

Strongly coupled fourth family and electroweak phase transition

Masaya Kohda (NTU)

甲田昌也

Based on work with

Y. Kikukawa (U. of Tokyo), J. Yasuda (Meijo U.)

H. Kohyama (CYCU)

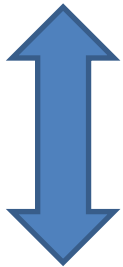
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The era of LHC

One of main task is to uncover

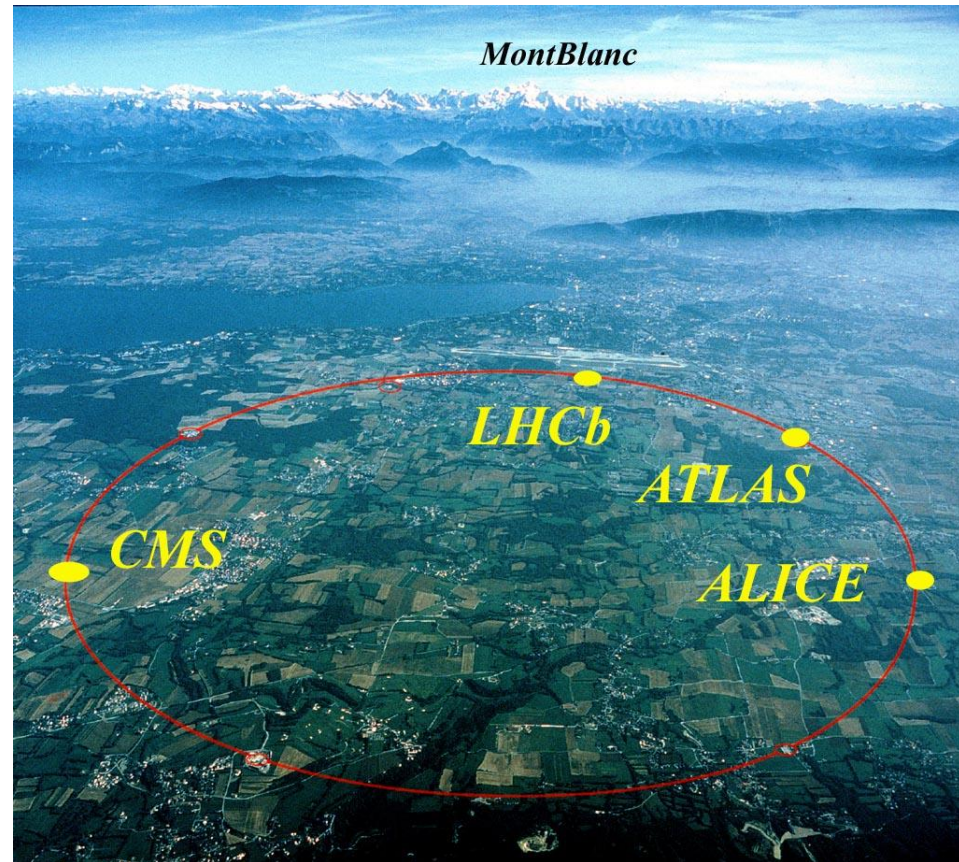
Higgs sector

origin of Electroweak
symmetry breaking
(*origin of particle masses*)



Electroweak phase transition

How Electroweak symmetry
was broken in early universe?
(*maybe related to origin of matter in the universe*)



Spontaneous Symmetry Breaking of EW gauge symmetry

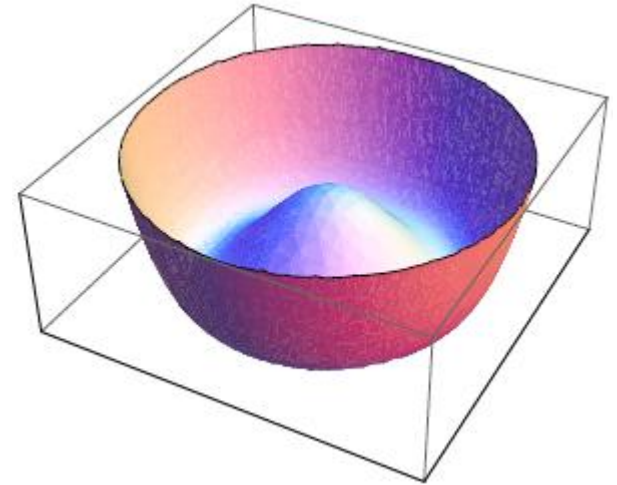
$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{EM}$$

--> W, Z acquire masses (Higgs mechanism)

Higgs sector : fields and interactions which trigger this SSB

In SM, SU(2) doublet-Higgs field H

--> SSB is realized by nonzero VEV of Higgs $\langle H \rangle$



◆ At finite T, EW symmetry is expected to be restored

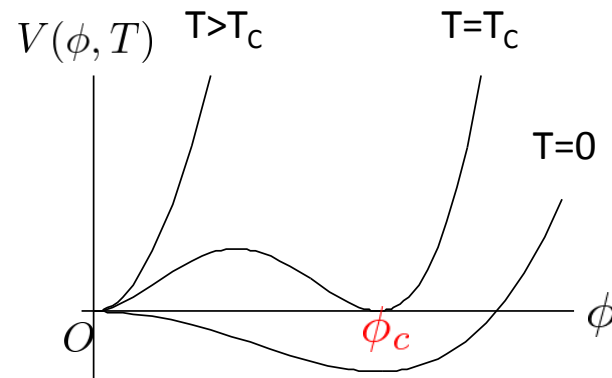
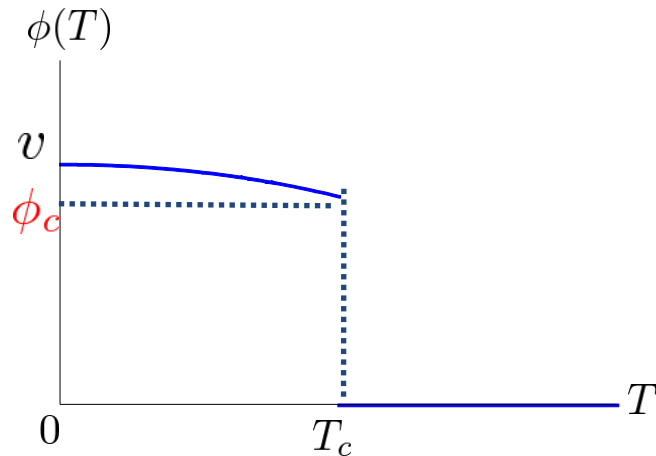
[Kirzhnits and Linde (1972)]

➤ pointed out based on analogy with superconductor

➤ early stage of universe is in unbroken phase

ElectroWeak Phase Transition

- ◆ breaking of $SU(2) \times U(1)$ at finite T : $\langle H \rangle = 0 \rightarrow \langle H \rangle \neq 0$
- ◆ EW baryogenesis requires 1st order phase transition



- ◆ For EW baryogenesis, phase transition should be also strongly 1st order (explain later): $\frac{\phi_c}{T_c} \gtrsim 1$
- ◆ In standard model, this condition is **not** satisfied
- ◆ In this talk, consider **4th family fermions**

Outline of my talk

1. Introduction

- Baryogenesis in SM, current status of 4th family

2. EW phase transition in SM

- one-loop analysis based on finite temp. effective potential

3. Strongly coupled 4th family

- 4th family as origin of EW symmetry breaking
- setup of our model

4. EWPT with strongly coupled 4th family

- numerical results for EWPT
- strongly 1st order PT ?

5. Summary and discussion

1. Introduction

Baryon Asymmetry of the Universe

- In current Universe: $N(\text{baryon}) \gg N(\text{antibaryon})$
- The universe is asymmetric between baryon and antibaryon
- Degree of baryon asymmetry

$$\eta \equiv \frac{n_B}{n_\gamma} = (4.7 - 6.5) \times 10^{-10} \quad [\text{PDG 2008}]$$

- required for big-bang nucleosynthesis
- also measured through Cosmic Microwave Background

Conditions for baryogenesis

$$\eta \equiv \frac{n_B}{n_\gamma} = (4.7 - 6.5) \times 10^{-10}$$

- ◆ Our goal is to explain this value based on particle physics and cosmology
- ◆ dynamical creation of baryon asymmetry in early universe (Baryogenesis)

$$n_B = 0 \quad \longrightarrow \quad n_B \neq 0$$

- ◆ **Sakharov's conditions** for baryogenesis

(1) B non-conservation (2) C, CP violation (3) Out of equilibrium

SM and Sakharov's conditions (1)

- ◆ SM can fulfill all Sakharov's conditions, in principle

(1) B non-conservation

- ◆ B is **violated** by quantum anomaly

Reaction rates of B-violating processes

- $T = 0$: *instanton* transition

$$\Gamma_{\cancel{B}} \sim e^{-2S_{\text{instanton}}} = e^{-4\pi/\alpha_2} \sim 10^{-170} \ll 1$$

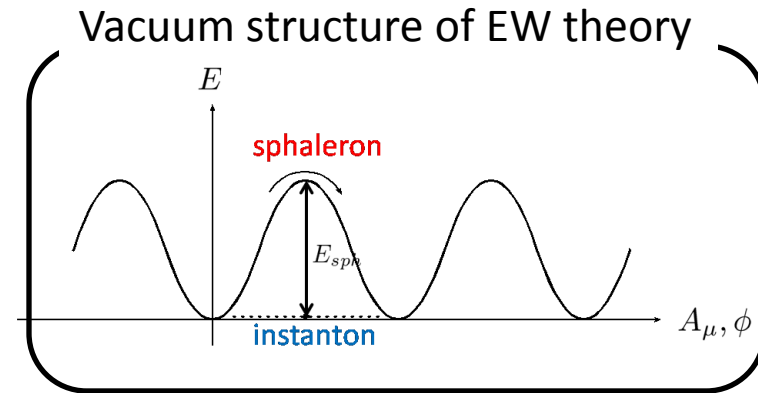
- $T > 0$: *sphaleron* transition

⇒ consistent with experiments

$$\Gamma_{\cancel{B}} \sim \begin{cases} T e^{-E_{sph}/T} & \text{for } T < T_c \text{ (} \langle H \rangle \neq 0 \text{)} \\ T & \text{for } T > T_c \text{ (} \langle H \rangle = 0 \text{)} \end{cases}$$

• sphaleron energy : $E_{sph} = \frac{4\pi\langle H \rangle}{g_2} \times (1.5 - 2.7)$

⇒ large B violation for $T > T_c \sim O(100) \text{ GeV}$!



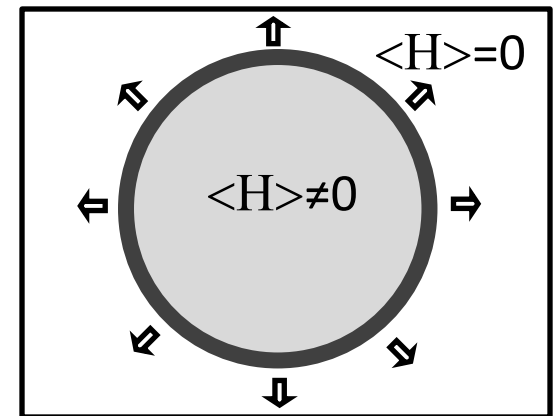
SM and Sakharov's conditions (2)

(2) C and CP violation

- ◆ $SU(2) \times U(1)$ gauge interaction is chiral $\Rightarrow$ C violation
- ◆ Kobayashi-Maskawa phase $\Rightarrow$ CP violation
(nicely explaining B,K physics)

(3) Out of equilibrium

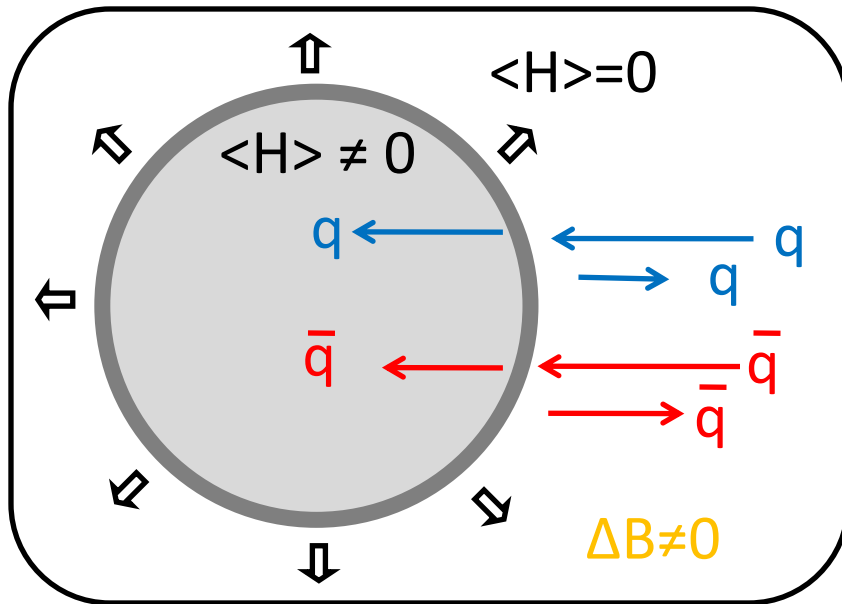
- ◆ At $T > T_c$, all gauge int. and sphaleron processes are in eq.
- ◆ **EWPT** $\rightarrow$ masses change suddenly
- ◆ If **1st order**
 - expanding bubble of broken phase $\Rightarrow$ out of eq.



