Flavor structure of the nucleon sea from lattice QCD

Jiunn-Wei Chen

National Taiwan U.

Collaborators: Huey-Wen Lin, Saul D. Cohen, Xiangdong Ji, Luchang Jin, Jianhui Zhang

arXiv: 1402.1462 + 1603.06664 + 1609.08102 + 1702.00008

Why is computational physics important ?

An Ultimate Question in Science

An Ultimate Question in Science

Life = Physical Laws ?

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Or more specifically,

Life = known Physical Laws?

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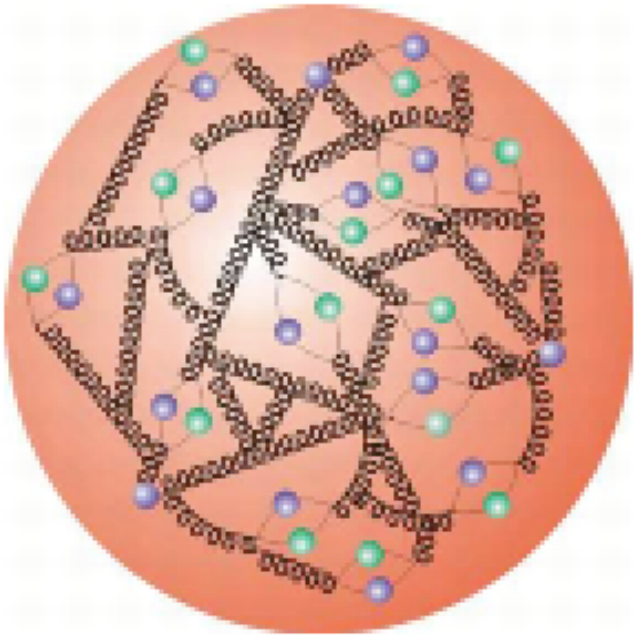
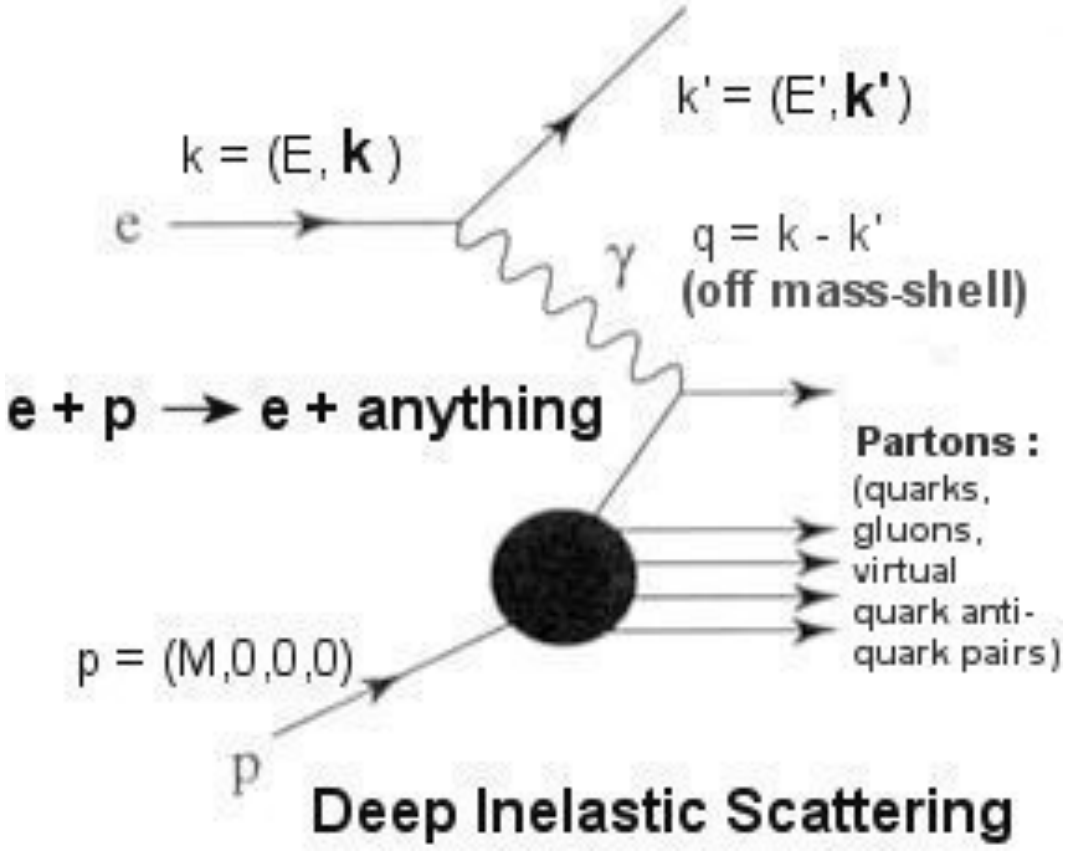
A computational problem!

Feynman's Parton Model

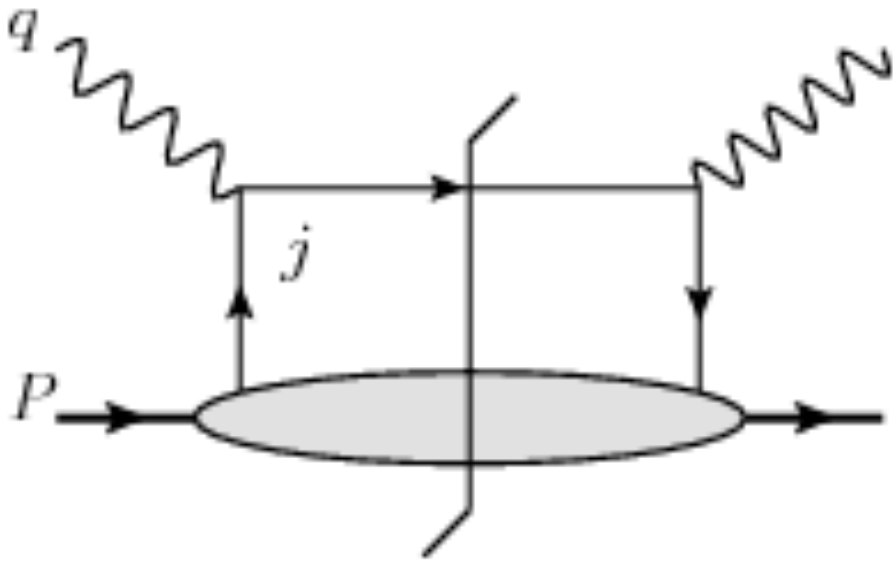


The momentum distributions of partons (quarks, antiquarks and gluons) become one dimensional distributions in the infinite momentum frame.

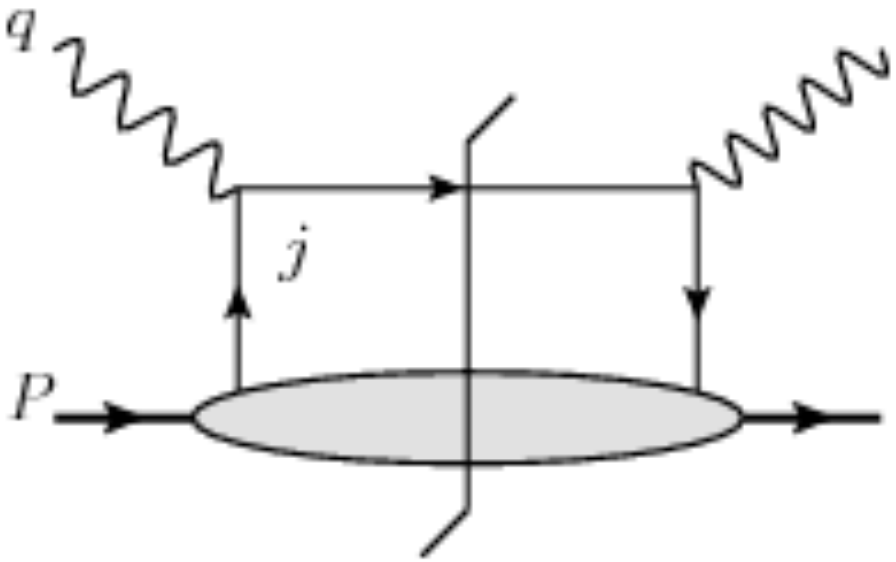
Measuring Parton Distributions Using DIS experiments



Parton Distribution Function (PDF) in QCD



Parton Distribution Function (PDF) in QCD



The struck parton moves on a light cone at the leading order in the twist-expansion.

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P \right\rangle$$

PDFs from QCD---why is it so hard?

- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P \right\rangle$$

- Non-perturbative, infinite dof, need lattice QCD
- Euclidean lattice: light cone operators cannot be distinguished from local operators
- Moments of PDF given by local twist-2 operators; limited to first few moments but carried out successfully

Beyond the first few moments

- Smeared sources: Davoudi & Savage
- Gradient flow: Monahan & Orginos
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin; QCDSF
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x -dependence directly.

