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Dealing with the large scale structure inhomogeneity

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The large scale structure

(Very) tentative definition :
"*Everything above galaxy scale that is sensitive to gravitational instability.*"

2dFGRS (2002) :

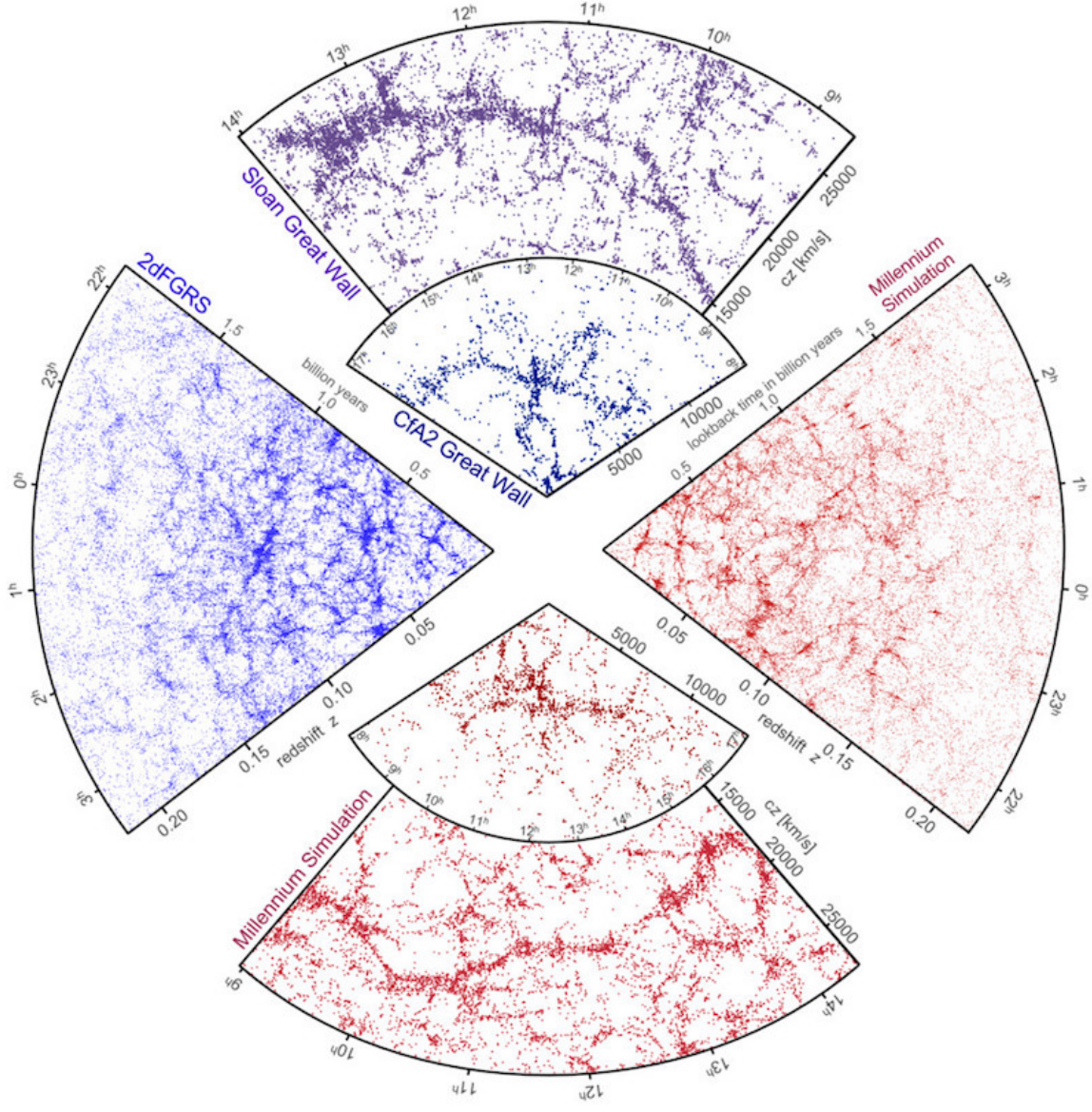
- 2.5 Gly depth on 2 slices
- $\sim$ 1500 sqdeg area
- spectra for $\sim 250k$ objects
- http ://www.2dfgrs.net/

Millennium Run (2005) :

- 10 G particles
- 2 Mly box
- $\sim$ 20 M galaxies

Sloan Digital Sky Survey :

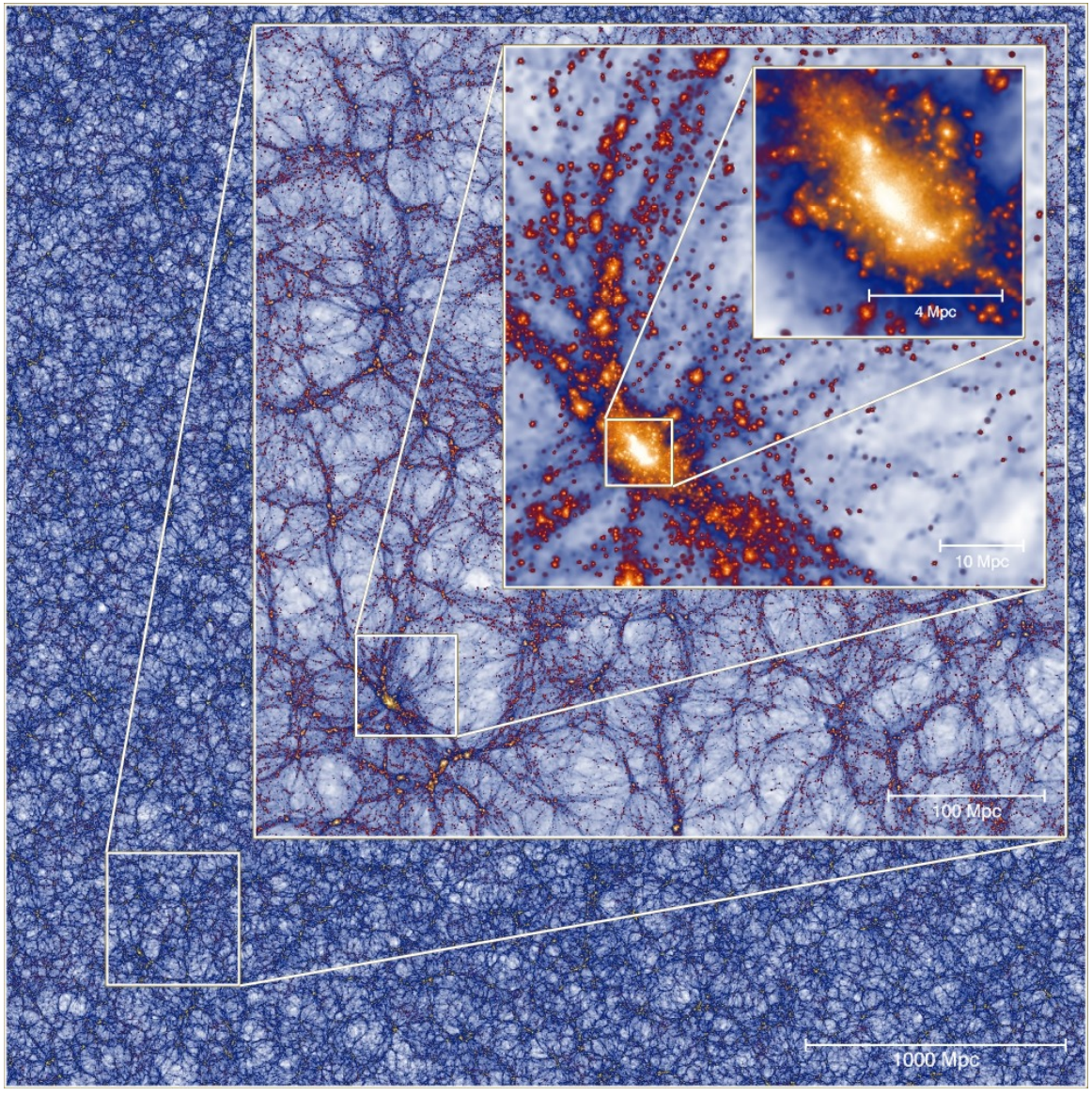
- 3 M spectra
- $\sim$ 35% of the sky



Millennium-XXL (2010)

- 300 billion particles whole Univ. to $z \sim 0.72$
- goal 1 : relation between optical richness, lensing mass, X-ray luminosity and thermal Sunyaev-Zeldovich (tSZ) signal from CMB
- goal 2 : mass of extreme galaxy **clusters**
- **useful of other probes :** BAOs, redshift space distortions (RSD), cluster number counts, weak gravitational lensing (WL), integrated Sachs-Wolfe (ISW) effect.
- halo mass function, power spectrum
- gives optical, lensing, X-ray, tSZ maps, galaxy clusters catalogues

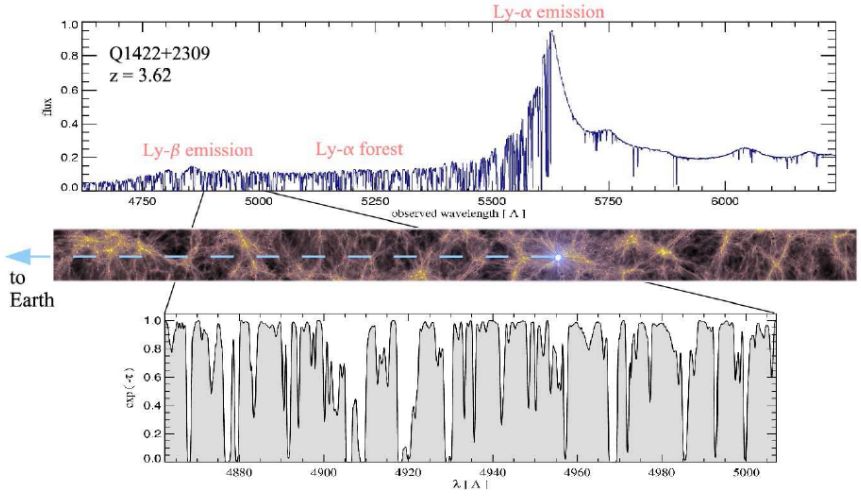
See Angulo *et al.* 2013.



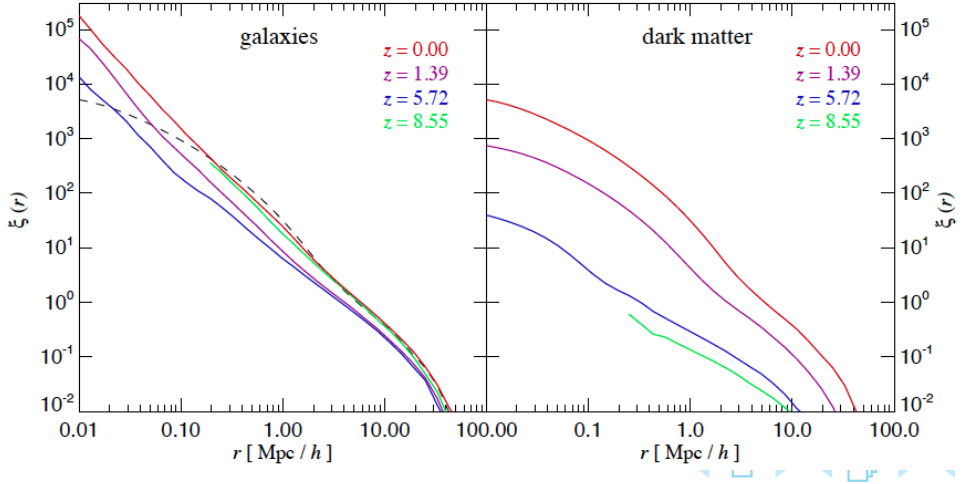
Examples of other probes

Source of images : Springel, Frenk, White 2006

- Intergalactic H absorption lines in quasar spectra ⇒ Lyman-α forest.



- 2pt-correlation (of galaxies or dark matter) ⇔ Power spectrum / BAO.



Relativistic corrections

Simple models of inflation ⇒ random independent phases and near-scale invariant **power spectrum**.

Context : Perturbations around a FLRW background using scalar perturbations (+ vector, tensor).

Impact on : luminosity distance (Hubble diagram), redshift, angles (gravitation lensing), volumes (number counts of galaxies). See recent papers by Yoo & Scaccabarozzi).
⇒ Very important for precision cosmology !

Gauge-invariant quantities are very important (see e.g. Yoo, Durrer, 2017).

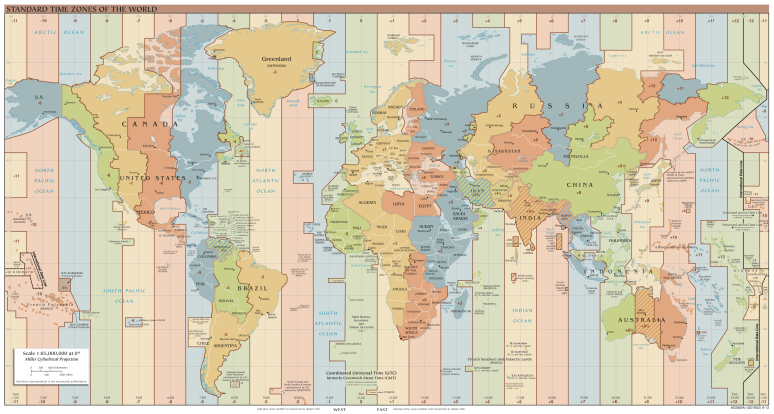
Two aspects of this talk

- **RELATIVISTIC CORRECTIONS DUE TO THE LSS :**
  how does the LSS affects cosmological observables, how we can use adapted coordinates which actually simplify calculations.

- **COMPARING SUPERNOVAE DATA AND LSS IN OUR LOCAL UNIVERSE :**
  what can we learn by comparing these probes, what it can say about the H_0 tension, supernovae or galaxy catalogues.

