Three-body hadronic B meson decays in PQCD

Wang Wen-Fei (王文飞)

wangwf@gate.sinica.edu.tw

Collaborators

Li Hsiang-nan (AS)
Lü Cai-Dian (IHEP)
Wang Wei (SJTU)
Hu Hao-Chung (NTU)

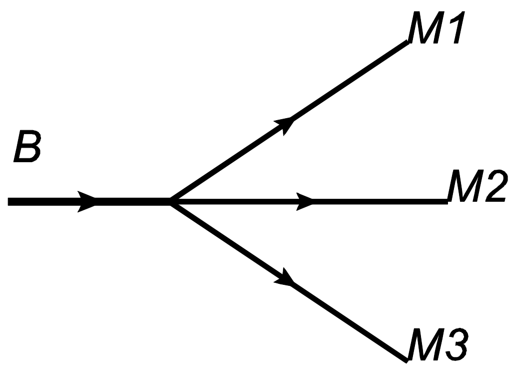
Chung Yuan Christian University

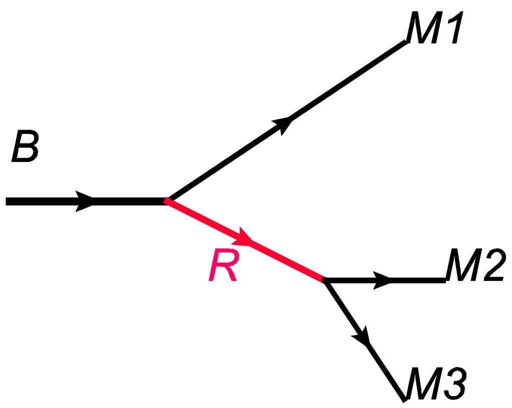
2016-05-17

Outline

- 3-body hadronic B meson decays
- PQCD approach
- Framework based on PQCD
- Quasi-2-body decay modes
- AcdaP – a package for PQCD
- Summary

3-body hadronic B meson decays



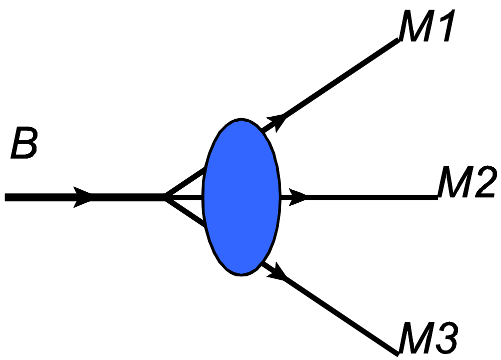


1 → 3 decay mode

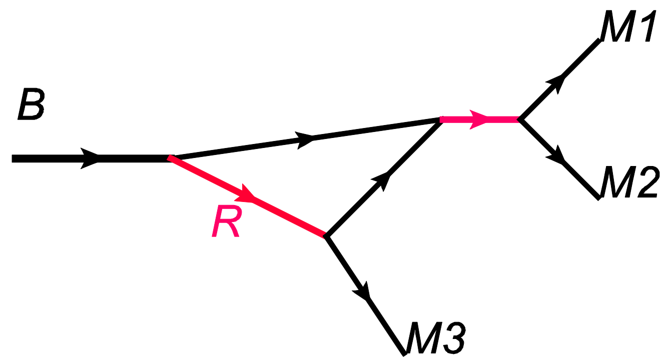
1 → 2 → 3 decay mode

quasi-two-body decay mode

3-body hadronic B meson decays



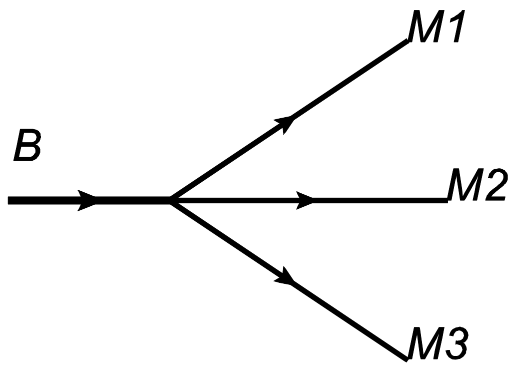
1 → 3 decay mode

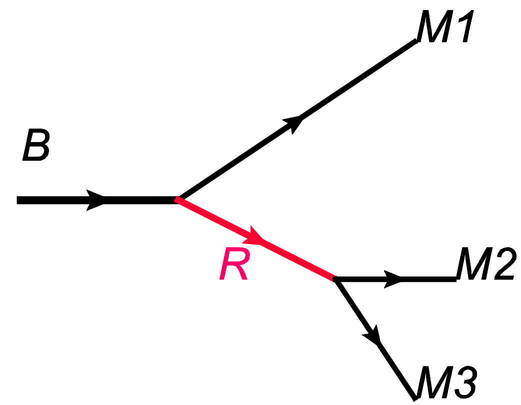


1 → 2 → 3 decay mode

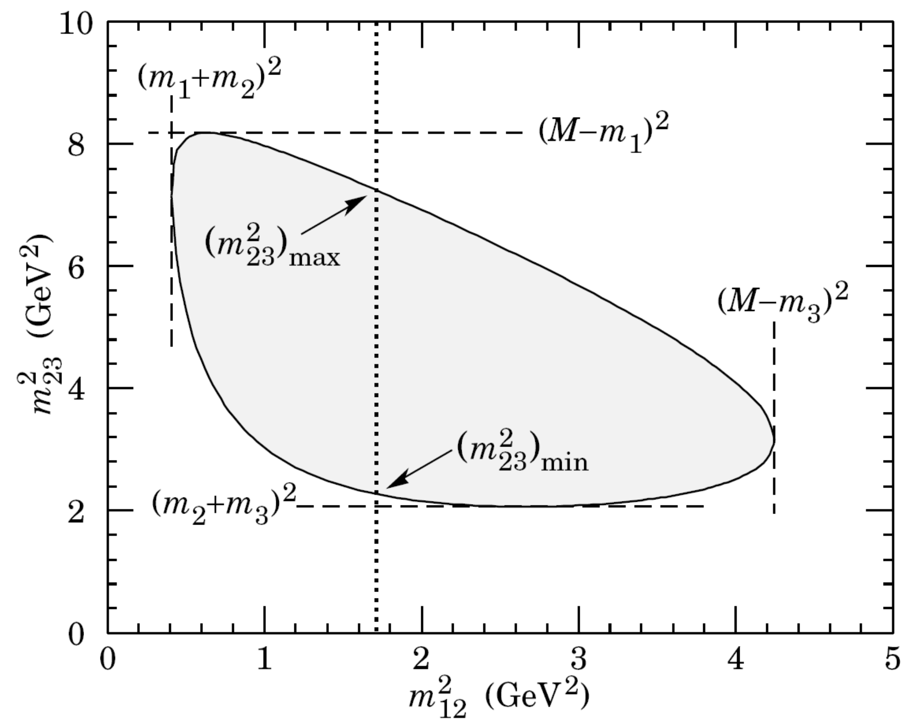
quasi-two-body decay mode

3-body hadronic B meson decays

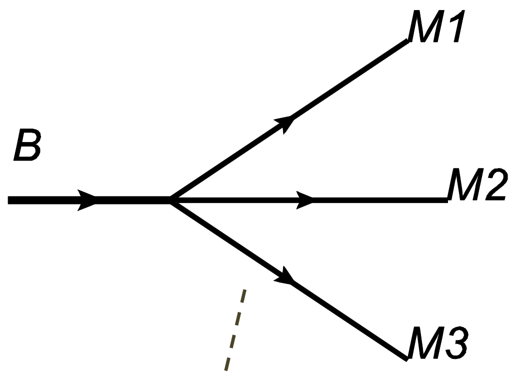


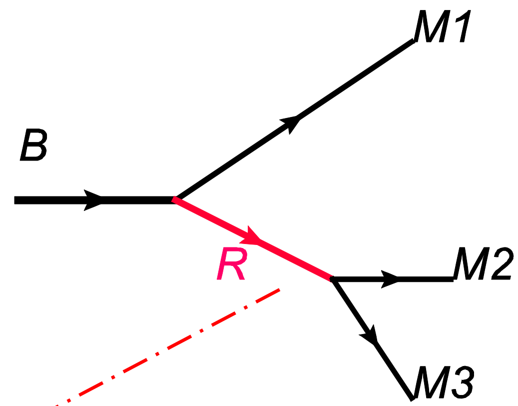


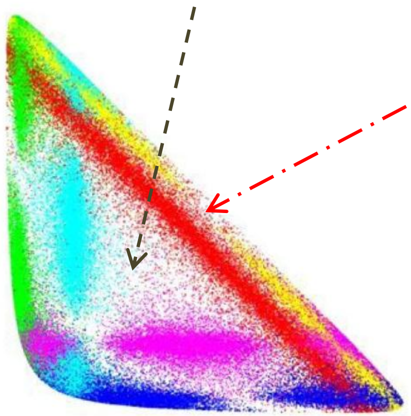
Dalitz Plot



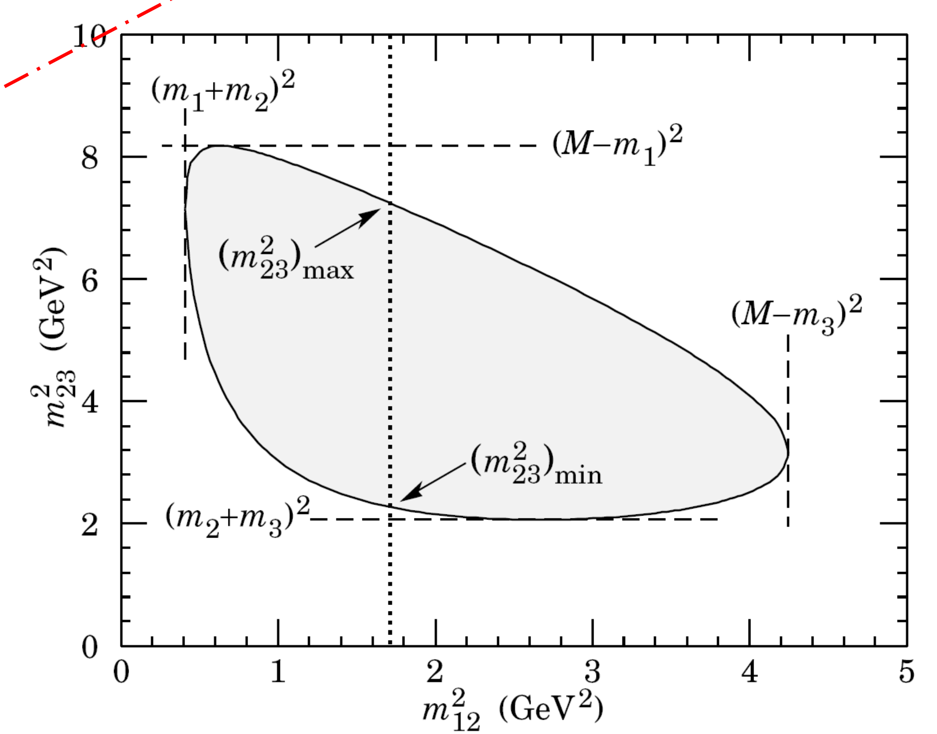
3-body hadronic B meson decays







Dalitz Plot



3-body hadronic B meson decays

Large amounts of data from CDF, D0, Belle, BABAR, LHCb *etc.* Collaborations

BABAR

- Evidence for CP violation in $B^+ \to K^*(892)^+\pi^0$ from a Dalitz plot analysis of $B^+ \to K_S^0\pi^+\pi^0$ decays — arXiv:1501.00705
- Dalitz plot analyses of $B^0 \to D^- D^0 K^+$ and $B^+ \to \overline{D}^0 D^0 K^+$ decays — arXiv:1412.6751
- Study of the $\boldsymbol{K^+}$ $\boldsymbol{K^-}$ invariant-mass dependence of CP asymmetry in $\boldsymbol{B^+ \to K^+K^-K^+}$ decays — arXiv:1305.4218
- Study of CP violation in Dalitz-plot analyses of $B^0 \to K^+K^-K_S^0$, $B^+ \to K^+K^-K^+$, and $B^+ \to K_S^0K_S^0K^+$ — arXiv:1201.5897

LHCb

- **First observation of the rare $B^{+}\to D^{+} K^{+} π^{-}$ decay** — arXiv:1512.02494
- **Studies of the resonance structure in $D^0\to K^0_S K^{\pm}π^{\mp}$ decays** — arXiv:1509.06628
- **Dalitz plot analysis of $B^0 \to \overline{D}^0 π^+π^-$ decays** — arXiv:1505.01710
- **Amplitude analysis of $B^0 \rightarrow \bar{D}^0 K^+ π^-$ decays** — arXiv:1505.01505
- **Observation of the decay $\overline{B}_s^0 \rightarrow ψ(2S) K^+ π^-$** — arXiv:1503.07112
- **First observation and amplitude analysis of the $B^{-}\to D^{+}K^{-}π^{-}$ decay** — arXiv:1503.02995

3-body hadronic B meson decays

QCDF

- **Hai-Yang Cheng**, C.-H. Chen, C.-K. Chua, C. Q. Geng, Y. K. Hsiao, A. Soni, Y. Li, …
- **Xin-Heng Guo**, Ya-Dong Yang, Zhen-Hua Zhang, Gang Lü, Jia-Qi Lei, …
- **Hossein Mehraban**, Mahboobeh Sayahi, …
- **A. Furman**, B.El-Bennich, R. Kaminski, L. Lesniak, B. Loiseau ….
- **Thomas Mannel**, Susanne Kränkl, Javier Virto

3-body hadronic B meson decays

QCDF

Hai-Yang Cheng, C.-H. Chen, C.-K. Chua, C. Q. Geng, Y. K. Hsiao, A. Soni, Y. Li, ...

Xin-Heng Guo, Ya-Dong Yang, Zhen-Hua Zhang, Gang Lü, Jia-Qi Lei, ...

Hossein Mehraban, Mahboobeh Sayahi, ...

A. Furman, B.El-Bennich, R. Kaminski, L. Lesniak, B. Loiseau ....

Thomas Mannel, Susanne Kränkl, Javier Virto

Symmetry

Xiao-Gang He, Guan-Nan Li, Dong Xu

$SU(3)$ **and Isospin Breaking Effects on** $B \to PPP$ **Amplitudes** arXiv:1410.0476

J.L.Rosner, B. Bhattacharyaa,M. Gronaub

U-spin ➡ CP asymmetries in three-body $B^{\pm}$ decays to charged pions and kaons PLB726(2013)337

PQCD

Hsiang-nan Li, Wen-Fei Wang, W. Wang, C.-D. Lü, Z.-J. Xiao, H.-C. Hu, C.-H. Chen, ...