Ji's idea

- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P \right\rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.
- Analogous to HQET: need power corrections & matching---LaMET

Review: Ji's LPDF (LaMET)

$$\widetilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{-ixzP^z} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(z\lambda) \right| P \right\rangle$$
$$\equiv \int \frac{dz}{2\pi} e^{-ixzP^z} h(zP^z) P^z$$

$$\lambda^\mu = (0,0,0,1)$$

- Taylor expansion yields

$$\overline{\psi}\lambda\cdot\gamma\Gamma\left(\lambda\cdot D\right)^n\psi = \lambda_{\mu_1}\lambda_{\mu_2}\cdots\lambda_{\mu_n}O^{\mu_1\cdots\mu_n}$$

op. symmetric but not traceless

Review: Ji's LPDF (LaMET)

$$\left\langle P \left| O^{(\mu_1 \cdots \mu_n)} \right| P \right\rangle = 2a_n P^{(\mu_1} \cdots P^{\mu_n)}$$

- LHS: trace, twist-4 $\mathcal{O}(\Lambda^2_{QCD}/(P^z)^2)$ corrections, parametrized in this work
- RHS: trace $\mathcal{O}(M^2/(P^z)^2)$
- One loop matching $\alpha_s \ln P^z$, OPE

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}, \frac{M^2}{P_z^2}\right) + \cdots.$$

First (isovector) LPDF Computation

- Lattice: $24^3 \times 64$

$$a \approx 0.12 \text{ fm} \quad L \approx 3 \text{ fm}$$

- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

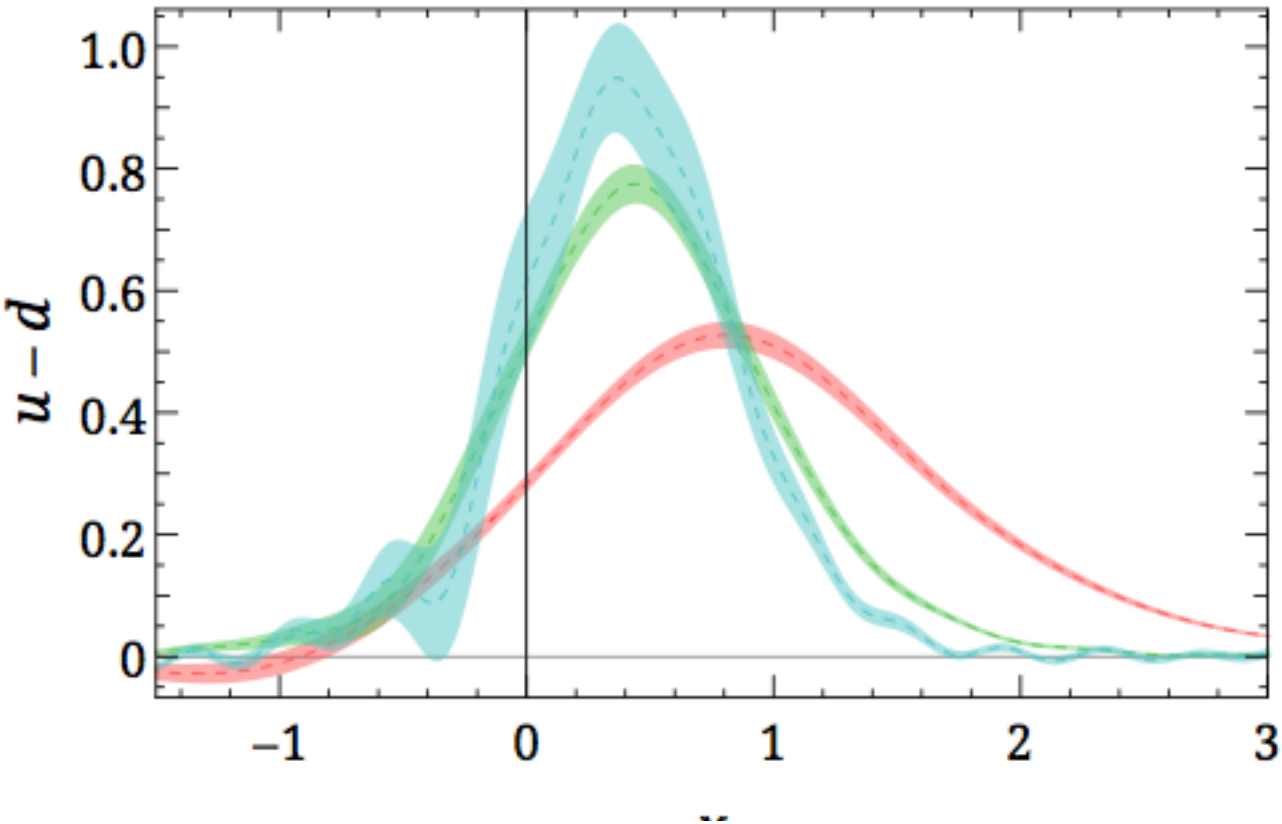
$$N_f = 2 + 1 + 1 \qquad M_\pi \approx 310 \text{ MeV}$$

- Gauge fields/links: hypercubic (HYP) smearing, 461 config.

- $$P^z = \frac{2\pi}{L} n = n \times 0.43 \; GeV$$ n = 1,2,3…

(high momentum smearing: Bali, Lang, Musch, Schafer)

Quasi-PDF (unpolarized)



$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV \qquad \text{n} = 1, 2, 3.$$

RG of Wilson Coefficient

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}, \frac{M_N^2}{P_z^2}\right) + \cdots.$$

Xiong, Ji, Zhang, Zhao (GPD: Ji, Schafer, Xiong, Zhang; Xiong, Zhang) Factorization (Ma, Qiu; Li), Linear divergence & LPT (Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang), RI (Monahan & Orginos; Yong & Stewart; Constantinou et al.), E vs. M spaces (Carlson et al.; Briceno et al.)

$\mathcal{O}(M^2/(P^z)^2)$ Corrections

$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$

- Computed to all orders in $\mathcal{O}(M^2/(P^z)^2)$

$$q(x) = \sqrt{1+4c} \sum_{n=0}^{\infty} \frac{f_-^n}{f_+^{n+1}} \left[(1+(-1)^n) \tilde{q}\left(\frac{f_+^{n+1} x}{2 f_-^n} \right) + (1-(-1)^n) \tilde{q}\left(\frac{-f_+^{n+1} x}{2 f_-^n} \right) \right]$$

$$f_\pm = \sqrt{1+4c} \pm 1 \qquad c = M^2/4P_z^2$$

$\mathcal{O}(\Lambda^2_{QCD}/(P^z)^2)$ Corrections

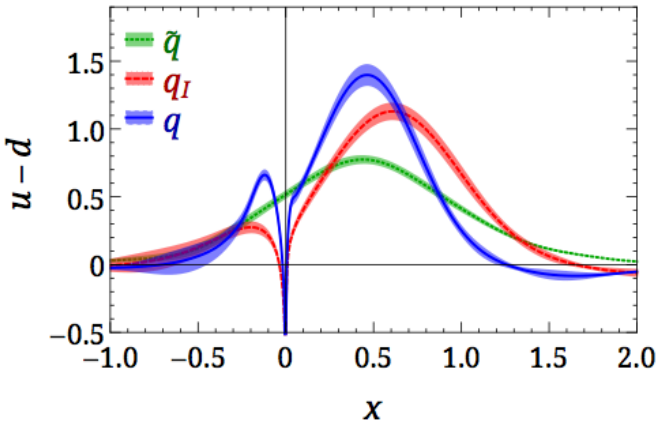
- Twist-4:

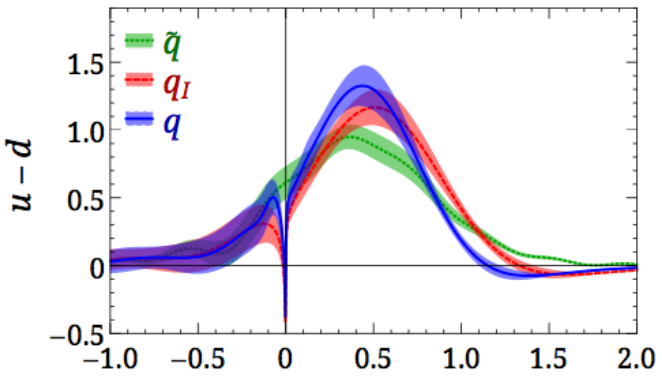
$$q_{tr}(x,\mu^2,P^z) = \frac{\lambda^2}{8\pi}\int_{-\infty}^{\infty}dz\int_0^1\frac{dt}{t}e^{i\frac{zk^z}{t}}\left\langle P\left|\widetilde{\mathcal{O}}_{tr}(z)\right|P\right\rangle$$

$$\widetilde{\mathcal{O}}_{tr}(z) = \int_0^z dz_1\bar{\psi}(0)\left[\gamma^\nu\Gamma\left(0,z_1\right)D_\nu\Gamma\left(z_1,z\right)\right.$$
$$\left.+\int_0^{z_1}dz_2\lambda\cdot\gamma\Gamma\left(0,z_2\right)D^\nu\Gamma\left(z_2,z_1\right)D_\nu\Gamma\left(z_1,z\right)\right]\psi(z\lambda)$$

⇨ Parameterized ($\alpha(x)+\beta(x)/P_z^2$)
Additional complications? E.g.
Radyushkin

Quasi-PDF (green) w/ loop (red) w/ loop + mass (blue)