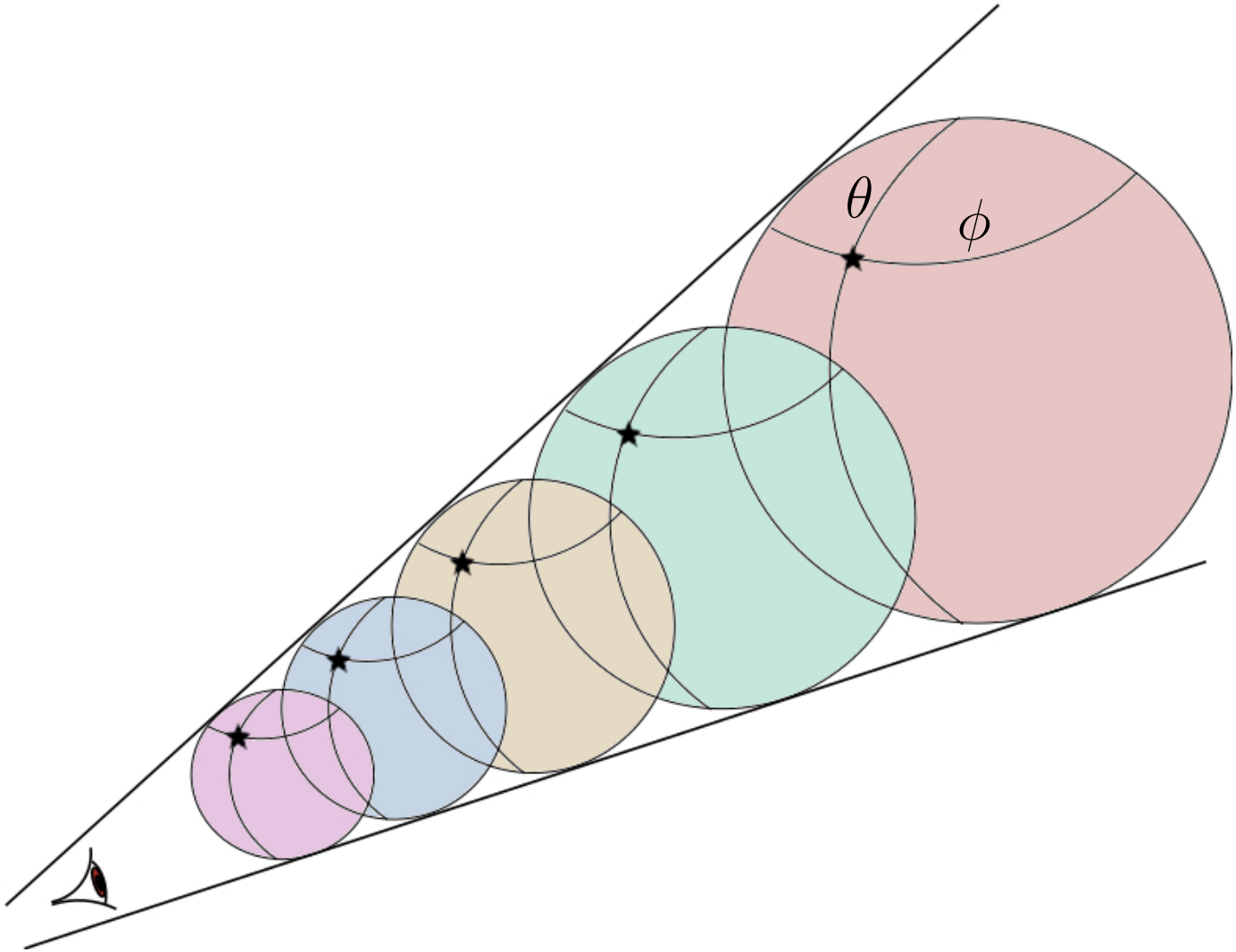
Adapted coordinates as a tool

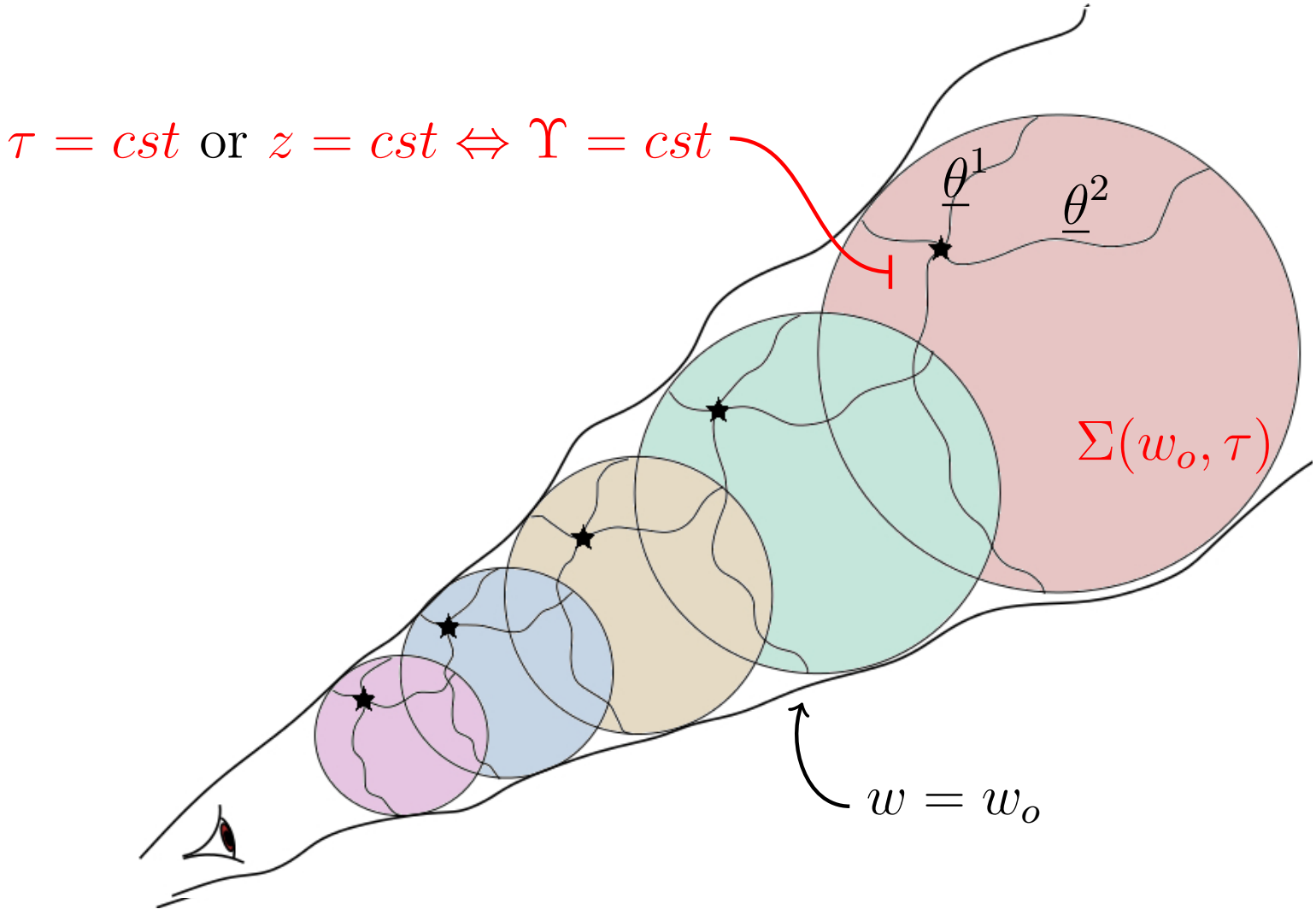
IDEA : Simplify relativistic calculations by working in coordinates defined from observable (and thus gauge-invariant) quantities.



- **1938** : Temple's **'optical co-ordinates"**
- **1958** : Joseph's **"optical co-ordinates"**
- **1968** : Saunders **"observer's polar coordinates"**
- **1984** : Maartens detailed **"observational coordinates"**
- **2011** : **"Geodesic light-cone coordinates"**
- Numerical implementations of Bester, Larena, Bishop '13 & '15

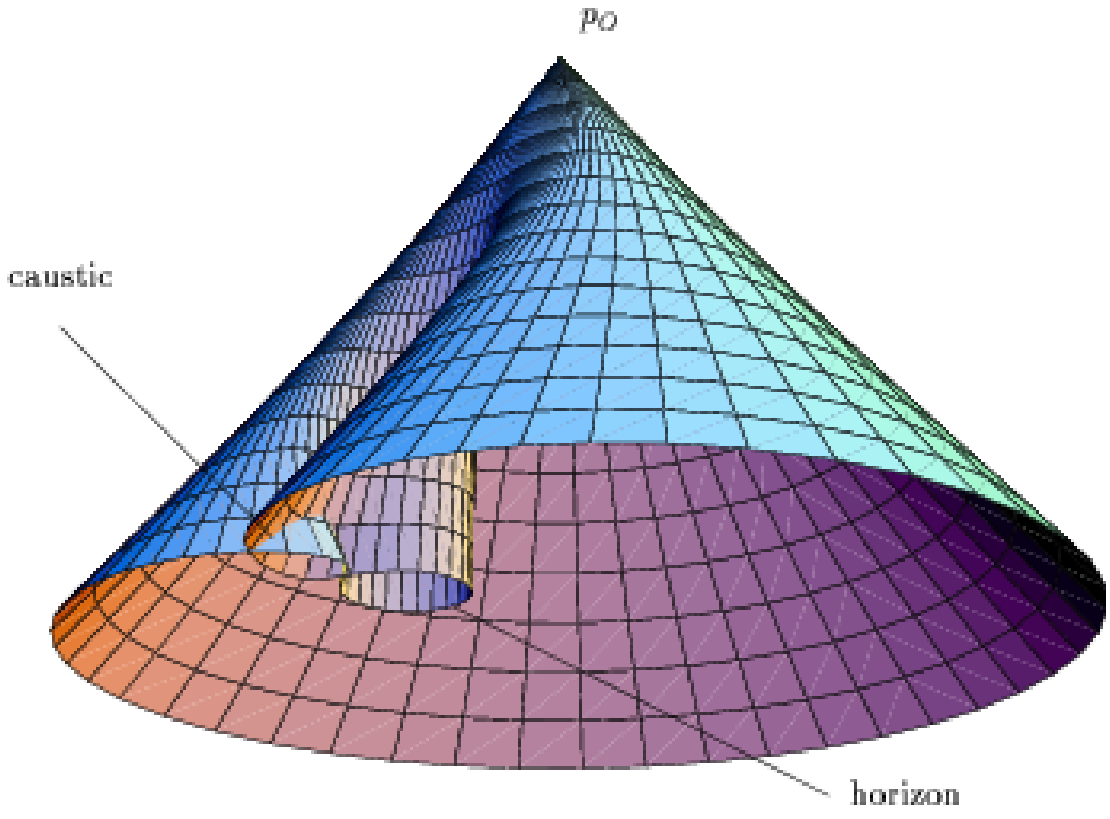
Geodesic Light Cone coordinates


θ
ϕ


$\tau = cst$ or $z = cst \Leftrightarrow \Upsilon = cst$
$\underline{\theta}^1$
$\underline{\theta}^2$
$\Sigma(w_o, \tau)$
$w = w_o$

NO CAUSTICS !

Way to generalization ?



Geodesic light-cone coordinates

$$\mathrm{d}s^2_{\mathrm{GLC}} = \Upsilon^2 \mathrm{d}w^2 - 2\Upsilon \mathrm{d}w\mathrm{d}\tau + \gamma_{ab}(\mathrm{d}\underline{\theta}^a - U^a \mathrm{d}w)(\mathrm{d}\underline{\theta}^b - U^b \mathrm{d}w)$$

(6 arbitrary functions : Υ, U^a, γ_{ab})

Properties :

w is a null coordinate, $\partial_\mu \tau$ defines a geodesic flow (from $g^{\tau\tau}_{GLC} = -1$), photons travel at $(w, \underline{\theta}^a) = \overrightarrow{cst}$ $\perp$ to $\Sigma(w, z)$.

Interpretation :

Υ is like an **inhomogeneous scale factor** (lapse function), U^a is a **shift-vector** and γ_{ab} the **metric inside the 2-sphere** $\Sigma(\tau, w)$.

FLRW :

$$w = \eta + r \ , \ \tau = t \ , \ (\underline{\theta}^1, \underline{\theta}^2) = (\theta, \phi) \ ,$$

$$\Upsilon = a(t) \quad , \quad U^a = 0 \quad , \quad \gamma_{ab} = a^2 r^2 \mathrm{diag}(1, \sin^2\theta) \quad .$$

Residual gauge freedoms

Relabelling lightcones : $\Upsilon' = \Upsilon \frac{\mathrm{d}w}{\mathrm{d}w'}$, $U_a \to U'_a = U_a \frac{\mathrm{d}w}{\mathrm{d}w'}$
($\Rightarrow$ *temporal gauge* : $w = \tau_o$).

Relabelling light rays :

$$\gamma^{ab} \to \gamma^{\alpha\beta} = \gamma^{ab}\partial_a\varphi^\alpha\partial_b\varphi^\beta \quad , \quad U^a \to U^\alpha = U^a\partial_a\varphi^\alpha - \partial_w\varphi^\alpha$$

Conditions : $\underline{\theta}^a$ are regular spherical angles on the observer's sky, observer is not rotating (*non-rotating observational gauge*).

Reparametrising light rays : $\partial_\mu w \to f(\tau, w, \underline{\theta}^a)\partial_\mu w$

Conformal transformations : $\mathrm{d}\tau = \Omega\mathrm{d}\tilde{\tau}$, $\Upsilon = \Omega\tilde{\Upsilon}$, $\gamma_{ab} = \Omega^2\tilde{\gamma}_{ab}$

See Fulvio Scaccabarozzi, Jaiyul Yoo, 1703.08553 giving a very rigorous treatment of observer angles reparametrization.

Direct simplifications

$$\mathrm{d}s^2_{GLC} = \Upsilon^2 \mathrm{d}w^2 - 2\Upsilon \mathrm{d}w\mathrm{d}\tau + \gamma_{ab}(\mathrm{d}\underline{\theta}^a - U^a \mathrm{d}w)(\mathrm{d}\underline{\theta}^b - U^b \mathrm{d}w)$$

$\Rightarrow$ **Redshift perturbation** (cf. Giuseppe's talk) :

$$\boxed{(1+z_s) = \frac{(k^\mu u_\mu)_s}{(k^\mu u_\mu)_o} = \frac{(\partial^\mu w \partial_\mu \tau)_s}{(\partial^\mu w \partial_\mu \tau)_o} = \frac{\Upsilon(w_o, \tau_o, \underline{\theta}^a)}{\Upsilon(w_o, \tau_s, \underline{\theta}^a)} \equiv \frac{\Upsilon_o}{\Upsilon_s}}$$

where $u_\mu = -\partial_\mu \tau$ is the peculiar velocity of the **comoving** observer/source and $k_\mu = \partial_\mu w$ is the photon momentum.

$\Rightarrow$ **(exact) Angular distance** (with homogeneous observer neighborhood) :

$$\boxed{d_A = \gamma^{1/4} \left(\sin \underline{\theta}^1\right)^{-1/2} \quad \text{with} \quad \gamma \equiv \det(\gamma_{ab}) = |\det(g_{\mathrm{GLC}})|/\Upsilon^2}$$

which, combined with redshift, gives the **distance-redshift relation**.

Hubble diagram

Magnitude :

$$m = -2.5 \log_{10}\left(\frac{\Phi}{\Phi_{\rm ref}}\right)$$

Flux :

$$\boxed{\Phi = \frac{L}{4\pi d_L^2}}$$

Absolute Mag. :

$$M = -2.5 \log_{10}\left(\frac{\Phi(10\text{pc})}{\Phi_{\rm ref(pc)}}\right)$$

Distance Modulus :

$$\boxed{\mu = m - M = 5\log_{10}(d_L) + cst}$$

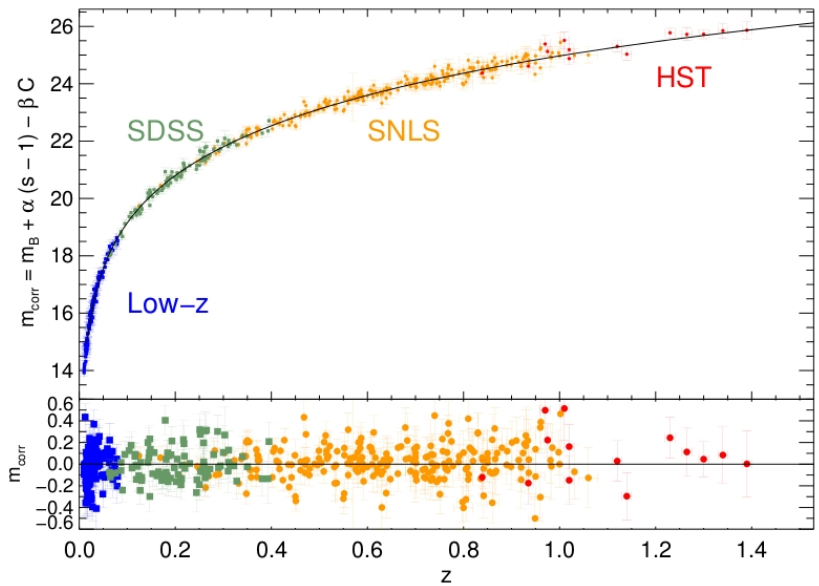
Two assumptions in SMC :

- GR valid on all scales,
- Isotropy + Homogeneity,

$\Rightarrow$ FLRW model.

Luminosity Distance (for $K = 0$) :

$$d_L^{FLRW}(z) = \frac{1+z}{H_0}\int_0^z \frac{dz'}{\left[\Omega_{\Lambda 0} + \Omega_{m0}(1+z')^3\right]^{1/2}}$$



Distance-redshift relation at $\mathcal{O}(2)$

The GLC metric allows to compute the $d_L(z)$ relation to $\mathcal{O}(2)$ in NG :

$$ds^2_{NG} = a^2(\eta)\left(-(1+2\Phi)\mathrm{d}\eta^2 + (1-2\Psi)(\mathrm{d}r^2 + \gamma^{(0)}_{ab}\mathrm{d}\theta^a\mathrm{d}\theta^b)\right)$$

with $\gamma^{(0)}_{ab} = r^2\mathrm{diag}\left(1, \sin^2\theta\right)$, and $\Phi = \psi + \frac{1}{2}\phi^{(2)}$, $\Psi = \psi + \frac{1}{2}\psi^{(2)}$ (Bardeen).

$$\psi^{(2)}, \phi^{(2)} \propto \nabla^{-2}(\partial_i\psi\partial^i\psi) \ , \ \partial_i\psi\partial^i\psi \quad \text{(cf. Bartolo, Matarrese, Riotto, 2005)}$$

FULL transformation GLC $\leftrightarrow$ NG at second order in PT :

$$(\tau, w, \widetilde{\theta}^1, \widetilde{\theta}^2) = f(\eta, r, \theta, \phi)$$
$$\Downarrow$$
$$(\Upsilon, U^a, \gamma^{ab}) = f(\psi, \psi^{(2)}, \phi^{(2)})$$
$$\Downarrow$$
$$d_L = (1+z)^2\,\gamma^{1/4}\left(\sin\tilde{\theta}^1\right)^{-1/2} \text{ up to } \mathcal{O}(2) \ :$$

$$\boxed{d_L(z_s, \theta^a) = d_L^{FLRW}(z_s)\left(1 + \delta_S^{(1)}(z_s, \theta^a) + \delta_S^{(2)}(z_s, \theta^a)\right)}$$

Compute the **distance-redshift relation** at $\mathcal{O}(2)$ in perturbations (from the Newtonian gauge, 1104.1167, 1209.4326, 1506.02003) :

$$d_L(z_s, \underline{\theta}^a) = d_L^{FLRW}(z_s)\left(1 + \delta_S^{(1)}(z_s, \underline{\theta}^a) + \delta_S^{(2)}(z_s, \underline{\theta}^a)\right)$$

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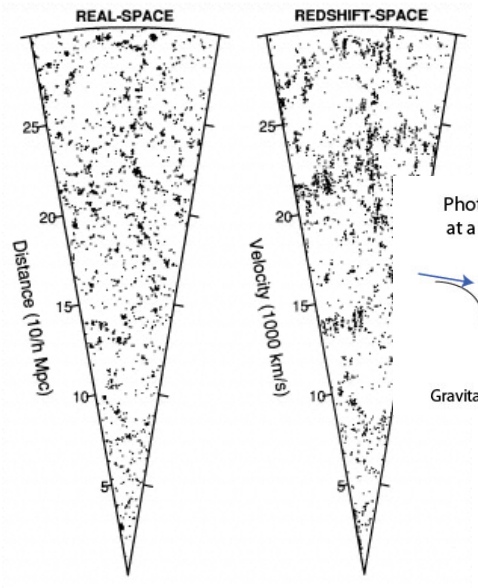
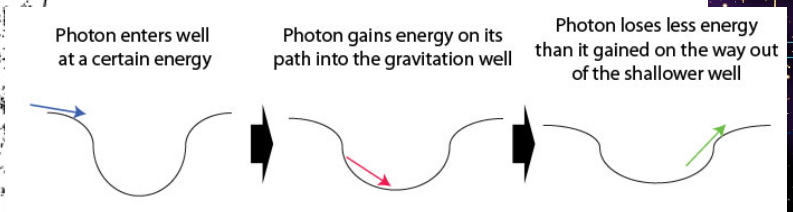




Qualitative agreement with O. Umeh, C. Clarkson and R. Maarten '14, Bonvin, Clarkson, Durrer, Maartens, Umeh '15, Kaiser, Peacock '15.