PQCD approach

➔ G. Lepage PRD22-2157(1980)

…. nonperturbative dynamics of a high-energy QCD process, characterized by a large scale Q, either cancel or is absorbed into hadron distribution amplitudes. The remaining part, being infrared finite, is calculable in perturbation theory …

$$\frac{1}{l^2}\cdot\frac{1}{p_1^2-m_B^2}\sim\frac{1}{m_B^4x_1x_2(1-x_2)}$$

$$\rightarrow\frac{1}{((1-x_2)m_B^2+k_{2T}^2)(x_1x_2m_B^2+(k_{1T}^2-k_{2T}^2)^2)}$$

PQCD approach

Sudakov factors in PQCD

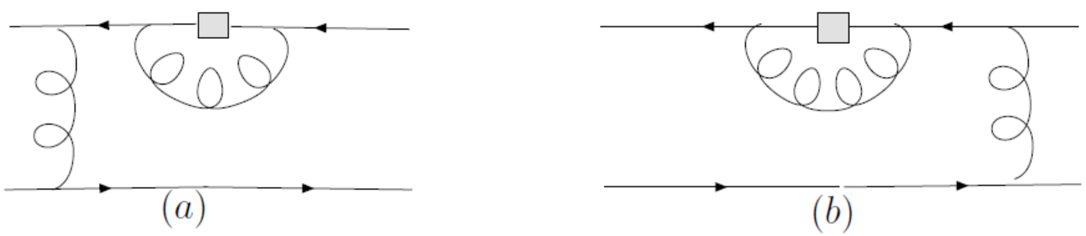
PRD66-094010, PLB369-137, NPB381-129

$$k_T \longrightarrow \exp[-s(x,b,Q)]$$

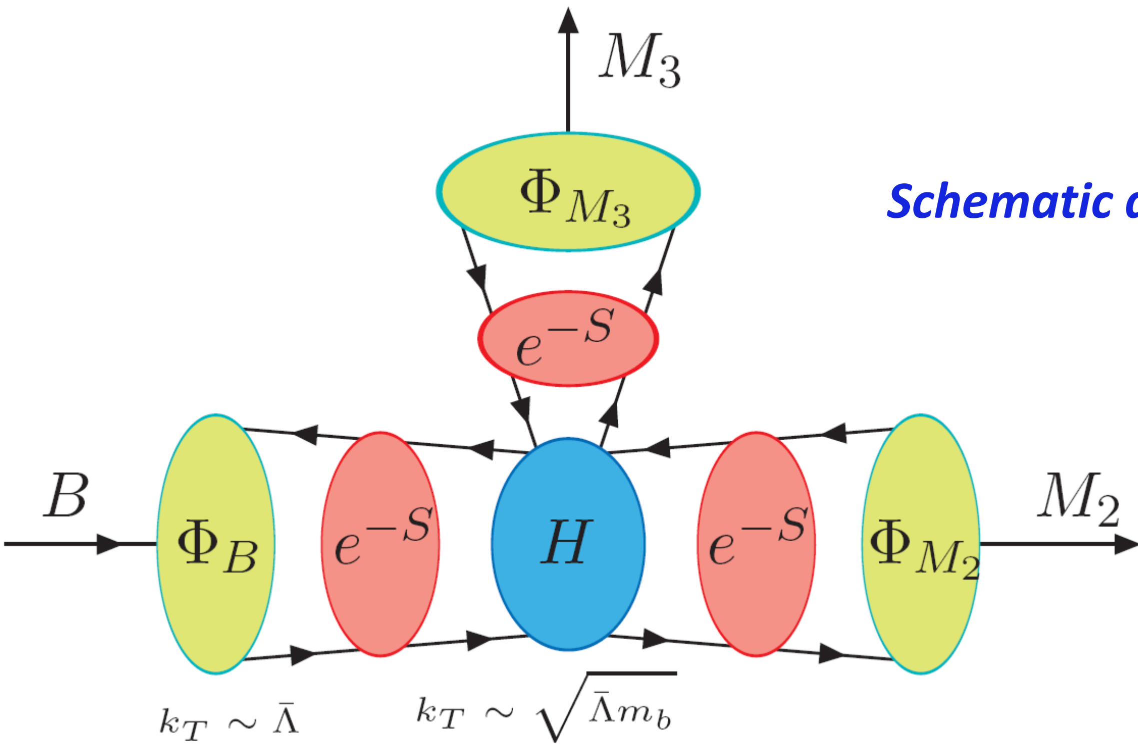
$$\begin{aligned} S_B &= s\left(x_B\frac{m_B}{\sqrt{2}}, b_B\right) + \frac{5}{3}\int_{1/b_B}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\ , \\ S_M &= s\left(z\frac{m_B}{\sqrt{2}}, b\right) + s\left((1-z)\frac{m_B}{\sqrt{2}}, b\right) + 2\int_{1/b}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\ , \end{aligned}$$

$$S_t(x) = \int_{a-i\infty}^{a+i\infty}\frac{dN}{2\pi i}\frac{S_t(N)}{N}(1-x)^{-N} = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}(1+c)}[x(1-x)]^c$$

c: 0.3～0.4



PQCD approach



Schematic diagram for PQCD

PQCD

1. **Semileptonic B meson decays**
2. **Two-body B meson decays (pure annihilation decays)**
3. **Three-body hadronic B meson decays**

Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Physics Letters B 561 (2003) 258–265

PHYSICS LETTERS B

www.elsevier.com/locate/npe

Three-body nonleptonic B decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li

Abstract

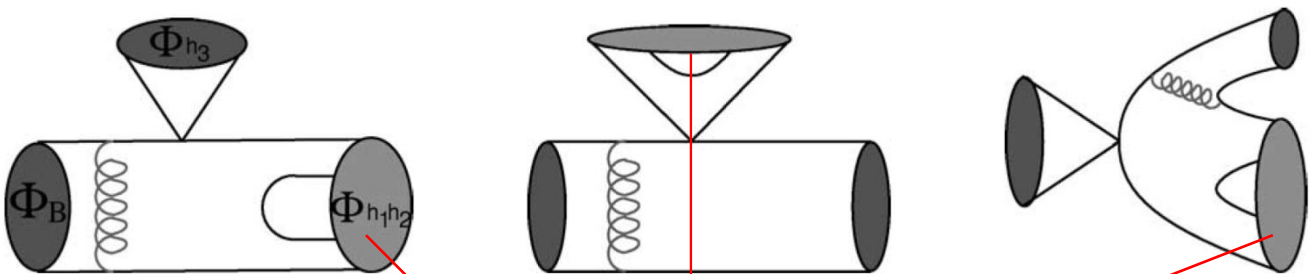
We develop perturbative QCD formalism for three-body nonleptonic B meson decays. Leading contributions are identified by defining power counting rules for various topologies of amplitudes. The analysis is simplified into the one for two-body decays by introducing two-meson distribution amplitudes. This formalism predicts both nonresonant and resonant contributions, and can be generalized to baryonic decays.

Framework (for 3-body hadronic decays)

Physics Letters B 561 (2003) 258–265

Three-body nonleptonic *B* decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li



two-meson distribution amplitudes

↓

TMDA

$$B \to h_1 h_2 h_3$$

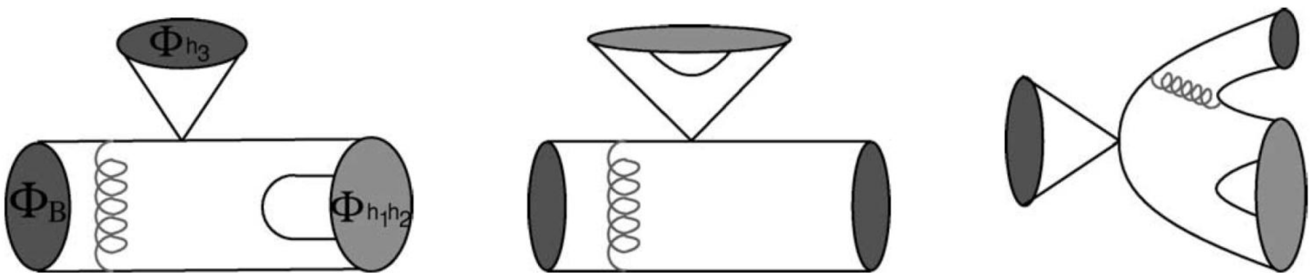
$$\Rightarrow \quad \mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1 h_2} \otimes \Phi_{h_3}$$

Framework (for 3-body hadronic decays)

Physics Letters B 561 (2003) 258–265

Three-body nonleptonic *B* decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li



$$\frac{1}{\sqrt{2N_c}}\left[\not{p}\,\Phi_v(z,\zeta,w^2)+w\Phi_s(z,\zeta,w^2)+\frac{\not{p}_1\not{p}_2-\not{p}_2\not{p}_1}{w(2\zeta-1)}\Phi_t(z,\zeta,w^2)\right]$$

← **TMDA**

$$\begin{cases}\Phi_{v,t}(z,\zeta,w^2)=\dfrac{3F_{\pi,t}(w^2)}{\sqrt{2N_c}}z(1-z)(2\zeta-1),\\[2ex] \Phi_s(z,\zeta,w^2)=\dfrac{3F_s(w^2)}{\sqrt{2N_c}}z(1-z),\end{cases}$$

$$F_\pi^{(nr)}(w^2)=\frac{m^2}{w^2+m^2},\qquad F_{s,t}^{(nr)}(w^2)=\frac{m_0m^2}{w^3+m_0m^2},$$

$$F_{\pi,s,t}^{(r)}(w^2)=\frac{M_V^2}{\sqrt{(w^2-M_V^2)^2+\Gamma_V^2w^2}}-\frac{M_V^2}{w^2+M_V^2},$$

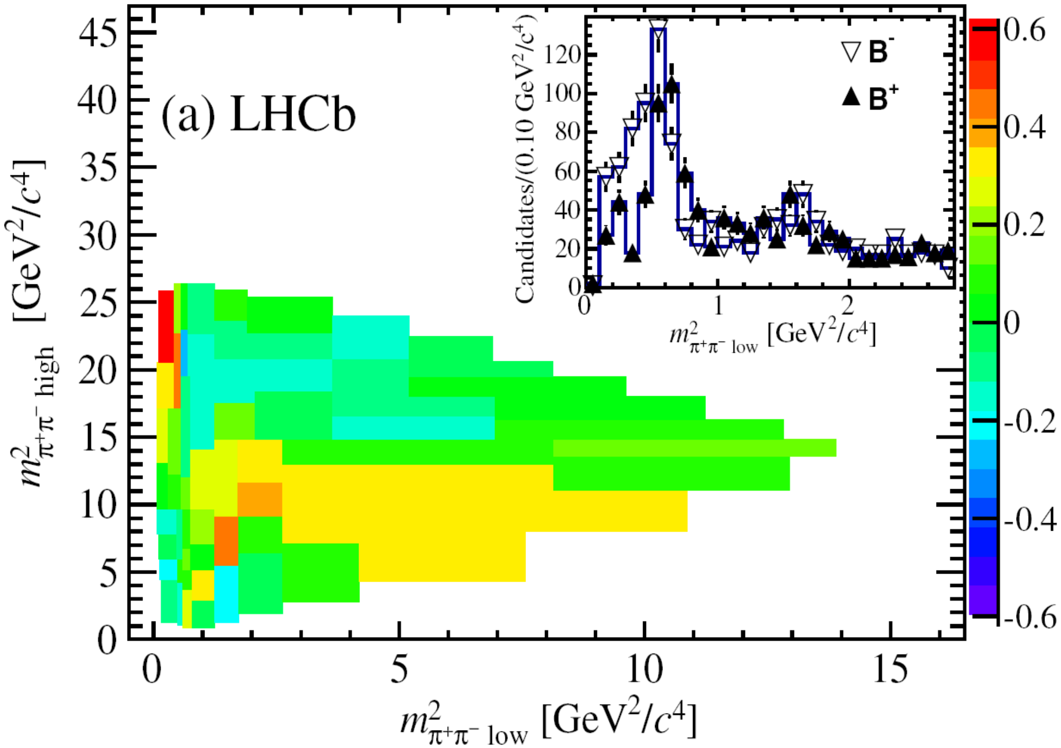
Framework (for PRD89-074031(2014))

PHYSICAL REVIEW D **89**, 074031 (2014)

Direct *CP* asymmetries of three-body *B* decays in perturbative QCD

Wen-Fei Wang,[1,*] Hao-Chung Hu,[2,3,†] Hsiang-nan Li,[3,4,5,‡] and Cai-Dian Lü[1,§]

LHCb PRL112-011801(2014)



Framework (for PRD89-074031(2014))

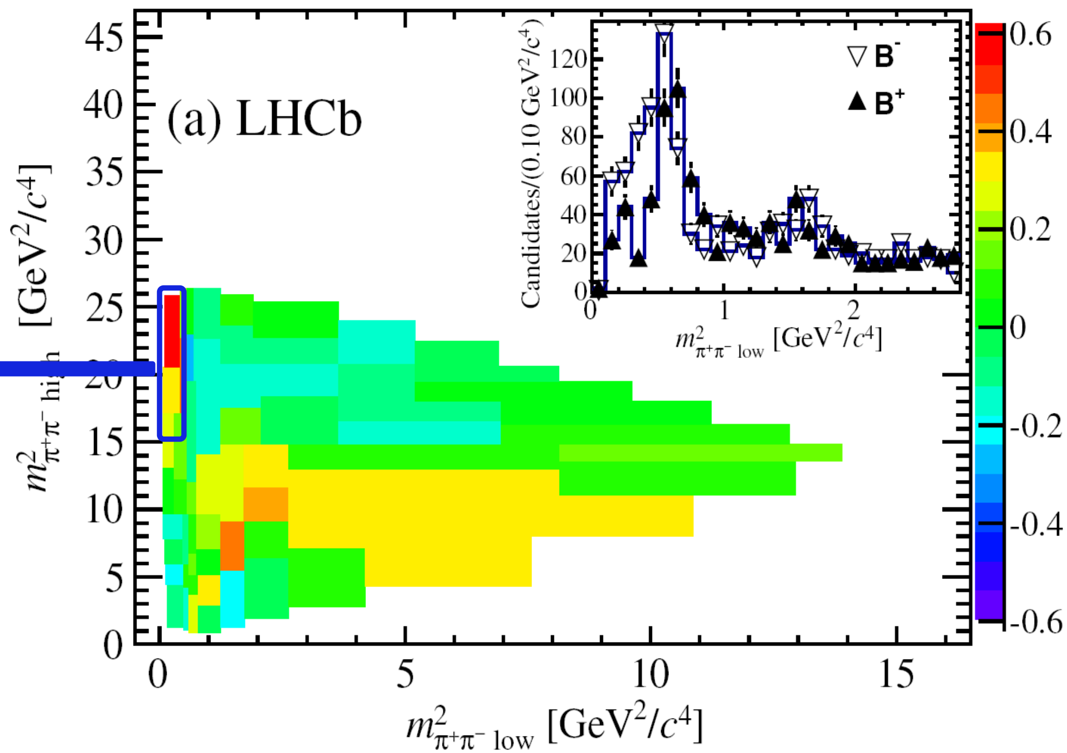
PHYSICAL REVIEW D **89**, 074031 (2014)