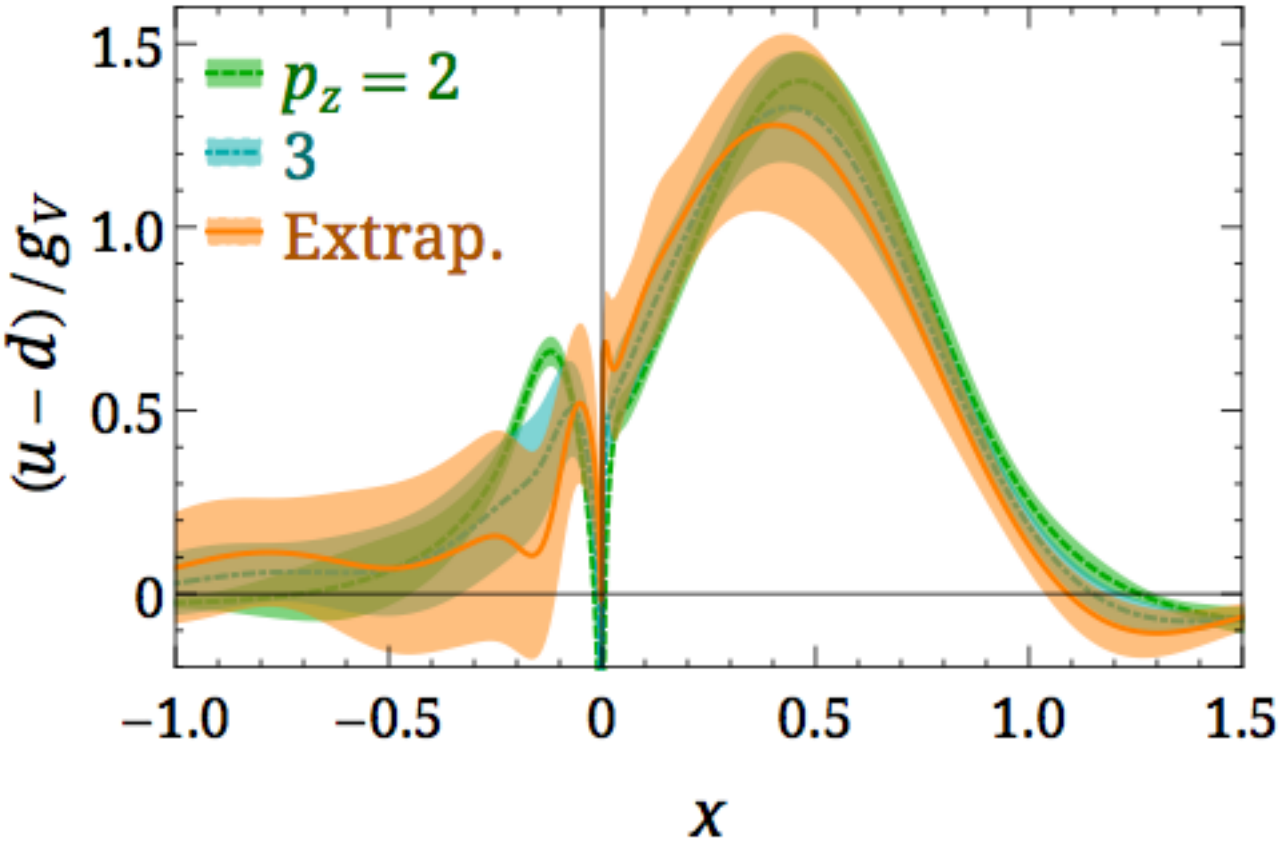


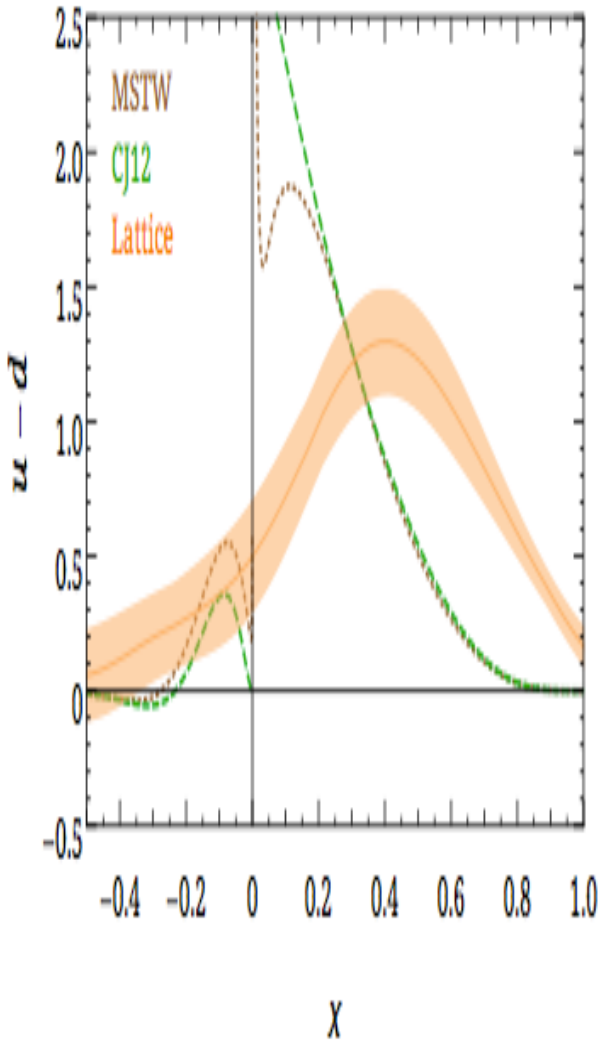


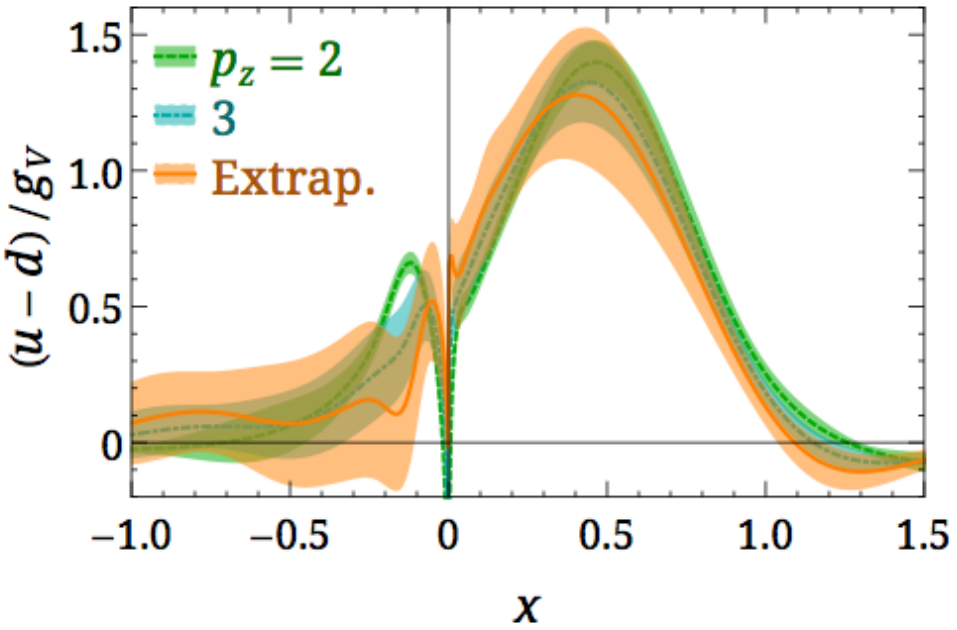
$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$