`Detailed comparison? Difficult task.` (see Yoo, Scaccabarozzi '16)

At $\mathcal{O}(1)$:

$$\delta_S^{(1)}(z_s,\theta^a) \sim \text{SW} + \text{ISW} + \text{Doppler} - \left(\psi_s^{(1)} + \int_{\eta_+}^{\eta_-} \mathrm{d}x\ \psi\right) - \text{Lensing}^{(1)}$$

$$\text{Lensing}^{(1)} = \frac{1}{2}\nabla_a\theta^{a(1)} = \int_{\eta_s^{(0)}}^{\eta_o} \frac{d\eta}{\Delta\eta}\,\frac{\eta-\eta_s^{(0)}}{\eta_o-\eta}\,\Delta_2\psi(\eta,\eta_o-\eta,\bar{\theta}^a)$$

$$\text{Doppler} = \left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)(\mathbf{v}_o-\mathbf{v}_s)\cdot\hat{n} \quad , \quad \mathbf{v}\equiv\int_{\eta_{\rm in}}^{\eta}\mathrm{d}\eta'\frac{a(\eta')}{a(\eta)}\nabla\psi(\eta',r,\theta^a)$$

At $\mathcal{O}(2)$, full calculation :

- **Dominant terms** : $(\text{Doppler})^2$, $(\text{Lensing})^2$!!!
- Combinations of $\mathcal{O}(1)$-terms : ψ_s^2, $(\text{[I]SW})^2$, [I]SW $\times$ Doppler, $(\psi_s, \int_{\eta_+}^{\eta_-} dx\ \psi)\times$ (Lensing, [I]SW, Doppler) ...
- Genuine $\mathcal{O}(2)$-terms : $\psi_s^{(2)}$, $\text{Lensing}^{(2)} = \frac{1}{2}\nabla_a\theta^{a(2)}$, $Q_s^{(2)}$...
- A LOT of other contributions : New integrated effects, Angle deformations, Redshift perturbations($\subset$ transverse peculiar velocity), Lens-Lens coupling, corrections to Born approximation, ... *See 1209.4326, also Umeh 1402.1933.*

The whole second order... (up to some observer terms)

4.2. DETAILED EXPRESSION FOR $D_L(Z,\theta^A)$

$$
\begin{aligned}
\bar{\delta}^{(2)}_{path} &= \Xi_s\Big\{-\frac{1}{4}\left(\phi_s^{(2)}-\phi_o^{(2)}\right)+\frac{1}{4}\left(\psi_s^{(2)}-\psi_o^{(2)}\right)+\frac{1}{2}(\psi_s-\psi_o)^2-\psi_o J_2^{(1)}\\
&+(\psi_o-\psi_s-J_2^{(1)})[\partial_+Q]_s+\frac{1}{4}(\gamma_0^{ab})_s\partial_aQ_s\partial_bQ_s+Q_s\left(-[\partial_+^2Q]_s+[\partial_+\psi]_s\right)+\frac{1}{\mathcal{H}_s}(\psi_o+[\partial_+Q]_s)[\partial_\eta\psi]_s\\
&-\frac{1}{2}\int_{\eta_{in}}^{\eta_s^{(0)}}d\eta'\frac{a(\eta')}{a(\eta_s^{(0)})}\partial_r\left[\phi^{(2)}-\psi^2\right](\eta',\Delta\eta,\tilde{\theta}^a)+\frac{1}{2}\int_{\eta_{in}}^{\eta_o}d\eta'\frac{a(\eta')}{a(\eta_o)}\partial_r\left[\phi^{(2)}-\psi^2\right](\eta',0,\tilde{\theta}^a)\\
&+\frac{1}{4}\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\;\partial_+\left[\hat{\phi}^{(2)}+\hat{\psi}^{(2)}+4\hat{\psi}\;\partial_+Q+\hat{\gamma}_0^{ab}\;\partial_aQ\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\\
&-\frac{1}{2}\partial_a(\psi_o+\partial_+Q_s)\left(\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\;\left[\hat{\gamma}_0^{ab}\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\Big\}\\
&-\frac{1}{2}\psi_s^{(2)}-\frac{1}{2}\psi_s^2-K_2+\psi_sJ_2^{(1)}+\frac{1}{2}(J_2^{(1)})^2+(J_2^{(1)}-\psi_o)\frac{Q_s}{\Delta\eta}-\frac{1}{\mathcal{H}_s\Delta\eta}\left(1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)\frac{1}{2}(\psi_o+[\partial_+Q]_s)^2\\
&-\frac{2}{\mathcal{H}_s\Delta\eta}\psi_s(\psi_o+[\partial_+Q]_s)+\frac{1}{2}\partial_a\left(\psi_s+J_2^{(1)}+\frac{Q_s}{\Delta\eta}\right)\left(\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\;\left[\hat{\gamma}_0^{ab}\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\\
&+\frac{1}{4}\partial_aQ_s\;\partial_+\left(\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\;\left[\hat{\gamma}_0^{ab}\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\\
&+\frac{1}{16}\partial_a\left(\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\;\left[\hat{\gamma}_0^{bc}\;\partial_cQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\partial_b\left(\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\;\left[\hat{\gamma}_0^{ad}\;\partial_dQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\\
&-\frac{1}{4\Delta\eta}\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\;\left[\hat{\phi}^{(2)}+\hat{\psi}^{(2)}+4\hat{\psi}\;\partial_+Q+\hat{\gamma}_0^{ab}\;\partial_aQ\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\\
&+\frac{1}{\mathcal{H}_s}(\psi_o+[\partial_+Q]_s)\left\{-[\partial_\eta\psi]_s+[\partial_r\psi]_s+\frac{1}{\Delta\eta^2}\int_{\eta_s^{(0)}}^{\eta_o}d\eta'\Delta_2\psi(\eta',\eta_o-\eta',\tilde{\theta}^a)\right\}\\
&+Q_s\Big\{[\partial_r\psi]_s+\partial_+\left(\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\frac{1}{\left(\eta_s^{(0)+}-x\right)^2}\int_{\eta_s^{(0)+}}^{x}dy\Delta_2\hat{\psi}(\eta_s^{(0)+},y,\tilde{\theta}^a)\right)\\
&+\frac{1}{2\Delta\eta^2}\int_{\eta_s^{(0)}}^{\eta_o}d\eta'\Delta_2\psi(\eta',\eta_o-\eta',\tilde{\theta}^a)\Big\}+\frac{1}{16\sin^2\tilde{\theta}}\left(\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\;\left[\hat{\gamma}_0^{1b}\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)^2\;, \qquad (4.67)
\end{aligned}
$$

$$
\begin{aligned}
\bar{\delta}^{(2)}_{pos} &= \frac{\Xi_s}{2}\Big\{([\partial_rP]_s-[\partial_rP]_o)^2+(\gamma_0^{ab})_s\partial_aP_s\partial_bP_s-\lim_{r\to0}\left[\gamma_0^{ab}\partial_aP\partial_bP\right]\\
&-\frac{2}{\mathcal{H}_s}([\partial_rP]_s-[\partial_rP]_o)\left(\mathcal{H}_s[\partial_rP]_s+[\partial_r^2P]_s\right)\\
&-\int_{\eta_{in}}^{\eta_s^{(0)}}d\eta'\frac{a(\eta')}{a(\eta_s^{(0)})}\partial_r\left[(\partial_rP)^2+\gamma_0^{ab}\partial_aP\partial_bP\right](\eta',\Delta\eta,\tilde{\theta}^a) \qquad (4.68)\\
&+\int_{\eta_{in}}^{\eta_o}d\eta'\frac{a(\eta')}{a(\eta_o)}\partial_r\left[(\partial_rP)^2+\gamma_0^{ab}\partial_aP\partial_bP\right](\eta',0,\tilde{\theta}^a)\Big\}-\frac{1}{2\mathcal{H}_s\Delta\eta}\left(1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)([\partial_rP]_s-[\partial_rP]_o)^2\;,
\end{aligned}
$$

$$
\begin{aligned}
\bar{\delta}^{(2)}_{mixed} &= \Xi_s\Big\{\left(2\psi_o-\psi_s+\partial_+Q_s-\frac{Q_s}{\Delta\eta}\right)[\partial_rP]_o-([\partial_rP]_s-[\partial_rP]_o)\left(\frac{1}{\mathcal{H}_s}[\partial_\eta\psi]_s-J_2^{(1)}\right)\\
&-(\gamma_0^{ab})_s\partial_aQ_s\partial_bP_s+\frac{1}{\mathcal{H}_s}(\psi_o+[\partial_+Q]_s)[\partial_r^2P]_s+Q_s[\partial_r^2P]_s\\
&+\frac{1}{2}\partial_a([\partial_rP]_s-[\partial_rP]_o)\left(\int_{\eta_s^{(0)+}}^{\eta_s^{(0)-}}dx\;\left[\hat{\gamma}_0^{ab}\;\partial_bQ\right](\eta_s^{(0)+},x,\tilde{\theta}^a)\right)\Big\}\\
&+\frac{1}{\Delta\eta}([\partial_rP]_s-[\partial_rP]_o)\left\{\frac{1}{\mathcal{H}_s}\left(1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\right)(\psi_o+[\partial_+Q]_s)+\frac{2}{\mathcal{H}_s}\psi_s+Q_s\right\}\\
&+\frac{1}{\mathcal{H}_s}([\partial_rP]_s-[\partial_rP]_o)\left\{[\partial_\eta\psi]_s-[\partial_r\psi]_s-\frac{1}{\Delta\eta^2}\int_{\eta_s^{(0)}}^{\eta_o}d\eta'\Delta_2\psi(\eta',\eta_o-\eta',\tilde{\theta}^a)\right\}\;. \qquad (4.69)
\end{aligned}
$$

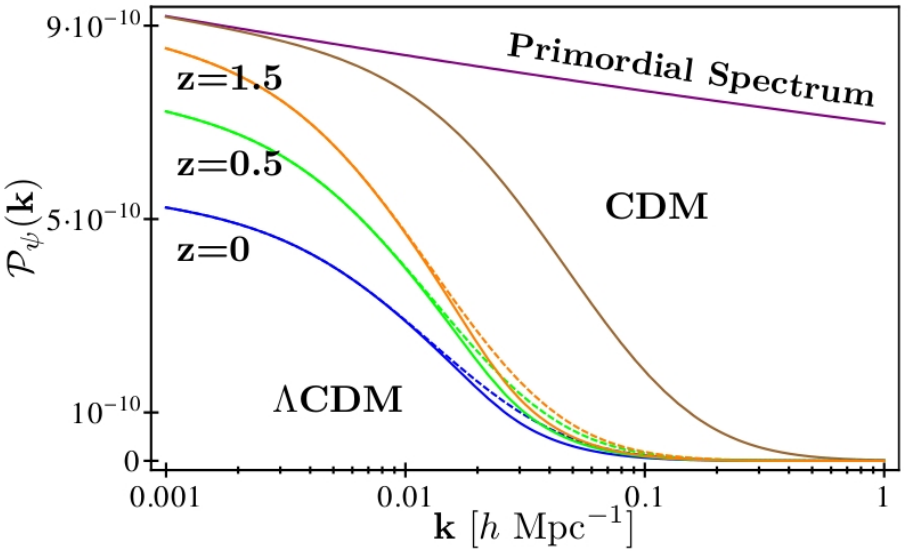
Stochastic average of inhomogeneous realizations

Inhomogeneities :

$$\psi(\eta,\vec{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\, e^{i\vec{k}\cdot\vec{x}}\psi_k(\eta)E(\vec{k})$$

with E a unit R.V. which is **homogeneous**$\left(E^*(\vec{k}) = E(-\vec{k})\right)$ and **gaussian**$\left(\overline{E(\vec{k})} = 0,\ \overline{E(\vec{k}_1)E(\vec{k}_2)} = \delta(\vec{k}_1+\vec{k}_2)\right)$.

Spectrum : $|\psi_k(\eta)|^2 = 2\pi^2\mathcal{P}_\psi(k)/k^3$



Light-cone average is combined with a stochastic average. In CDM :

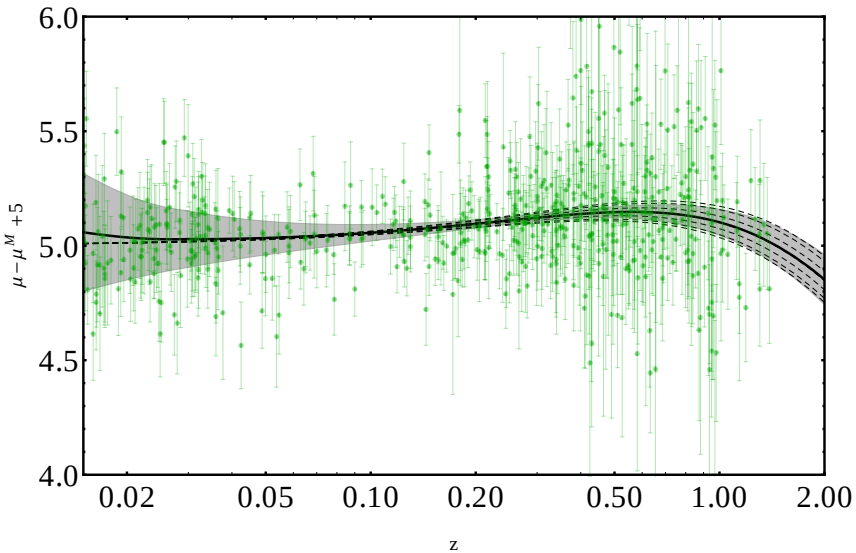
$$\boxed{\overline{\langle d_L\rangle} = \int_0^\infty \frac{\mathrm{d}k}{k}\mathcal{P}_\psi(k)C(k\Delta\eta)}$$

We do the same $\forall$ terms in $\overline{\left\langle \delta_S^{(1)}\right\rangle}$ and $\overline{\left\langle \delta_S^{(2)}\right\rangle}$ in ΛCDM... with approximations.

Kaiser & Peacock 2015 for precise discussion on 'directional' / 'source' averaging.