 **Electroweak baryogenesis**

EW Baryogenesis in SM (1)

[Farrar and Shaposhnikov (1993)]



- CP violating scattering of quarks with bubble wall
 - Transmission rate is CP asymmetric
 $\Rightarrow B \neq 0$ is stored in **broken phase**
 - B in **symmetric phase** is erased by rapid sphaleron process
- To avoid washout of generated baryon asymmetry, sphaleron processes should **decouple inside bubble**

$$\Gamma_{\cancel{B}} \sim e^{-E_{sph}/T} \big|_{T=T_c} \lesssim H(T_c) \quad E_{sph} = \frac{4\pi\phi}{g_2} \times (1.5 - 2.7)$$

$$\Rightarrow \frac{\phi_c}{T_c} \gtrsim 1 \quad \Rightarrow \text{strongly 1st order EWPT is required}$$

EW Baryogenesis in SM (2)

◆ EW baryogenesis does not work in SM, because of two reasons :

(1) CP violation from *KM phase is not sufficient*

$$\left| \frac{n_B}{s} \right| < 10^{-26} \quad \text{from Huet and Sather (1995)} \\ \text{[Similar bound by Gavela et al. (1994)]}$$

15 orders of magnitude smaller than $\left(\frac{n_B}{s} \right)_{obs.} \sim 10^{-11}$

(2) For $m_h > 114 \text{ GeV}$ (LEP), *EWPT is not 1st order* (explained later)

--> To explain baryon asymmetry, *physics beyond SM* is required!

4th family revive EW Baryogenesis ?

4th family fermions : t' , b' , ν' , τ'

(1) **extra CP violating phases** appear in 4×4 CKM matrix

4th family enhances CP asymmetry

[Shaposhnikov (1987); W.-S. Hou (2008)]

◆ smallness of CPV in SM

➤ Jarlskog invariant: nonzero if and only if CPV exists

$$J = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)2A$$

A: area of unitarity triangle

➤ size of CPV at $T_c \sim 100\text{GeV}$ $\frac{J}{T_c^{12}} \sim 10^{-20}$ <-- too small to explain BAU

◆ In 4th family case, similar quantity for 2-3-4 generation

$$J_{(2,3,4)}^{sb} = (m_{t'}^2 - m_t^2)(m_{t'}^2 - m_c^2)(m_t^2 - m_c^2)(m_{b'}^2 - m_b^2)(m_{b'}^2 - m_s^2)(m_b^2 - m_s^2)2A_{234}^{sb}$$
$$\sim \frac{m_{t'}^4}{m_t^2 m_c^2} \frac{m_{b'}^4}{m_b^2 m_s^2} \frac{A_{234}^{sb}}{A} J$$

--> 10^{15} enhancement is possible for $m_{t'} \sim m_{b'} \sim 600\text{GeV}$!

4th family revive EW Baryogenesis ?

4th family fermions : t' , b' , ν' , τ'

(1) **extra CP violating phases** appear in 4×4 CKM matrix

◆ dimensional analysis suggests **large enough enhancement**

4th family revive EW Baryogenesis ?

4th family fermions : t' , b' , ν' , τ'

(1) **extra CP violating phases** appear in 4×4 CKM matrix

◆ dimensional analysis suggests **large enough enhancement**

(2) In this talk, I will concentrate on another problem of EWBG

◆ 4th family fermions can induce strongly 1st order EWPT?

Experimental constraints on 4th family

$N_\nu \sim 3$

$$Z \not\rightarrow \nu' \bar{\nu}'$$

◆ from measurement of Z decay width --> $m_{\nu'} > m_Z/2$

Direct search experiments

[Particle Data Group 2010]

$m_{t'} > 256 \text{ GeV}$, $m_{b'} > 199 \text{ GeV}$, $m_{\tau'} > 102 \text{ GeV}$, $m_{\nu'} > 90 \text{ GeV}$

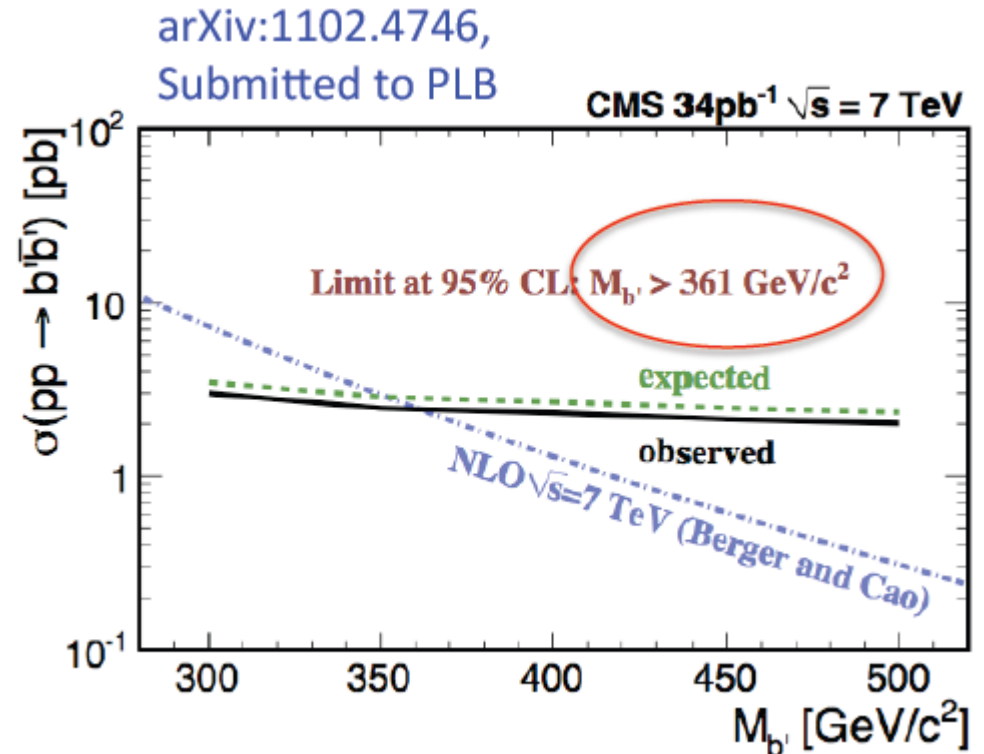
--> $m_{t'} > 335 \text{ GeV}$ (CDF@4.6fb⁻¹), $m_{b'} > 372 \text{ GeV}$ (CDF@4.8fb⁻¹)

LHC already gives constraint on 4th family

◆ CMS b' search

- **Pair produced $b' \rightarrow tW \rightarrow WWb$**
- Like-sign dilepton and trilepton (e, μ) decays + jets (BR=7.3%)
- $N_{\text{background}} = 0.3 \pm 0.2$ events ($t\bar{t}$ +jets)
- 0 events observed

Similar to a new CDF limit
of 372 GeV, arXiv:1101.5728 (4.8 fb^{-1})



From Santanastasio's talk slide @ Moriond2011

Already comparable to Tevatron bound!

Experimental constraints on 4th family

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--> $m_{t'} > 335 \text{ GeV}$ (CDF@4.6fb⁻¹), $m_{b'} > 372 \text{ GeV}$ (CDF@4.8fb⁻¹)

EW precision data (indirect measurement)

◆ ρ -parameter constraint

$$\delta\rho \sim 0 \Rightarrow |m_{t'} - m_{b'}| \lesssim m_W \quad \rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W}$$

◆ also consistent with other EW data (by S,T-parameter analysis)

[Kribs , Plehn, Spannowsky and Tait (2007)]

4th family is still consistent with experiments!

2. EW phase transition in SM

Finite Temperature Effective Potential

- ◆ tool for analyzing phase transition
- ◆ FTEP = free energy density
- ◆ obtained from partition function

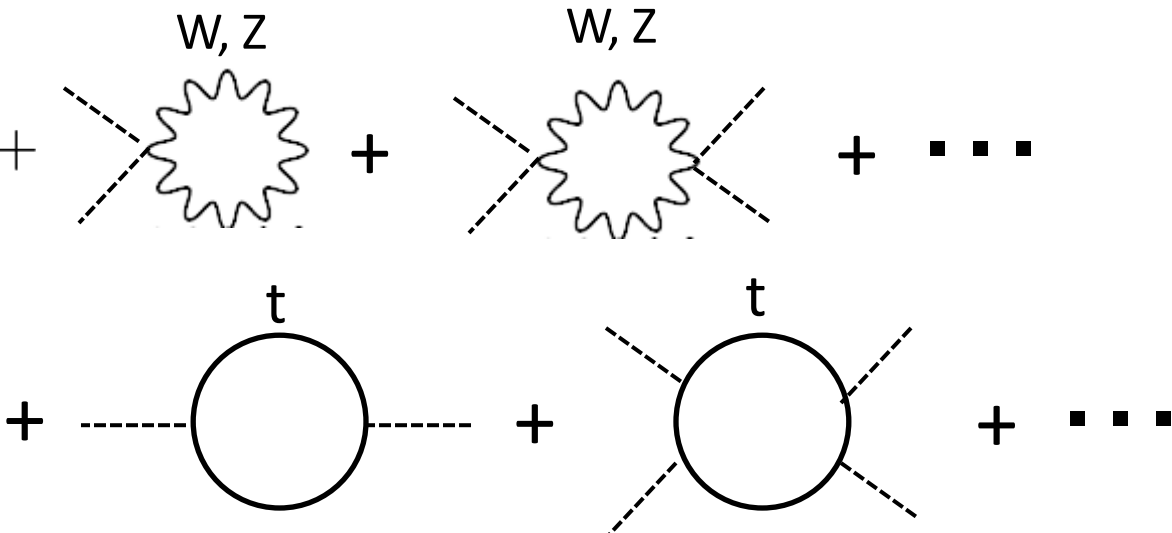
$$V(\phi, T) = -\frac{1}{V}T \log Z \quad \left\{ \begin{array}{l} \text{partition function: } Z = \text{Tr}[e^{-\beta H}] \quad \beta \equiv 1/T \\ \text{order parameter: } \langle H(x) \rangle_T = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix} \end{array} \right.$$

- ◆ calculated based on the finite-temperature field theory

One-loop FTEP in SM (1)

◆ FTEP at one loop

tree level effective potential : $V_0(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$

$$V(\phi, T) = V_0(\phi) +$$


The diagram illustrates the one-loop corrections to the effective potential. It shows two rows of Feynman diagrams. The first row contains two diagrams with a wavy loop (representing W or Z bosons) and two external dashed lines, followed by a plus sign and an ellipsis. The second row contains two diagrams with a fermion loop (representing top quarks, labeled 't') and two external dashed lines, followed by a plus sign and an ellipsis.