n = 2 (upper) & 3

Unpolarized Isovector Proton PDF



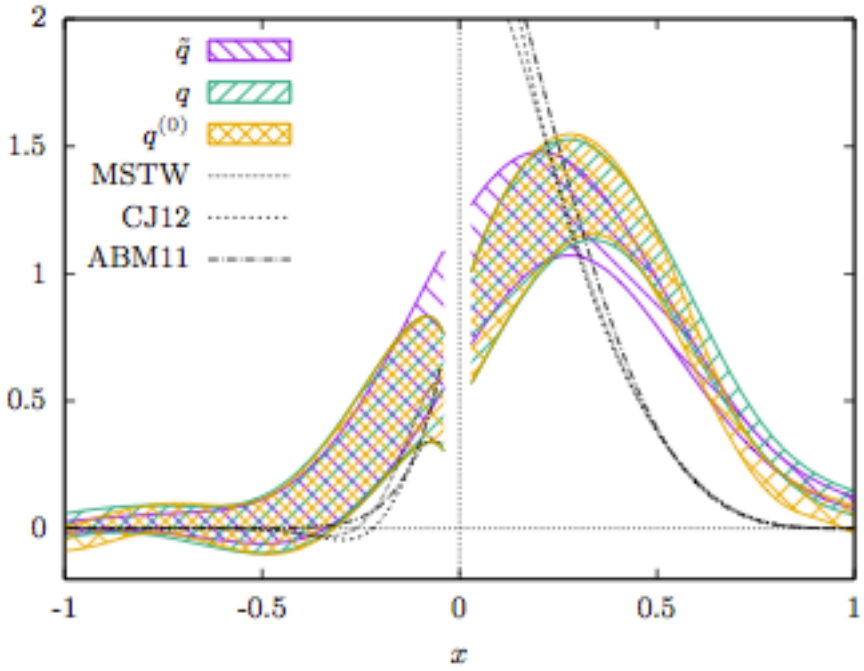


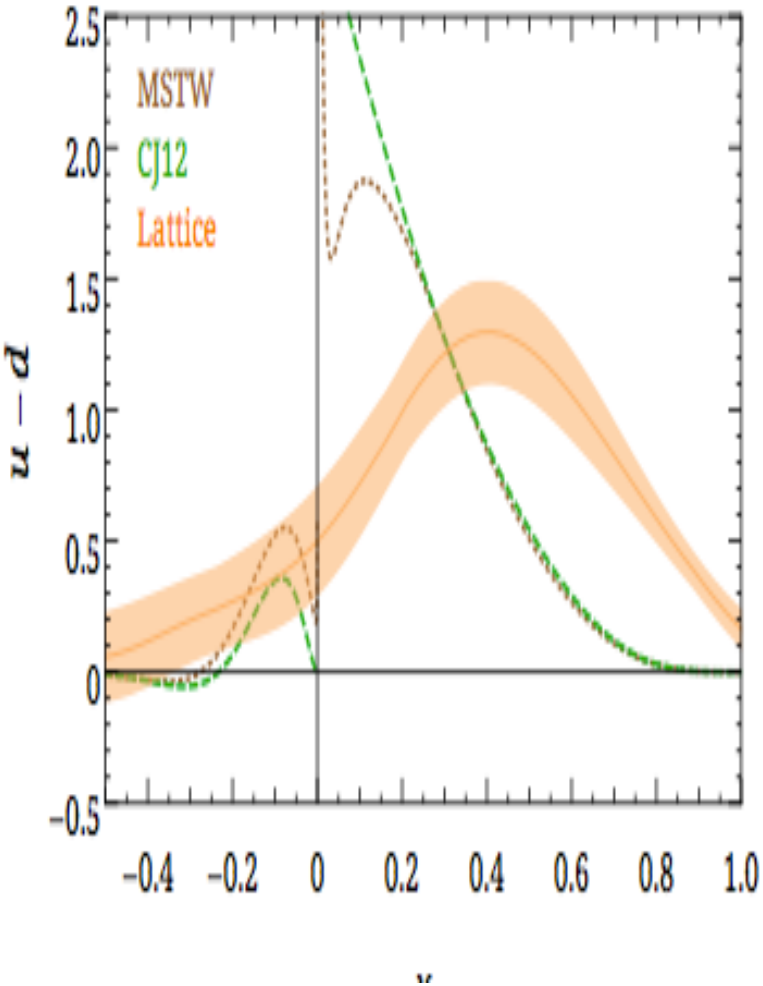


Quark mass effect!

Follow-up works

(Alexandrou et. al.:1504.07455+1610.03689)





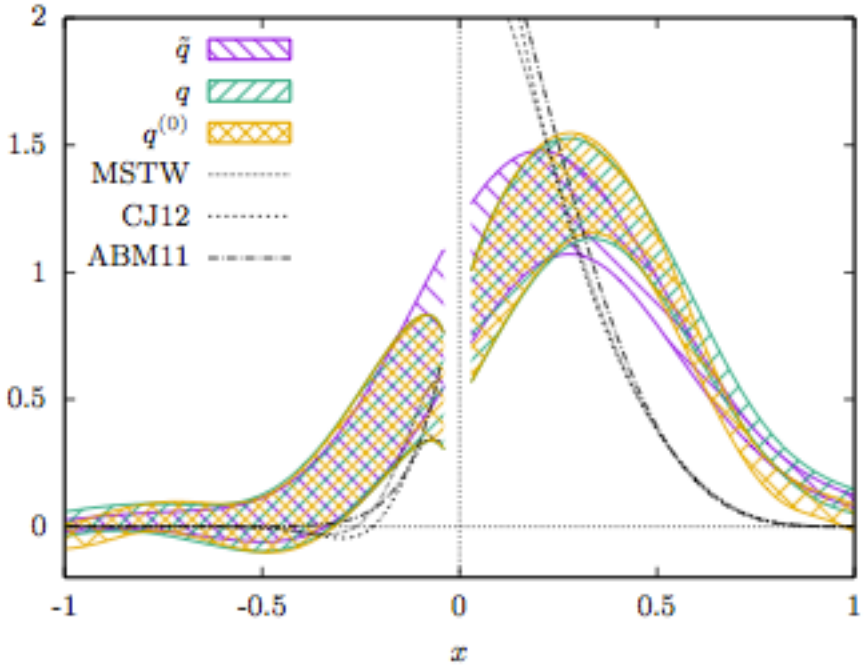
Follow-up works

(Alexandrou et. al.)