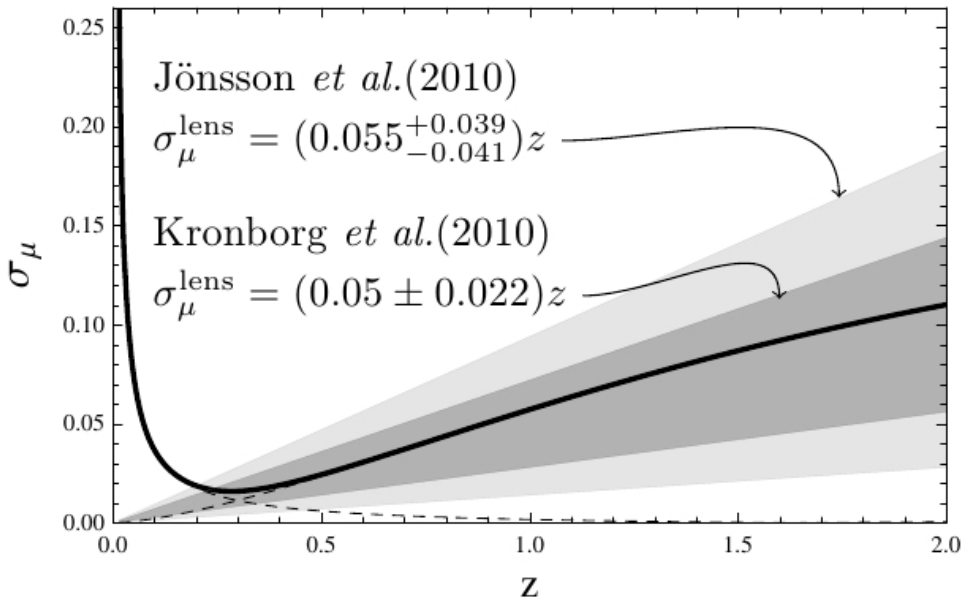
The averaged modulus $\overline{\langle\mu\rangle}$ depends on $\overline{\langle(\Phi_1/\Phi_0)^2\rangle}$ while the standard deviation $\sigma_\mu = \sqrt{\overline{\langle\mu^2\rangle} - \overline{\langle\mu\rangle}^2} = 2.5(\log_{10} e)\sqrt{\overline{\langle(\Phi_1/\Phi_0)^2\rangle}}$ with

$$\overline{\langle(\Phi_1/\Phi_0)^2\rangle} \sim \overline{\langle(\text{Doppler})^2\rangle} + \overline{\left\langle\left(\text{Lensing}^{(1)}\right)^2\right\rangle}$$



With the Union 2 dataset :

- small z : Velocities explain quite well the scatter.
- large z : Lensing is too weak to explain data's scatter ($\sim\%\,\Omega_{\Lambda 0}$).



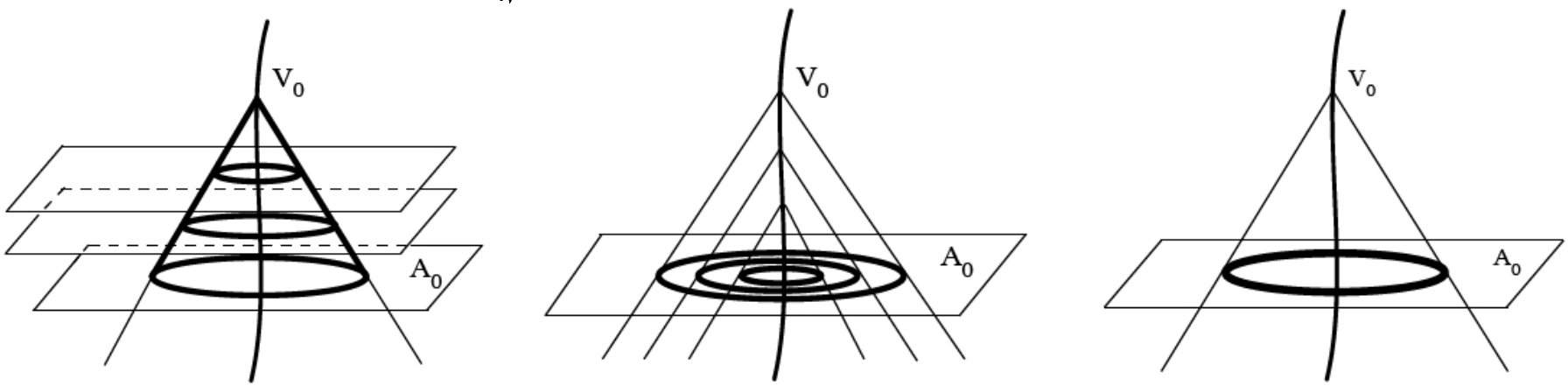
- The total effect is well approximated by Doppler ($z \leq 0.2$) + Lensing ($z > 0.3$),
- Lensing prediction is in great agreement with experiments so far !

Cf. Ben-Dayan 2014, 1401.7973 for effect on H_o.

Simplify **averages on the past lightcone** (1207.1286, 1302.0740) :

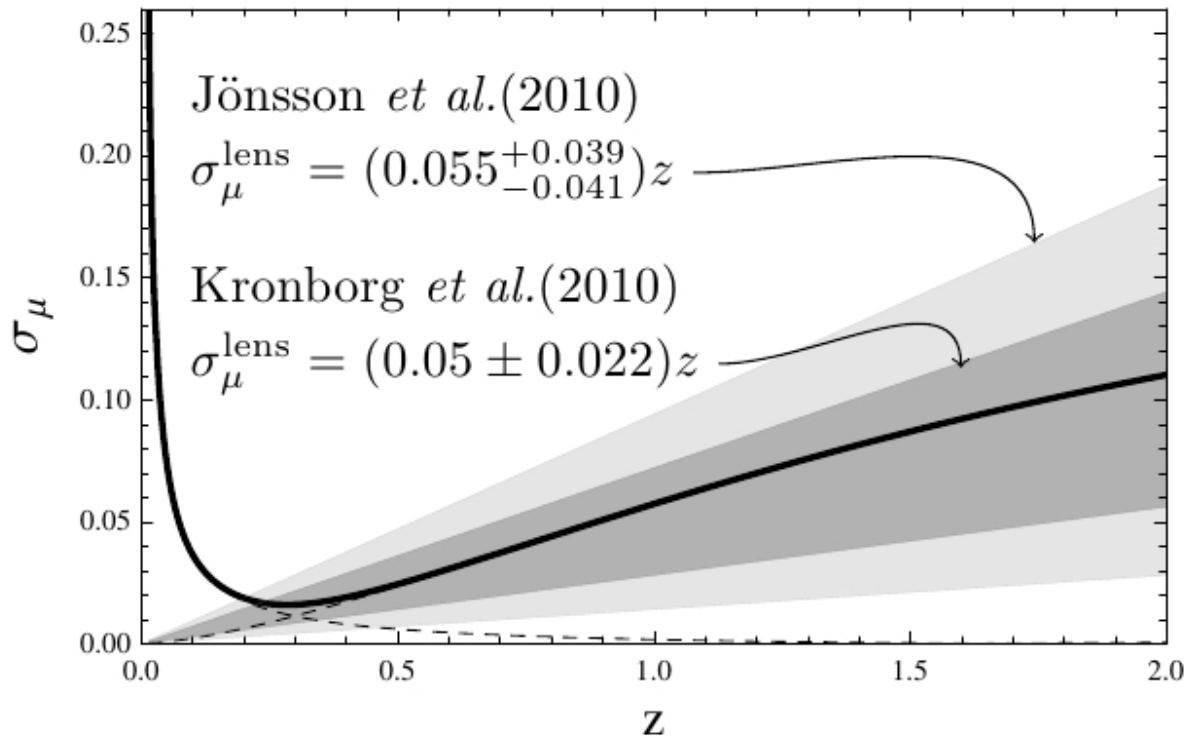
$$\langle S\rangle_{w_o,\tau_s} = \frac{\int_\Sigma d^4x\sqrt{-g}\,\delta_{\rm D}(w-w_o)\delta_{\rm D}(\tau-\tau_s)\,|\partial_\mu\tau\partial^\mu w|\,S(\tau,w,\underline{\theta}^a)}{\int_\Sigma d^4x\sqrt{-g}\,\delta_{\rm D}(w-w_o)\delta_{\rm D}(\tau-\tau_s)\,|\partial_\mu\tau\partial^\mu w|}$$

$$= \frac{\int d^2\underline{\theta}\,\sqrt{\gamma(w_o,\tau_s,\underline{\theta}^a)}\,S(w_o,\tau_s,\underline{\theta}^a)}{\int d^2\widetilde{\theta}\sqrt{\gamma(w_o,\tau_s,\theta^a)}}$$



$$V_0 \sim w_0 \quad , \quad A_0 \sim \tau_s$$

Estimate the effect of **large scale structure** on the **Hubble diagram** : average and dispersion of the distance modulus (my thesis, 1309.6542, and d_L references above.).



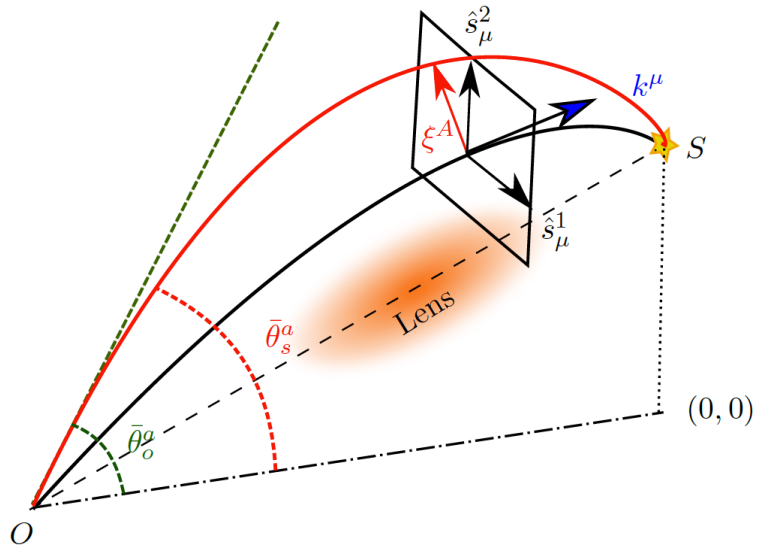
Measurements to confirm this prediction?

Other applications of GLC

Amplification matrix $((\ldots)^{\cdot} \equiv \partial_\tau(\ldots))$:

$$\mathcal{A}^A_B(\lambda_s,\lambda_o) = \frac{s^A_a(\lambda_s)\left[2u_\tau(\dot\gamma_{ab})^{-1}\right]_o s^B_b(\lambda_o)}{\bar d_A(\lambda_s)} = \begin{pmatrix} 1-\kappa-\hat\gamma_1 & -\hat\gamma_2+\hat\omega \\ -\hat\gamma_2-\hat\omega & 1-\kappa+\hat\gamma_1 \end{pmatrix}$$

Fanizza, Nugier 2014, 1408.1604



The **angular distance** and **lensing quantities** become :

$$d_A \propto (\gamma\gamma_o)^{1/4} \quad , \quad \hat\mu = (\det\mathcal{A})^{-1} = \left(\frac{\bar d_A}{d_A}\right)^2 \quad ,$$

involving $\bar d_A = a(\tau)r$ with $r = w - \int a^{-1}(\tau)\mathrm{d}\tau$ measured from the observer,

$$\left\{ \begin{matrix} (1-\kappa)^2+\hat\omega^2 \\ \hat\gamma_1^2+\hat\gamma_2^2 \end{matrix} \right\} = \left(\frac{u_{\tau_o}}{\bar d_A}\right)^2 \left\{ \left[\frac{\gamma\,\dot\gamma_{ab}\gamma^{bc}\dot\gamma_{cd}}{\left(\det{}^{ab}\,\dot\gamma_{ab}\right)^2}\right]_o \gamma\,\gamma^{ad} \pm 2\frac{\sqrt{\gamma\,\gamma_o}}{\left(\det{}^{ab}\,\dot\gamma_{ab}\right)_o} \right\}$$

- Evaluate the **galaxy number counts** at $\mathcal{O}(2)$ in perturbations (Di Dio, Durrer, Marozzi, Montanari 1407.0376, 1510.04202) $\Rightarrow$ Bispectrum !

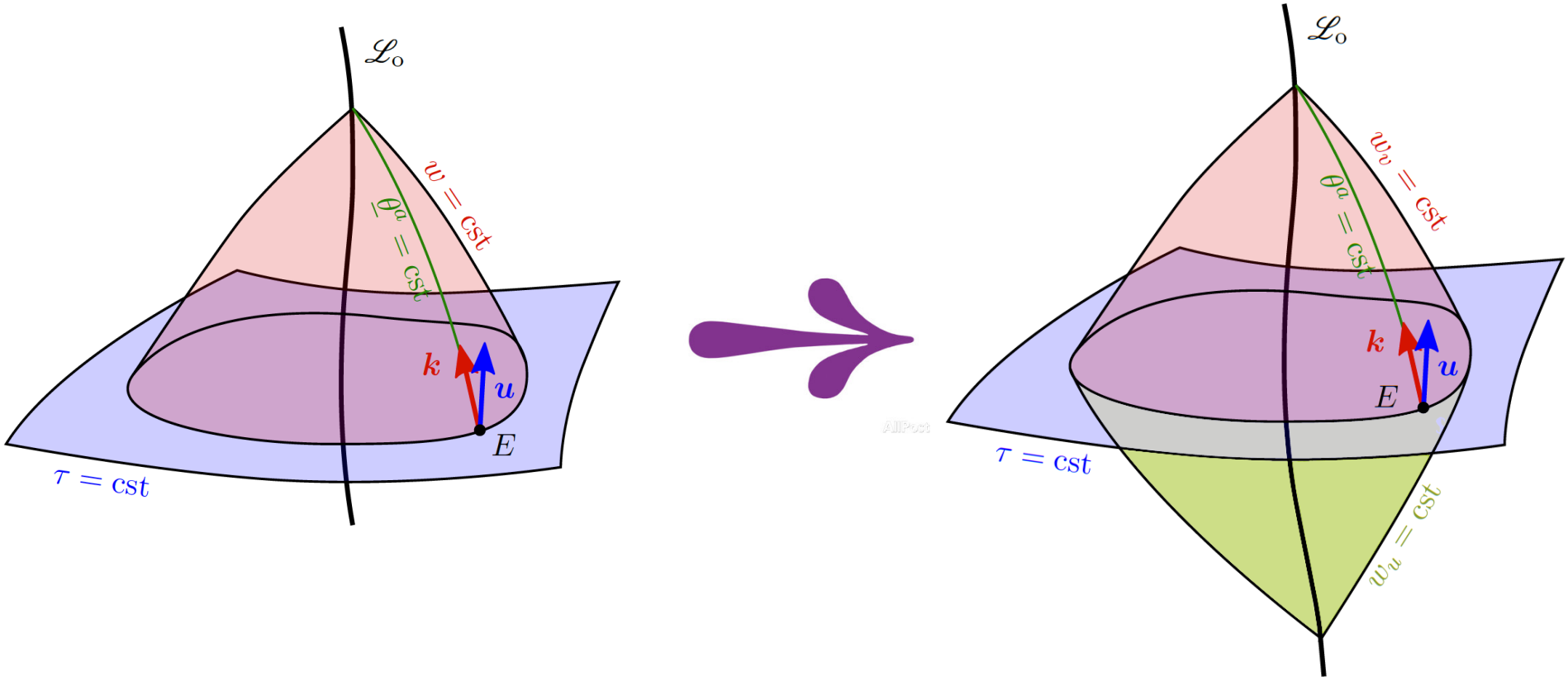
- **Inhomogeneous spacetime :** Lemaître-Tolman-Bondi with off-center observer and no curvature (*Fanizza, Nugier 2014*, 1408.1604), lensing quantities for over/under dense regions.

- Application to an **Anisotropic Bianchi I spacetime** *Fleury, Nugier, Fanizza 2016*, 1602.04461). $\Rightarrow$ we find that the **anisotropy of the Bianchi I spacetime violates** $\langle \mu^{-1} \rangle_\Omega = 1$ **!** (as for inhomogeneous Swiss-Cheese).

- Application to the **time-of-flight of UR particles** (*Fanizza, Gasperini, Marozzi, Veneziano*, 1512.08489).

Which other application can we find ?

Turning TIME into FUTURE NULL coordinate!