Direct *CP* asymmetries of three-body *B* decays in perturbative QCD

Wen-Fei Wang,[1,*] Hao-Chung Hu,[2,3,†] Hsiang-nan Li,[3,4,5,‡] and Cai-Dian Lü[1,§]

$$A_{CP}^{\text{reg}}(B^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm})$$
$$= 0.584 \pm 0.082 \pm 0.027 \pm 0.007$$

$$\left\{ \begin{array}{l} m^2_{\pi^+\pi^-\ \text{low}} < 0.4\ \text{GeV}^2/c^4 \\ m^2_{\pi^+\pi^-\ \text{high}} > 15\ \text{GeV}^2/c^4 \end{array} \right\}$$



LHCb PRL112-011801(2014)

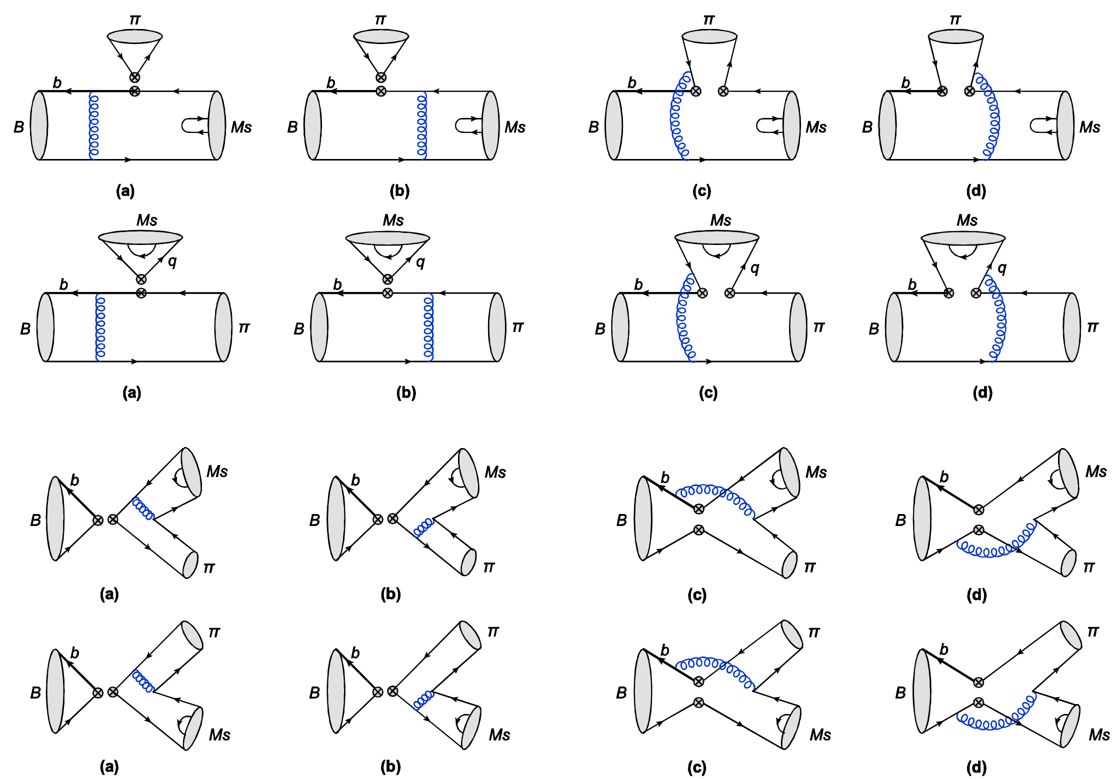
Framework (for PRD89-074031(2014))

PHYSICAL REVIEW D **89**, 074031 (2014)

Direct *CP* asymmetries of three-body *B* decays in perturbative QCD

Wen-Fei Wang,[1,*] Hao-Chung Hu,[2,3,†] Hsiang-nan Li,[3,4,5,‡] and Cai-Dian Lü[1,§]

$$B^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$$



Framework (for PRD89-074031(2014))

PHYSICAL REVIEW D **89,** 074031 (2014)

Direct *CP* asymmetries of three-body *B* decays in perturbative QCD

Wen-Fei Wang,[1,*] Hao-Chung Hu,[2,3,†] Hsiang-nan Li,[3,4,5,‡] and Cai-Dian Lü[1,§]

$$\frac{1}{\sqrt{2N_c}}\left[\not{p}\,\Phi_v(z,\zeta,w^2) + w\Phi_s(z,\zeta,w^2) + \frac{\not{p}_1\not{p}_2 - \not{p}_2\not{p}_1}{w(2\zeta-1)}\Phi_t(z,\zeta,w^2)\right] \longleftarrow \textbf{TMDA}$$

$$\begin{cases} \phi^{v,t}_{\pi\pi}(z,\zeta,\omega^2) = \dfrac{3F_{\pi,t}(\omega^2)}{\sqrt{2N_c}} z(1-z)(2\zeta-1), \\ \phi^{s}_{\pi\pi}(z,\zeta,\omega^2) = \dfrac{3F_{s}(\omega^2)}{\sqrt{2N_c}} z(1-z), \end{cases}$$

$$\begin{cases} F_\pi(w^2) = \dfrac{m^2\exp[i\delta^1_1(w)]}{w^2+m^2}, \\ F_t(w^2) = \dfrac{m^\pi_0 m^2\exp[i\delta^1_1(w)]}{w^3+m^\pi_0 m^2}, \\ F_s(w^2) = \dfrac{m^\pi_0 m^2\exp[i\delta^0_0(w)]}{w^3+m^\pi_0 m^2}, \end{cases}$$

$$2m_\pi < w < 0.7\ \text{GeV} \Longrightarrow \delta^0_0(w) = \pi(w-2m_\pi), \qquad \delta^1_1(w) = 1.4\pi(w-2m_\pi)^2,$$

Framework (for PRD89-074031(2014))

PHYSICAL REVIEW D **89**, 074031 (2014)

Direct *CP* asymmetries of three-body *B* decays in perturbative QCD

Wen-Fei Wang,[1,*] Hao-Chung Hu,[2,3,†] Hsiang-nan Li,[3,4,5,‡] and Cai-Dian Lü[1,§]

$$\frac{1}{\sqrt{2N_c}}\left[\not{p}\Phi_v(z,\zeta,w^2)+w\Phi_s(z,\zeta,w^2)+\frac{\not{p}_1\not{p}_2-\not{p}_2\not{p}_1}{w(2\zeta-1)}\Phi_t(z,\zeta,w^2)\right]$$ ← **TMDA**

$$A_{CP}^{\rm reg}(B^\pm\to\pi^+\pi^-\pi^\pm)$$

$$=0.519^{+0.124}_{-0.219}(\omega_B)^{+0.108}_{-0.091}(a_2^\pi)^{+0.027}_{-0.032}(m_0^\pi),$$ PQCD

$$=0.584\pm0.082\pm0.027\pm0.007$$ LHCb

$$\begin{cases} F_\pi(w^2)=\dfrac{m^2\exp[i\delta_1^1(w)]}{w^2+m^2}, \\ F_t(w^2)=\dfrac{m_0^\pi m^2\exp[i\delta_1^1(w)]}{w^3+m_0^\pi m^2}, \\ F_s(w^2)=\dfrac{m_0^\pi m^2\exp[i\delta_0^0(w)]}{w^3+m_0^\pi m^2}, \end{cases}$$

$2m_\pi < w < 0.7$ GeV ⟹ $\delta_0^0(w)=\pi(w-2m_\pi)$, $\delta_1^1(w)=1.4\pi(w-2m_\pi)^2$,

Framework (for PRD89-074031(2014))

PHYSICAL REVIEW D **89**, 074031 (2014)

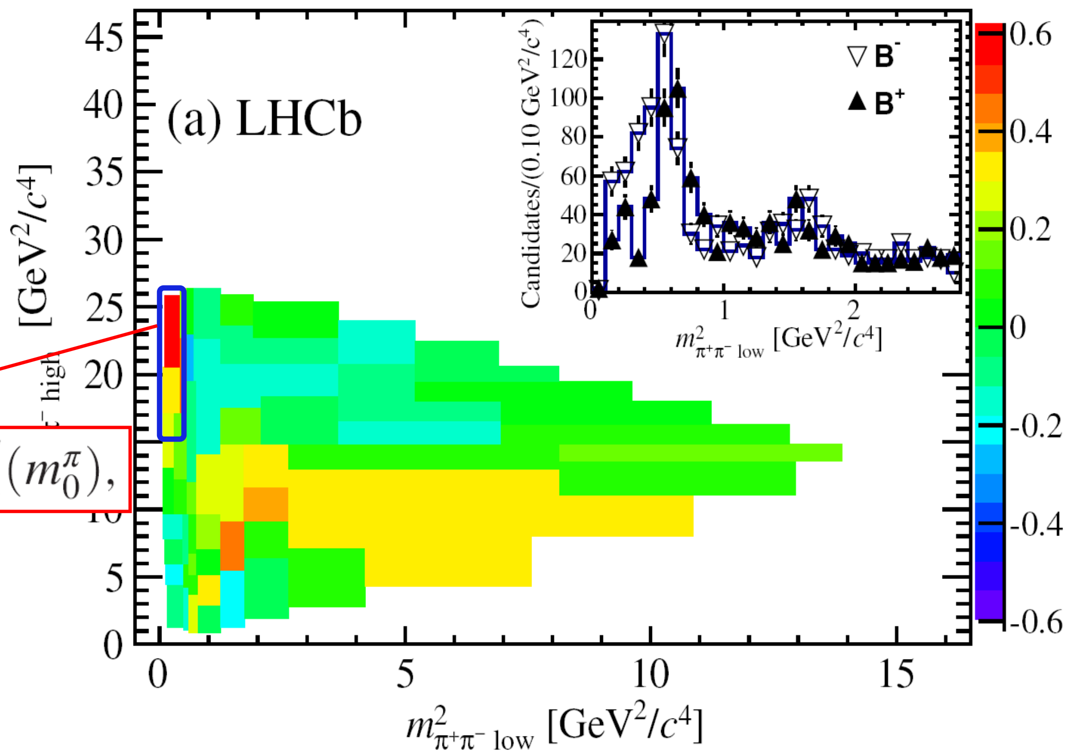
Direct *CP* asymmetries of three-body *B* decays in perturbative QCD

Wen-Fei Wang,[1,*] Hao-Chung Hu,[2,3,†] Hsiang-nan Li,[3,4,5,‡] and Cai-Dian Lü[1,§]

$$A_{CP}^{\rm reg}(B^\pm \to \pi^+\pi^-\pi^\pm)$$

$$= 0.519^{+0.124}_{-0.219}(\omega_B)^{+0.108}_{-0.091}(a_2^\pi)^{+0.027}_{-0.032}(m_0^\pi),$$

$$= 0.584 \pm 0.082 \pm 0.027 \pm 0.007$$



Framework (for PRD89-074031(2014))

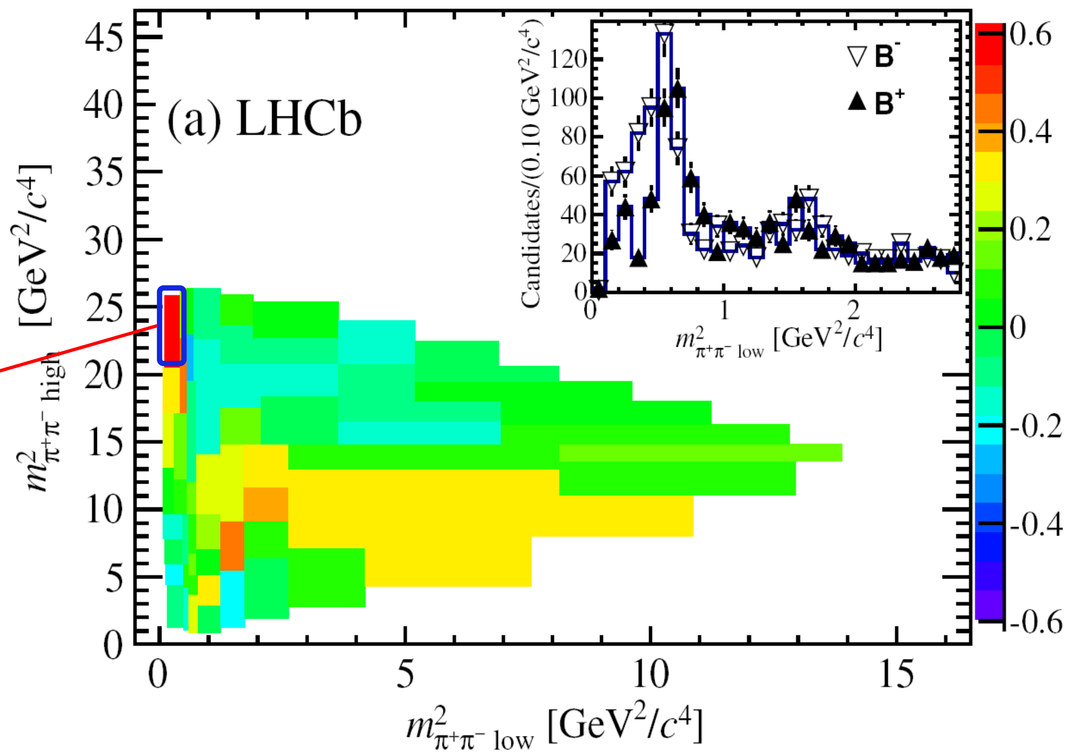
PHYSICAL REVIEW D **89,** 074031 (2014)

Direct *CP* asymmetries of three-body *B* decays in perturbative QCD

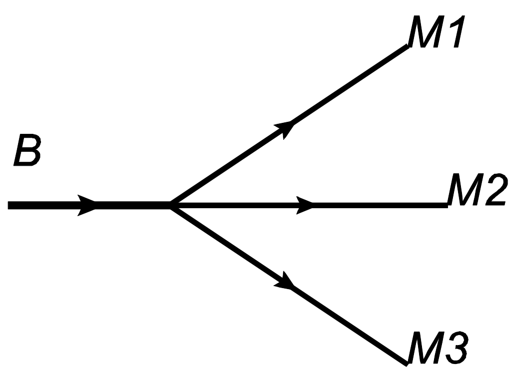
Wen-Fei Wang,[1,*] Hao-Chung Hu,[2,3,†] Hsiang-nan Li,[3,4,5,‡] and Cai-Dian Lü[1,§]