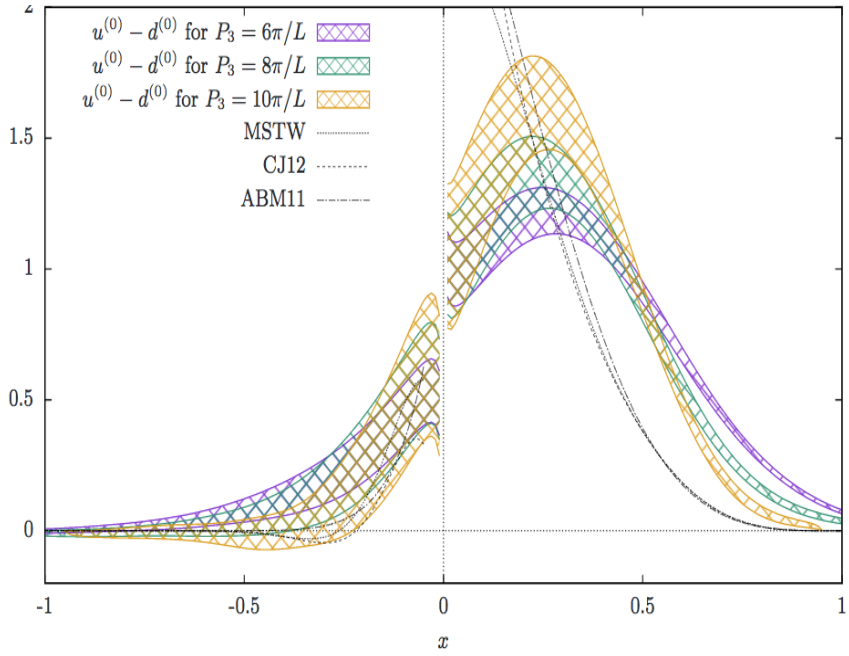
1504.07455

Pz=6π/L

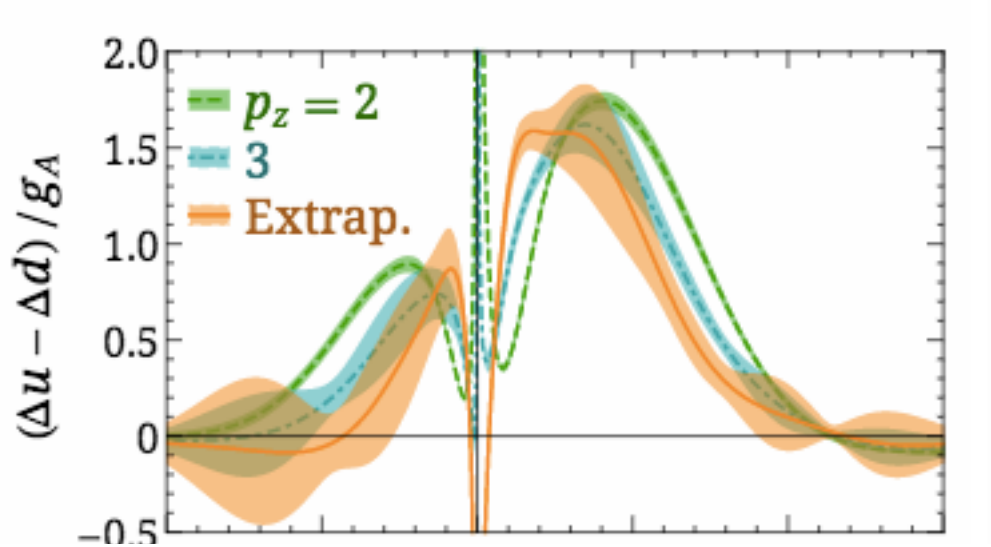
5 steps HYP smearing

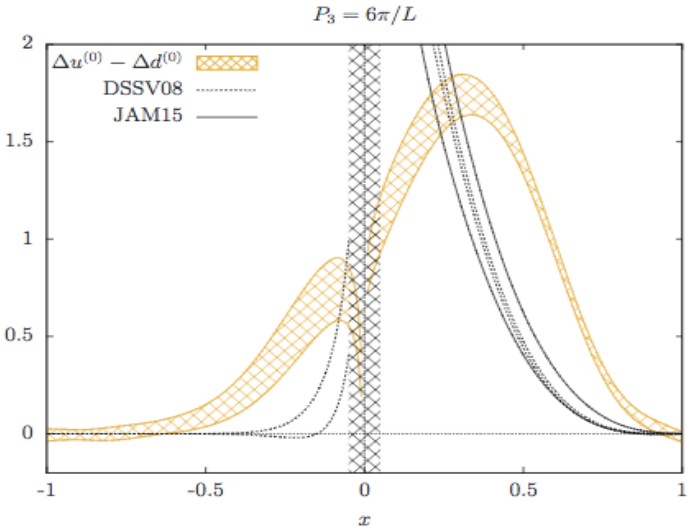


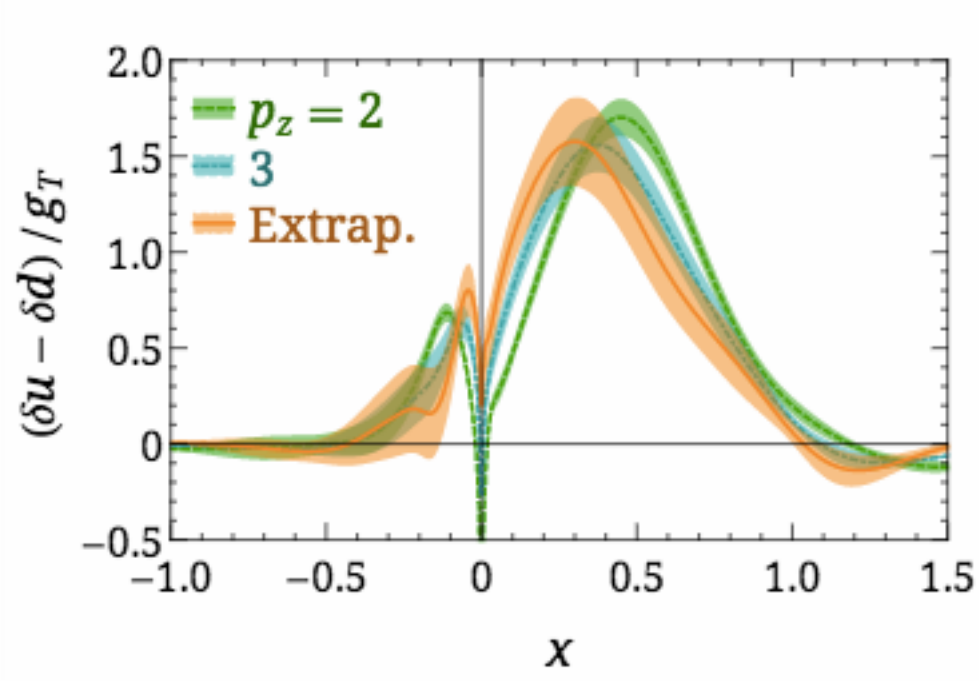
1610.03689

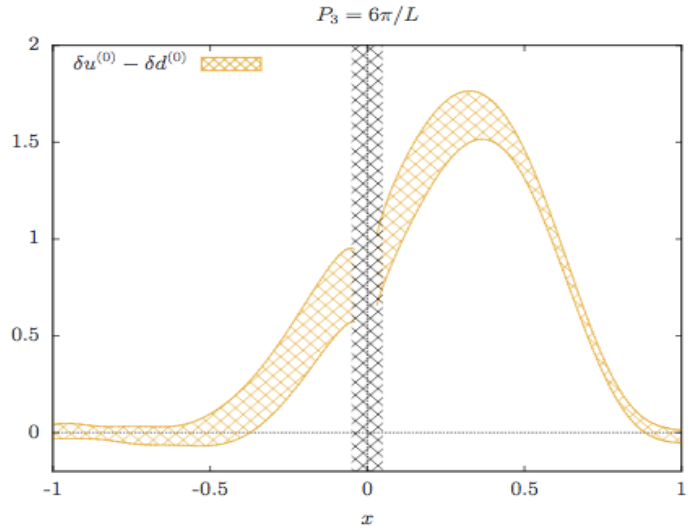


Isovector Proton Helicity and Transversity



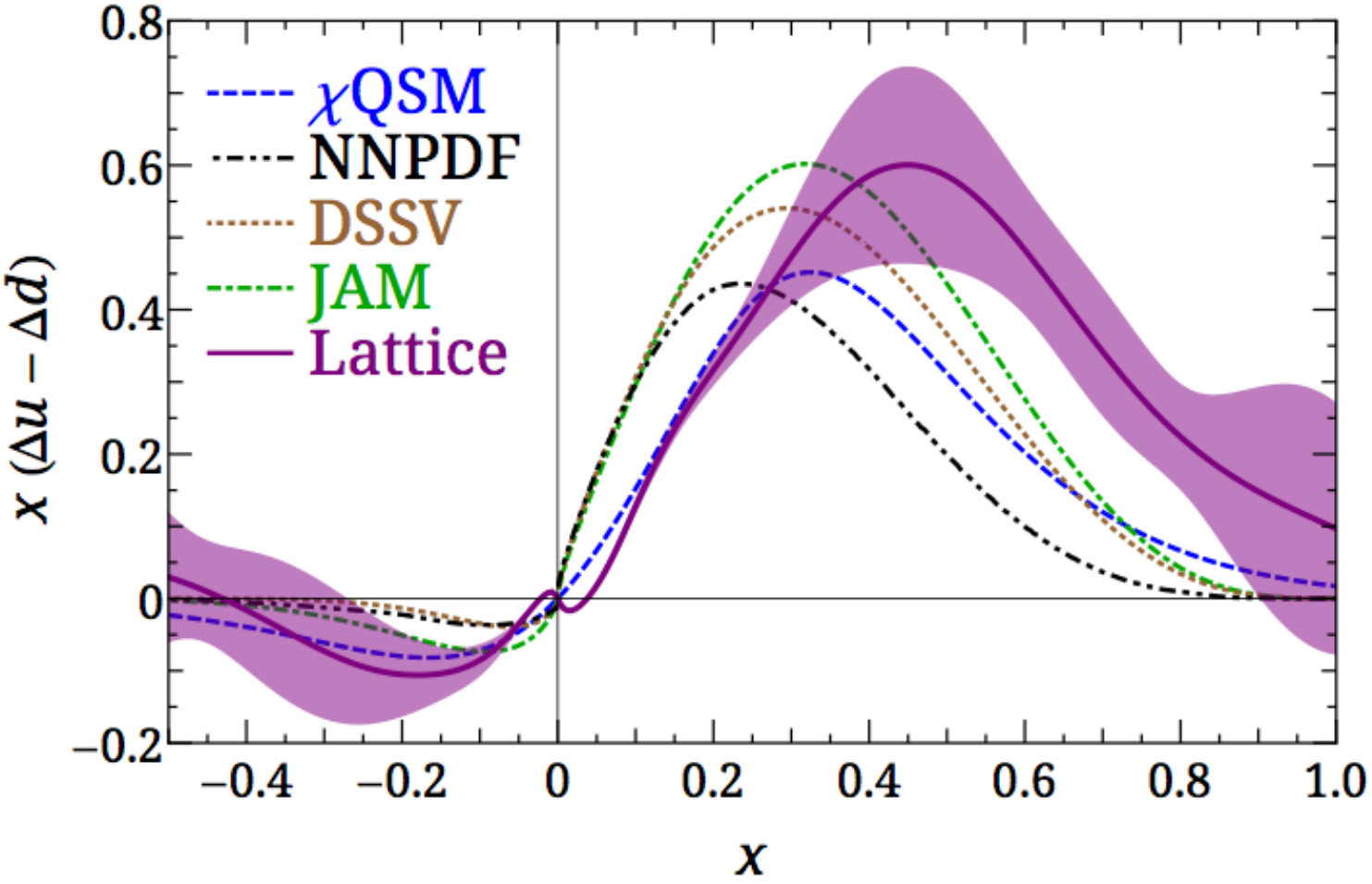




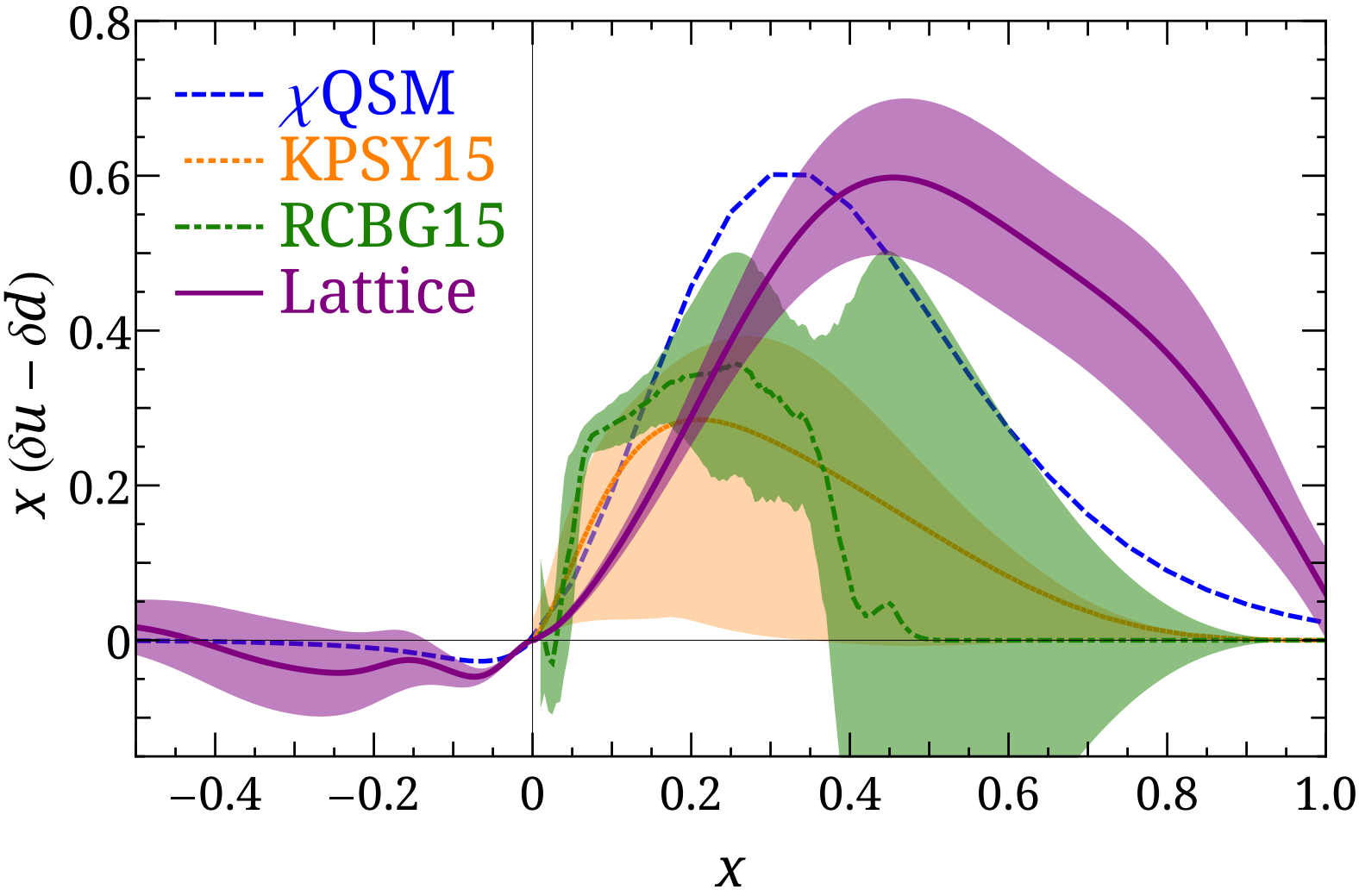


(Alexandrou et.al., 1609.00172)