$x^\mu \equiv (\tau, w, \underline{\theta}^a)$ $\qquad$ $y^\mu \equiv (w_u, w_v, \theta^a)$

"(Geodesic → Double) Light-Cone coordinates"

Nugier 2016, 1606.08296

$$y^\mu \equiv (w_u, w_v, \theta^a) \quad \rightarrow \quad g^{\rm DLC}_{\mu\nu}(y) = \frac{\partial x^\alpha}{\partial y^\mu}\frac{\partial x^\beta}{\partial y^\nu} g^{\rm GLC}_{\alpha\beta}(x) \quad \leftarrow \quad x^\mu \equiv (\tau, w, \underline{\theta}^a)$$

- $w = w_v \quad \Rightarrow \quad \partial w/\partial w_v = 1$
- w_u independent from $w_v \quad \Rightarrow \quad \partial w/\partial w_u = 0$
- in GLC $\underline{\theta}^a$ is independent from $w \quad \Rightarrow \quad \partial\underline{\theta}^a/\partial w_v = 0$
- light rays independent from $w_u \quad \Rightarrow \quad \partial\underline{\theta}^a/\partial w_u = 0.$

$$g^{\rm DLC}_{\mu\nu} = \begin{pmatrix} 0 & g^{\rm DLC}_{w_u w_v} & \vec{0} \\ g^{\rm DLC}_{w_u w_v} & g^{\rm DLC}_{w_v w_v} & g^{\rm DLC}_{w_v a} \\ \vec{0}^T & (g^{\rm DLC}_{w_v a})^T & g^{\rm DLC}_{ab} \end{pmatrix} \quad \text{with} \quad \begin{cases} g^{\rm DLC}_{w_u w_v} = -\Upsilon \frac{\partial \tau}{\partial w_u} \\ g^{\rm DLC}_{w_v w_v} = (\Upsilon^2 + U^2) - 2\Upsilon \frac{\partial \tau}{\partial w_v} \end{cases},$$

- $\theta^a = \underline{\theta}^a$ (residual gauge freedom on $\Sigma(w_u, w_v)$) $\quad \Rightarrow \quad \partial\underline{\theta}^a/\partial\theta^b \equiv \delta^a_b$
- natural consequence $\quad \Rightarrow \quad \partial w/\partial\theta^a = 0$

$$g^{\mu\nu}_{\rm DLC} = \begin{pmatrix} g^{w_u w_u}_{\rm DLC} & -2/\widetilde{\Upsilon}^2 & -2\widetilde{U}^b/\widetilde{\Upsilon}^2 \\ -2/\widetilde{\Upsilon}^2 & 0 & \vec{0} \\ -2(\widetilde{U}^a)^T/\widetilde{\Upsilon}^2 & \vec{0}^T & \gamma^{ab} \end{pmatrix} \quad \text{from} \quad \begin{cases} g^{\rm DLC}_{ab} = \gamma_{ab} \\ g^{\rm DLC}_{w_v a} = -U_a - \Upsilon \frac{\partial \tau}{\partial \theta^a} \equiv -\widetilde{U}_a \end{cases},$$

We introduced new metric quantities :

$$\widetilde{U}^a \equiv \gamma^{ab}\widetilde{U}_b = U^a + \Upsilon\gamma^{ab}\frac{\partial\tau}{\partial\theta^b} \quad , \quad \widetilde{\Upsilon} = \sqrt{2\Upsilon\frac{\partial\tau}{\partial w_u}}$$

We make w_u **null by imposing** $g^{w_u w_u}_{\rm DLC} = 0$. So we need the conditions on τ :

$$\frac{\partial\tau}{\partial w_u} = \frac{\widetilde{\Upsilon}^2}{2\Upsilon} \quad , \quad \frac{\partial\tau}{\partial w_v} = \frac{\Upsilon}{2} - \widetilde{U}^a\frac{\partial\tau}{\partial\theta^a} + \frac{\Upsilon}{2}\gamma^{ab}\frac{\partial\tau}{\partial\theta^a}\frac{\partial\tau}{\partial\theta^b}$$

Once satisfied, we get the result :

$$g^{\rm DLC}_{\mu\nu} = \begin{pmatrix} 0 & -\widetilde{\Upsilon}^2/2 & \vec{0} \\ -\widetilde{\Upsilon}^2/2 & \widetilde{U}^2 & -\widetilde{U}_b \\ \vec{0}^T & -\widetilde{U}^T_a & \gamma_{ab} \end{pmatrix}$$

The line element in DLC coordinates is :

$$\boxed{{\rm d}s^2_{\rm DLC} = -\widetilde{\Upsilon}^2{\rm d}w_u{\rm d}w_v + \gamma_{ab}({\rm d}\theta^a - \widetilde{U}^a{\rm d}w_v)({\rm d}\theta^b - \widetilde{U}^b{\rm d}w_v)}$$

Summary GLC/DLC

DLC = GLC for which **we replace τ by a future null coordinate w_u !**

$$\mathrm{d}s^2_{\rm GLC} = \Upsilon^2 \mathrm{d}w^2 - 2\Upsilon \mathrm{d}w\mathrm{d}\tau + \gamma_{ab}(\mathrm{d}\underline{\theta}^a - U^a \mathrm{d}w)(\mathrm{d}\underline{\theta}^b - U^b \mathrm{d}w)$$

$$\Downarrow$$

$$\boxed{\mathrm{d}s^2_{\rm DLC} = -\widetilde{\Upsilon}^2 \mathrm{d}w_u \mathrm{d}w_v + \gamma_{ab}(\mathrm{d}\theta^a - \widetilde{U}^a \mathrm{d}w_v)(\mathrm{d}\theta^b - \widetilde{U}^b \mathrm{d}w_v)}$$

$$\widetilde{U}^a \equiv \gamma^{ab}\widetilde{U}_b = U^a + \Upsilon \gamma^{ab} \frac{\partial \tau}{\partial \theta^b} \quad ,$$

$$\frac{\partial \tau}{\partial w_u} = \frac{\widetilde{\Upsilon}^2}{2\Upsilon} \quad , \quad \frac{\partial \tau}{\partial w_v} = \frac{\Upsilon}{2} - \widetilde{U}^a \frac{\partial \tau}{\partial \theta^a} + \frac{\Upsilon}{2}\gamma^{ab} \frac{\partial \tau}{\partial \theta^a}\frac{\partial \tau}{\partial \theta^b} \quad .$$

$$g^{\rm GLC}_{\mu\nu} = \begin{pmatrix} 0 & -\Upsilon & \vec{0} \\ -\Upsilon & \Upsilon^2 + U^2 & -U_b \\ \vec{0}^{\,T} & -U_a^T & \gamma_{ab} \end{pmatrix} \quad , \quad g^{\mu\nu}_{\rm GLC} = \begin{pmatrix} -1 & -\Upsilon^{-1} & -U^b/\Upsilon \\ -\Upsilon^{-1} & 0 & \vec{0} \\ -(U^a)^T/\Upsilon & \vec{0}^{\,T} & \gamma^{ab} \end{pmatrix} \quad ,$$

$$g^{\rm DLC}_{\mu\nu} = \begin{pmatrix} 0 & -\widetilde{\Upsilon}^2/2 & \vec{0} \\ -\widetilde{\Upsilon}^2/2 & \widetilde{U}^2 & -\widetilde{U}_b \\ \vec{0}^{\,T} & -\widetilde{U}_a^T & \gamma_{ab} \end{pmatrix} \quad , \quad g^{\mu\nu}_{\rm DLC} = \begin{pmatrix} 0 & -2/\widetilde{\Upsilon}^2 & -2\widetilde{U}^b/\widetilde{\Upsilon}^2 \\ -2/\widetilde{\Upsilon}^2 & 0 & \vec{0} \\ -2(\widetilde{U}^a)^T/\widetilde{\Upsilon}^2 & \vec{0}^{\,T} & \gamma^{ab} \end{pmatrix} \quad .$$

Simple – but mixed – quantities in DLC

Photon momentum :

$$k_\mu \equiv \partial_\mu w_v = \delta^{w_v}_\mu \qquad \Rightarrow \qquad k^\mu = -\frac{2}{\widetilde{\Upsilon}^2}\delta^\mu_{w_u}$$

Observer velocity defined in GLC by $u_\mu \equiv \partial_\mu \tau$ gives :

$$u^\mu = -\frac{\Upsilon}{\widetilde{\Upsilon}^2}\left[1+\frac{(\widetilde{U}_a - U_a)(\widetilde{U}^a - U^a)}{\Upsilon^2}\right]\delta^\mu_{w_u} - \frac{1}{\Upsilon}\delta^\mu_{w_v} - \frac{U^a}{\Upsilon}\delta^\mu_a$$

u^{w_v} and u^a are identical to GLC.

We still have $k_\mu u^\mu = -1/\Upsilon$ and thus the **redshift** :

$$1+z_s \equiv \frac{(k_\mu u^\mu)_s}{(k_\mu u^\mu)_o} = \frac{\Upsilon_o}{\Upsilon_s}$$

and the **lensing quantities** are not changed.

DLC = double-null coordinates !

Line element in the **double-null coordinates** :

$$\mathrm{d}s^2 = e^{\lambda}\eta_{AB}\mathrm{d}w^A\mathrm{d}w^B + g_{ab}(\mathrm{d}\theta^a + s^a_A\mathrm{d}w^A)(\mathrm{d}\theta^b + s^b_B\mathrm{d}w^B)$$

So we get back DLC coordinates by taking :

$$\mathrm{d}x^0 = \mathrm{d}w_u \quad , \quad \mathrm{d}x^1 = \mathrm{d}w_v \quad , \quad \mathrm{d}x^a = s^a_{w_u}\mathrm{d}w_u + (s^a_{w_v} + \widetilde{U}^a)\mathrm{d}w_v + \mathrm{d}\theta^a$$

$$\lambda = \ln\left(\widetilde{\Upsilon}^2/2\right) \quad , \quad s^a_{w_u} = 0 \quad , \quad s^a_{w_v} = -\widetilde{U}^a \qquad \text{(gauge fixing)}$$

$\Rightarrow \widetilde{U}^a$ **is** a **shift vector** in the (2+2) decomposition.

They are **equivalent to the double-null coordinates** of *Brady, Droz, Israel and Morsink 1995* after a gauge fixing !

Probing the local homogeneity with standard candles

Collaboration with Hsu-Wen Chiang (蔣序文),
Enea Romano, Pisin Chen (陳丕燊).

Motivations

- Estimate how standard candles can probe the density contrast.
- Investigate on H_0 as *Riess 2016* re-evaluates $H_0 = 73.24 \pm 1.74\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, raising the **tension to 3.4** σ against $66.93 \pm 0.62\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ from Planck.
- Isotropic inhomogeneity extending very far should not exist, but anisotropic inhomogeneity may. Keenan, Barger, Cowie '13 find a super-void extending to $z \sim 0.07$ ($\sim 300\, h_{70}^{-1}\mathrm{Mpc}$).
- We know that being inside an underdense region may alleviate H_0 tension (see Ben-Dayan, Durrer, Marozzi and Schwarz, 2014 and Romano 2016).

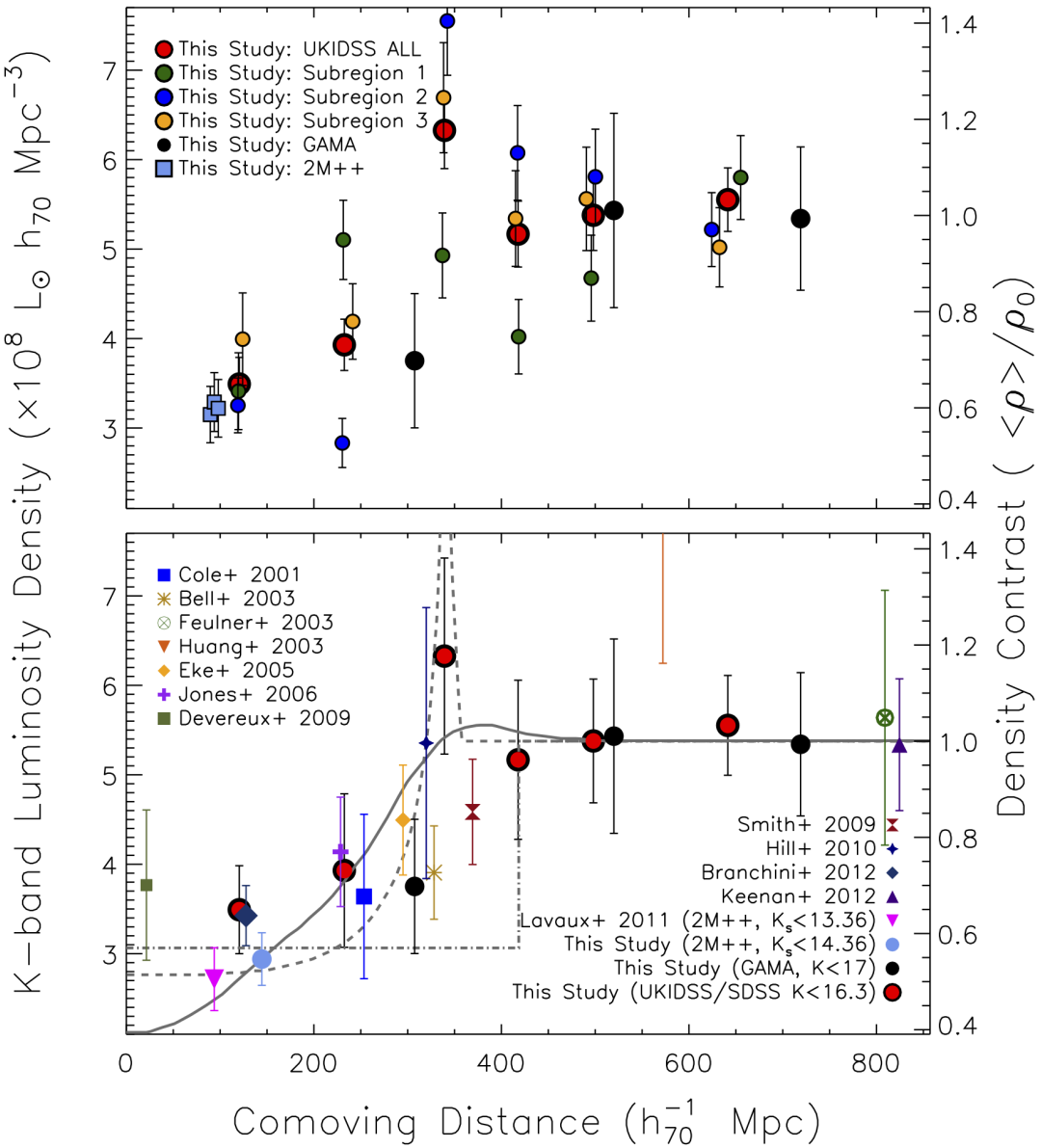
Method

Model independent fit of luminosity distance observations (standard candles)
+ **Inversion** based on LTB to reconstruct the local radial density profile)
assuming Planck background, along two different directions of the sky.

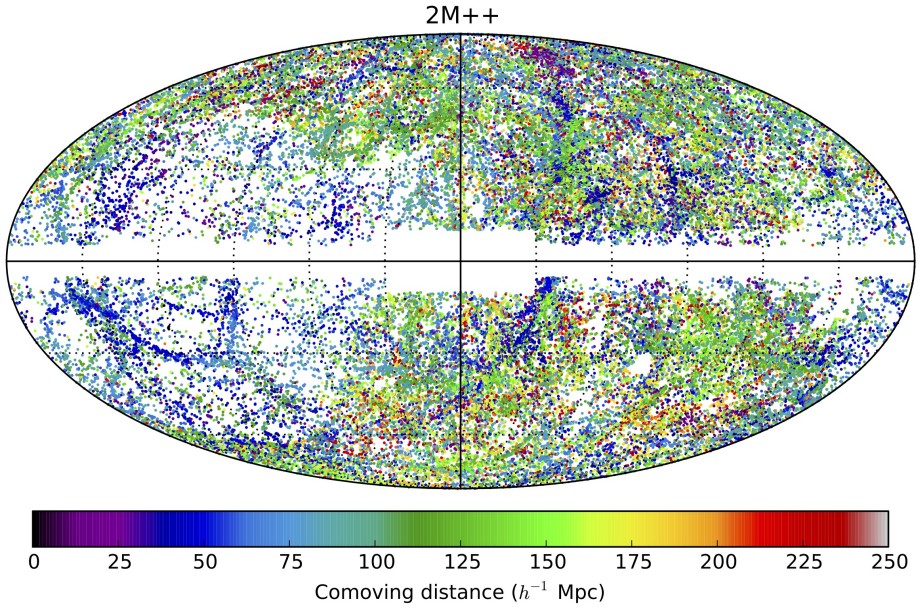
Comparison with : density maps obtained from luminosity density, in particular Keenan et al. 2013 and the 2M++ galaxy catalogue.

Remark : See estimation of cosmic variance on H_0 in Ben-Dayan, Durrer, Marozzi and Schwarz, '14.

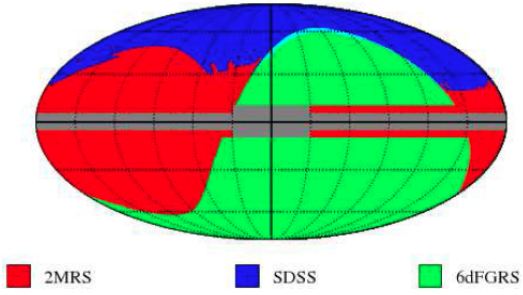
Keenan etal 2013, Figure 11



2M++ (Lavaux & Hudson 2011, Carrick *et al.*2015)



Redshifts are from 2MRS, SDSS-DR7, and 6dFGRS.