One-loop FTEP in SM (2)

◆ FTEP at one loop

$$V(\phi, T) = V_0(\phi) + \underline{V_1^{(0)}(\phi)} + \underline{V_1^{(T)}(\phi, T)} + \dots$$

1. T-independent correction (Coleman-Weinberg potential)

$$V_1^{(0)}(\phi) = \sum_{i=W,Z,t} \frac{m_i^4(\phi)}{64\pi^2} \left[\ln \frac{m_i^2(\phi)}{\mu^2} + \text{const.} \right]$$

2. T-dependent correction

$$V_1^{(T)}(\phi, T) = \frac{T^4}{2\pi^2} (6J_B[m_W^2(\phi)/T^2] + 3J_B[m_Z^2(\phi)/T^2] - 6J_F[m_t^2(\phi)/T^2])$$

$$J_{B,F}(m^2/T^2) = \int_0^\infty dx \, x^2 \log[1 \mp e^{-\sqrt{x^2+m^2/T^2}}]$$

One-loop FTEP in SM (3)

$$J_{B,F}(m^2/T^2) = \int_0^\infty dx \, x^2 \log[1 \mp e^{-\sqrt{x^2+m^2}/T^2}]$$

◆ high temperature expansion : $m(\phi)/T \ll 1$

$$\begin{cases} J_B(m^2/T^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12} \frac{m^2}{T^2} - \frac{\pi}{6} \left(\frac{m^2}{T^2} \right)^{3/2} + \mathcal{O}\left(\frac{m^4}{T^4}\right) \\ J_F(m^2/T^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24} \frac{m^2}{T^2} + \mathcal{O}\left(\frac{m^4}{T^4}\right) \end{cases}$$

◆ For $T > m_W(\phi), m_Z(\phi), m_t(\phi)$, $m_i^2(\phi) \sim \phi^2$

$$V(\phi, T) \simeq \left[\left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v^2} \right) T^2 - \frac{\mu^2}{2} \right] \phi^2 - ET |\phi|^3 + \frac{\lambda_T}{4} \phi^4$$

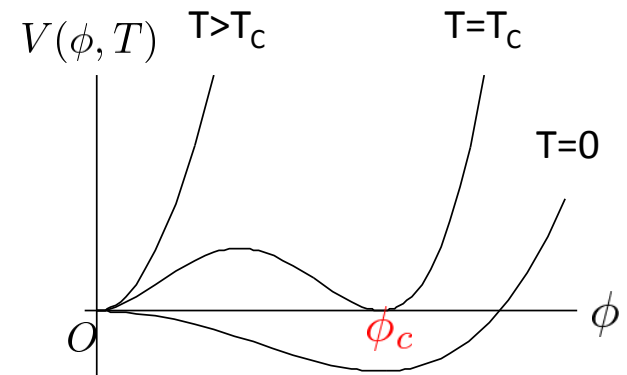
$$\begin{cases} E = \frac{2m_W^3 + m_Z^3}{4\pi v^3} \sim 10^{-2} \\ \lambda_T = \lambda + \text{log-terms} \end{cases}$$

EW phase transition in SM (1)

Behavior of EWPT

$$V(\phi, T) \simeq \left[\left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v^2} \right) T^2 - \frac{\mu^2}{2} \right] \phi^2 - ET|\phi|^3 + \frac{\lambda_T}{4} \phi^4$$

- ◆ For high T , quadratic term > 0 --> restoration of EW symmetry
- ◆ cubic term induce 1st order phase transition



EW phase transition in SM (2)

Behavior of EWPT

$$V(\phi, T) \simeq \left[\left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v^2} \right) T^2 - \frac{\mu^2}{2} \right] \phi^2 - ET|\phi|^3 + \frac{\lambda_T}{4} \phi^4$$

- ◆ For high T , quadratic term > 0 --> restoration of EW symmetry
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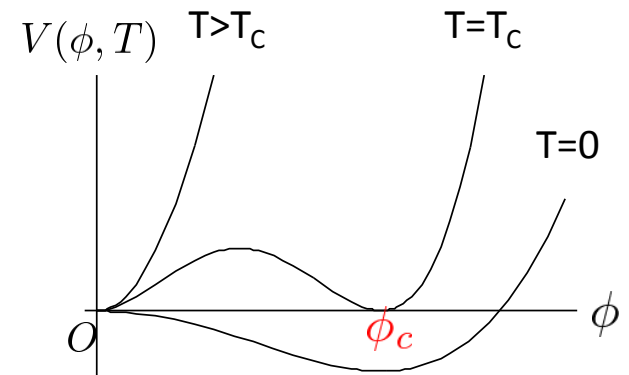
$$\frac{\phi_c}{T_c} = \frac{2E}{\lambda_{T_c}} \xrightarrow{\frac{\phi_c}{T_c} \gtrsim 1} m_h \lesssim \sqrt{4E}v \sim 49\text{GeV}$$



LEP bound : $m_h > 114\text{GeV}$



EWPT is not strongly 1st order in SM



- ◆ Lattice : crossover for $m_h \gtrsim 80\text{GeV}$

[Kajantie et al. (1996)]

How to strengthen EWPT

- ◆ Introduce **new bosons** strongly coupled to Higgs

 $\Delta V \sim \Delta E T \phi^3$

 strengthen 1st order phase transition
ex. light stop scenario in MSSM

- ◆ 4th family fermions themselves do not play role

- ◆ Extra bosons are required

➤ SUSY 4th family model [Fok and Kribs(2008)]

- ◆ Naturally realized when 4th family is responsible for EW symmetry breaking (strongly coupled 4th family scenario)

3. Strongly coupled 4th family

4th family and EW symmetry breaking [Holdom (1984)]

4th family fermions : t', b', ν', τ'

- ◆ extremely heavy : $m \gtrsim 246 \text{ GeV}$
- ◆ can be origin of EW symmetry breaking

order parameters

$$\langle H \rangle \longrightarrow \langle \bar{t}' t' \rangle, \langle \bar{b}' b' \rangle, \langle \bar{\tau}' \tau' \rangle, \langle \bar{\nu}'_{\tau} \nu'_{\tau} \rangle$$

➤ Higgs sector : 4th family fermions w/ new int.

- ◆ 4th family version of top-quark condensate model

Nambu – Jona-Lasinio model of 4th family (1)

◆ SM without Higgs field + 4th family

◆ introduce 4-Fermi operator among 4th family quarks

$$\mathcal{L}_{4f} = G_{q'} (\bar{q}'_{Li} q'_{Rj}) (\bar{q}'_{Rj} q'_{Li}) \quad q'_i = \begin{pmatrix} t' \\ b' \end{pmatrix}$$

with cutoff Λ_{4f}

➤ If $G_{q'} > G_{q',c} = 8\pi^2 / (N_c \Lambda_{4f}^2)$

$$\langle \bar{t}' t' \rangle = \langle \bar{b}' b' \rangle \neq 0 \quad \Rightarrow \quad \begin{cases} \text{break EW symmetry} \\ m_{t'} = m_{b'} \neq 0 \end{cases}$$

➤ **2 Higgs doublets** appear as the bound state of 4th quarks

$$\phi_1 = \begin{pmatrix} \phi_1^0 \\ \phi_1^- \end{pmatrix} \sim \begin{pmatrix} \bar{t}'_R t'_L \\ \bar{t}'_R b'_L \end{pmatrix} \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \sim \begin{pmatrix} \bar{b}'_R t'_L \\ \bar{b}'_R b'_L \end{pmatrix}$$

3NGBs + 5 physical Higgs bosons --> **can induce 1st EWPT!**

Nambu – Jona-Lasinio model of 4th family (2)

Other 4 Fermi operators

- among 4th family leptons $\ell' = (\nu', \tau')$

ex.) $G_{\ell' \bar{\ell}'_L \ell'_R \bar{\ell}'_R \ell'_L}$

⇒ extra 2 Higgs doublet is generated (4 doublet Higgs in total)

- between 4th family and first 3 family fermions

ex.) $G_{t' t \bar{t}'_L t'_R \bar{t}_R t_L} \quad \rightarrow \text{mass of top}$

◆ For simplicity,

- assume that **only** 4th family quarks condense
- neglect effects from τ', ν' and top
- switch off $SU(2) \times U(1)$ gauge couplings

Low-energy effective model of NJL (1)

◆ The effects of composite Higgs should be included

➤ low-energy effective model at $\mu_{EW}=246$ GeV

➤ Lagrangian --Yukawa theory of 4th quarks and 2HD--

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - y(\bar{q}'_L \phi_1 t'_R + \bar{q}'_L \phi_2 b'_R + h.c.) - V(\phi_1, \phi_2)$$

$$\left[\begin{array}{l} \mathcal{L}_{\text{kin}} : \text{kinetic term of 4th family quarks and Higgs} \\ V(\phi_1, \phi_2) : \text{Higgs potential} \end{array} \right.$$

➤ assume $\Lambda_{4f} \gg \mu_{EW} \Rightarrow$ higher dim. operator can be neglected
can use renormalizable Lagrangian

Low-energy effective model of NJL (2)

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - y(\bar{q}'_L \phi_1 t'_R + \bar{q}'_L \phi_2 b'_R + h.c.) - V(\phi_1, \phi_2)$$

➤ to satisfy $\delta\rho \sim 0$,

assume $m_{t'} = m_{b'}$ and impose $SU(2)_R$ symmetry

➤ symmetry of this model: $SU(2)_L \times SU(2)_R \times U(1)_A$

$$U(1)_A : q'_L \rightarrow e^{-i\theta} q'_L, \quad q'_R \rightarrow e^{i\theta} q'_R, \quad \phi_{1,2} \rightarrow e^{-2i\theta} \phi_{1,2}$$

➤ use matrix notation for Higgs fields

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (2, 2^*)$$

Low-energy effective model of NJL (3)

Higgs potential for bi-fundamental Higgs $\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$

$$V(\Phi) = m_\Phi^2 \text{tr}(\Phi^\dagger \Phi) + \frac{\lambda_1}{2} \text{tr}(\Phi^\dagger \Phi) \text{tr}(\Phi^\dagger \Phi) + \frac{\lambda_2}{2} \text{tr}(\Phi^\dagger \Phi \Phi^\dagger \Phi)$$

◆ This model is equivalent to NJL model [Bardeen, Hill and Lindner (1990)]

➤ If running coupling satisfy boundary condition

$$\bar{\lambda}_1(\mu) \rightarrow 0, \quad \bar{\lambda}_2(\mu) \rightarrow \infty, \quad \bar{g}(\mu) \rightarrow \infty \quad \text{when } \mu \rightarrow \Lambda_{4f}$$

(**compositeness condition**)

at some high energy scale Λ_{4f} (composite scale)

◆ One can obtain value of λ_1, λ_2 corresponding to NJL by RG

◆ In following analysis, we first consider **whole** of parameter space **without** compositeness condition, and then discuss about effects of compositeness

Low-energy effective model of NJL (4)

Higgs potential

$$V(\Phi) = m_{\Phi}^2 \text{tr}(\Phi^\dagger \Phi) + \frac{\lambda_1}{2} \text{tr}(\Phi^\dagger \Phi) \text{tr}(\Phi^\dagger \Phi) + \frac{\lambda_2}{2} \text{tr}(\Phi^\dagger \Phi \Phi^\dagger \Phi)$$

bi-fundamental Higgs : $\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}$

◆ massless NG boson of $U(1)_A$ symmetry breaking

➤ add explicit $U(1)_A$ (soft) breaking mass term

$$\Delta V(\Phi) = -c(\det \Phi + h.c.)$$

➤ examine effect of nonzero pseudo NG boson mass

Low-energy property of model (1)

◆ scalar sector is special case of 2 Higgs doublet model

◆ $SU(2)_R$ symmetry $\Rightarrow \langle \Phi \rangle = \frac{1}{2} \begin{pmatrix} \phi & 0 \\ 0 & \phi \end{pmatrix} \Rightarrow \tan \beta = 1$

◆ tree-level effective potential

$$V_0(\phi) = \frac{1}{2}(m_{\Phi}^2 - c)\phi^2 + \frac{1}{8} \left(\lambda_1 + \frac{\lambda_2}{N_f} \right) \phi^4$$

$N_f = 2 :$

of 4th family quark flavors

➤ $(\lambda_1 + \lambda_2/N_f)$ corresponds to λ in SM

➤ For the stability of potential : $\lambda_1 + \lambda_2/N_f > 0$

◆ 4 free parameters

scalar self couplings : λ_1, λ_2 , Yukawa coupling : y

$U(1)_A$ breaking mass : c

Low-energy property of model (2)