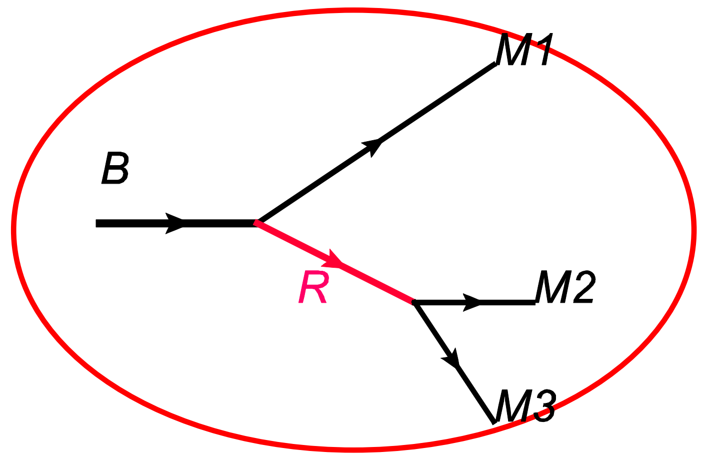
$$A_{CP}^{\rm reg}(B^\pm \to \pi^+\pi^-\pi^\pm)$$

$$= 0.63$$



Quasi-2-body decays



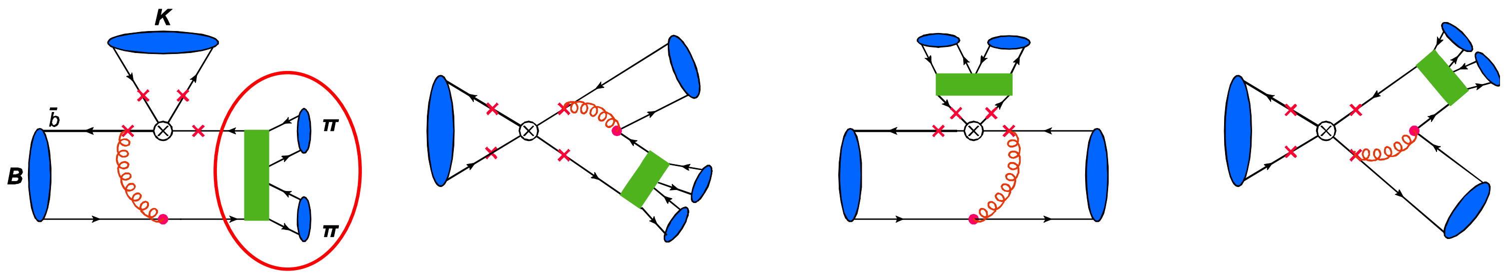


1 → 3 decay mode

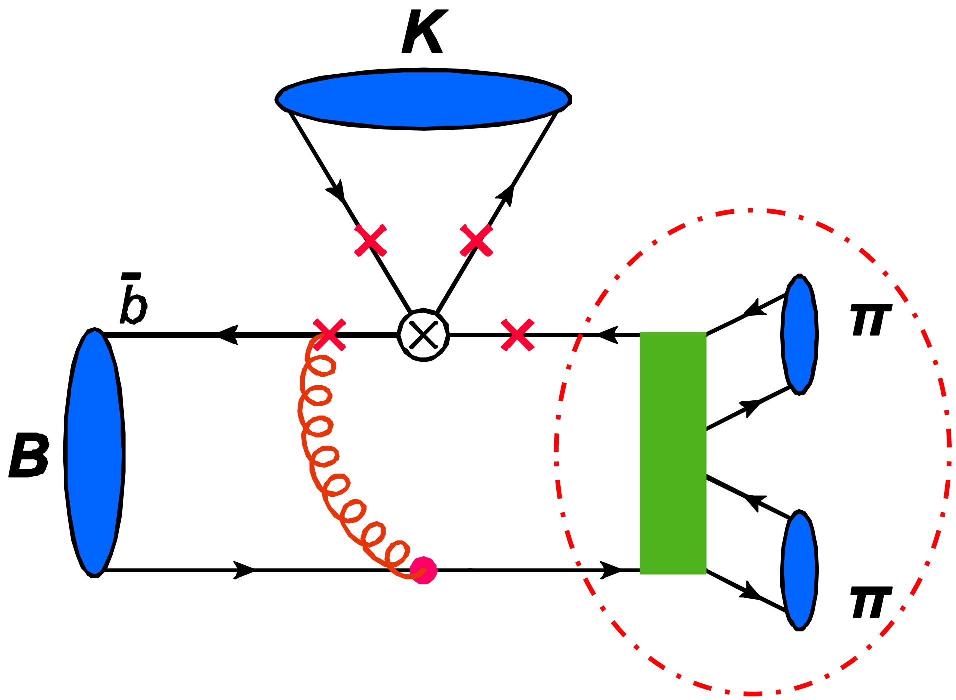
1 → 2 → 3 decay mode

quasi-two-body decay mode

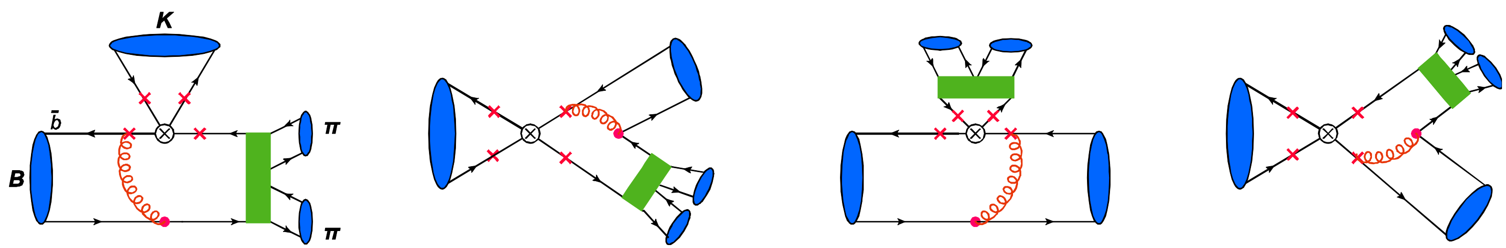
Quasi-2-body decays



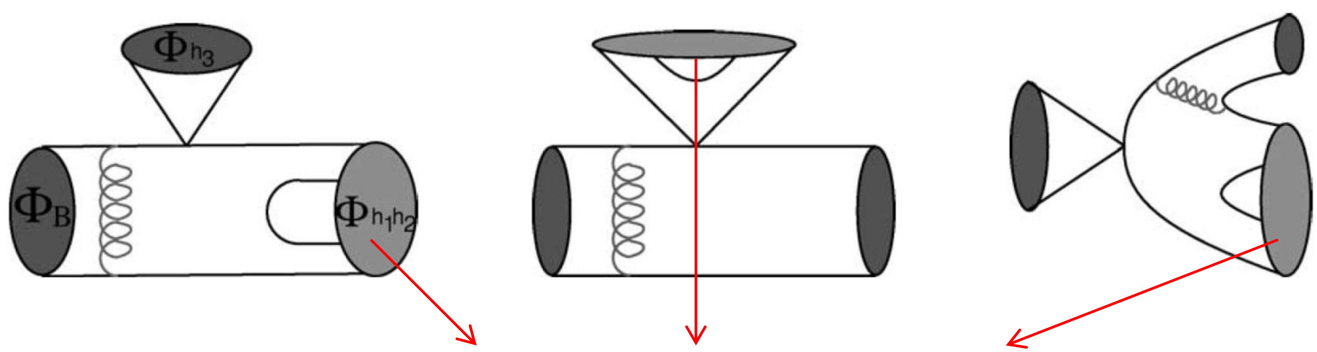
Quasi-2-body decays



Quasi-2-body decays

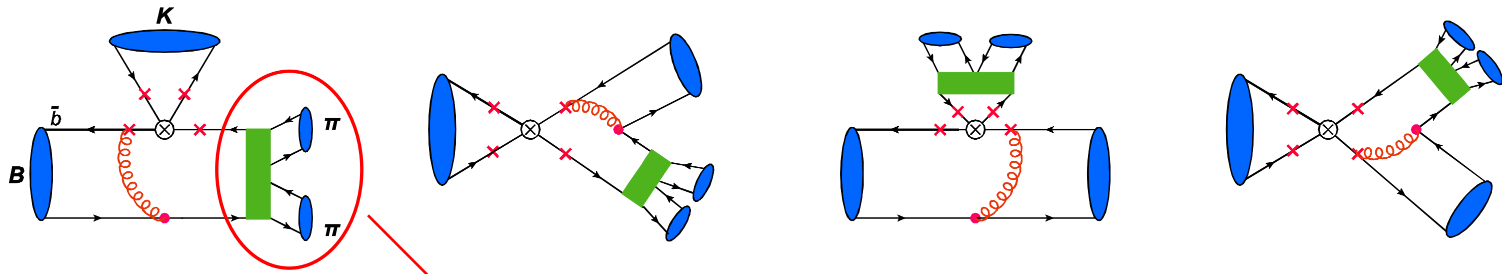


Physics Letters B 561 (2003) 258–265



two-meson distribution amplitudes

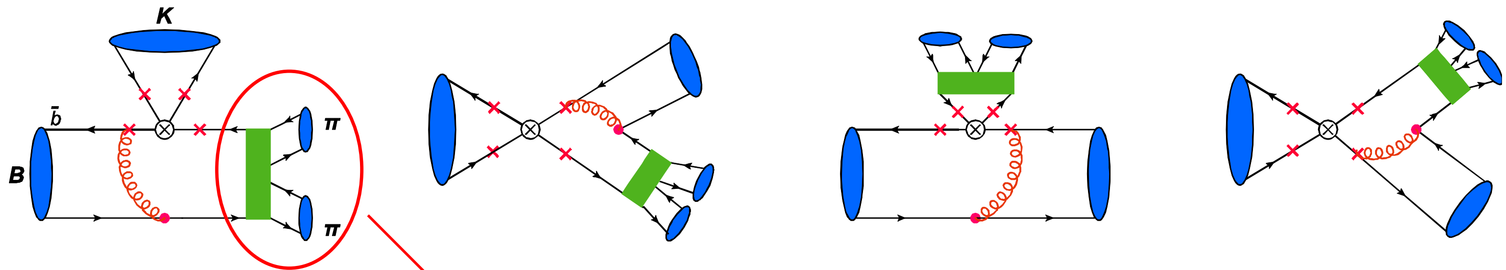
Quasi-2-body decays



resonant state distribution amplitude

RSDA

Quasi-2-body decays



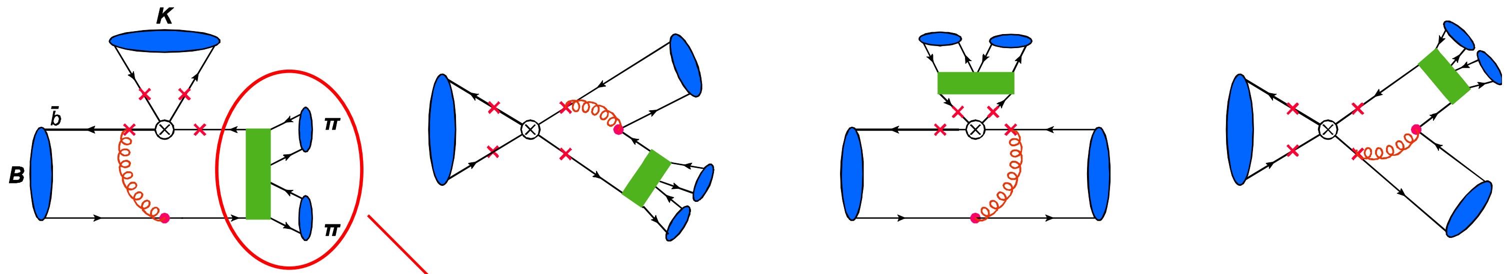
resonant state distribution amplitude

RSDA

$$\Phi^{||}_{\pi\pi}(z,\zeta,w) = \frac{F_p^{n\bar{n}^{(\prime)}}(w^2)}{\sqrt{2N_c}} \left\{ \not\epsilon_L \left[w\phi^0 + \not p \phi^t \right] + w\phi^s \right\}$$

$$\begin{cases} \phi^0(z) = \dfrac{3f_\rho}{\sqrt{6}} z(1-z)\left[1 + a^0_\rho C_2^{3/2}(1-2z)\right] , \\ \phi^t(z) = \dfrac{3f^T_\rho}{2\sqrt{6}}(1-2z)^2\left[1 + a^t_\rho C_2^{3/2}(1-2z)\right] , \\ \phi^s(z) = \dfrac{3f^T_\rho}{2\sqrt{6}}(1-2z)\left[1 + a^s_\rho\left(1 - 10z + 10z^2\right)\right] \end{cases}$$

Quasi-2-body decays



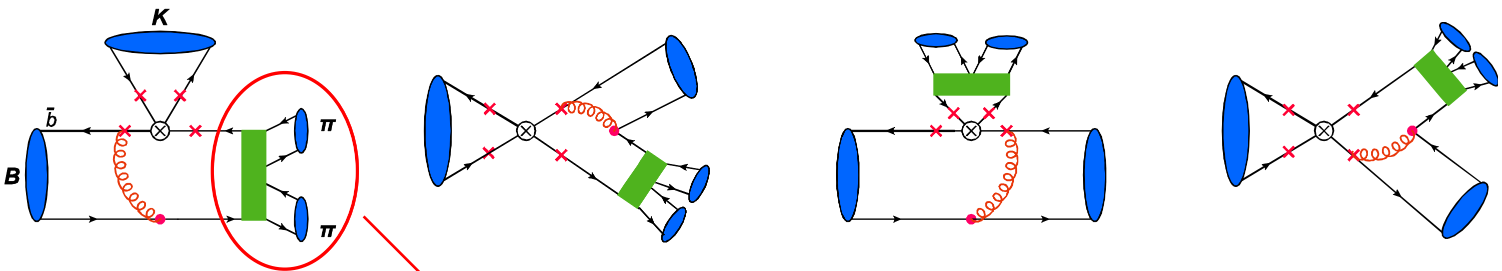
resonant state distribution amplitude

RSDA

$$\Phi^{||}_{\pi\pi}(z,\zeta,w) = \frac{F_p^{n\bar{n}^{(\prime)}}(w^2)}{\sqrt{2N_c}} \left\{ \not\epsilon_L \left[w\phi^0 + \not p \phi^t \right] + w\phi^s \right\}$$

$$F_\pi(s) = \frac{\mathrm{GS}_\rho(s, m_\rho, \Gamma_\rho)\frac{1+c_\omega \mathrm{BW}_\omega(s,m_\omega,\Gamma_\omega)}{1+c_\omega} + \sum c_i \mathrm{GS}_i(s, m_i, \Gamma_i)}{1+\sum c_i}$$

