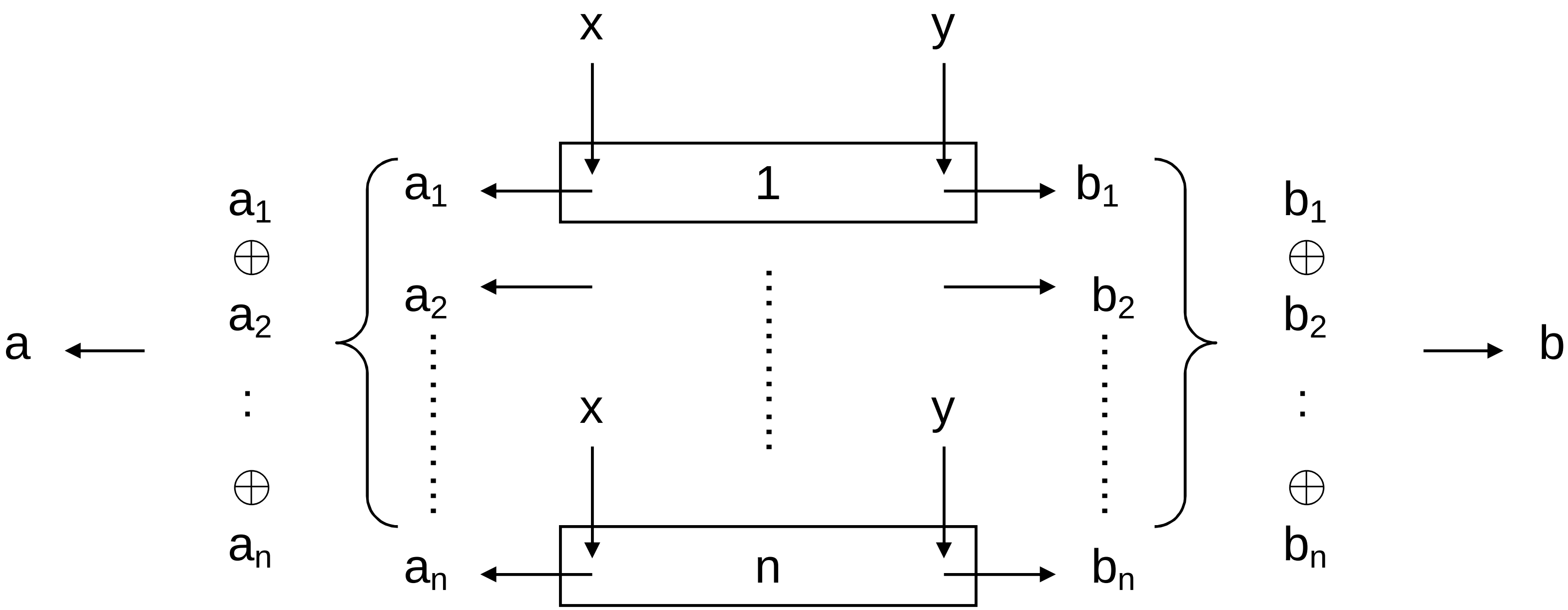
- For spin-singlet state: $\langle \vec{x}\vec{y} \rangle = -\vec{x}\cdot\vec{y}$
- In example 1, the CHSH (Clauser-Horne-Shimony-Holt) nonlocality [2] is

$$\left| \langle \vec{x}_0 \vec{y}_0 \rangle + \langle \vec{x}_0 \vec{y}_1 \rangle + \langle \vec{x}_1 \vec{y}_0 \rangle - \langle \vec{x}_1 \vec{y}_1 \rangle \right| = \left| -3\cos(\pi/8) + \cos(3\pi/8) \right| \approx 2.389$$