◆ mass spectra of Higgs bosons ($\phi_0 = 246\text{GeV}$)

h : SM like Higgs

$$m_h = \sqrt{\lambda_1 + \lambda_2/N_f}\phi_0$$

$\xi = (H^0, H^\pm)$: extra Higgs

$$m_\xi = \sqrt{2c + (\lambda_2/N_f)\phi_0^2}$$

➤ **degenerate** due to $SU(2)_R$

η : pseudo scalar Higgs

$$m_\eta = \sqrt{2c}$$

⇒ pNGB of $U(1)_A$ breaking

π : would be NG bosons

$$m_\pi = 0$$

⇒ eaten by W,Z

◆ experimental bound on Yukawa coupling

$$m_{q'} \gtrsim 256\text{GeV} \quad \Rightarrow \quad y \gtrsim 2.1 \quad \text{cf.} \quad m_{q'} = \frac{y}{\sqrt{2N_f}}\phi_0$$

Cutoff scale of effective model

- ◆ We define cutoff as energy scale where one of running coupling
 - **diverges**
 - or
 - **becomes negative**
- ⇒ suggest breakdown of theory

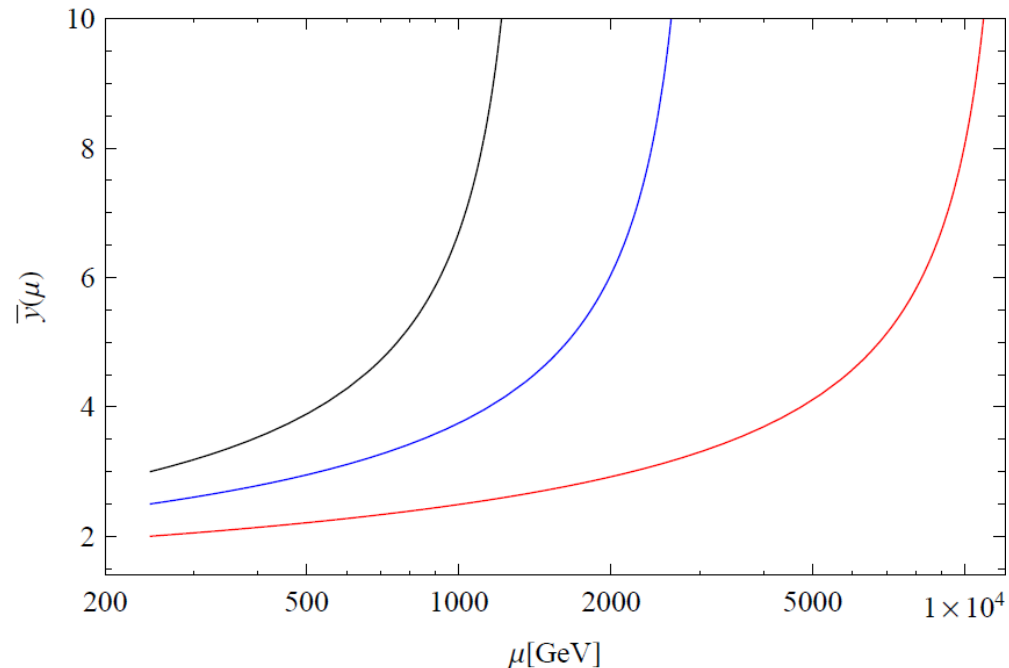
➤ Yukawa coupling

- 1-loop RGE

$$\mu \frac{\partial}{\partial \mu} \bar{y} = \frac{1}{16\pi^2} (N_f + N_c) \bar{y}^3$$

- boundary condition:
 $y > 2.1$ at $\mu = 246$ GeV

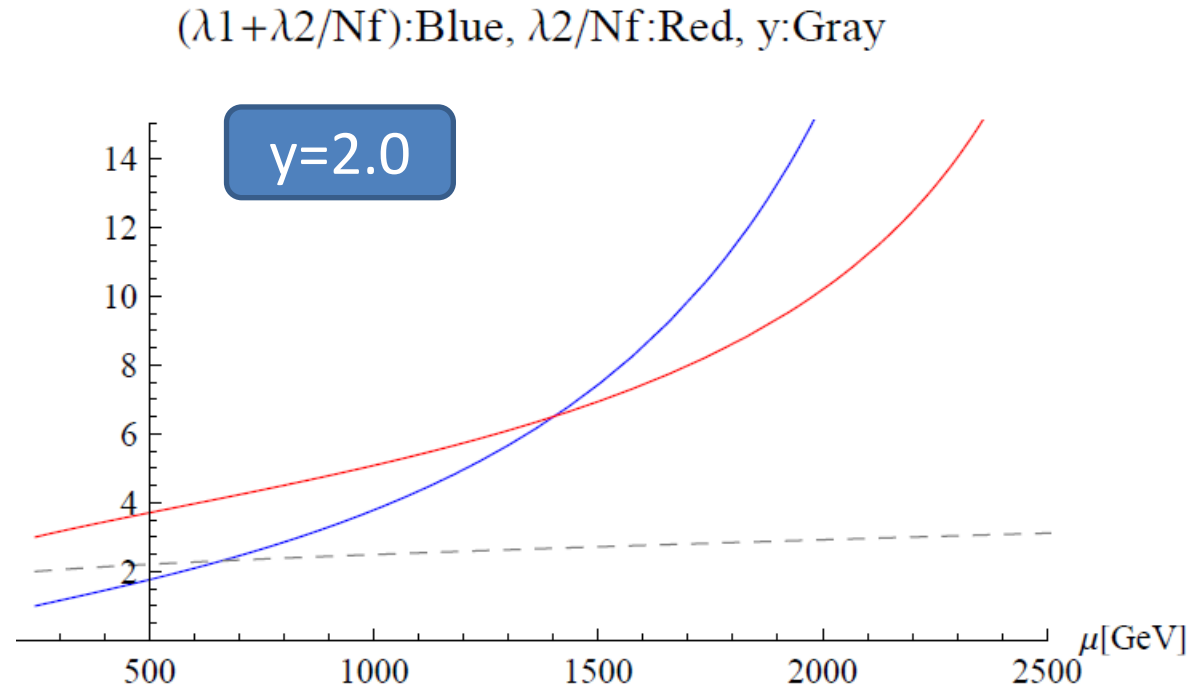
- blow up around $1 \sim 10$ TeV



➤ Higgs self-couplings

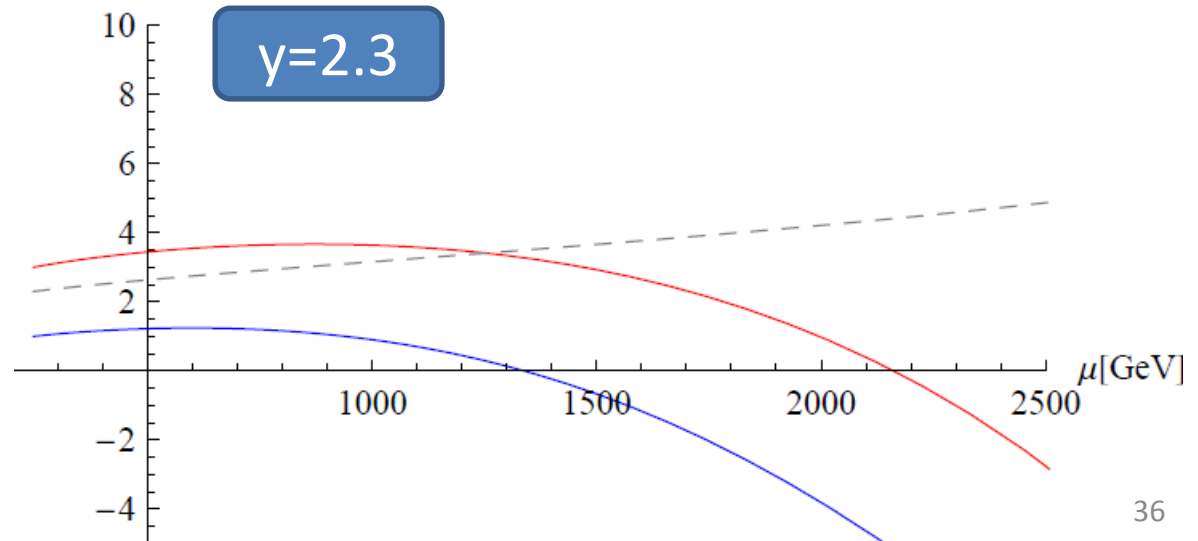
Case 1.

Higgs self-couplings
blow up
(Landau pole)



Case 2.

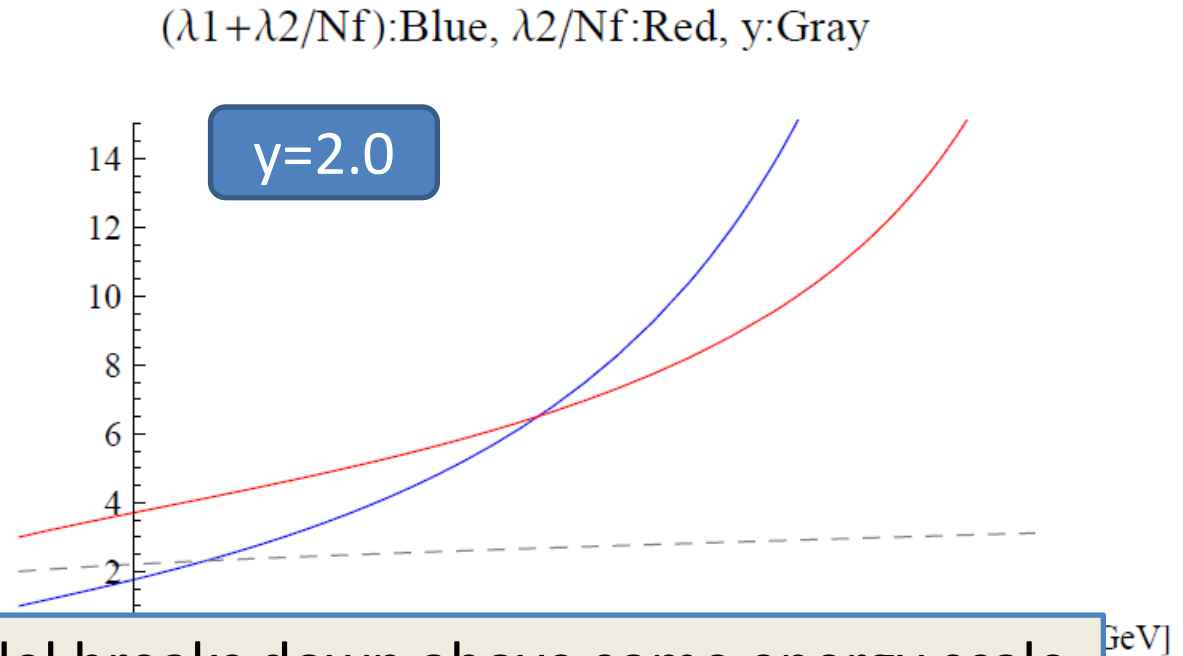
Higgs self-couplings
are pulled down to
negative
(vacuum instability)



➤ Higgs self-couplings

Case 1.

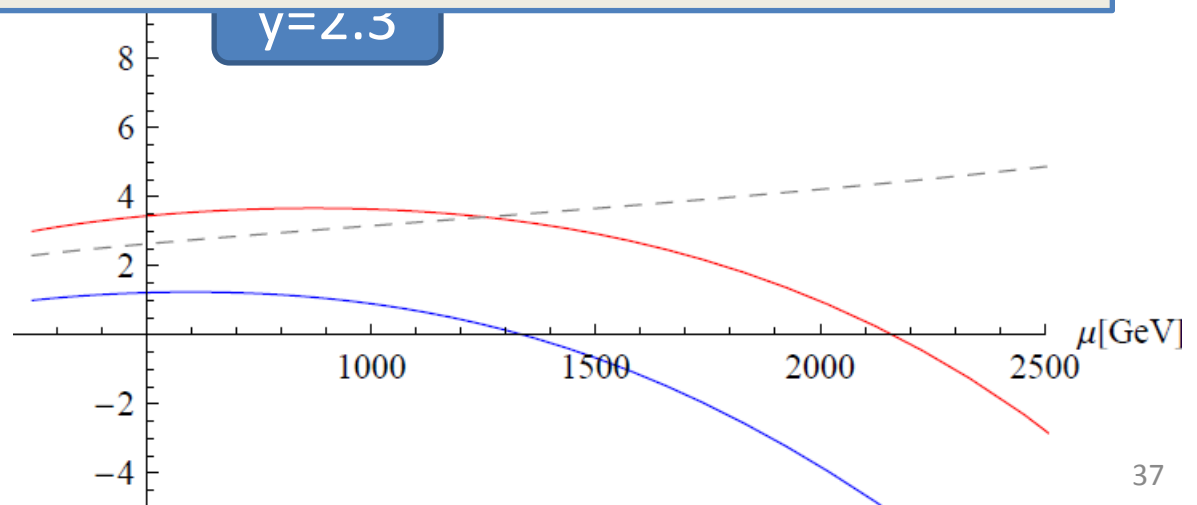
Higgs self-couplings
blow up
(Landau pole)



The effective model breaks down above some energy scale
Cutoff Λ should be introduced

Case 2.