Quasi-2-body decays



resonant state distribution amplitude

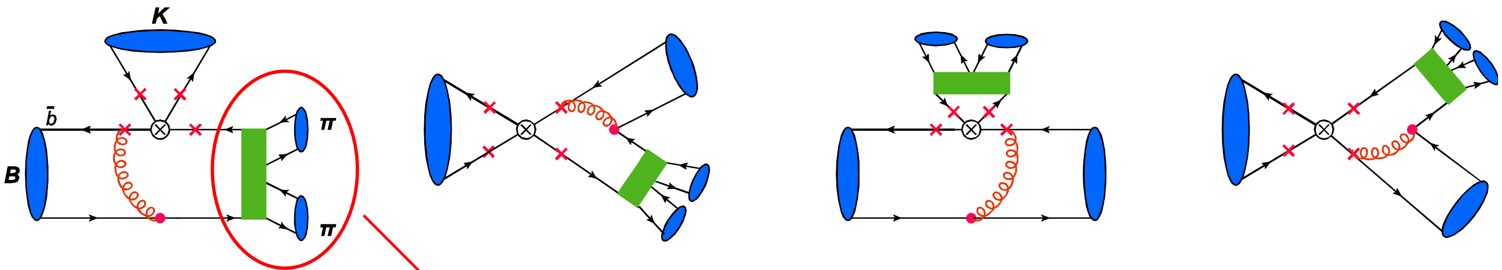
RSDA

$$\Phi^{||}_{\pi\pi}(z,\zeta,w) = \frac{F_p^{n\bar{n}^{(\prime)}}(w^2)}{\sqrt{2N_c}}\left\{\not{\epsilon}_L\left[w\phi^0 + \not{p}\phi^t\right] + w\phi^s\right\}$$

$$F_p^{n\bar{n}^{(\prime)}}(w^2) = CF_\pi(s)$$

$$F_\pi(s) = \frac{\mathrm{GS}_\rho(s,m_\rho,\Gamma_\rho)\frac{1+c_\omega \mathrm{BW}_\omega(s,m_\omega,\Gamma_\omega)}{1+c_\omega} + \sum c_i \mathrm{GS}_i(s,m_i,\Gamma_i)}{1+\sum c_i}$$

Quasi-2-body decays



resonant state distribution amplitude

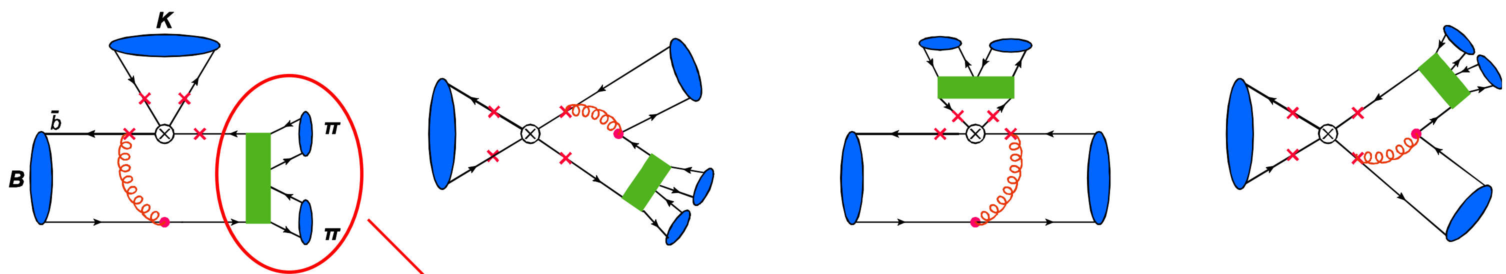
RSDA

$$F_\pi(s) = \frac{\mathrm{GS}_\rho(s, m_\rho, \Gamma_\rho)\frac{1+c_\omega \mathrm{BW}_\omega(s,m_\omega,\Gamma_\omega)}{1+c_\omega} + \sum c_i \mathrm{GS}_i(s, m_i, \Gamma_i)}{1 + \sum c_i}$$

$$\frac{m_\rho{}^2 + d m_\rho \Gamma_\rho}{(m_\rho{}^2 - s) + \Gamma_\rho (m_\rho{}^2/k_\rho{}^3)\{k^2[h(s) - h(m_\rho{}^2)] + k_\rho{}^2 h'(m_\rho{}^2)(m_\rho{}^2 - s)\} - i m_\rho \Gamma_\rho (k/k_\rho)^3 (m_\rho/\sqrt{s})},$$

G. Gounaris, J.J. Sakurai
PRL21-244(1968)

Quasi-2-body decays



resonant state distribution amplitude

RSDA

$$\Phi^{||}_{\pi\pi}(z,\zeta,w) = \frac{F_p^{n\bar{n}^{(\prime)}}(w^2)}{\sqrt{2N_c}} \{\not{\epsilon}_L \left[w\phi^0 + \not{p}\phi^t\right] + w\phi^s\}$$

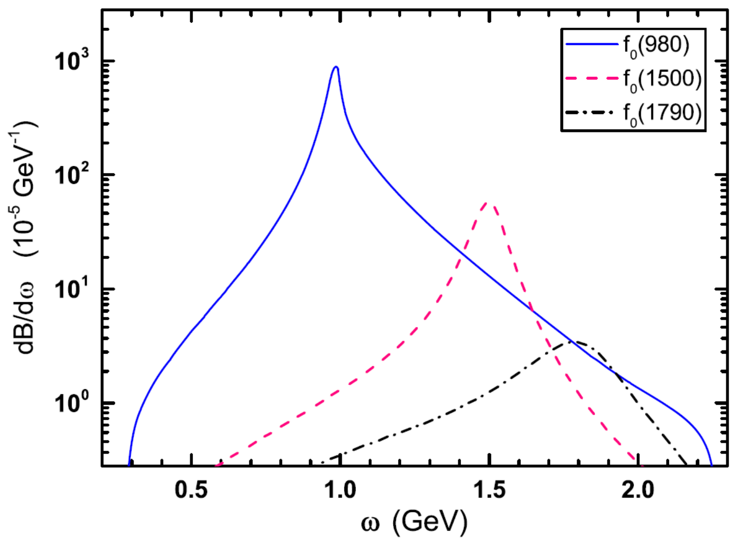
$$F_p^{n\bar{n}^{(\prime)}}(w^2) = CF_\pi(s) \quad \longleftrightarrow \quad \int ds \frac{|\overrightarrow{p_\pi}|^3 |\overrightarrow{p_K}|^3}{12\pi^2 m_B^2} |CF_\pi(s)|^2 \simeq 1$$

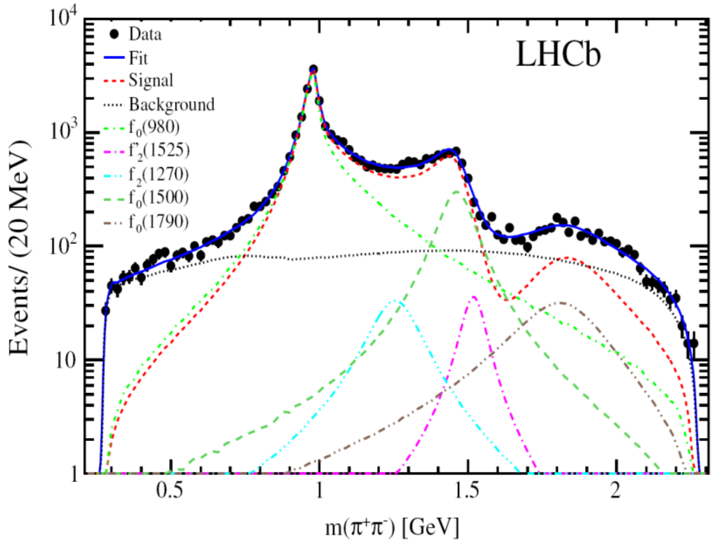
Quasi-2-body decays

PHYSICAL REVIEW D **91**, 094024 (2015)

S-wave resonance contributions to the $B^0_{(s)} \to J/\psi\pi^+\pi^-$ and $B_s \to \pi^+\pi^-\mu^+\mu^-$ decays

Wen-Fei Wang,[1,*] Hsiang-nan Li,[2,†] Wei Wang,[3,4,‡] and Cai-Dian Lü[1,4,§]





$$\mathcal{B}(B_s \to J/\psi f_0(980)[f_0(980) \to \pi^+\pi^-]) = (1.15^{+0.49}_{-0.38}(\omega_{B_s})^{+0.18}_{-0.15}(a_2^{I=0})^{+0.02}_{-0.01}(m_c)) \times 10^{-4},$$

$$\mathcal{B}(B_s \to J/\psi f_0(1500)[f_0(1500) \to \pi^+\pi^-]) = (1.62^{+0.54}_{-0.42}(\omega_{B_s})^{+0.28}_{-0.20}(a_2^{I=0})^{+0.03}_{-0.02}(m_c)) \times 10^{-5},$$

$$\mathcal{B}(B_s \to J/\psi f_0(1790)[f_0(1790) \to \pi^+\pi^-]) = (3.26^{+0.82}_{-0.71}(\omega_{B_s})^{+0.54}_{-0.38}(a_2^{I=0})^{+0.05}_{-0.04}(m_c)) \times 10^{-6},$$

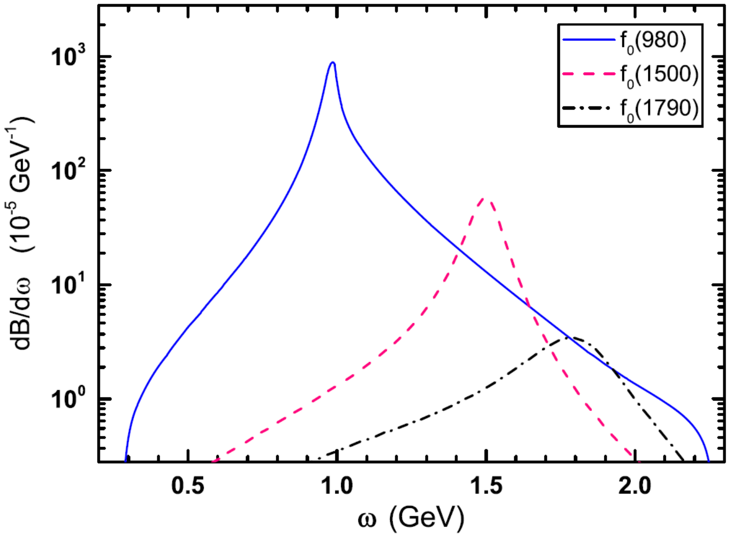
$$\mathcal{B}(B^0 \to J/\psi f_0(500)[f_0(500) \to \pi^+\pi^-]) = (6.91^{+3.37}_{-2.16}(\omega_B)^{+1.58}_{-1.15}(a_2^{I=0})^{+0.09}_{-0.08}(m_c)) \times 10^{-6},$$

Quasi-2-body decays

PHYSICAL REVIEW D **91**, 094024 (2015)

S-wave resonance contributions to the $B^0_{(s)} \to J/\psi\pi^+\pi^-$ and $B_s \to \pi^+\pi^-\mu^+\mu^-$ decays

Wen-Fei Wang,[1,*] Hsiang-nan Li,[2,†] Wei Wang,[3,4,‡] and Cai-Dian Lü[1,4,§]



$$F_s^{s\bar{s}}(\omega^2) = \frac{c_1 m^2_{f_0(980)} e^{i\theta_1}}{m^2_{f_0(980)} - \omega^2 - i m_{f_0(980)}(g_{\pi\pi}\rho_{\pi\pi} + g_{KK}\rho_{KK})} + \frac{c_2 m^2_{f_0(1500)} e^{i\theta_2}}{m^2_{f_0(1500)} - \omega^2 - i m_{f_0(1500)}\Gamma_{f_0(1500)}(\omega^2)} + \frac{c_3 m^2_{f_0(1790)} e^{i\theta_3}}{m^2_{f_0(1790)} - \omega^2 - i m_{f_0(1790)}\Gamma_{f_0(1790)}(\omega^2)},$$

$$\mathcal{B}(B_s \to J/\psi f_0(980)[f_0(980) \to \pi^+\pi^-]) = (1.15^{+0.49}_{-0.38}(\omega_{B_s})^{+0.18}_{-0.15}(a_2^{I=0})^{+0.02}_{-0.01}(m_c)) \times 10^{-4},$$

$$\mathcal{B}(B_s \to J/\psi f_0(1500)[f_0(1500) \to \pi^+\pi^-]) = (1.62^{+0.54}_{-0.42}(\omega_{B_s})^{+0.28}_{-0.20}(a_2^{I=0})^{+0.03}_{-0.02}(m_c)) \times 10^{-5},$$

$$\mathcal{B}(B_s \to J/\psi f_0(1790)[f_0(1790) \to \pi^+\pi^-]) = (3.26^{+0.82}_{-0.71}(\omega_{B_s})^{+0.54}_{-0.38}(a_2^{I=0})^{+0.05}_{-0.04}(m_c)) \times 10^{-6},$$

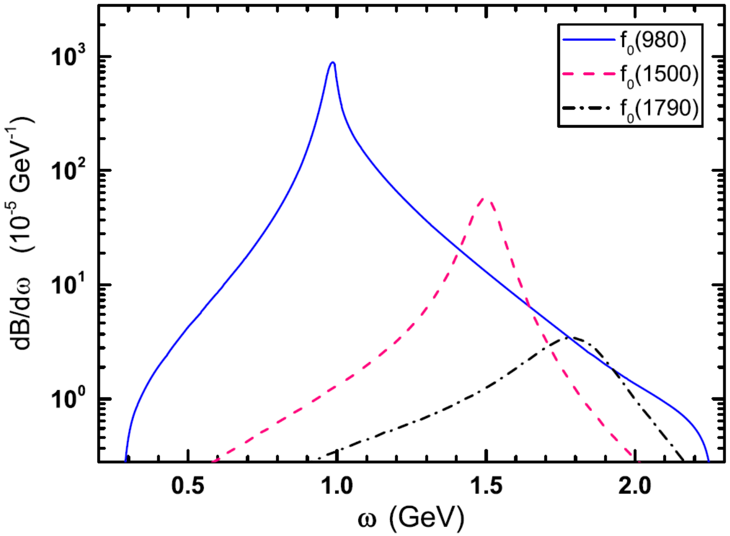
$$\mathcal{B}(B^0 \to J/\psi f_0(500)[f_0(500) \to \pi^+\pi^-]) = (6.91^{+3.37}_{-2.16}(\omega_{B})^{+1.58}_{-1.15}(a_2^{I=0})^{+0.09}_{-0.08}(m_c)) \times 10^{-6},$$

Quasi-2-body decays

PHYSICAL REVIEW D **91**, 094024 (2015)

S-wave resonance contributions to the $B^0_{(s)} \to J/\psi\pi^+\pi^-$ and $B_s \to \pi^+\pi^-\mu^+\mu^-$ decays

Wen-Fei Wang,[1,*] Hsiang-nan Li,[2,†] Wei Wang,[3,4,‡] and Cai-Dian Lü[1,4,§]