Peculiar velocities are obtained from the galaxy density :

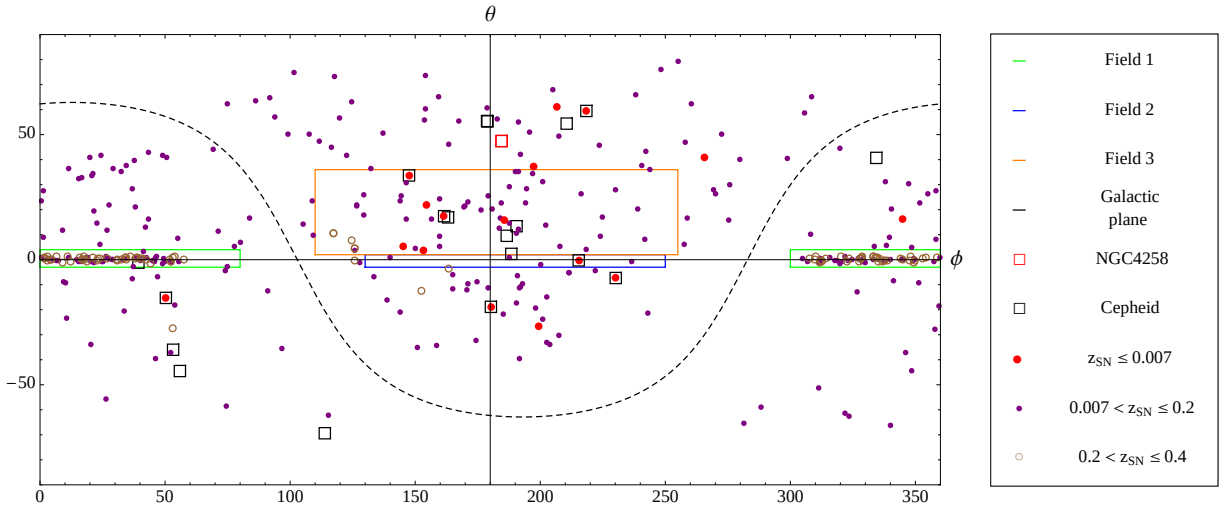
$$\vec{v}(\vec{r}) = \frac{\beta^*}{4\pi} \int_0^{R_{\rm max}} \mathrm{d}^3\vec{r}'\, \delta_g^*(\vec{r}')\frac{\vec{r}'}{r'^3} \quad , \quad \bar{z} = z_{\rm obs} - \vec{v}\cdot\vec{n}$$

where $\beta^* = 0.43$ is a best fit value and the upper limit of integration is the depth of the survey $R_{\rm max} = 200\,h^{-1}$Mpc, i.e. $z = 0.067$.

$\Rightarrow$ limited to **200** Mpc + **an external bulk flow** (that we remove).

Data

Cepheids are from *Riess 2016* (z_{host} from NED) and **SNe Ia** from UNION 2.1 (with $z < 0.2$ or 0.4 and positions from SIMBAD), corrected to Riess H_0.



Velocity dispersion : 250 km s^{-1} for SNe, 0, 40, 250 km s^{-1} for Cepheids. Implies a change in μ by $\Delta\mu_{\text{v.d.}} \approx \frac{5}{\log 10}\frac{\Delta v}{cz}$.

Field N°	ICRS coordinates (deg)		Number of SNe		Cepheids-hosting galaxies of Riess
	R.A.	Dec.	$z_{\max} = 0.2$	$z_{\max} = 0.4$	
1	$[300, 360] \cup [0, 80]$	$[-3, 4]$	69	144	1
2	$[130, 250]$	$[-3, 2]$	4+1	4+1	1
3	$[110, 255]$	$[2, 36]$	47+2	52+2	6
/	Whole Sky		288	372	20

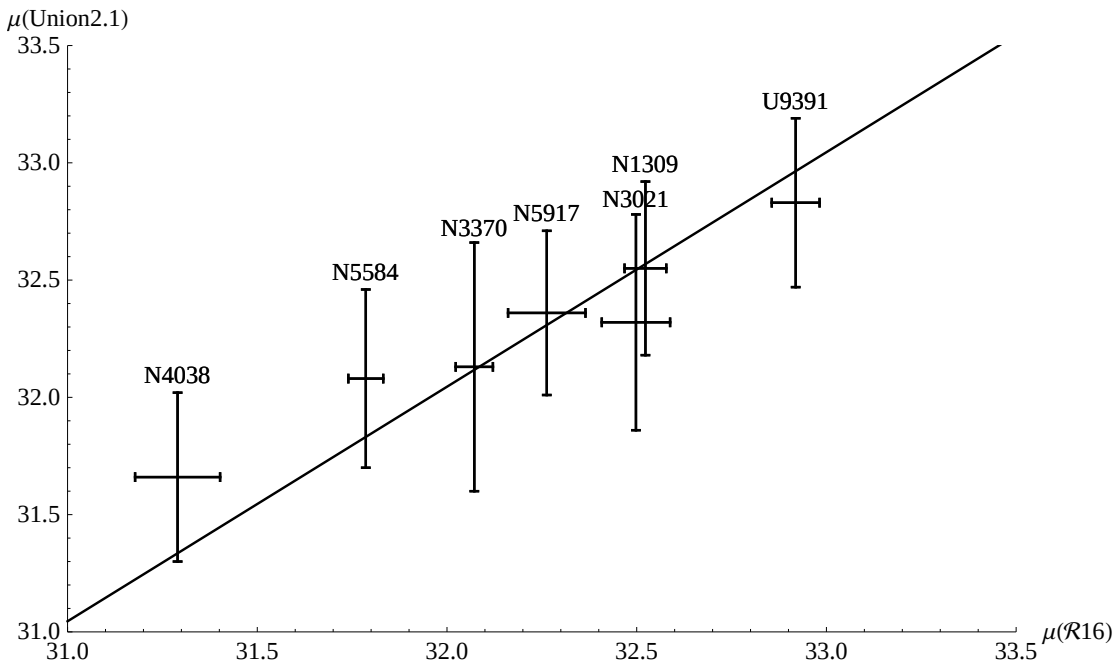
Initial rescaling

At small z :

$$\mu \equiv m - M = 25 - 5\log_{10} H_0^{\rm loc} + 5\log_{10}\left(H_0^{\rm loc} D_L\right) \approx 25 - 5\log_{10} H_0^{\rm loc} + 5\log_{10} cz \quad ,$$

where $H_0^{\rm loc}$ is the local Hubble parameter.

$$\Rightarrow \mu(\mathcal{R}16) = \mu(\text{Union 2.1}) - 5\log_{10}\left(\frac{73.24}{70}\right)$$



7 host galaxies common to $\mathcal{R}16$ Cepheids and UNION 2.1 SNe

1D Fitting

Fits of the distance modulus data $(z_i, \mu_i, \Delta\mu_i)$ by minimizing χ^2 of the deviation from a homogeneous model :

$$\chi^2 = \sum_i \left(\frac{f(z_i) - \left(\mu_i - \mu^{\text{Planck}}(z_i)\right)}{\Delta\mu_i} \right)^2 \quad , \quad f(z) = \left(\mu^{\text{obs}} - \mu^{\text{Planck}}\right)(z)$$

where $\mu^{\text{Planck}}(z)$ is the ΛCDM theoretical value of distance modulus at z.

Model independent by decomposing the fitting function $f(z)$ wrt a set of *radial basis functions* (RBFs NN) :

$$f(z) = w_0 + w_{-1}\, z + \sum_{m=1}^{N_{\text{NL}}} w_m\, \Phi\left(|z - p_m|\right)$$

where Φ are chosen to be $\Phi(r) = r^3$ (N_{NL} RBFs), p_m are the **non-linear parameters** or "centers" of the RBFs, w_m the linear parameters, w_0 (intercept) and/or w_{-1} (slope) parameters.

Best fit and confidence bands

Linear parameters $\mathbf{w} \equiv (w_{-1}, \ldots, w_{N_{\rm NL}})$: we use the simple Moore-Penrose pseudo-inverse method.

Non-linear parameters $\mathbf{p} \equiv (p_1, \ldots, p_{N_{\rm NL}})$: we use a Monte Carlo (MC) random sampling method and a LO algorithm (Gauss-Newton).

To speed up the MC process and fill up confidence band : we use a Monte Carlo Markov Chain (MCMC) algorithm to explore the non-linear parameter space (see paper for illustration).

A fitting model is classified by a set of parameters $(N_0, N_{-1}, N_{\rm NL})$.

We use a F-test to determine the **best model parameters**.

Density contrast (Romano, Chiang and Chen, 2013)

LTB metric and EFEs :

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2\left[\left(1+\frac{a_{,r}\,r}{a}\right)^2\frac{\mathrm{d}r^2}{1-k(r)r^2}+r^2\mathrm{d}\Omega_2^2\right]$$

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{k(r)}{a^2}+\frac{\rho_0(r)}{3a^3} \quad , \quad \rho(t,r)=\frac{(\rho_0 r^3)_{,r}}{3a^2r^2(ar)_{,r}} \, ,$$

Solution of the EFEs can be expressed in conformal time $\eta = \int^t dt'/a\,(t',r)$:

$$a\,(\eta,r) = \frac{\rho_0}{6k(r)}\left[1-\cos\left(\sqrt{k(r)}\eta\right)\right]$$

$$t\,(\eta,r) = \frac{\rho_0}{6k(r)}\left[\eta-\frac{1}{\sqrt{k(r)}}\sin\left(\sqrt{k(r)}\eta\right)\right]+(t_b(r)\equiv 0)$$

$$\Rightarrow D_L(z) = (1+z)^2\, r(z)\, a\,(\eta\,(z)\,,r\,(z)) \, ,$$

where $\eta\,(z)$ and $r\,(z)$ are the solutions of the radial ingoing null geodesic equations, and the redshift z is defined by $\frac{1+z_s}{1+z_o} = \exp\left(\partial_{t_s}\int_s^o \frac{\mathrm{d}t}{\mathrm{d}r}\right)$.

To obtain $r\,(z)$, $\eta\,(z)$ and $k\,(z)$, **the relation of D_L needs to be inverted and solved together with the radial null geodesic equations** $\Rightarrow \rho(z, D_L(z))$.

Inversion (Romano, Chiang and Chen, 2013), i.e. what we solve :

$$a(z=0)=a_0\ ,\ H_{\rm LTB}(z=0)\equiv[a^{-1}\partial_t a](z=0)=H_0\ , \qquad \text{(I.C.)}$$

$$q_{\rm LTB}(z=0)\equiv\left[-a\left(\partial_t a\right)^{-2}\partial_t\partial_t a\right](z=0)=q_0\ ,\ r(z=0)=0\ , \qquad \text{(I.C.)}$$

$$\frac{{\rm d}k}{{\rm d}z}=\frac{\sqrt{1-S^2}}{3(1+z)S}\frac{2k\tan(\tau/2)\mathcal{A}}{3-\tau\csc(\tau)(2+\cos(\tau))}\ ,$$

$$\frac{{\rm d}\eta}{{\rm d}z}=\frac{1}{(1+z)\sqrt{k}}\left(\csc(\tau)\mathcal{B}-\frac{\sqrt{1-S^2}}{3S}\mathcal{A}\right)\ ,$$

$$\frac{{\rm d}r}{{\rm d}z}=\frac{\sqrt{1-S^2}}{(1+z)\sqrt{k}}\left(\frac{\cos(\tau)+3\tau\csc(\tau)-4}{3-\tau\csc(\tau)(2+\cos(\tau))}\frac{\csc(\tau)\mathcal{A}}{3}+\tan(\tau/2)\right)\ ,$$

where we have defined

$$\tau\equiv\sqrt{k}\eta\quad,\quad S\equiv\sqrt{k}r\quad,\quad \mathcal{A}=1-\cos(\tau)+\mathcal{B}$$

$$\mathcal{B}=\frac{2}{S}\frac{(a_0H_0)^{-3}(1+z)k^{3/2}}{1-\tan(\tau/2)\sqrt{1-S^2}/S}\left(1-\frac{1+2q_0}{4(1+q_0)^2}\right)^{-1}\frac{\rm d}{{\rm d}z}\left(\frac{H_0D_L(z)}{(1+z)^2}\right)\ .$$

The density is then given

$$\rho(z)=\rho\left(t(z),r(z)\right)=\frac{a^{-3}\rho_0}{1+r\left(\partial_r\ln a|_\eta-\frac{\partial t}{a\partial r}\partial_\eta\ln a\Big|_r\right)}\quad,\quad \rho_0\equiv3H_0^2\left[1-\frac{1+2q_0}{4}(1+q_0)^{-2}\right]$$

Density contrast :

$$\delta_C=f^{-1}\left(\frac{\rho_{\rm inv}(D_L,z)}{\rho_{\rm inv}\left(D_L^{\rm Planck},z\right)}-1\right)\quad,\quad f^{-1}=\Omega_{m0}^{-0.55}\ .$$

Full sky fitting

Applying peculiar velocity correction (PVC) from 2M++ and velocity dispersion (VD) of 250 $\mathrm{km\,s^{-1}}$ for SNe (Riess 2016 applies 250 $\mathrm{km\,s^{-1}}$ also for Cepheids).

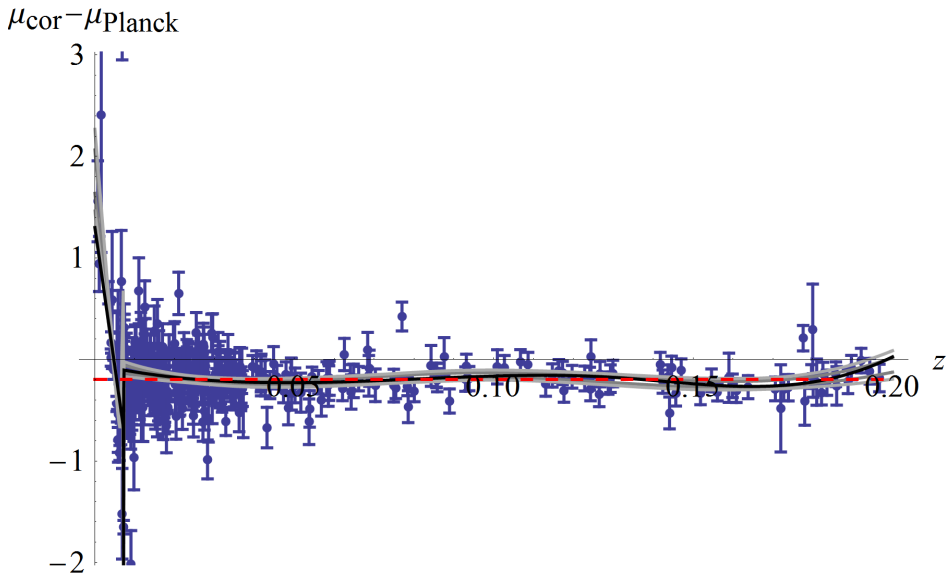
250 $\mathrm{km\,s^{-1}}$ VD for Cepheids :

$\Rightarrow$ Preferred model is **(1,0,0)** : $f(z) = \mu^{\mathrm{obs}}(z) - \mu^{\mathrm{Planck}}(z) = w_0$, i.e. homogeneous model with an apparent value of the Hubble parameter :

$$H_0^{\mathrm{loc}} \equiv H_0^{\mathrm{Planck}} 10^{-f(z=0)/5} = H_0^{\mathrm{Planck}} 10^{-w_0/5} = 10^{-w_0/5} (\,66.93\ \mathrm{km\,s^{-1}\,Mpc^{-1}}\,) .$$

We find : $H_0^{\mathrm{loc}} = 73.06 \pm 1.61\,(\mathrm{stat.})\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, in good agreement with $73.24 \pm 1.61\,(\mathrm{stat.}) \pm 0.66\,(\mathrm{sys.})\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ of Riess 2016 (we have $\chi^2_R = 1.49$).

40 $\mathrm{km\,s^{-1}}$ VD for Cepheids :
(see Tully 2007)

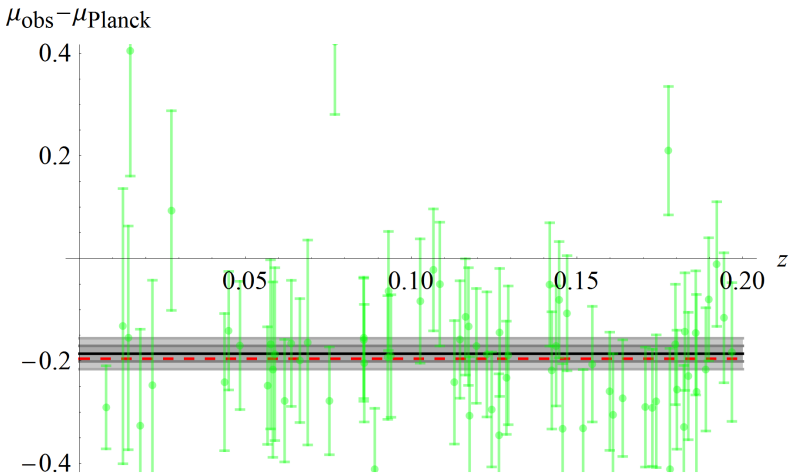


Best fit is a $(0, 1, 7)$ model !
$\chi^2_R = 4.18$

$\Rightarrow$ there must be structure not accounted for by 2M++.