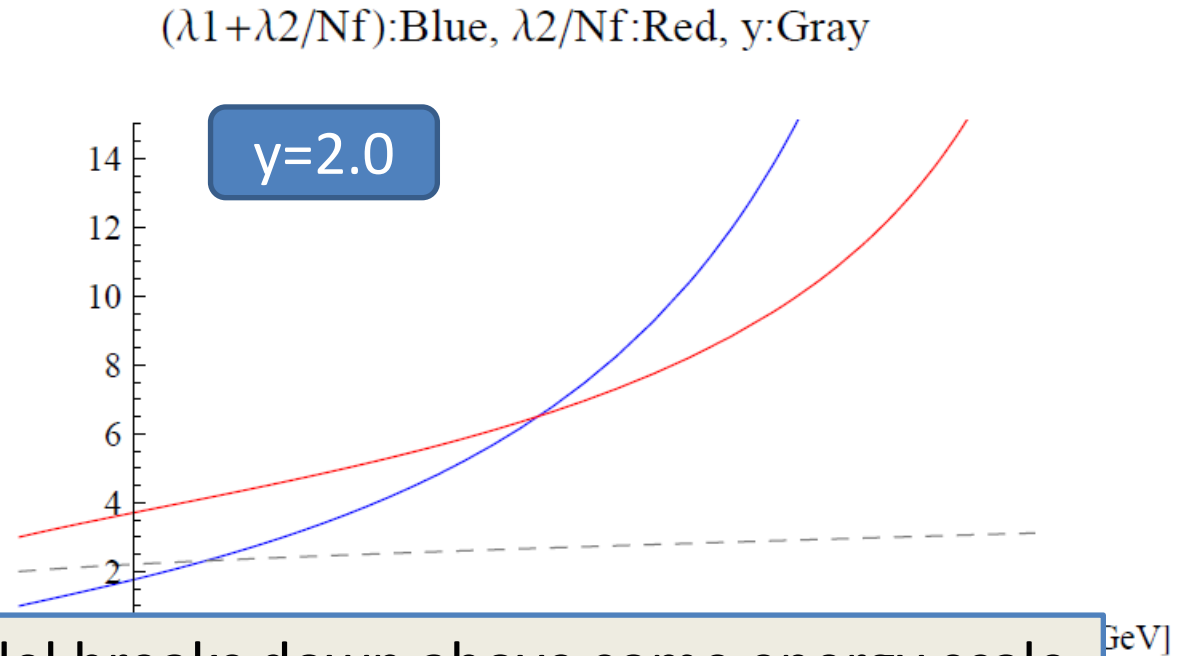
Higgs self-couplings
are pulled down to
negative
(vacuum instability)



➤ Higgs self-couplings

Case 1.

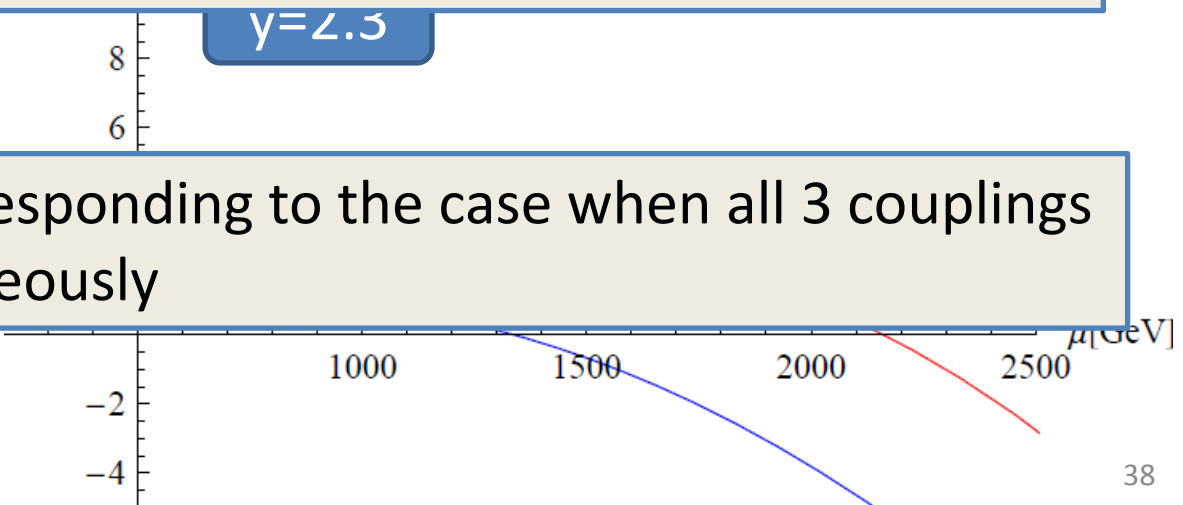
Higgs self-couplings
blow up
(Landau pole)



The effective model breaks down above some energy scale
Cutoff Λ should be introduced

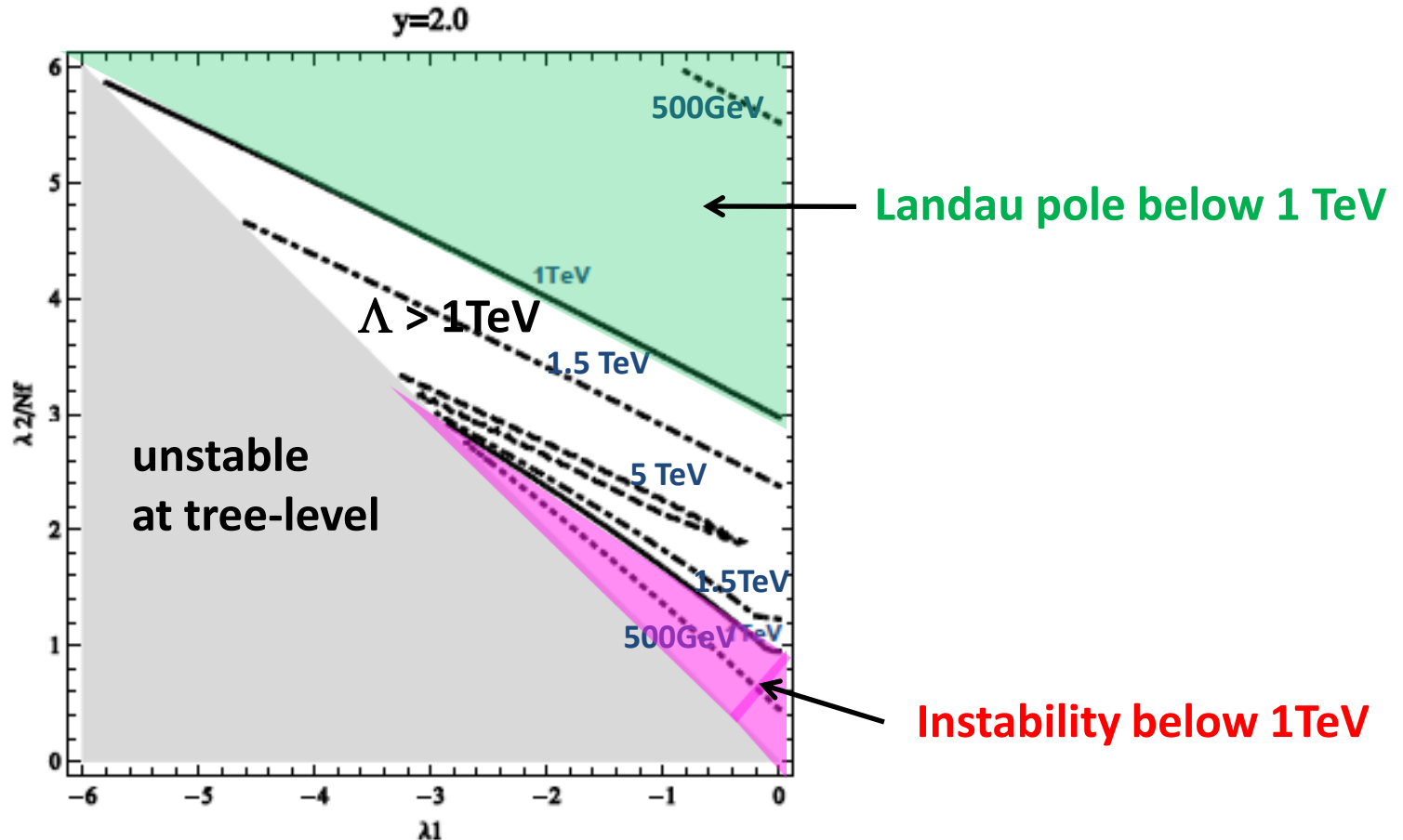
Case 2.

Higgs self-couplings
are pulled down to
negative values
(vacuum instability)



NJL model is corresponding to the case when all 3 couplings
blow up simultaneously

◆ contour plot of cutoff Λ



To ensure renormalizability $\rightarrow \Lambda \gg m_h$, $T_c \sim O(100)\text{GeV}$

- In the following, we require $\Lambda > 1\text{TeV}$
 $\leftrightarrow y < 3.0$ ($m_{q'} < 480\text{ GeV}$ (on-shell))

4. EWPT with strongly coupled 4th family

Y. Kikukawa , M.K., J.Yasuda, Prog. Theor. Phys. 122, No. 2, 2009

One-loop effective potential (with ring improvement)

$$\begin{aligned}
 V(\phi, T) = & \frac{1}{2}(m_{\Phi}^2 - c)\phi^2 + \frac{1}{8} \left(\lambda_1 + \frac{\lambda_2}{N_f} \right) \phi^4 \\
 & + \sum_{i=h,\xi,\eta,\pi,q'} n_i \frac{\mathcal{M}_i^4(\phi, T)}{64\pi^2} \left[\ln \frac{\mathcal{M}_i^2(\phi, T)}{\mu^2} - \frac{3}{2} \right] + \frac{1}{2} A \phi^2 \\
 & + \sum_{i=h,\xi,\eta,\pi,q'} n_i \frac{T^4}{2\pi^2} \int_0^\infty dx \, x^2 \ln \left[1 \mp \exp \left(-\sqrt{x^2 + \mathcal{M}_i^2(\phi, T)/T^2} \right) \right]
 \end{aligned}$$

$$\begin{cases} \mathcal{M}_i^2(\phi, T) = m_i^2(\phi) + [(N_f^2 + 1)\lambda_1 + 2N_f\lambda_2 + y^2] \frac{T^2}{12} \\ \mathcal{M}_{q'}^2(\phi, T) = m_{q'}^2(\phi) \end{cases}$$

- including higher order contributions (ring diagram)

$$V_{\text{ring}}(\phi, T) = \text{[1-loop ring diagram]} + \text{[2-loop ring diagram]} + \text{[3-loop ring diagram]} + \dots$$

$$\text{At high } T, \text{ [1-loop ring diagram]} \sim \lambda_i T^2$$

High temperature expansion

- ◆ extra Higgs ξ induce cubic term ($m_\xi \gg m_h$)

$$\Delta V \sim \Delta E T \phi^3 \quad \Delta E \sim (\lambda_2/N_f)^{3/2} \sim m_\xi^3$$