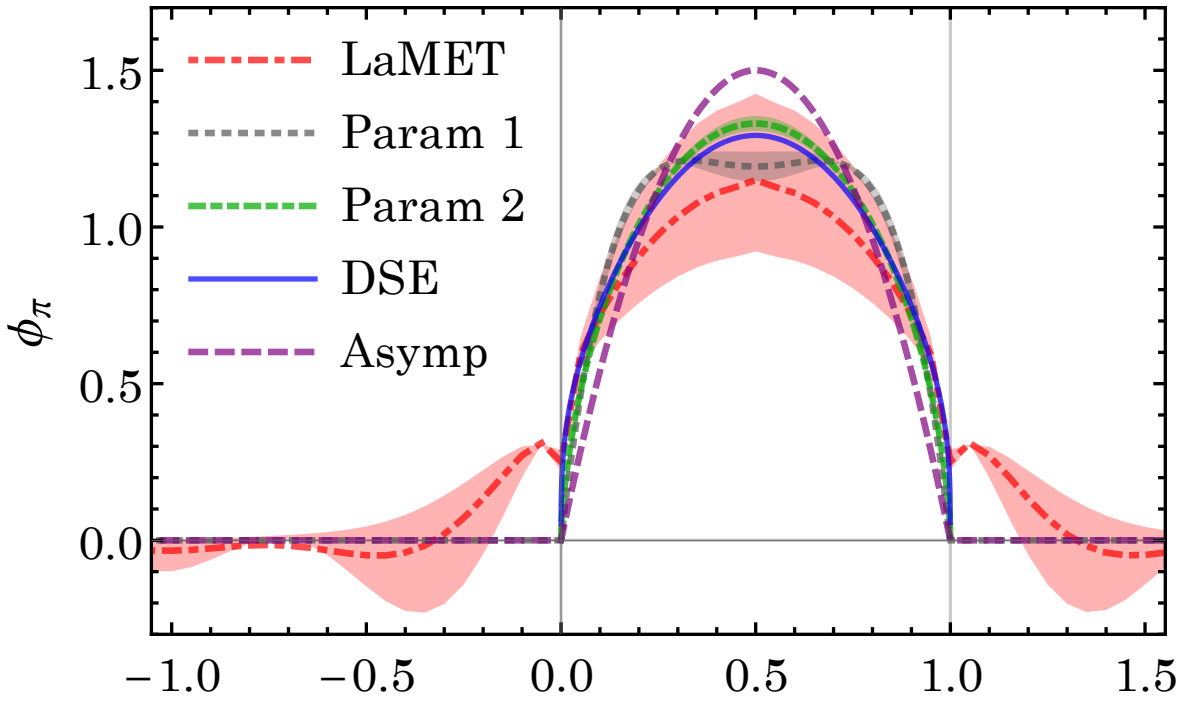
Isovector Proton Helicity



Isovector Proton Transversity



Pion Light Cone DA-Zhang, JWC, Ji, Jin, Lin



$$\delta m \simeq -260 \pm 200 \text{ MeV}$$

$$\phi_\pi(x,\mu) + 3\phi_\eta(x,\mu) = 2[\phi_{K^+}(x,\mu) + \phi_{K^-}(x,\mu)]$$
$$= 2[\phi_{K^0}(x,\mu) + \phi_{\overline{K}^0}(x,\mu)],$$

No leading chiral log

JWC, Iain W. Stewart, Phys.Rev.Lett. 92 (2004) 202001

Outlook

- Further tests (non-singlet): wee partons (smaller quark mass, momentum smearing, NP RI/MOM renormalization), small x: larger boxes, large x: twist-4; factorization proof.

  Know whether it works within 5 years (~20%)?

- Singlet PDF's: s, c, b and gluons

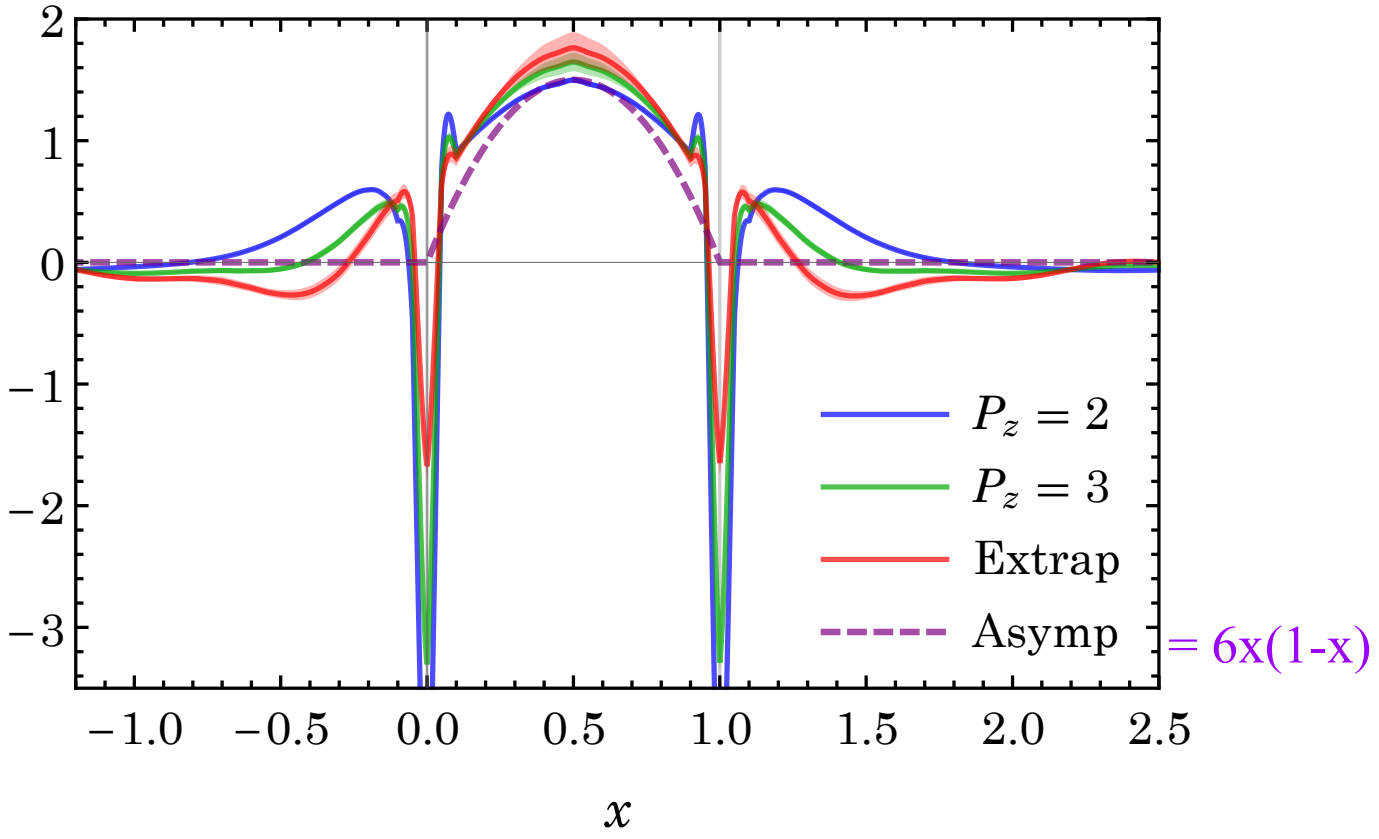
  Additional 3-5 yrs?