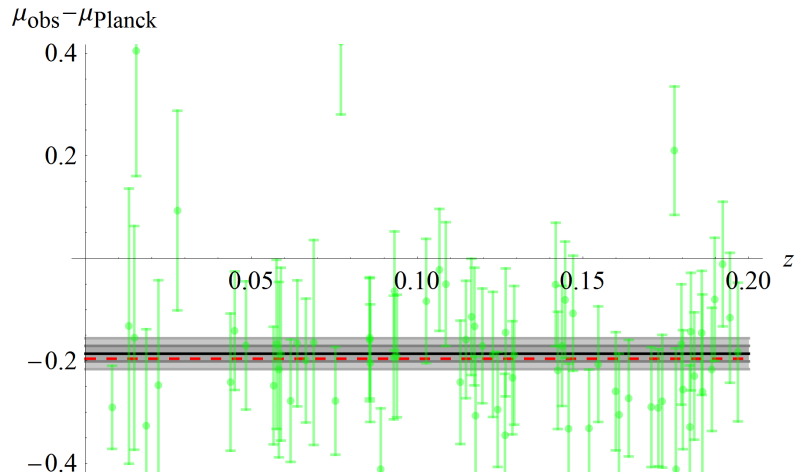
Directional fitting in F1

With PVC from 2M++ :



- best fit we get is from a simple $(1, 0, 0)$ model with $H_0^{loc} = 72.89 \pm 0.50\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\chi^2_R = 1.05$ (see figure),
- next best fit (Threshold $< 33\%$) is given by a $(0, 1, 5)$ model with $\chi^2_R = 0.88$.

Without PVC from 2M++ :



- best fit model is $(1, 0, 0)$ with $H_0^{loc} = 72.90 \pm 0.51\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ with F-test *Threshold* $> 36\%$ and $\chi^2_R \sim 1.07$ (see figure),
- second best model is an inhomogeneous $(1, 1, 13)$ model with $\chi^2_R \sim 0.59$ but low threshold.

Directional fitting in F3 with PVC

Applying PVC from 2M++ and a **velocity dispersion** (VD) of 250 $\mathrm{km\,s^{-1}}$ for SNe Ia (host rotation), and no VD for Cepheids.

It is necessary to remove some "outliers" to have invertible fits.
We use a F-test *Threshold* around 95% to compare a given model with a constant fit model.

$z_{\max} = 0.2$			
χ^2_R	*Threshold* (%)	*Param.*	Removal
19.5	Not Preferred	76.40 ± 2.90	
1.59	$81 \sim 100\%$	$(0, 1, 6)$	
5.92	$95.8 \sim 100\%$	$(0, 0, 0)$	NGC 4536
2.06	$90.7 \sim 95.7\%$	$(0, 1, 3)$	Same
2.05	$73 \sim 100\%$	70.56 ± 0.93	+NGC 4424

$z_{\max} = 0.4$			
χ^2_R	*Threshold* (%)	*Param.*	Removal
17.9	Not Preferred	76.36 ± 2.75	
1.60	$74 \sim 100\%$	$(0, 1, 6)$	
5.58	$96.9 \sim 100\%$	$(0, 0, 0)$	NGC 4536
2.03	$85 \sim 96.8\%$	$(0, 1, 3)$	Same
2.00	$72 \sim 100\%$	70.65 ± 0.91	+NGC 4424

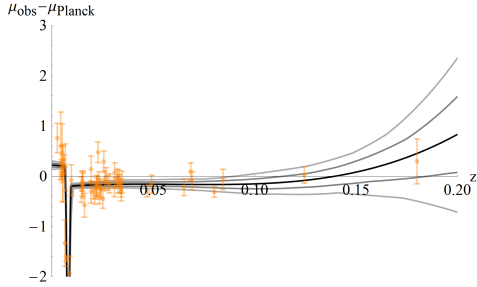
Directional fitting in F3 without PVC

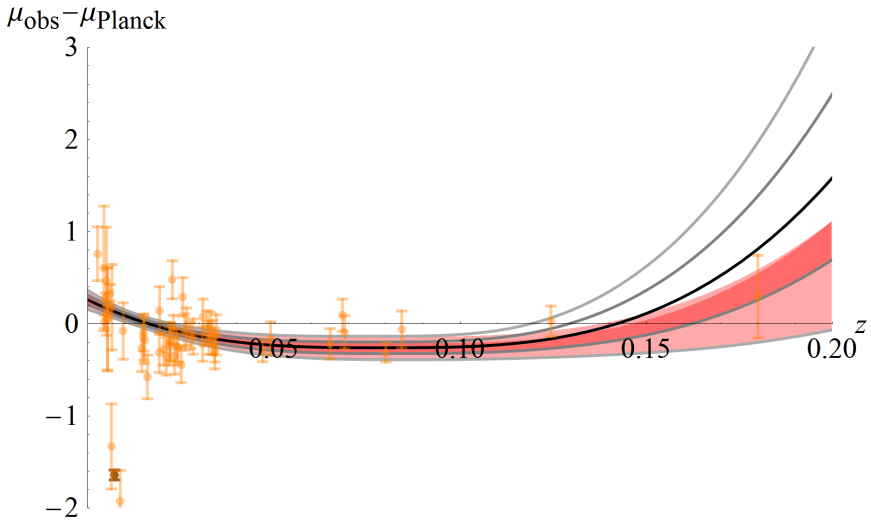
We don't apply PVC since we want to see the whole contribution from SNe Ia and Cepheids.

Just apply a VD of 250 $\mathrm{km\,s^{-1}}$ for SNe Ia (host rotation).

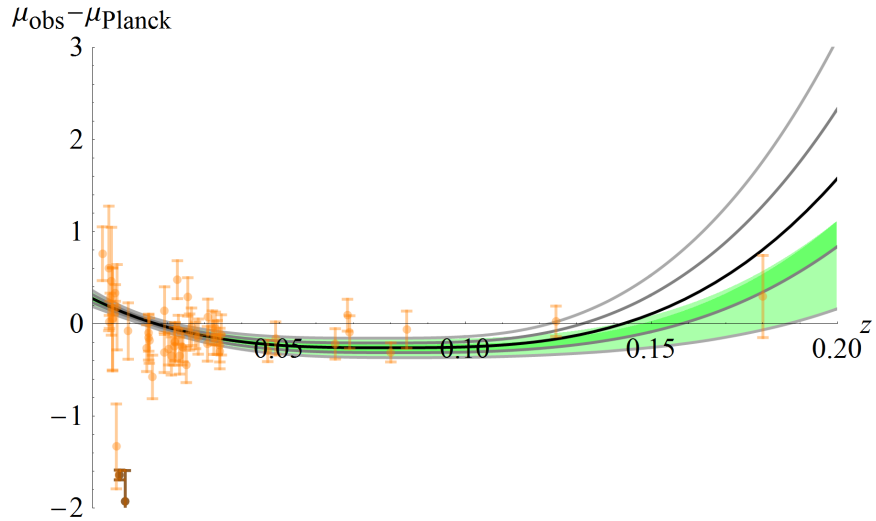
$z_{\max} = 0.2$			
χ^2_R	*Threshold* (%)	*Param.*	Removal
1.40	$39 \sim 100$	$(0, 0, 5)$	
3.45	$97.5 \sim 100$	$(0, 0, 0)$	NGC 4536
2.26	$89 \sim 97.4$	$(1, 0, 1)$	Same
2.88	$99.5 \sim 100$	$(0, 0, 0)$	+1999cl
1.55	$94.1 \sim 99.4$	$(1, 0, 1)$	Same
1.47	**$47 \sim 94.0$**	**(0, 0, 2)**	**Same**

$z_{\max} = 0.4$			
χ^2_R	*Threshold* (%)	*Param.*	Removal
1.43	$38 \sim 100$	$(0, 1, 5)$	
3.31	$92.6 \sim 100$	$(0, 0, 0)$	NGC 4536
2.20	$92.1 \sim 92.5$	$(0, 1, 2)$	Same
2.80	$96.3 \sim 100$	$(0, 0, 0)$	+1999cl
1.96	$89 \sim 96.2$	$(1, 0, 1)$	Same
1.37	**$76 \sim 88$**	**(0, 0, 4)**	**Same**

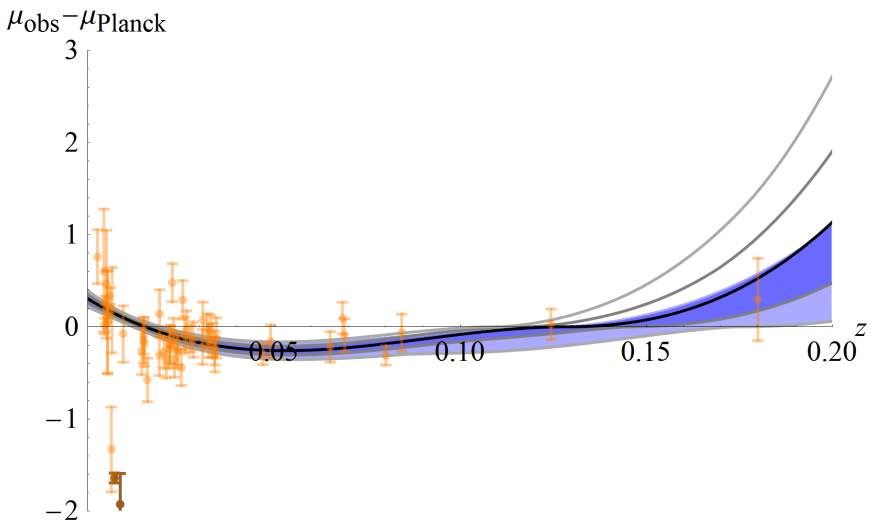




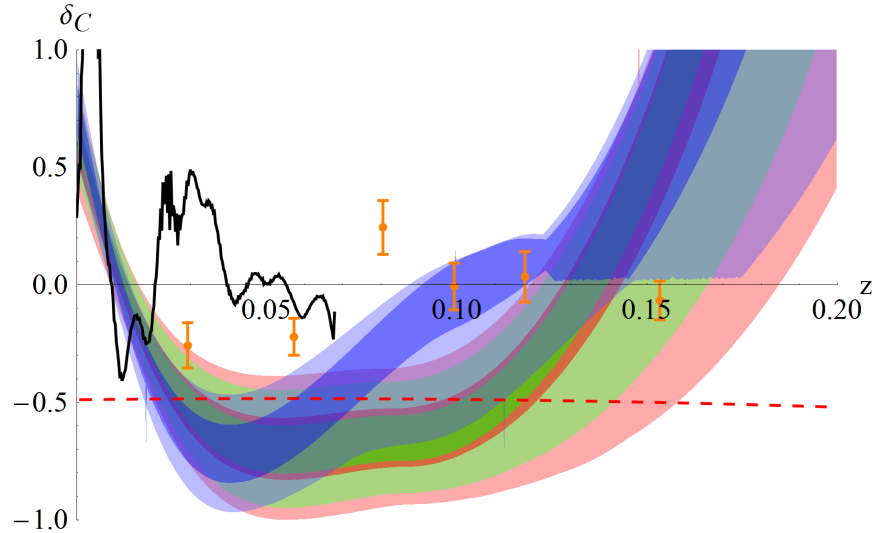
(a) NGC 4536 removed $(1, 0, 1)$



(b) NGC 4536, 1999cl removed $(1, 0, 1)$

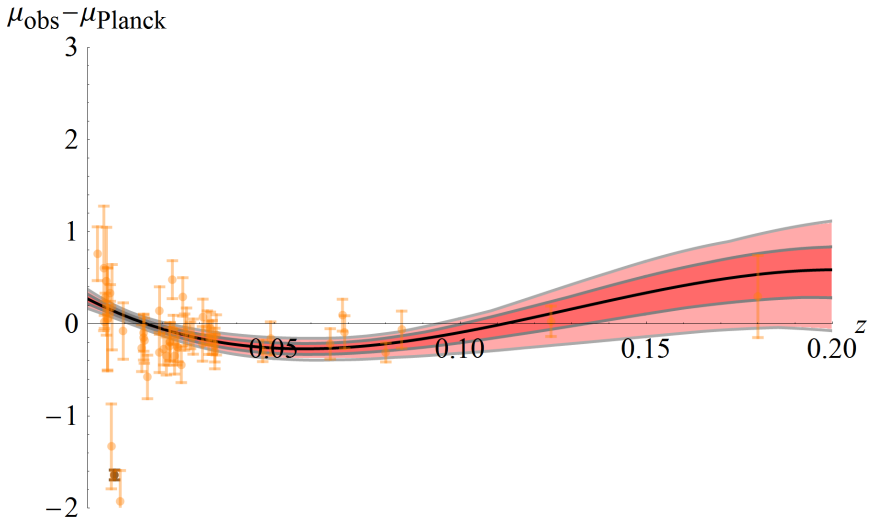


(c) **NGC 4536, 1999cl removed $(0, 0, 2)$**

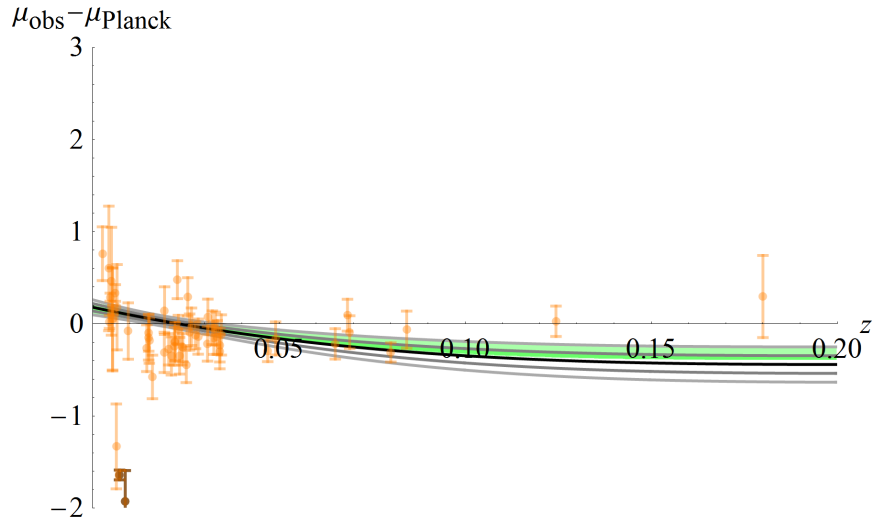


(d) Inverted density

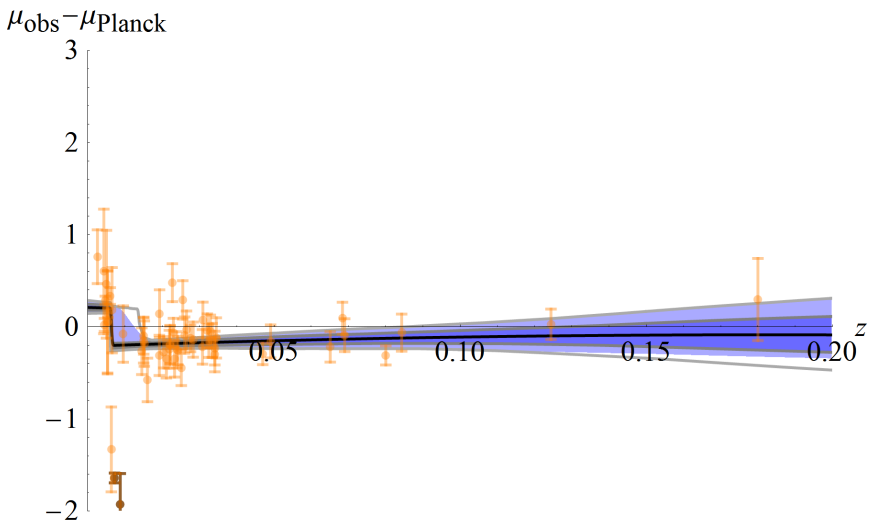
FIGURE: Distance modulus best fit models are plotted for F3 with $z_{max} = 0.2$, without peculiar velocity corrections and with a 250 km s^{-1} velocity dispersion for SNe.