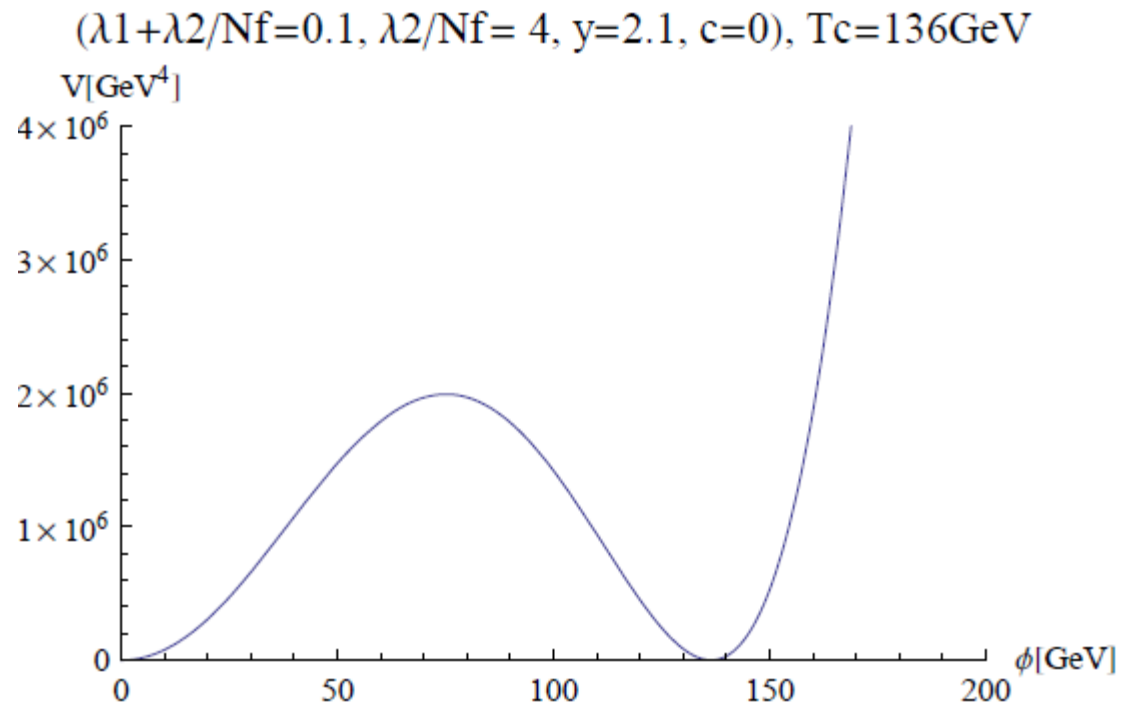

$$\frac{\phi_c}{T_c} \sim \frac{(\lambda_2/N_f)^{3/2}}{(\lambda_1 + \lambda_2/N_f)} \sim \frac{m_\xi^3}{m_h^2}$$

 strongly 1st order for $m_\xi \gg m_h$

- ◆ make sense for $\lambda_i, y \ll 1$
- ◆ cannot rely on high-T expansion for 4th family quarks

 estimate T-dependent loop integral numerically

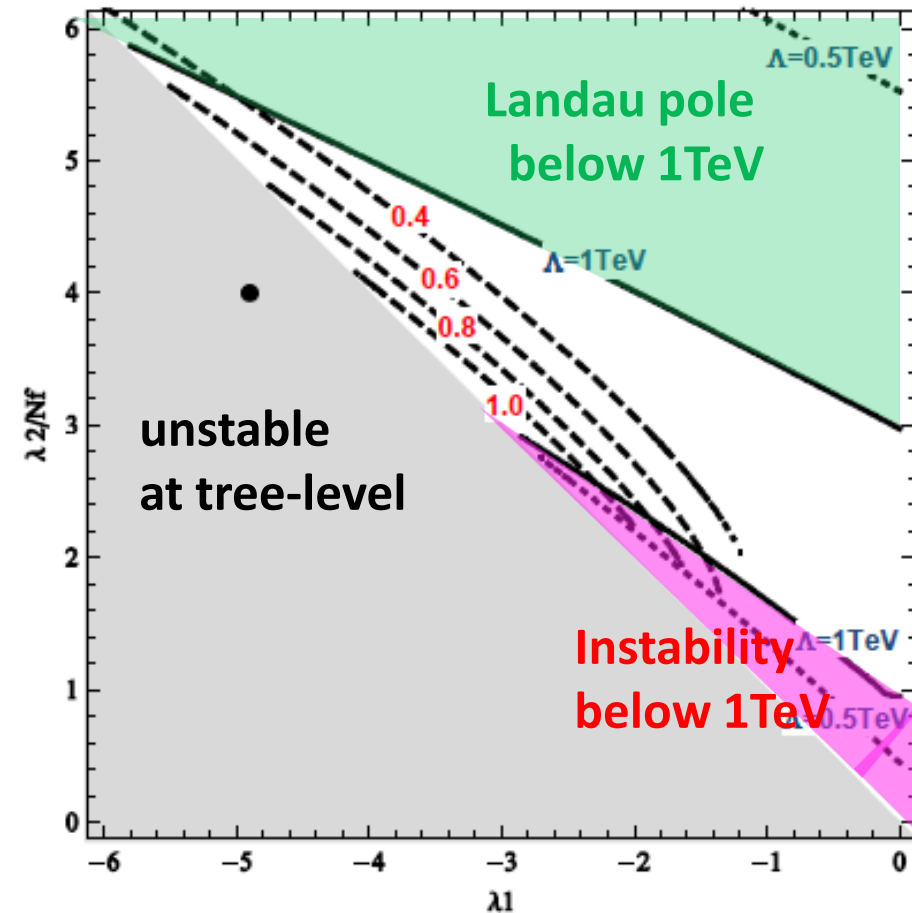
Effective potential at T_c



Numerical results (1)

◆ Contour plot for ϕ_c/T_c on λ_1 - λ_2/N_f plane

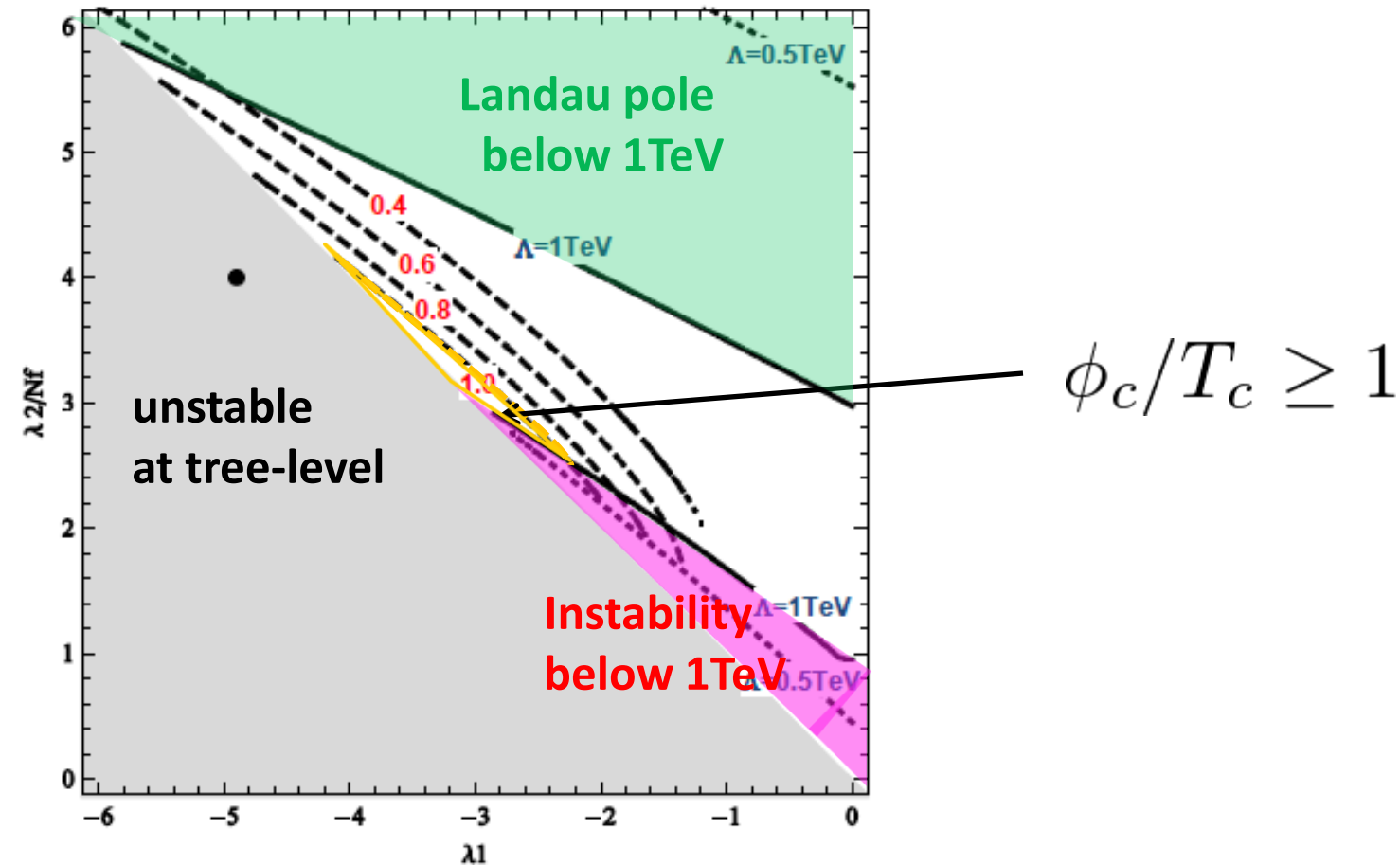
$\gamma=2$ ($m_{q'}=246$ GeV) and $m_\eta=0$



Numerical results (1)

◆ Contour plot for ϕ_c/T_c on λ_1 - λ_2/N_f plane

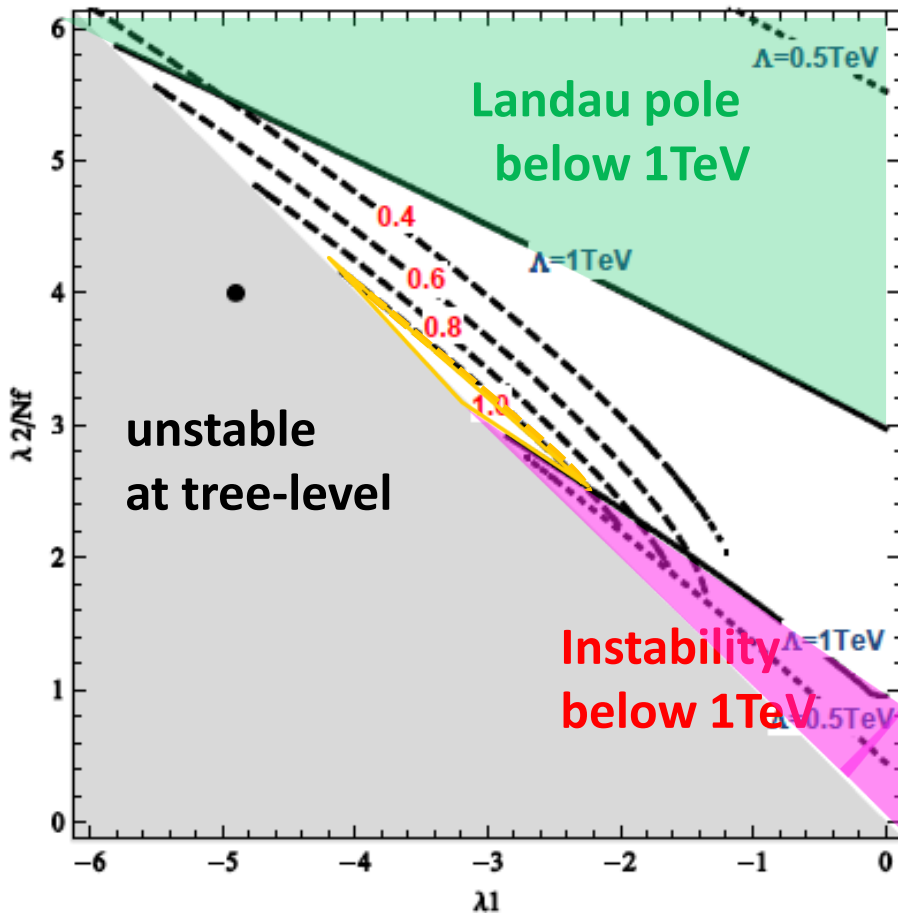
$\gamma=2$ ($m_{q'}=246$ GeV) and $m_\eta=0$