- If it works, complimentary to exp.: PDF (isov. sea, small and large x's, non-valence partons), DA, GPD, TMD …

Backup slides

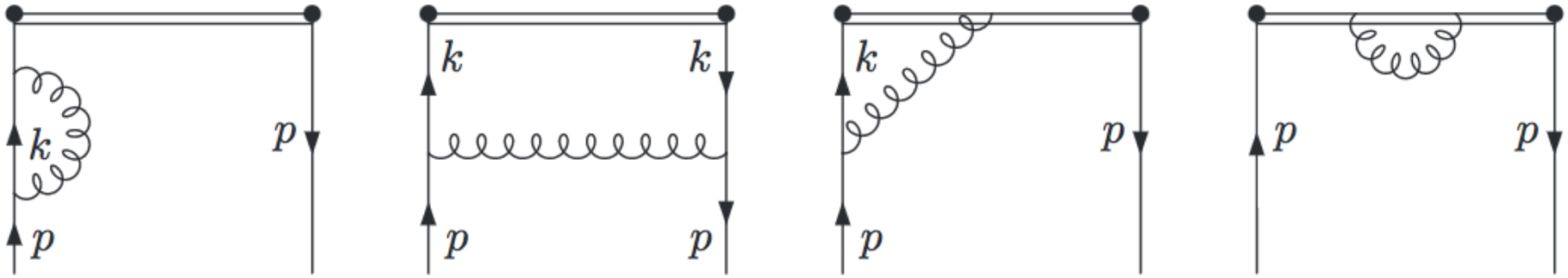
Pion Light Cone Distribution Amplitude

Zhang, JWC, Ji, Jin, Lin



Improved Quasi-PDF's

(Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang)



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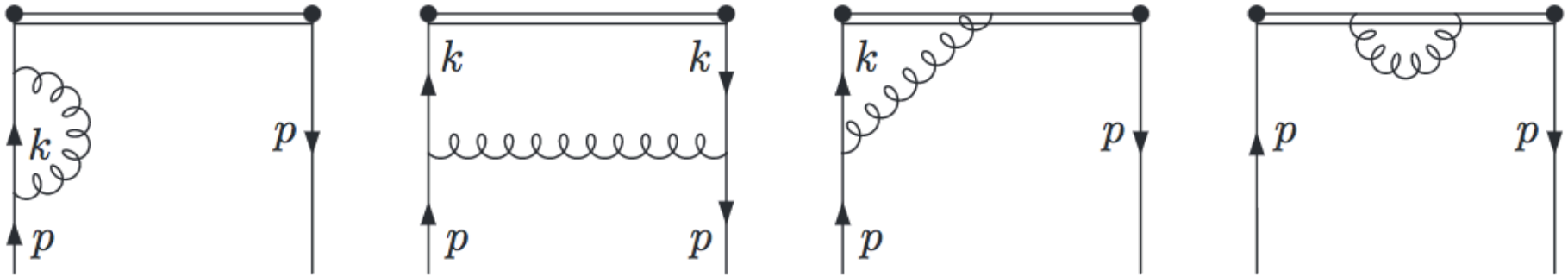
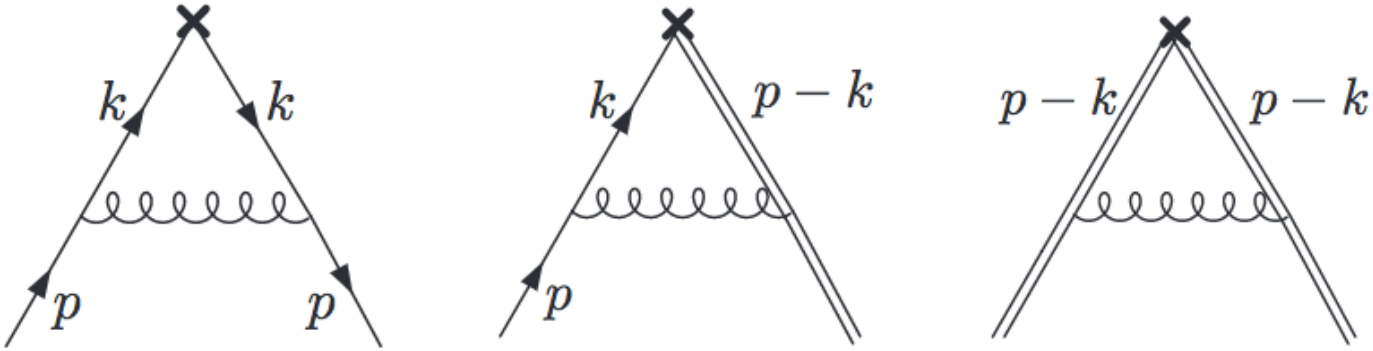


Equivalent to



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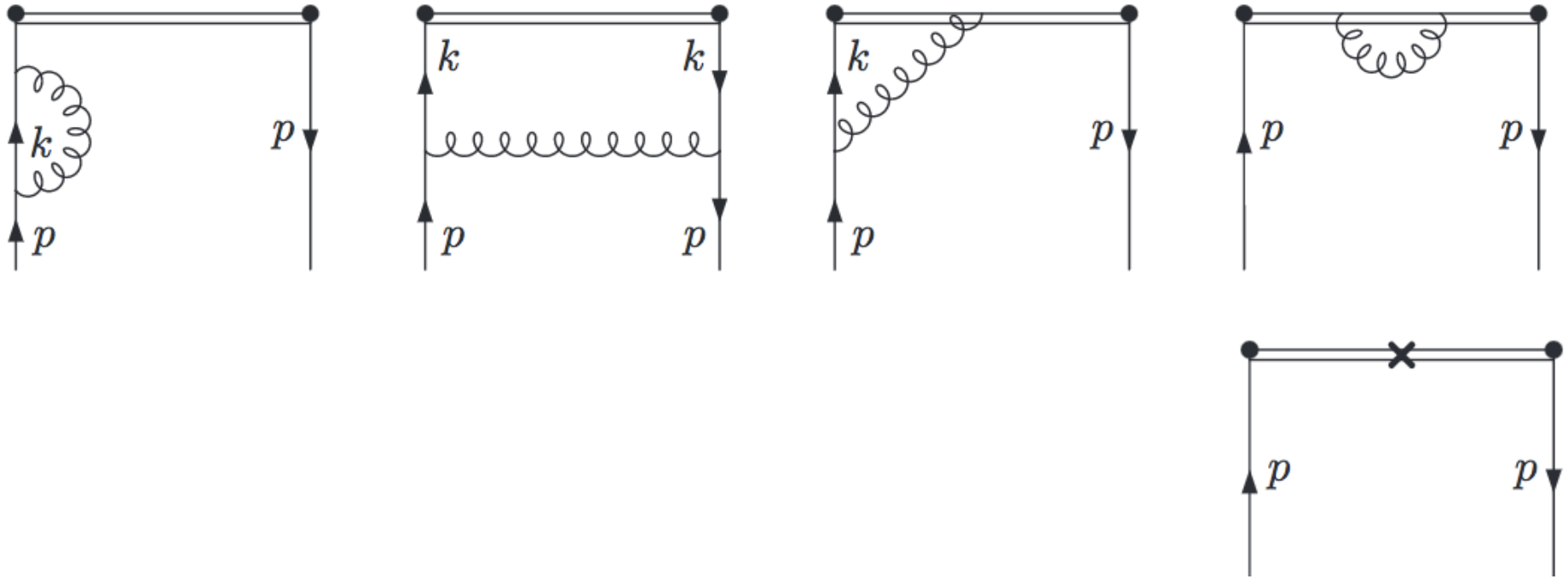


$$\tilde{q}_{\rm imp}(x,\Lambda,p^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z - \delta m|z|} \langle p|\overline{\psi}(0,0_\perp,z)\gamma^z L(z,0)\psi(0)|p\rangle$$

Improved Quasi-PDF's

Ishikawa, Ma, Qiu, Yoshida: x-space

JWC, Ji, Zhang: p-space

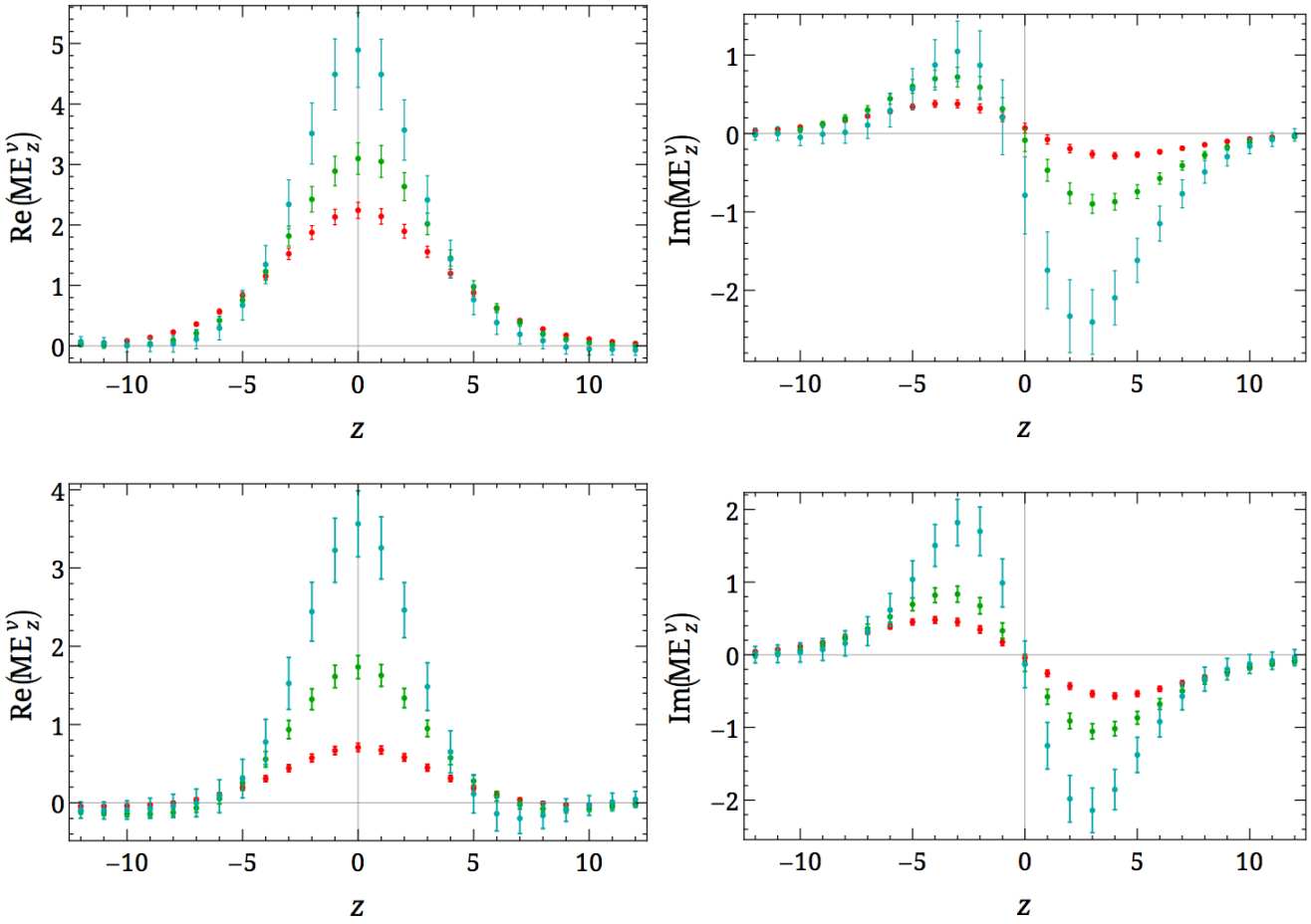
$$\tilde{q}_{\rm imp}(x,a_L,p^z)=\int_{-1}^{1}\frac{dy}{|y|}Z\left(\frac{x}{y},p^z a_L,\frac{\mu}{p^z}\right)q(y,\mu)+\mathcal{O}\left(\Lambda^2_{\rm QCD}/(p^z)^2,M^2/(p^z)^2\right)$$