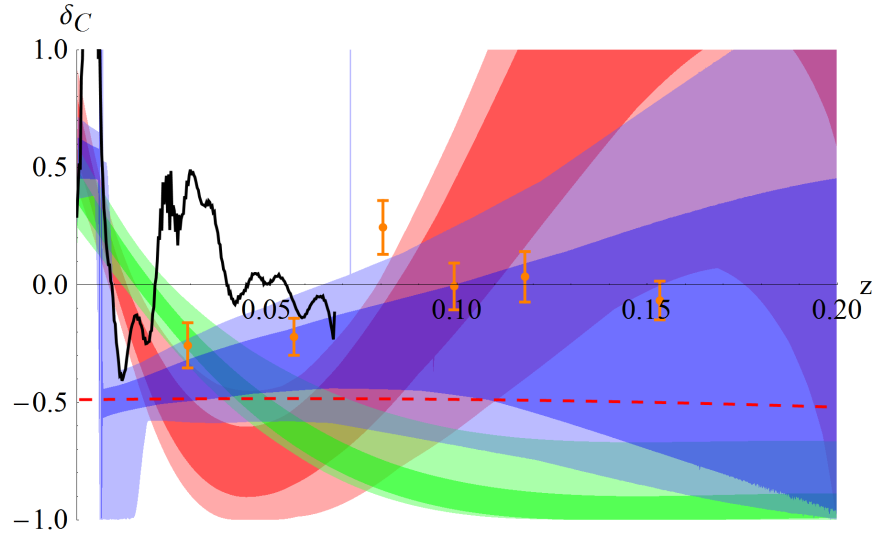
(a) NGC 4536 removed $(0, 1, 2)$



(b) NGC 4536, 1999cl removed $(1, 0, 1)$



(c) **NGC 4536, 1999cl removed $(0, 0, 4)$**



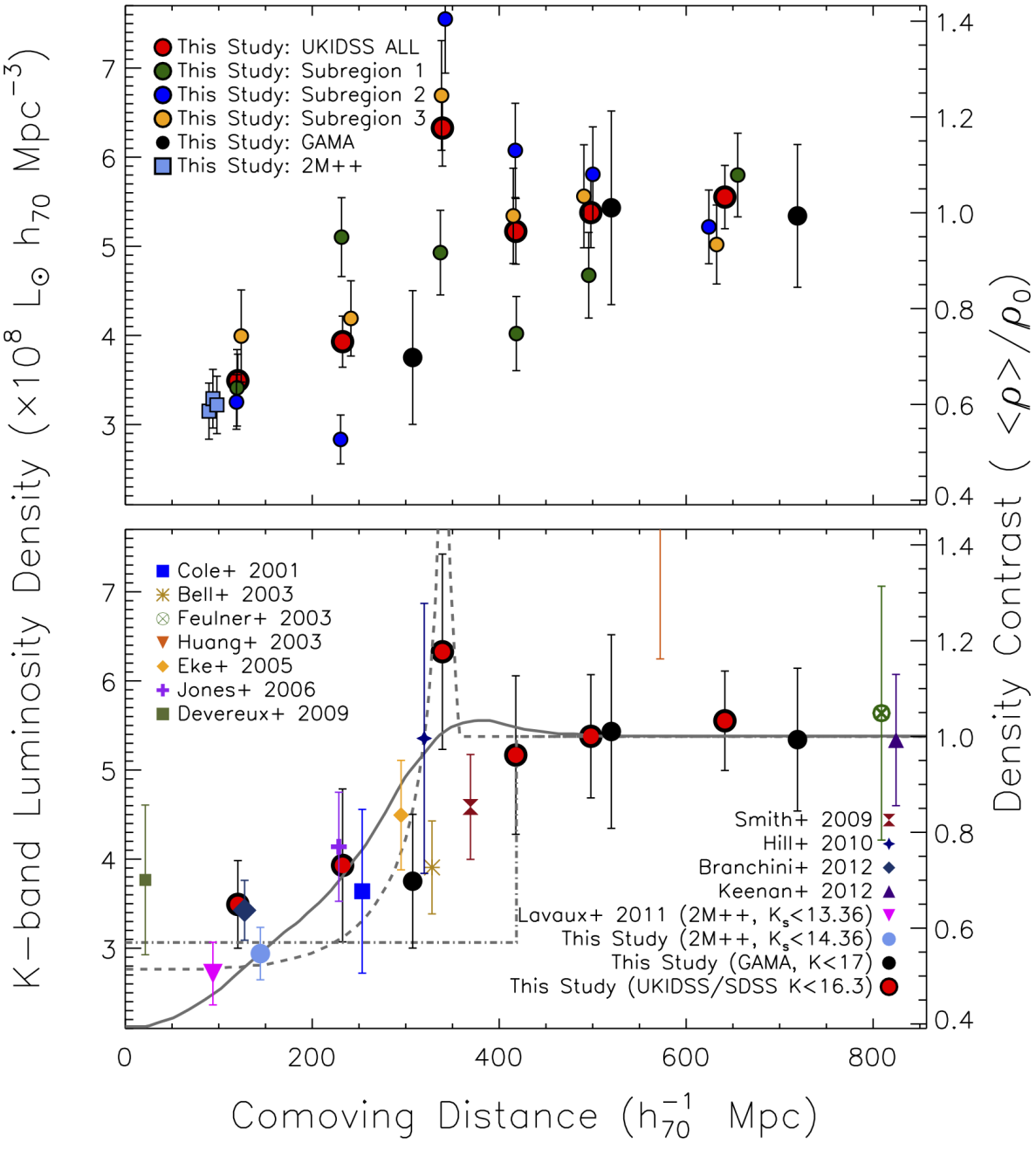
(d) Inverted density

FIGURE: Distance modulus best fit models are plotted for F3 with $z_{max} = 0.4$, without peculiar velocity corrections and with a 250 km s^{-1} velocity dispersion for SNe.

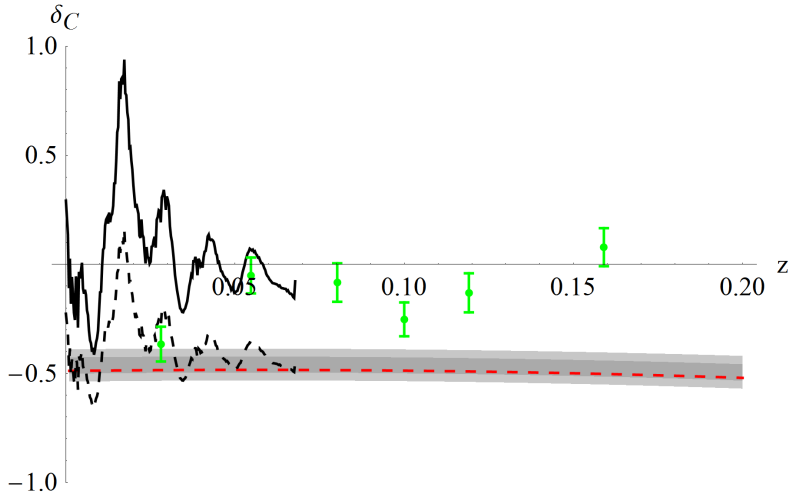
Comparison with density maps obtained from luminosity density of Keenan et al. 2013.

- Green : Field 1
- Blue : Field 2
- Orange : Field 3

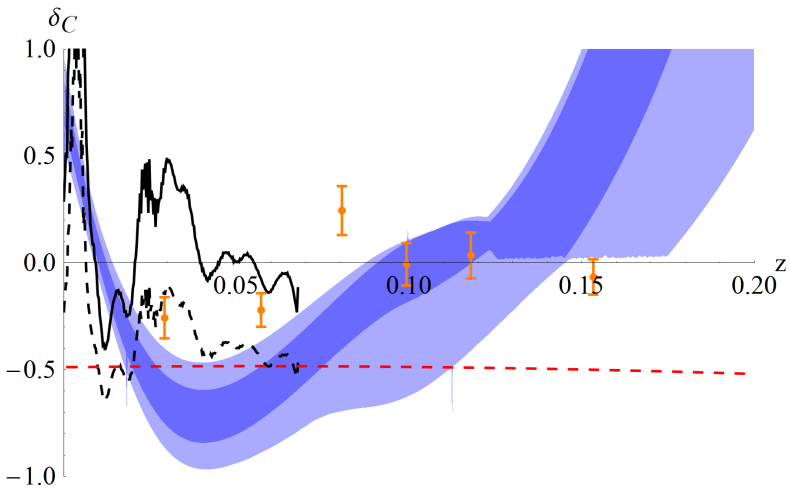
Keenan '13 suggests 2M++ **rescaling of** $\sim 0.6 \Rightarrow$ our density matches quite well with 2M++ for such a rescaling in F1, F3.



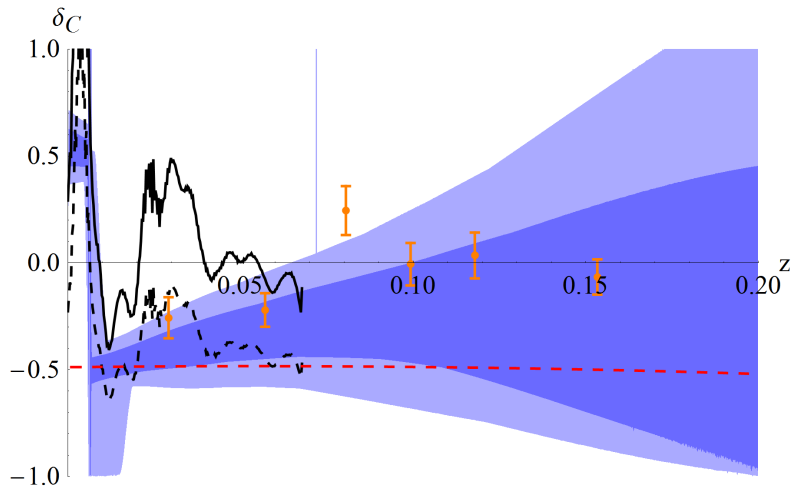
Main results



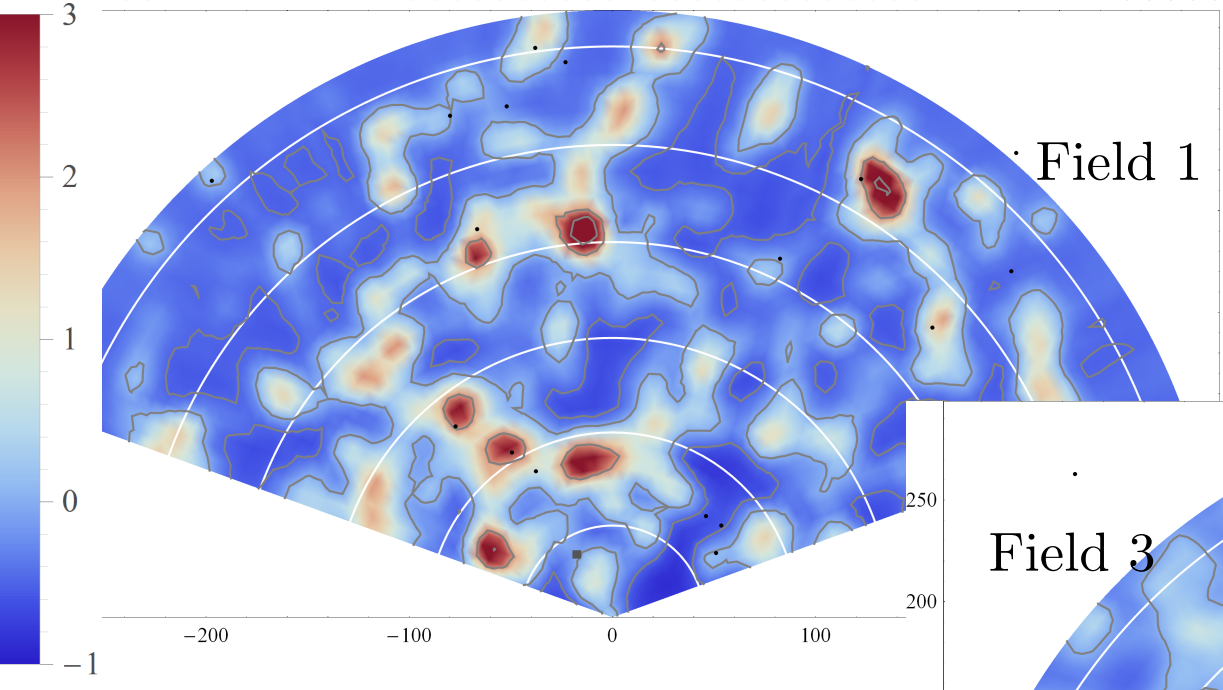
(a) Field 1, $(1, 0, 0)$

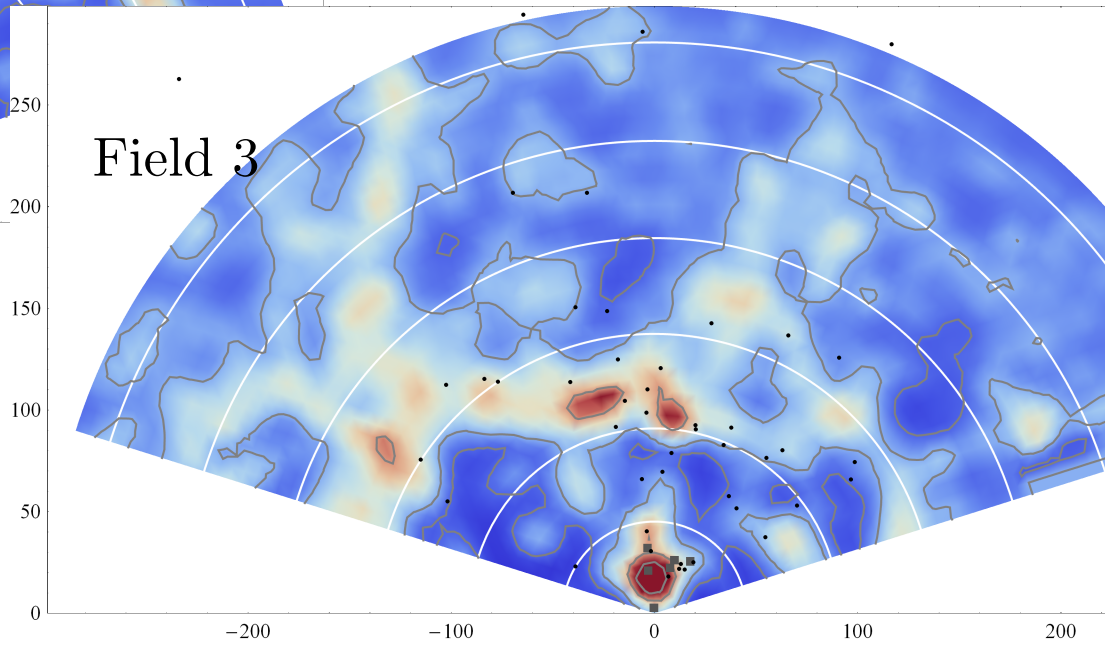


(b) Field 3, $z_{\max} = 0.2$ with NGC 4536 and 1999cl removed $(0, 0, 2)$



(c) Field 3, $z_{\max} = 0.4$ with NGC 4536 and 1999cl removed $(0, 0, 4)$





Comparison with 2M++ density map (averaged along declination direction in ICRS coordinates).

White arcs correspond to $z = 0.01, 0.02, \ldots, 0.06$, gray contours indicate iso-density lines of $\delta_C = -0.5, 0, 2, 4$. Depth of the survey $R_{\max} = 200\,h^{-1}\mathrm{Mpc}$, i.e. $z = 0.067$.

Rescaling 2M++

- 2M++ is normalized wrt the average within its depth ⇒ its normalization can be wrong if 2M++ is embedded in a larger structure.
- Same with Keenan 2013 with background equal to the averaged luminosity density over the data set.
- **Our reconstructed density profile is normalized wrt the background** since we are assuming cosmological background parameters obtained from large scale observations (Planck).

If we take $\mathcal{K}13$ background density we would have to rescale 2M++ as

$$\delta_C^{\rm cor} = \frac{\tilde{\rho}_{\rm 2M++}}{\tilde{\rho}_{\mathcal{K}13}}(1+\delta_C) - 1,$$

where $\delta_C^{\rm cor}$ is the rescaled density contrast, while $\tilde{\rho}_{\rm 2M++}$ and $\tilde{\rho}_{\mathcal{K}13}$ are the assumed background density of 2M++ and $\mathcal{K}13$ ⇒ **factor 0.6 rescaling**.

Our results :

- independently confirms the existence of inhomogeneities,
- to some extent in qualitative agreement with Keenan 2013 (claiming ~ 300 Mpc void), but normalization of background seems crucial,
- based on 1D fit in windows of the sky, LTB inversion model, with SNe Ia and Cepheids data $\Rightarrow$ different sources of uncertainty,
- SNe Ia could be useful to correctly normalize density maps from galaxy surveys with respect to the average density of the Universe,
- could clarify the apparent discrepancy between local and large scale estimations of the H_0 (especially between Planck and Riess, which uses 2M++).

Conclusions

We have tools to work with **relativistic effects** in the large scale structure. Adapted coordinates is one of them :

- GLC coordinates are convenient for many applications,
- they can be related to double-null coordinates (via DLC coordinates),
- there are still applications to explore,

in order to go below **percent accuracy** in cosmology.

But tools are not everything, we also have to **understand what we measure** :

- SNe are messy (and not classified !),
- H_0 tension is a big problem to solve ! (before/with DE ?),
- we need to better understand our local Universe,

as discoveries now hide **into the details**.


Thank You!