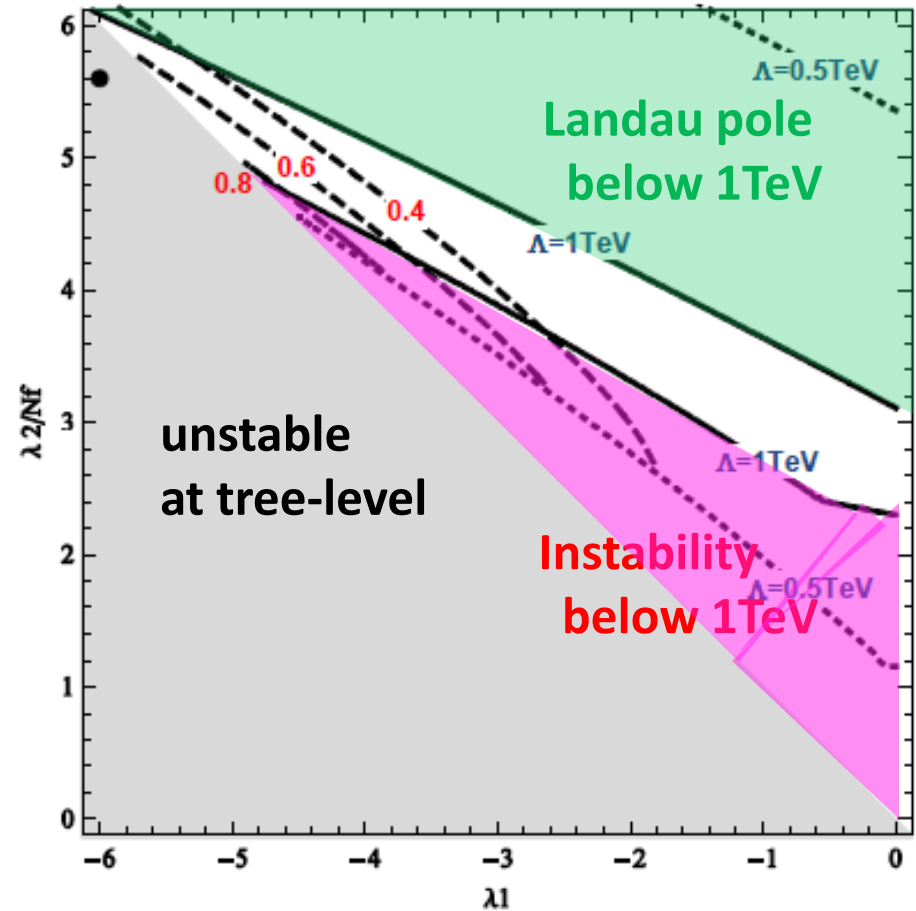
Numerical results (1)

◆ Contour plot for ϕ_c/T_c on λ_1 - λ_2/N_f plane

$y=2$ ($m_{q'}=246$ GeV) and $m_\eta=0$



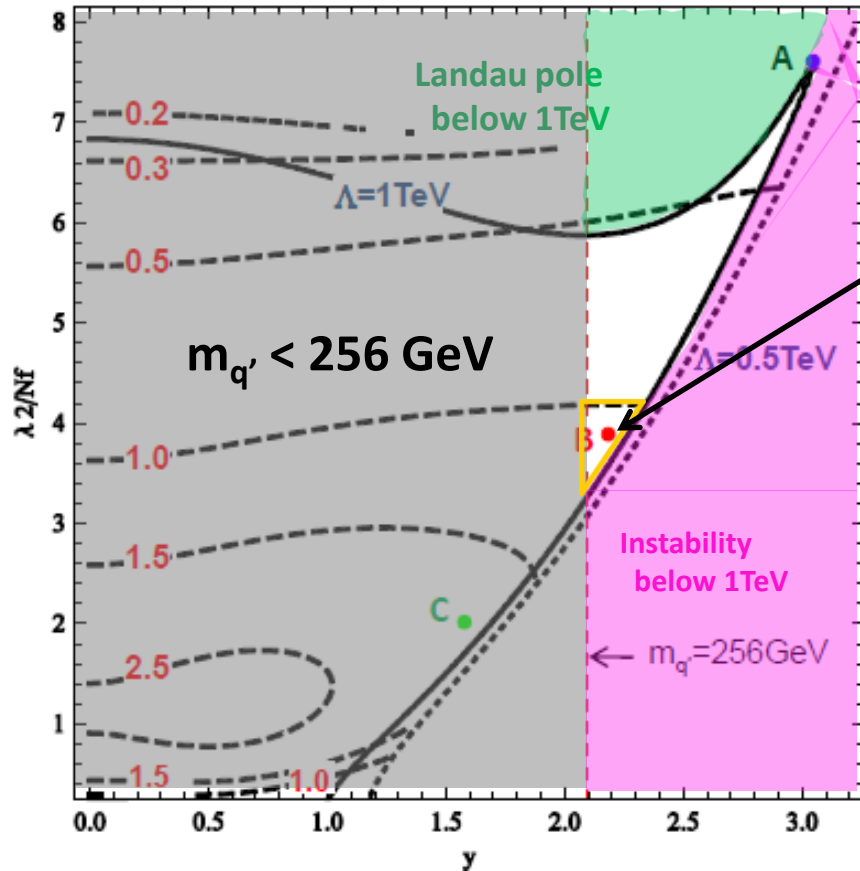
$y=2.5$ ($m_{q'}=310$ GeV) and $m_\eta=0$



Numerical results (2)

◆ Contour plot for ϕ_c/T_c on y - λ_2/N_f plane

$\lambda_1 + \lambda_2/N_f \sim 0$ and $m_\eta = 0$



$$\phi_c/T_c \geq 1$$

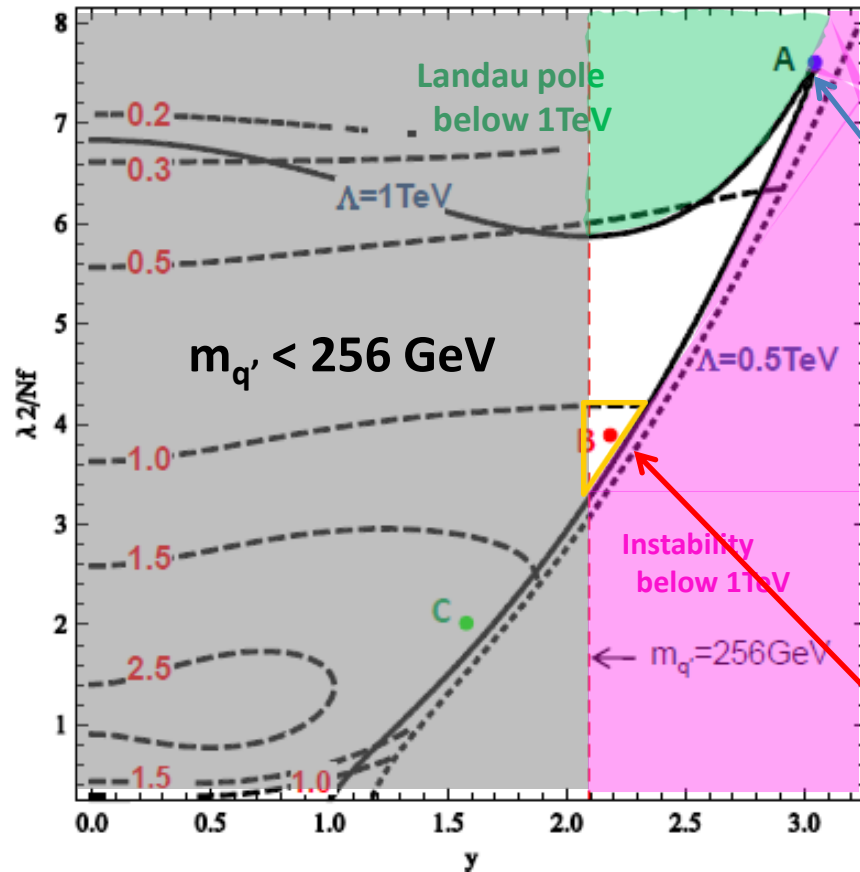
- upper bound on λ_2/N_f and y
 $\Rightarrow$ upper bound on m_ξ and $m_{q'}$

$$\begin{aligned} 256 \text{ GeV} &< m_{q'} < 290 \text{ GeV} \\ 430 \text{ GeV} &< m_\xi < 500 \text{ GeV} \\ 200 \text{ GeV} &< m_h(1\text{-loop}) < 300 \text{ GeV} \end{aligned}$$

Numerical results (2) --Prediction of NJL model--

◆ Contour plot for ϕ_c/T_c on y - λ_2/N_f plane

$\lambda_1 + \lambda_2/N_f \sim 0$ and $m_\eta = 0$



compositeness condition

$$\bar{\lambda}_1(\Lambda_{4f}) = 0, \quad \bar{\lambda}_2(\Lambda_{4f}) = \frac{16\pi^2}{N_f}, \quad \bar{y}(\Lambda_{4f})^2 = \frac{16\pi^2}{N_c}$$

$$\Lambda_{4f} = 1 \text{ TeV}$$

$$\Rightarrow \phi_c/T_c \sim 0$$

$$\bar{\lambda}_1(\Lambda_{4f}) = 0, \quad \bar{\lambda}_2(\Lambda_{4f}) = \frac{8\pi^2}{N_f}, \quad \bar{y}(\Lambda_{4f})^2 = \frac{8\pi^2}{N_c}$$

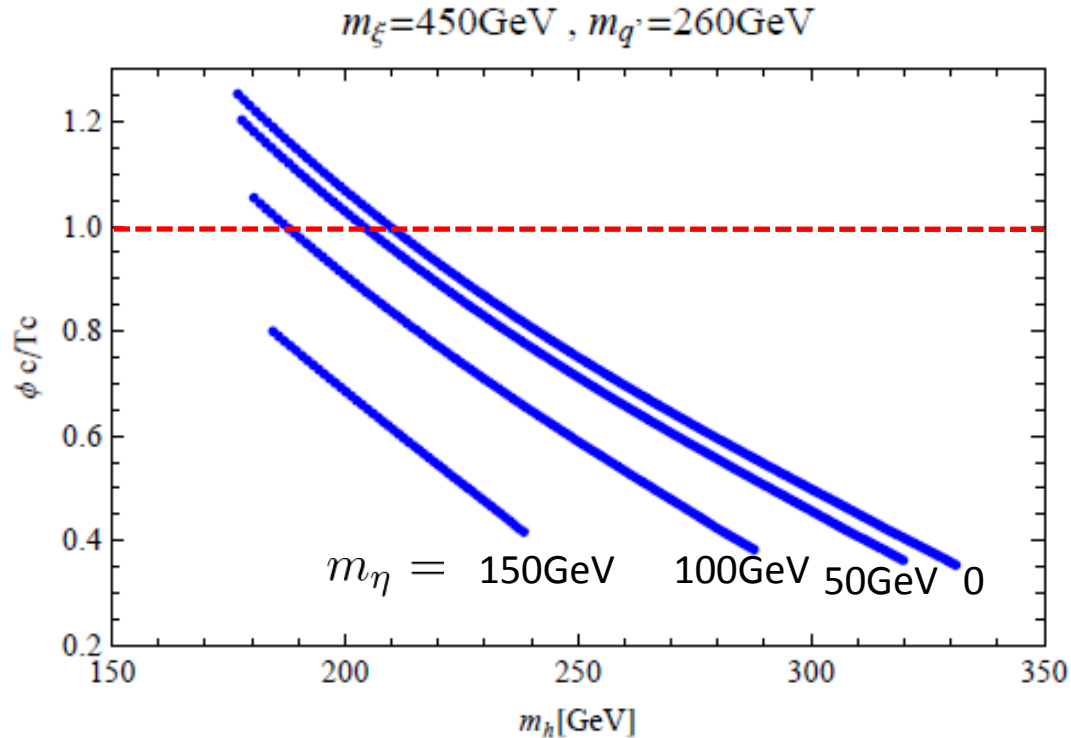
$$\Lambda_{4f} = 4 \text{ TeV}$$

Prediction is not so definite

$$\Rightarrow \phi_c/T_c > 1$$

Numerical results (3)

- ◆ effect of nonzero pseudo scalar Higgs mass : m_η



- for larger m_η , ϕ_c/T_c becomes smaller
- for strongly 1st order PT, $m_\eta \lesssim 100 \text{ GeV}$