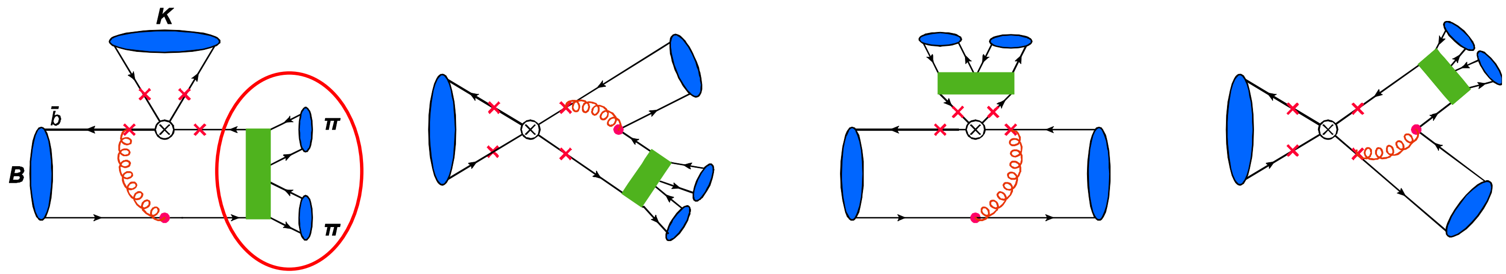


$$c_1 = 0.900, \qquad c_2 = 0.106, \qquad c_3 = 0.066,$$
$$\theta_1 = -\frac{\pi}{2}, \qquad \theta_2 = \frac{\pi}{4}, \qquad \theta_3 = 0,$$

$$\mathcal{B}(B_s \to J/\psi f_0(980)[f_0(980) \to \pi^+\pi^-]) = (1.15^{+0.49}_{-0.38}(\omega_{B_s})^{+0.18}_{-0.15}(a_2^{I=0})^{+0.02}_{-0.01}(m_c)) \times 10^{-4},$$
$$\mathcal{B}(B_s \to J/\psi f_0(1500)[f_0(1500) \to \pi^+\pi^-]) = (1.62^{+0.54}_{-0.42}(\omega_{B_s})^{+0.28}_{-0.20}(a_2^{I=0})^{+0.03}_{-0.02}(m_c)) \times 10^{-5},$$
$$\mathcal{B}(B_s \to J/\psi f_0(1790)[f_0(1790) \to \pi^+\pi^-]) = (3.26^{+0.82}_{-0.71}(\omega_{B_s})^{+0.54}_{-0.38}(a_2^{I=0})^{+0.05}_{-0.04}(m_c)) \times 10^{-6},$$
$$\mathcal{B}(B^0 \to J/\psi f_0(500)[f_0(500) \to \pi^+\pi^-]) = (6.91^{+3.37}_{-2.16}(\omega_B)^{+1.58}_{-1.15}(a_2^{I=0})^{+0.09}_{-0.08}(m_c)) \times 10^{-6},$$

Quasi-2-body decays

$$B \to K\rho^0 \to K\pi^+\pi^-$$



$$\Phi_{\pi\pi}^{||}(z,\zeta,w) = \frac{F_p^{n\bar{n}^{(\prime)}}(w^2)}{\sqrt{2N_c}}\{\not{\epsilon}_L\left[w\phi^0 + \not{p}\phi^t\right] + w\phi^s\}$$

$$\begin{cases} \phi^0(z) = \dfrac{3f_\rho}{\sqrt{6}} z(1-z)\left[1 + a_\rho^0 C_2^{3/2}(1-2z)\right] , \\ \phi^t(z) = \dfrac{3f_\rho^T}{2\sqrt{6}}(1-2z)^2\left[1 + a_\rho^t C_2^{3/2}(1-2z)\right] , \\ \phi^s(z) = \dfrac{3f_\rho^T}{2\sqrt{6}}(1-2z)\left[1 + a_\rho^s\left(1 - 10z + 10z^2\right)\right] \end{cases}$$

$\Rightarrow$ $a_\rho^0 = 0.25, a_\rho^t = -0.60, a_\rho^s = 0.75$

Quasi-2-body decays

Mode		Quasi-two-body	Data [93]
$B^+ \to K^+[\rho^0 \to]\pi^+\pi^-$	$\mathcal{B}\ (10^{-6})$	$3.59^{+0.87}_{-0.60}(\omega_B)^{+0.48}_{-0.43}(a^t_\rho)^{+0.43}_{-0.40}(m^K_0)^{+0.35}_{-0.32}(a^0_\rho)^{+0.35}_{-0.32}(a^s_\rho)$	3.7 ± 0.5
	$\mathcal{A}_{CP}$	$0.43^{+0.04}_{-0.06}(\omega_B) \pm 0.07(a^t_\rho) \pm 0.03(m^K_0)^{+0.01}_{-0.02}(a^0_\rho)^{+0.00}_{-0.01}(a^s_\rho)$	0.37 ± 0.10
$B^+ \to K^0[\rho^+ \to]\pi^+\pi^0$	$\mathcal{B}\ (10^{-6})$	$7.88^{+2.19}_{-1.44}(\omega_B)^{+1.82}_{-1.63}(a^t_\rho)^{+1.08}_{-0.98}(m^K_0)^{+0.64}_{-0.57}(a^0_\rho)^{+0.59}_{-0.53}(a^s_\rho)$	8.0 ± 1.5
	$\mathcal{A}_{CP}$	$0.16^{+0.03}_{-0.04}(\omega_B)^{+0.02}_{-0.03}(a^t_\rho) \pm 0.01(m^K_0) \pm 0.00(a^0_\rho) \pm 0.00(a^s_\rho)$	-0.12 ± 0.17
$B^0 \to K^+[\rho^- \to]\pi^-\pi^0$	$\mathcal{B}\ (10^{-6})$	$6.97^{+2.06}_{-1.28}(\omega_B)^{+0.74}_{-0.61}(a^t_\rho)^{+0.87}_{-0.81}(m^K_0)^{+0.68}_{-0.60}(a^0_\rho)^{+0.57}_{-0.52}(a^s_\rho)$	7.0 ± 0.9
	$\mathcal{A}_{CP}$	$0.31^{+0.01}_{-0.02}(\omega_B)^{+0.10}_{-0.09}(a^t_\rho)^{+0.03}_{-0.02}(m^K_0)^{+0.01}_{-0.02}(a^0_\rho) \pm 0.01(a^s_\rho)$	0.20 ± 0.11
$B^0 \to K^0[\rho^0 \to]\pi^+\pi^-$	$\mathcal{B}\ (10^{-6})$	$4.03^{+1.21}_{-0.81}(\omega_B)^{+0.74}_{-0.67}(a^t_\rho)^{+0.54}_{-0.50}(m^K_0)^{+0.30}_{-0.27}(a^0_\rho)^{+0.26}_{-0.23}(a^s_\rho)$	4.7 ± 0.6
	$\mathcal{A}_{CP}$	$0.06^{+0.00}_{-0.01}(\omega_B)^{+0.01}_{-0.00}(a^t_\rho) \pm 0.00(m^K_0) \pm 0.00(a^0_\rho) \pm 0.00(a^s_\rho)$	$-$

2-body

$B^+ \to K^+\rho^0$
$$\begin{cases} \mathcal{B} = (3.60^{+0.69}_{-0.46}(\omega_B)^{+0.42}_{-0.35}(a^t_\rho)^{+0.43}_{-0.39}(m^K_0)^{+0.49}_{-0.44}(a^0_\rho)^{+0.26}_{-0.24}(a^s_\rho)) \times 10^{-6}\,, \\ \mathcal{A}_{CP} = 0.55^{+0.02}_{-0.04}(\omega_B)^{+0.09}_{-0.08}(a^t_\rho) \pm 0.03(m^K_0)^{+0.00}_{-0.01}(a^0_\rho) \pm 0.01(a^s_\rho)\,, \end{cases}$$

$B^+ \to K^0\rho^+$
$$\begin{cases} \mathcal{B} = (7.83^{+1.83}_{-1.22}(\omega_B)^{+1.73}_{-1.47}(a^t_\rho)^{+1.06}_{-0.97}(m^K_0)^{+0.86}_{-0.75}(a^0_\rho)^{+0.44}_{-0.41}(a^s_\rho)) \times 10^{-6}\,, \\ \mathcal{A}_{CP} = 0.22 \pm 0.03(\omega_B)^{+0.03}_{-0.05}(a^t_\rho) \pm 0.01(m^K_0) \pm 0.00(a^0_\rho) \pm 0.00(a^s_\rho)\,, \end{cases}$$

$B^0 \to K^+\rho^-$
$$\begin{cases} \mathcal{B} = (7.07^{+1.62}_{-1.06}(\omega_B)^{+0.69}_{-0.54}(a^t_\rho)^{+0.88}_{-0.82}(m^K_0)^{+0.94}_{-0.82}(a^0_\rho)^{+0.43}_{-0.40}(a^s_\rho)) \times 10^{-6}\,, \\ \mathcal{A}_{CP} = 0.34^{+0.00}_{-0.01}(\omega_B)^{+0.13}_{-0.12}(a^t_\rho)^{+0.03}_{-0.02}(m^K_0)^{+0.01}_{-0.02}(a^0_\rho)^{+0.01}_{-0.02}(a^s_\rho)\,. \end{cases}$$

$B^0 \to K^0\rho^0$
$$\begin{cases} \mathcal{B} = (4.10^{+1.10}_{-0.72}(\omega_B)^{+0.72}_{-0.64}(a^t_\rho)^{+0.56}_{-0.51}(m^K_0)^{+0.41}_{-0.36}(a^0_\rho) \pm 0.19(a^s_\rho)) \times 10^{-6}\,, \\ \mathcal{A}_{CP} = 0.04 \pm 0.01(\omega_B) \pm 0.00(a^t_\rho) \pm 0.00(m^K_0)^{+0.00}_{-0.01}(a^0_\rho) \pm 0.00(a^s_\rho)\,. \end{cases}$$

Quasi-2-body decays

Mode		Quasi-two-body	Data [93]
$B^+ \to K^+[\rho^0 \to]\pi^+\pi^-$	$\mathcal{B}\ (10^{-6})$	$3.59^{+0.87}_{-0.60}(\omega_B)^{+0.48}_{-0.43}(a^t_\rho)^{+0.43}_{-0.40}(m_0^K)^{+0.35}_{-0.32}(a^0_\rho)^{+0.35}_{-0.32}(a^s_\rho)$	3.7 ± 0.5
	$\mathcal{A}_{CP}$	$0.43^{+0.04}_{-0.06}(\omega_B) \pm 0.07(a^t_\rho) \pm 0.03(m_0^K)^{+0.01}_{-0.02}(a^0_\rho)^{+0.00}_{-0.01}(a^s_\rho)$	0.37 ± 0.10
$B^+ \to K^0[\rho^+ \to]\pi^+\pi^0$	$\mathcal{B}\ (10^{-6})$	$7.88^{+2.19}_{-1.44}(\omega_B)^{+1.82}_{-1.63}(a^t_\rho)^{+1.08}_{-0.98}(m_0^K)^{+0.64}_{-0.57}(a^0_\rho)^{+0.59}_{-0.53}(a^s_\rho)$	8.0 ± 1.5
	$\mathcal{A}_{CP}$	$0.16^{+0.03}_{-0.04}(\omega_B)^{+0.02}_{-0.03}(a^t_\rho) \pm 0.01(m_0^K) \pm 0.00(a^0_\rho) \pm 0.00(a^s_\rho)$	-0.12 ± 0.17
$B^0 \to K^+[\rho^- \to]\pi^-\pi^0$	$\mathcal{B}\ (10^{-6})$	$6.97^{+2.06}_{-1.28}(\omega_B)^{+0.74}_{-0.61}(a^t_\rho)^{+0.87}_{-0.81}(m_0^K)^{+0.68}_{-0.60}(a^0_\rho)^{+0.57}_{-0.52}(a^s_\rho)$	7.0 ± 0.9
	$\mathcal{A}_{CP}$	$0.31^{+0.01}_{-0.02}(\omega_B)^{+0.10}_{-0.09}(a^t_\rho)^{+0.03}_{-0.02}(m_0^K)^{+0.01}_{-0.02}(a^0_\rho) \pm 0.01(a^s_\rho)$	0.20 ± 0.11
$B^0 \to K^0[\rho^0 \to]\pi^+\pi^-$	$\mathcal{B}\ (10^{-6})$	$4.03^{+1.21}_{-0.81}(\omega_B)^{+0.74}_{-0.67}(a^t_\rho)^{+0.54}_{-0.50}(m_0^K)^{+0.30}_{-0.27}(a^0_\rho)^{+0.26}_{-0.23}(a^s_\rho)$	4.7 ± 0.6
	$\mathcal{A}_{CP}$	$0.06^{+0.00}_{-0.01}(\omega_B)^{+0.01}_{-0.00}(a^t_\rho) \pm 0.00(m_0^K) \pm 0.00(a^0_\rho) \pm 0.00(a^s_\rho)$	$-$

2-body

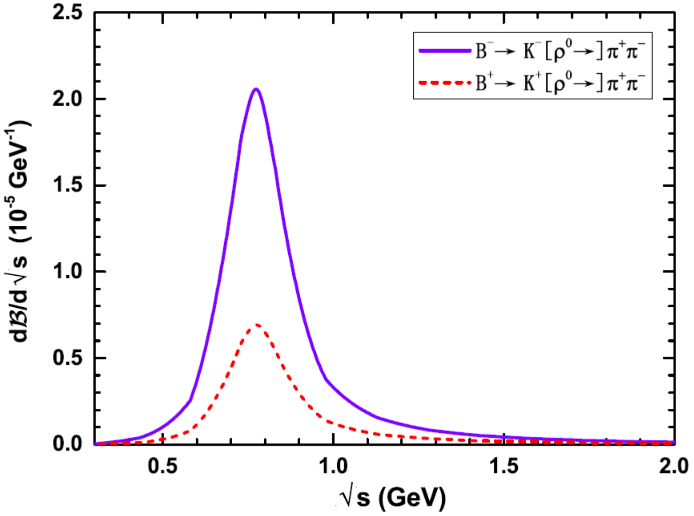
$B^+ \to K^+\rho^0$
$$\begin{cases} \mathcal{B} = (3.60^{+0.69}_{-0.46}(\omega_B)^{+0.42}_{-0.35}(a^t_\rho)^{+0.43}_{-0.39}(m_0^K)^{+0.49}_{-0.44}(a^0_\rho)^{+0.26}_{-0.24}(a^s_\rho)) \times 10^{-6}, \\ \mathcal{A}_{CP} = 0.55^{+0.02}_{-0.04}(\omega_B)^{+0.09}_{-0.08}(a^t_\rho) \pm 0.03(m_0^K)^{+0.00}_{-0.01}(a^0_\rho) \pm 0.01(a^s_\rho), \end{cases}$$