$$Z\left(\xi\right)=\delta(\xi-1)+\frac{\alpha_s}{2\pi}\left[Z^{(1)}\left(\xi\right)-\int dy Z^{(1)}\left(y\right)\delta(\xi-1)\right]+\ldots$$

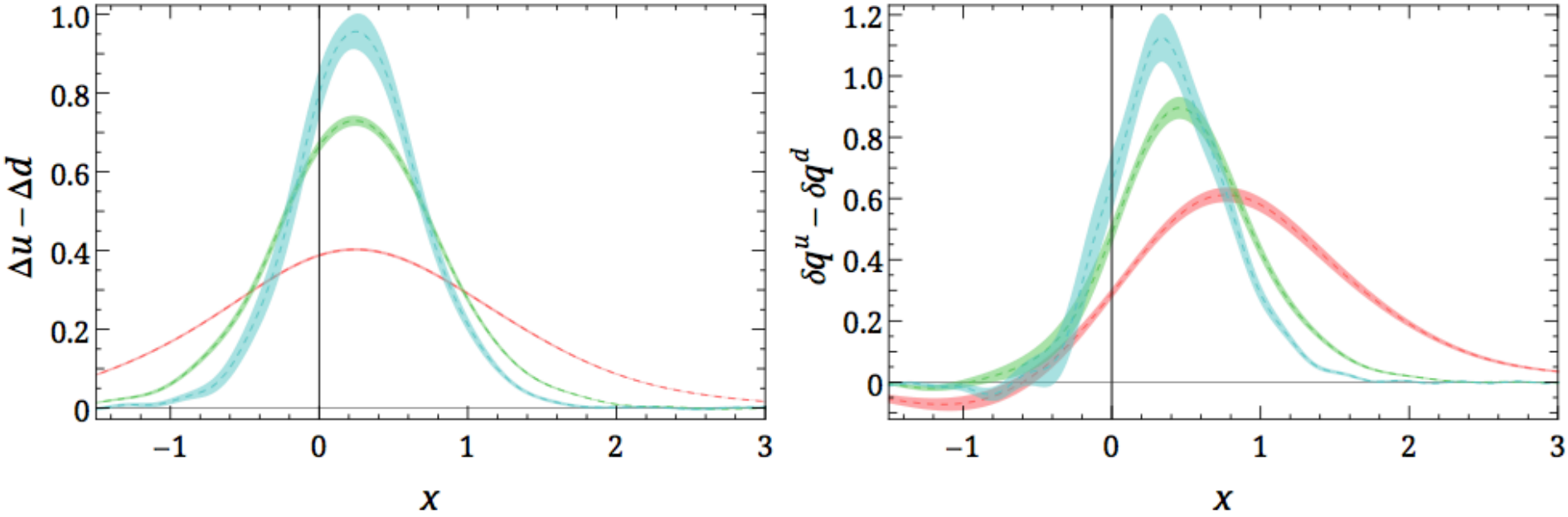
$$Z^{(1)}/C_F=\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1}+1\,, & \xi>1\,,\\ \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(p^z)^2}{\mu^2}+\left(\frac{1+\xi^2}{1-\xi}\right)\ln\left[4\xi(1-\xi)\right]-\frac{2\xi}{1-\xi}+1\,, & 0<\xi<1\,,\\ \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi-1}{\xi}-1\,, & \xi<0\,,\end{cases}$$

In progress: Stewart & Zhang, NP RI/MOM renorm.
+ one-loop RI/MOM MS-bar matching

Helicity and Transversity (isovector)

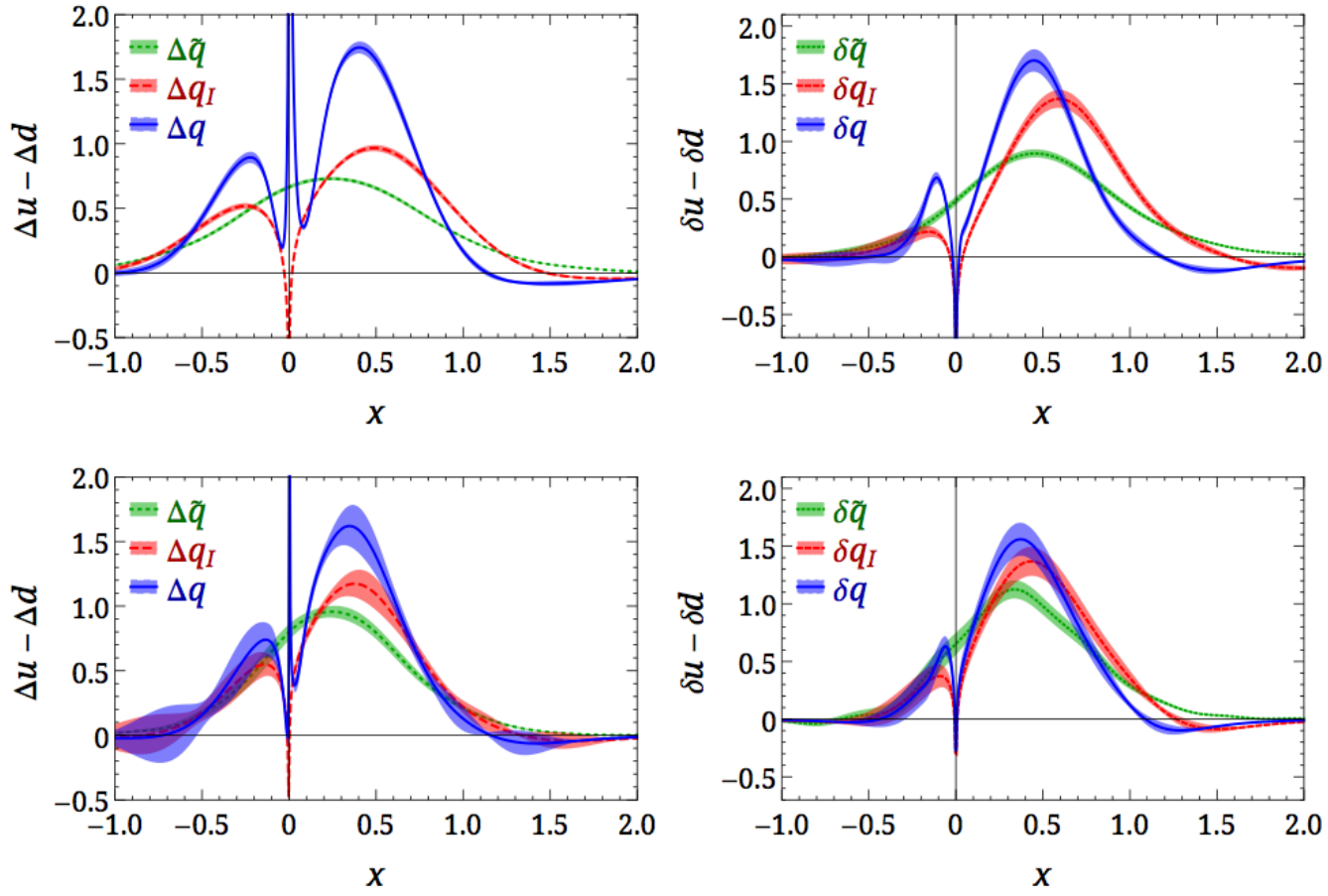


Quasi-PDF (Helicity and Transversity)



$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV \qquad \text{n} = 1, 2, 3.$$

Quasi-PDF (green) w/ loop (red) w/ loop + mass (blue)



$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$

n = 2 (upper) & 3