- For n = 2, FWW protocol on 2 resource systems, the nonlocality becomes

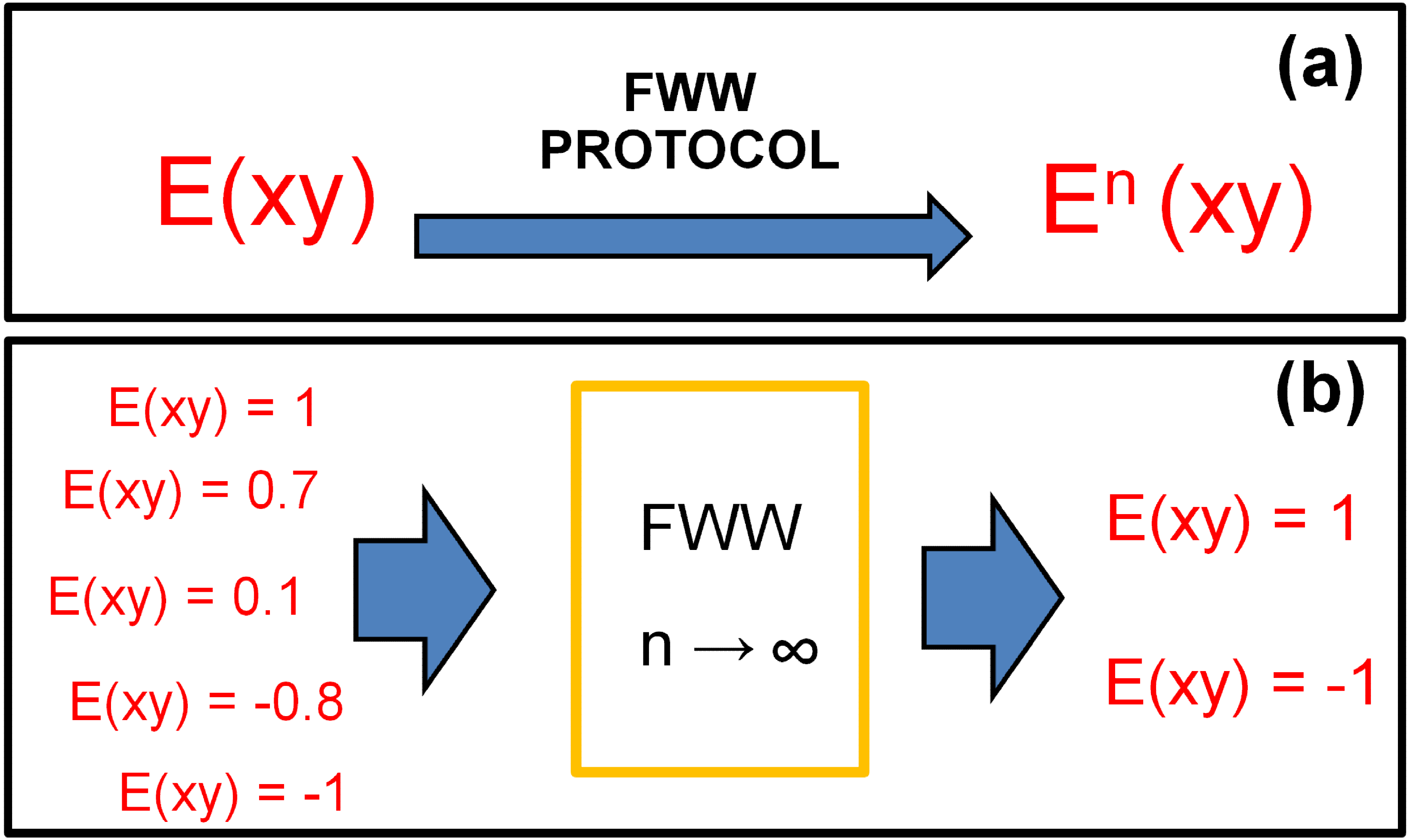
$$\left| \langle \vec{x}_0 \vec{y}_0 \rangle^2 + \langle \vec{x}_0 \vec{y}_1 \rangle^2 + \langle \vec{x}_1 \vec{y}_0 \rangle^2 - \langle \vec{x}_1 \vec{y}_1 \rangle^2 \right| = \left| 3\cos^2(\pi/8) - \cos^2(3\pi/8) \right| \approx 2.414$$

- In this example, the nonlocality is amplified from ***2.389*** to ***2.414***



$$\begin{pmatrix} P'(00|xy) \\ P'(01|xy) \\ P'(10|xy) \\ P'(11|xy) \end{pmatrix} = \begin{pmatrix} P(00|xy) & P(01|xy) & P(10|xy) & P(11|xy) \\ P(01|xy) & P(00|xy) & P(11|xy) & P(10|xy) \\ P(10|xy) & P(11|xy) & P(00|xy) & P(01|xy) \\ P(11|xy) & P(10|xy) & P(01|xy) & P(00|xy) \end{pmatrix}^{n-1} \begin{pmatrix} P(00|xy) \\ P(01|xy) \\ P(10|xy) \\ P(11|xy) \end{pmatrix}$$

$$\boldsymbol{E(xy)\rightarrow E(xy)^{n}}$$


(a)
FWW
PROTOCOL
E(xy)
Eⁿ (xy)
(b)
E(xy) = 1
E(xy) = 0.7
E(xy) = 0.1
E(xy) = -0.8
E(xy) = -1
FWW
n → ∞
E(xy) = 1
E(xy) = -1

Nonlocality distillation (amplification)

$$E(xy) \to E(xy)^n \begin{cases} E(xy) = \mathbf{1} \ \to \ \lim_{n\to\infty} E(xy)^n = \mathbf{1} \\ E(xy) = -\mathbf{1} \ \to \ \lim_{\substack{n\to\infty \\ \text{n is odd}}} E(xy)^n = -\mathbf{1} \\ -\mathbf{1} < E(xy) < \mathbf{1} \ \to \ \lim_{n\to\infty} E(xy)^n = \mathbf{0} \end{cases}$$