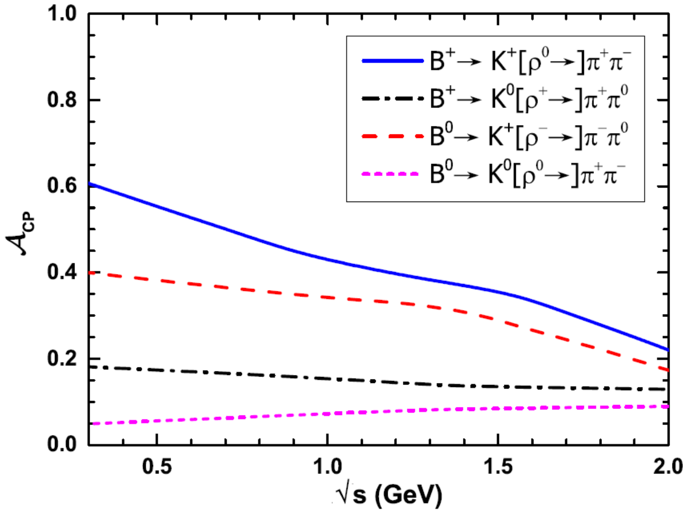
$B^+ \to K^0\rho^+$
$$\begin{cases} \mathcal{B} = (7.83^{+1.83}_{-1.22}(\omega_B)^{+1.73}_{-1.47}(a^t_\rho)^{+1.06}_{-0.97}(m_0^K)^{+0.86}_{-0.75}(a^0_\rho)^{+0.44}_{-0.41}(a^s_\rho)) \times 10^{-6}, \\ \mathcal{A}_{CP} = 0.22 \pm 0.03(\omega_B)^{+0.03}_{-0.05}(a^t_\rho) \pm 0.01(m_0^K) \pm 0.00(a^0_\rho) \pm 0.00(a^s_\rho), \end{cases}$$

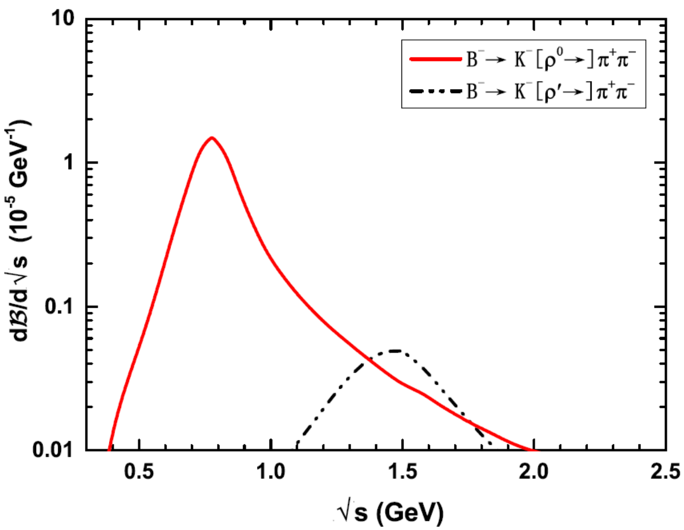
$B^0 \to K^+\rho^-$
$$\begin{cases} \mathcal{B} = (7.07^{+1.62}_{-1.06}(\omega_B)^{+0.69}_{-0.54}(a^t_\rho)^{+0.88}_{-0.82}(m_0^K)^{+0.94}_{-0.82}(a^0_\rho)^{+0.43}_{-0.40}(a^s_\rho)) \times 10^{-6}, \\ \mathcal{A}_{CP} = 0.34^{+0.00}_{-0.01}(\omega_B)^{+0.13}_{-0.12}(a^t_\rho)^{+0.03}_{-0.02}(m_0^K)^{+0.01}_{-0.02}(a^0_\rho)^{+0.01}_{-0.02}(a^s_\rho). \end{cases}$$

$B^0 \to K^0\rho^0$
$$\begin{cases} \mathcal{B} = (4.10^{+1.10}_{-0.72}(\omega_B)^{+0.72}_{-0.64}(a^t_\rho)^{+0.56}_{-0.51}(m_0^K)^{+0.41}_{-0.36}(a^0_\rho) \pm 0.19(a^s_\rho)) \times 10^{-6}, \\ \mathcal{A}_{CP} = 0.04 \pm 0.01(\omega_B) \pm 0.00(a^t_\rho) \pm 0.00(m_0^K)^{+0.00}_{-0.01}(a^0_\rho) \pm 0.00(a^s_\rho). \end{cases}$$

Quasi-2-body decays







Quasi-2-body decays

$$F_\pi(s) = \frac{\mathrm{GS}_\rho(s, m_\rho, \Gamma_\rho)\frac{1+c_\omega \mathrm{BW}_\omega(s,m_\omega,\Gamma_\omega)}{1+c_\omega} + \sum c_i \mathrm{GS}_i(s, m_i, \Gamma_i)}{1+\sum c_i}$$

Mode		Quasi-two-body
$B^+ \to K^+[\rho'^0 \to]\pi^+\pi^-$	$\mathcal{B}\ (10^{-7})$	$2.86^{+0.84}_{-0.59}(\omega_B)^{+0.14}_{-0.21}(a^0_\rho)^{+0.56}_{-0.45}(a^t_\rho)^{+0.51}_{-0.40}(a^s_\rho)^{+0.32}_{-0.26}(m^K_0)$
	$\mathcal{A}_{CP}$	$0.33^{+0.01}_{-0.02}(\omega_B)^{+0.02}_{-0.03}(a^0_\rho) \pm 0.03(a^t_\rho) \pm 0.01(a^s_\rho)^{+0.01}_{-0.00}(m^K_0)$
$B^+ \to K^0[\rho'^+ \to]\pi^+\pi^0$	$\mathcal{B}\ (10^{-7})$	$7.05^{+2.28}_{-1.83}(\omega_B)^{+0.26}_{-0.49}(a^0_\rho)^{+1.97}_{-1.83}(a^t_\rho)^{+0.86}_{-0.88}(a^s_\rho)^{+0.68}_{-0.76}(m^K_0)$
	$\mathcal{A}_{CP}$	$0.11^{+0.02}_{-0.03}(\omega_B)^{+0.01}_{-0.03}(a^0_\rho)^{+0.01}_{-0.03}(a^t_\rho)^{+0.01}_{-0.02}(a^s_\rho) \pm 0.02(m^K_0)$
$B^0 \to K^+[\rho'^- \to]\pi^-\pi^0$	$\mathcal{B}\ (10^{-7})$	$5.20^{+1.76}_{-1.26}(\omega_B)^{+0.27}_{-0.38}(a^0_\rho)^{+0.84}_{-0.62}(a^t_\rho)^{+0.84}_{-0.67}(a^s_\rho)^{+0.71}_{-0.66}(m^K_0)$
	$\mathcal{A}_{CP}$	$0.27 \pm 0.02(\omega_B)^{+0.02}_{-0.03}(a^0_\rho)^{+0.05}_{-0.08}(a^t_\rho)^{+0.02}_{-0.03}(a^s_\rho)^{+0.00}_{-0.01}(m^K_0)$
$B^0 \to K^0[\rho'^0 \to]\pi^+\pi^-$	$\mathcal{B}\ (10^{-7})$	$3.23^{+1.09}_{-0.92}(\omega_B)^{+0.19}_{-0.25}(a^0_\rho)^{+0.64}_{-0.69}(a^t_\rho)^{+0.33}_{-0.38}(a^s_\rho)^{+0.34}_{-0.35}(m^K_0)$
	$\mathcal{A}_{CP}$	$0.07^{+0.01}_{-0.02}(\omega_B)^{+0.01}_{-0.02}(a^0_\rho)^{+0.00}_{-0.03}(a^t_\rho)^{+0.02}_{-0.03}(a^s_\rho)^{+0.01}_{-0.02}(m^K_0)$

Quasi-2-body decays

$$F_\pi(s) = \frac{\mathrm{GS}_\rho(s, m_\rho, \Gamma_\rho)\frac{1+c_\omega \mathrm{BW}_\omega(s,m_\omega,\Gamma_\omega)}{1+c_\omega} + \sum c_i \mathrm{GS}_i(s, m_i, \Gamma_i)}{1+\sum c_i}$$

$\tau^- \to \pi^-\pi^0\nu_\tau$

$$\mathcal{B}(\rho^- \to \pi^-\pi^0) \simeq C_{\pi\pi}\int ds\sqrt{(1-4m_\pi^2/s)}\cdot|\mathrm{GS}_\rho(s, m_\rho, \Gamma_\rho)|^2 \ ,$$

$$\mathcal{B}(\rho' \to \pi^-\pi^0) \simeq C_{\pi\pi}\int ds\sqrt{(1-4m_\pi^2/s)}\cdot|c_{\rho'}\cdot \mathrm{GS}_{\rho'}(s, m_{\rho'}, \Gamma_{\rho'})|^2 \ ,$$

$$\mathcal{B}(\rho' \to \pi\pi) \simeq \frac{\int ds\sqrt{(1-4m_\pi^2/s)}\cdot|c_{\rho'}\cdot \mathrm{GS}_{\rho'}(s, m_{\rho'}, \Gamma_{\rho'})|^2}{\int ds\sqrt{(1-4m_\pi^2/s)}\cdot|\mathrm{GS}_\rho(s, m_\rho, \Gamma_\rho)|^2} = 7.23\%.$$

A. B. Clegg, A. Donnachie, Z. Phys. C 62 (1994) 455

$\mathcal{B}(\rho' \to \pi\pi) = 4.56\% \sim 10.0\%$

AcdaP: a package for PQCD

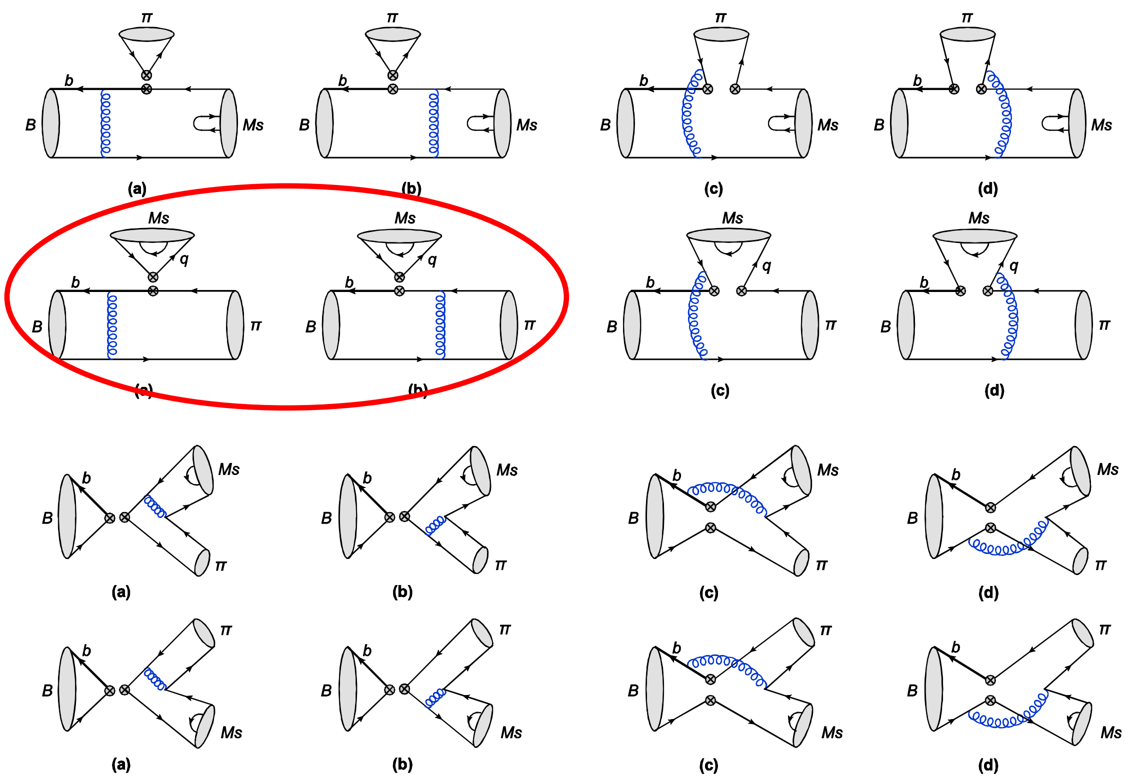
AcdaP: Automated calculations of the decay amplitudes for PQCD

$$B^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$$

(V-A)(V-A)

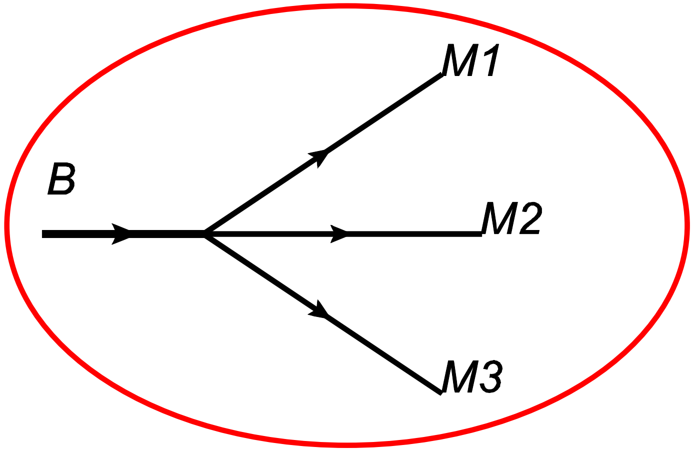
(V-A)(V+A)

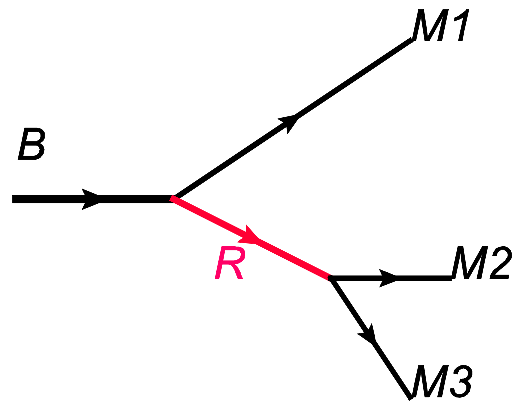
(S-P)(S+P)



AcdaP: a package for PQCD

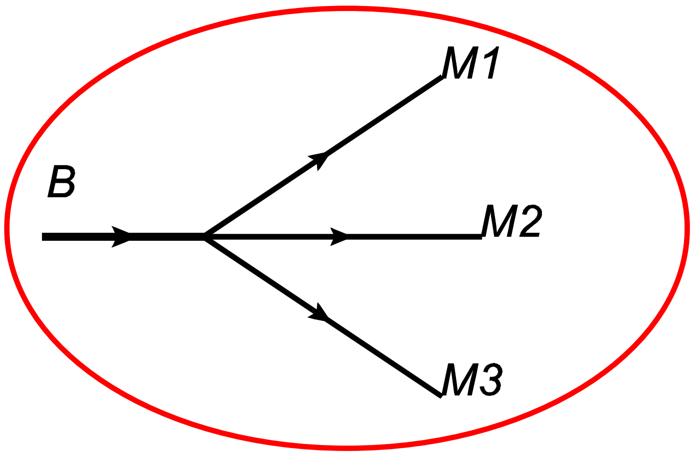
AcdaP: Automated calculations of the decay amplitudes for PQCD

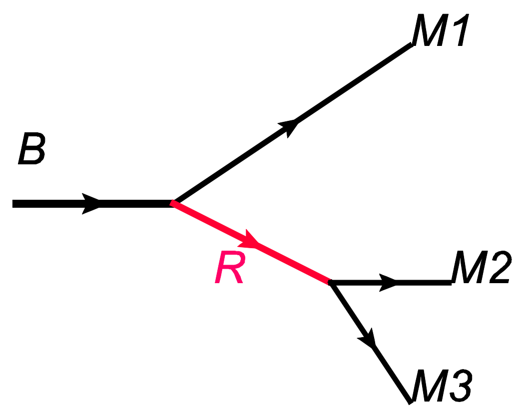


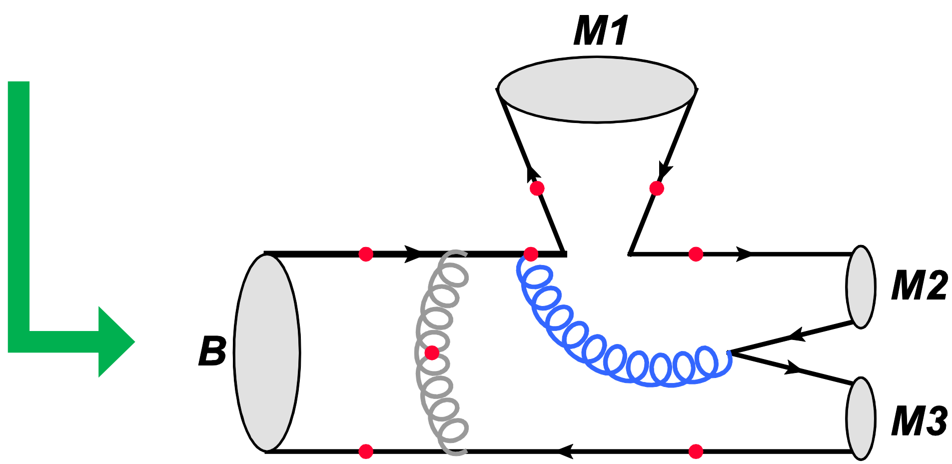


AcdaP: a package for PQCD

AcdaP: Automated calculations of the decay amplitudes for PQCD







>200

AcdaP: a package for PQCD

AcdaP: Automated calculations of the decay amplitudes for PQCD

Input:

initial state + final states

Output:

$\mathcal{B}$ Acp …

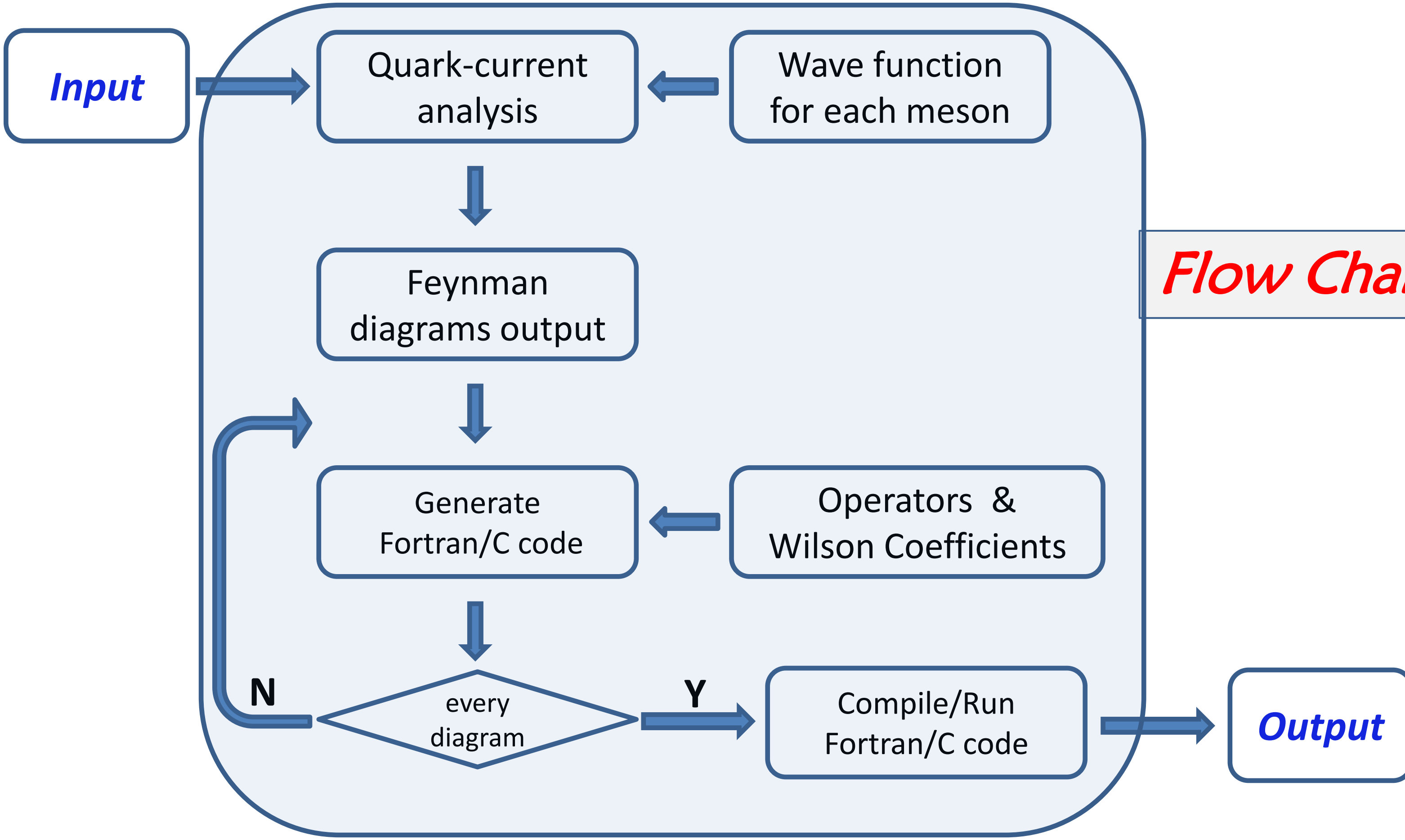
***Example*:**

```
AcdaP[{B^0},{K^*+,pi^-}]
```

$\mathcal{B}$(B^0->K^*+pi^-)=8.4*10^-6

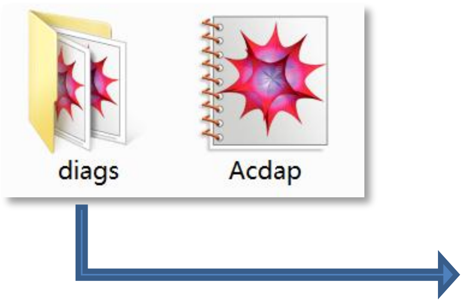
Acp(B^0->K^*+pi^-)=-0.22

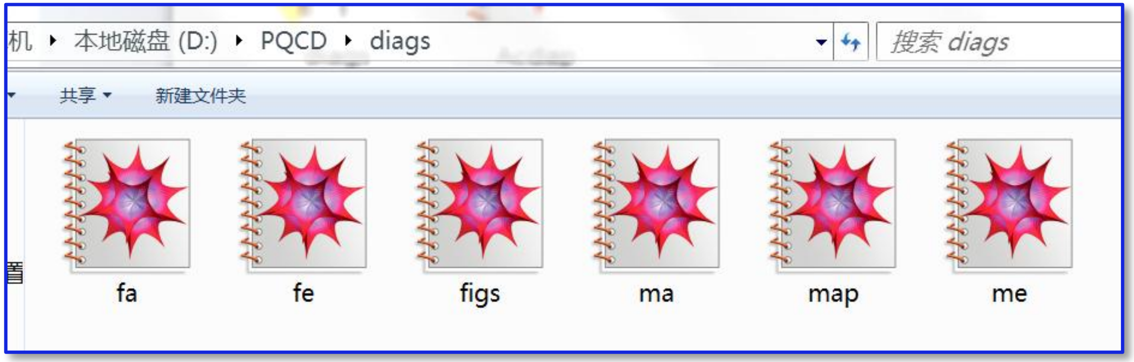
AcdaP: a package for PQCD

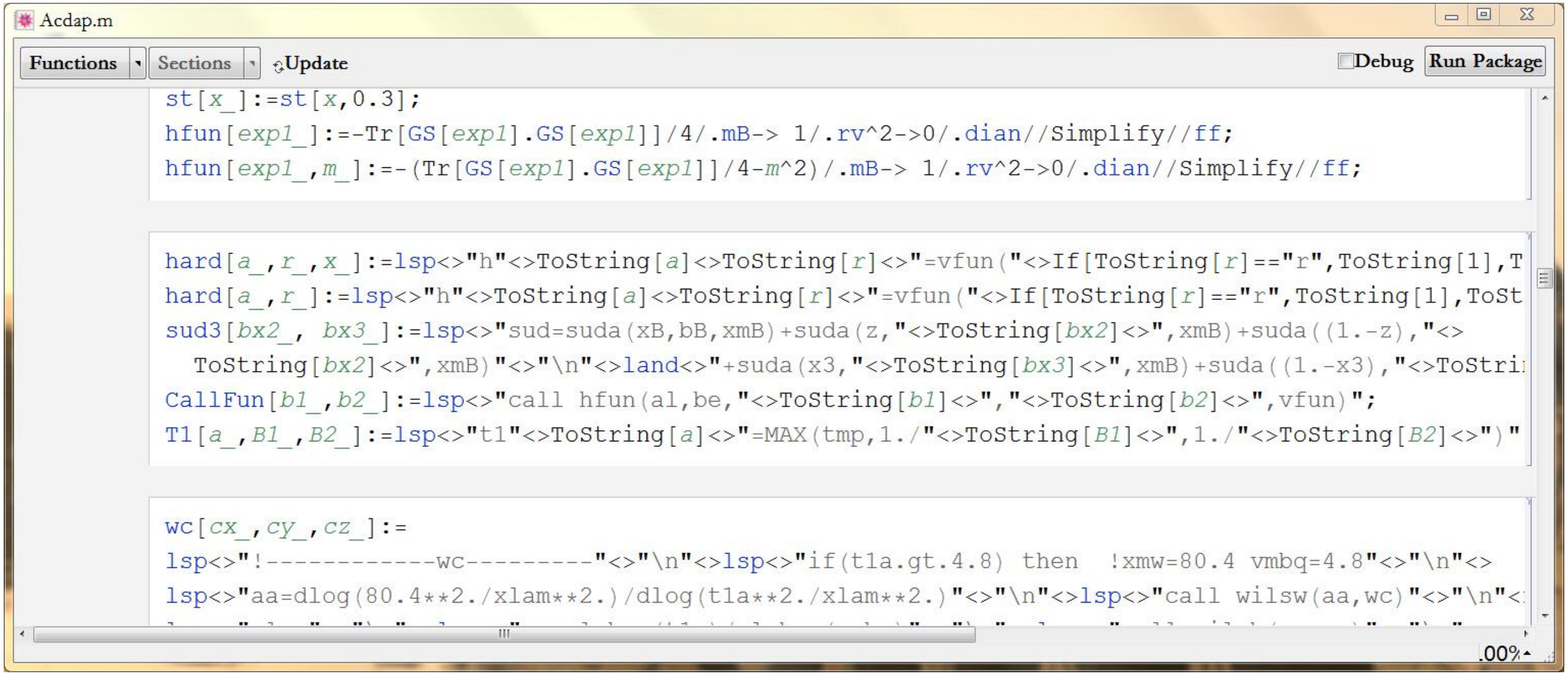


Flow Chart

AcdaP: a package for PQCD







AcdaP: a package for PQCD

```
<< HighEnergyPhysics`FeynCalc`;
Loading FeynCalc from C:\Program Files\Wolfram
Research\Mathematica\7.0\AddOns\ExtraPackages\HighEnergyPhysics
FeynCalc 8.2.0 For help, type ?FeynCalc, open FeynCalcRef8.nb or visit www.feyncalc.org
Loading FeynArts, see www.feynarts.de for documentation
FeynArts 3.7 patched for use with FeynCalc
```

```
<< PQCD`Acdap`
```

Acdap (Automatic calculation for the decay amplitudes of pQCD approach)

```
Mafig[s, u, u]
```

$$\frac{GF}{\sqrt{2}}\{Vub^* Vus[c1]MLL-Vtb^* Vts[(c3+c9)MLL+(c5+c7)MLR]\}$$

AcdaP: a package for PQCD

fig[a, b]

```
      subroutine Figab(xB,z,x3,bB,b,b3,v1ab)      ! Fig1-ab
      implicit double precision(a-h,o-z)
      common/con1/xlamb,xlam,xmw,pi,xmB,vmbq,fB,wB,xNB,ckA,ckl,ckr,cke
      common/con2/xmP,xm0P,xmc,xmV,fP,fV,fTV,xtmp,xyz,xvv,xeta,xqq,eta
      double precision wc(10),v1ab(2),vfun (2),vpK (3)
      r=xmP/xmB
      rc=xmc/xmB
      rv=xmV/xmB
      r0=xm0P/xmB

                           al=x3*(-1+z)
                         be=1+(-1+x3+xB)*z
          tmp=MAX(xmB*dsqrt(dabs(al)),xmB*dsqrt(dabs(be))) !fig-a
                      t1a=MAX(tmp,1./b,1./bB)
                     call hfun(al,be,b,bB,vfun)
                           har=vfun(1)
                           hai=vfun(2)

                           al=x3*(-1+z)
                         be=(x3-xB)*(-1+z)
          tmp=MAX(xmB*dsqrt(dabs(al)),xmB*dsqrt(dabs(be))) !fig-b
                      t1b=MAX(tmp,1./b,1./bB)
                     call hfun(al,be,b,bB,vfun)
                           hbr=vfun(1)
                           hbi=vfun(2)
```

....

......

Summary

✓ **Status for the 3-body hadronic B meson decays**

Summary

✓ **Status for the 3-body hadronic B meson decays**

✓ **Framework for the 3-body (quasi-2-body) hadronic B meson decays in PQCD approach**

Summary

- ✓ Status for the 3-body hadronic B meson decays
- ✓ Framework for the 3-body (quasi-2-body) hadronic B meson decays in PQCD approach
- ✓ AcdaP: a package for the decay amplitudes calculations in PQCD approach

Summary

- ✓ Status for the 3-body hadronic B meson decays
- ✓ Framework for the 3-body (quasi-2-body) hadronic B meson decays in PQCD approach
- ✓ AcdaP: a package for the decay amplitudes calculations in PQCD approach

Thank You