E(00)=1　E(01)=1　E(10)=1　E(11)=0.99

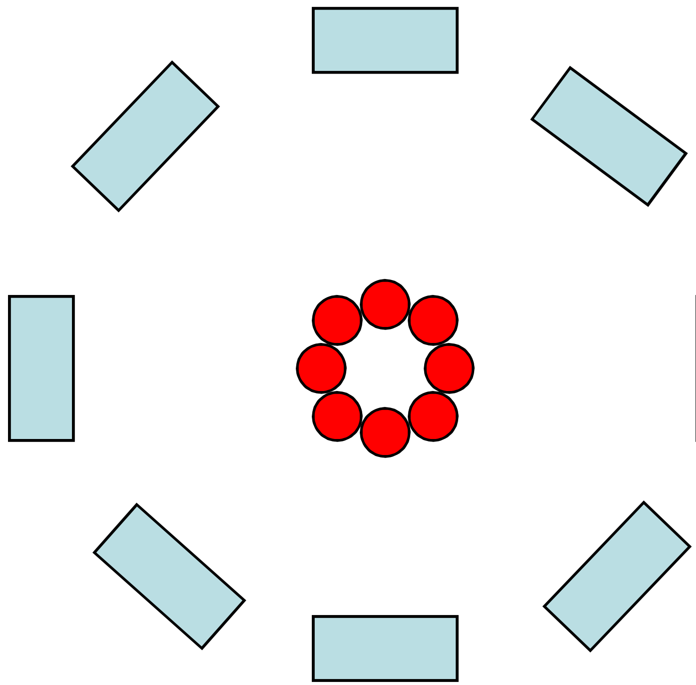
2.01　　　3

n=1　　　n=∞

$$\left|E(\mathbf{00}) + E(\mathbf{01}) + E(\mathbf{10}) - E(\mathbf{11})\right| = \mathbf{2.01}$$

$$\lim_{n\to\infty}\left|E(\mathbf{00})^n + E(\mathbf{01})^n + E(\mathbf{10})^n - E(\mathbf{11})^n\right| = \mathbf{3}$$

Multipartite nonlocality distillation



For Multipartite scenario,
the correlations have the same transformation.

$$E(XY\cdots Z) \rightarrow E(XY\cdots Z)^n$$

Conclusion

1. Quantum theory is a nonlocal theory. Its nonlocality can be demonstrated by entangled states.
2. Quantum mechanics can not be explained by local hidden variable model.
3. FWW protocol, implemented with n resources, can transform the correlation into itself to the power of n.
4. Nonlocality of a bipartite binary system can possibly be distilled up to 3 by FWW protocol.
5. Multipartite nonlocality distillation is possible.

每週二中午，系辦旁期刊室，書報討論

量子資訊科學，熱情招生中

徐立義（ L.Y. Hsu ）

吳耿碩 (K.S. Wu)

謝盛琦

Thank You !

$$
\begin{pmatrix} P'(00|xy) \\ P'(01|xy) \\ P'(10|xy) \\ P'(11|xy) \end{pmatrix} = \begin{pmatrix} P(00|xy) & P(01|xy) & P(10|xy) & P(11|xy) \\ P(01|xy) & P(00|xy) & P(11|xy) & P(10|xy) \\ P(10|xy) & P(11|xy) & P(00|xy) & P(01|xy) \\ P(11|xy) & P(10|xy) & P(01|xy) & P(00|xy) \end{pmatrix}^{n-1} \begin{pmatrix} P(00|xy) \\ P(01|xy) \\ P(10|xy) \\ P(11|xy) \end{pmatrix}
$$

$$T = \begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{pmatrix}$$

$$T = UVU^+,$$

$$U = U^+ = U^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix},$$

$$V = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A-B+C-D & 0 & 0 \\ 0 & 0 & A+B-C-D & 0 \\ 0 & 0 & 0 & A-B-C+D \end{pmatrix}$$

$$\begin{pmatrix} A' \\ B' \\ C' \\ D' \end{pmatrix} = (T)^{n-1} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}$$

$$\langle xy \rangle' = (\langle xy \rangle)^n$$

$$NLP' = max_{xy}\left|\langle xy\rangle' + \langle \bar{x}y\rangle' + \langle x\bar{y}\rangle' - \langle \bar{x}\bar{y}\rangle'\right|$$

$$= max_{xy}\left|\langle xy\rangle^n + \langle \bar{x}y\rangle^n + \langle x\bar{y}\rangle^n - \langle \bar{x}\bar{y}\rangle^n\right|$$

Since the value of correlation functions $\langle xy\rangle$ is between 1 and -1, the optimized situation for the distillable NLP' is $\langle xy\rangle = \langle \bar{x}y\rangle = \langle x\bar{y}\rangle = 1$ and $0 < \langle \bar{x}\bar{y}\rangle < 1$ for some specific xy, leading to $\langle xy\rangle^n = \langle \bar{x}y\rangle^n = \langle x\bar{y}\rangle^n = 1$ and $\langle \bar{x}\bar{y}\rangle^n < \langle \bar{x}\bar{y}\rangle$. Considering the optimized situation mentioned above, it is obvious NLP' is increasing for larger n and tends to 3.