Summary of effective theory's results

- ◆ NJL prediction for low energy parameters are not definite
- ◆ In order to realize $\phi_c/T_c > 1$, there are upper bounds on masses

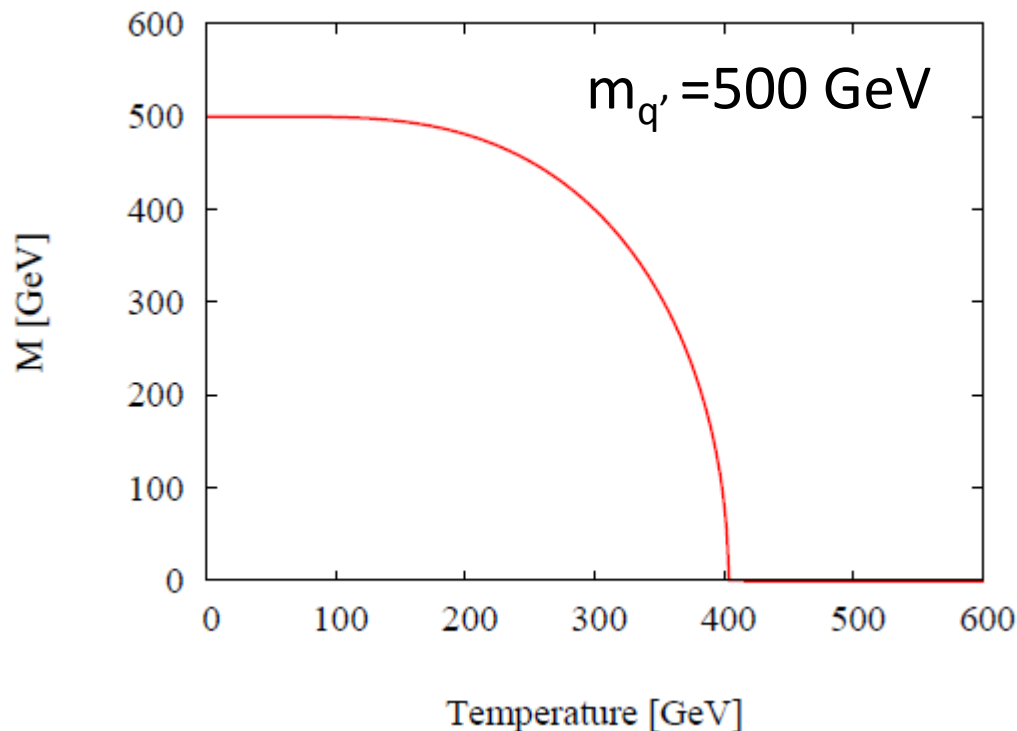
$$\underline{m_{q'} < 290 \text{ GeV}}, \quad m_\xi < 500 \text{ GeV}, \quad m_\eta < 100 \text{ GeV}$$

<-- already excluded by recent data by CDF, CMS

- ◆ Our effective theory is valid for $\Lambda < 1\text{TeV}$
--> $m_{q'} < 480 \text{ GeV}$ (on-shell)
- ◆ Need alternative method for $m_{q'} > 500 \text{ GeV}$

Direct analysis of NJL not relying on effective theory

- ◆ NJL was originally invented in hadron physics M.K., H. Kohyama
- Usually PT is investigated by using gap equation with $1/N_c$ -expansion



2nd order for $m_{q'} > 500$ GeV

⇒ Two complementary results suggest EWPT is not strong 1st order

5. Summary and discussion

- ◆ In the strongly coupled 4th family model, EWPT can be strongly 1st order due to the loop effects of the extra composite Higgs
- ◆ However, there is upper bound on 4th quark mass $m_{q'} < 300 \text{ GeV}$
 - <-- excluded by recent experimental data
- ◆ Result from $1/N_c$ expansion suggests EWPT is 2nd order for $m_{q'} > 500 \text{ GeV}$
- ◆ 4-fermion interaction of 4th quarks can not cause strongly 1st order EWPT for experimentally allowed $m_{q'}$
- ◆ Possible way out: 4-fermion int. of 4th leptons
 - bound states of τ', ν' provide extra 2 Higgs doublets
 - enhance 1st order?

Back Up Slides

t' (4th Generation) Quark, Searches for

Mass $m > 256$ GeV, CL = 95% ($p\bar{p}$, $t'\bar{t}'$ prod., $t' \rightarrow Wq$)

b' (4th Generation) Quark, Searches for

Mass $m > 190$ GeV, CL = 95% ($p\bar{p}$, quasi-stable b')

Mass $m > 199$ GeV, CL = 95% ($p\bar{p}$, neutral-current decays)

Mass $m > 128$ GeV, CL = 95% ($p\bar{p}$, charged-current decays)

Mass $m > 46.0$ GeV, CL = 95% (e^+e^- , all decays)

Heavy Charged Lepton Searches

$L^\pm$ – **charged lepton**

Mass $m > 100.8$ GeV, CL = 95% ^[h] Decay to νW .

$L^\pm$ – **stable charged heavy lepton**

Mass $m > 102.6$ GeV, CL = 95%

Heavy Neutral Leptons, Searches for

For excited leptons, see Compositeness Limits below.

Stable Neutral Heavy Lepton Mass Limits

Mass $m > 45.0$ GeV, CL = 95% (Dirac)

Mass $m > 39.5$ GeV, CL = 95% (Majorana)

Neutral Heavy Lepton Mass Limits

Mass $m > 90.3$ GeV, CL = 95%

(Dirac ν_L coupling to e, μ, τ ; conservative case(τ))

Mass $m > 80.5$ GeV, CL = 95%

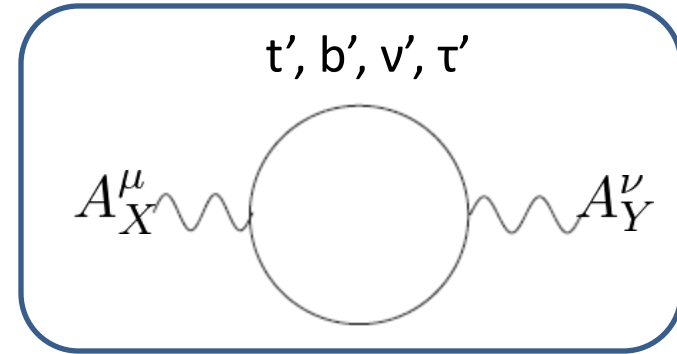
(Majorana ν_L coupling to e, μ, τ ; conservative case(τ))

Constraints from S, T parameter

◆ vacuum polarization of gauge bosons

$$\Pi_{XY}^{\mu\nu}(q^2) \equiv g^{\mu\nu} \Pi_{XY}(q^2) + (q^\mu q^\nu\text{-terms})$$

$$\Pi_{XY}(q^2) = \Pi_{XY}^{sm}(q^2) + \Pi_{XY}^{new}(q^2)$$



◆ definition for S and T

$$\alpha S \equiv 4e^2 [\Pi_{33}'^{new}(0) - \Pi_{3Q}'^{new}(0)]$$

$$\alpha T \equiv \frac{e^2}{s_W^2 c_W^2 m_Z^2} [\Pi_{11}^{new}(0) - \Pi_{33}^{new}(0)]$$

◆ each doublet (N, E) contribute (when $m_Z \ll m_N, m_E$)

$$S = \frac{1}{6\pi} \left[1 - Y \log \frac{m_N^2}{m_E^2} \right]$$

$$T = \frac{1}{16\pi s_W^2 c_W^2 m_Z^2} \left[m_N^2 + m_E^2 - \frac{2m_N^2 m_E^2}{m_N^2 - m_E^2} \log \frac{m_N^2}{m_E^2} \right]$$

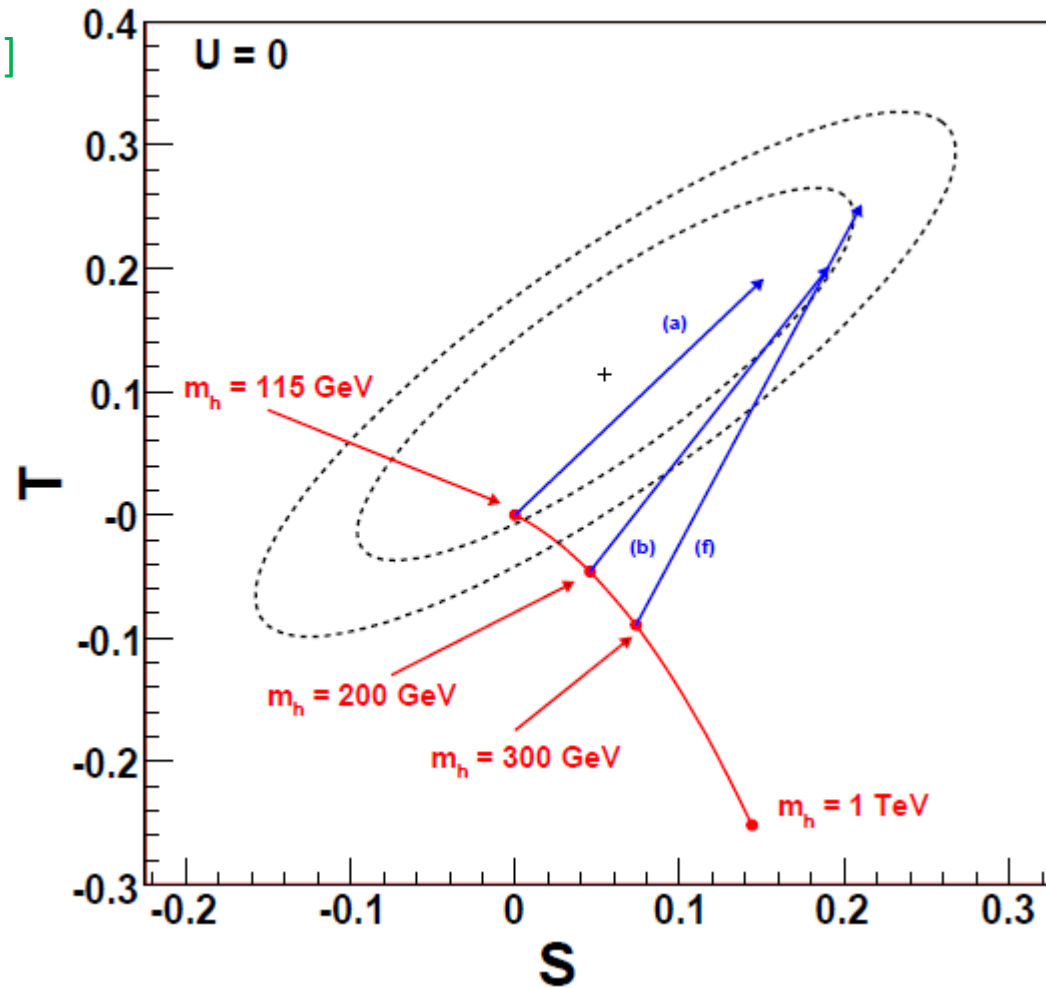
Constraints from S, T parameter (cont'd)

[Kribs , Plehn, Spannowsky and Tait (2007)]

parameter set	m_{u_4}	m_{d_4}	m_H	ΔS_{tot}	ΔT_{tot}
(a)	310	260	115	0.15	0.19
(b)	320	260	200	0.19	0.20
(c)	330	260	300	0.21	0.22
(d)	400	350	115	0.15	0.19
(e)	400	340	200	0.19	0.20
(f)	400	325	300	0.21	0.25

$$m_{\nu_4} = 100 \text{ GeV and } m_{\ell_4} = 155 \text{ GeV}$$

“relatively heavy Higgs is allowed”



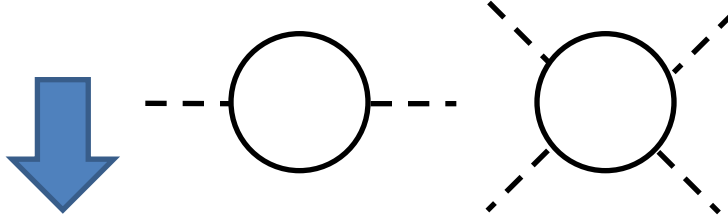
Construction of effective theory [Bardeen, Hill and Lindner]

$$\mu = \Lambda_{4f} \quad \mathcal{L}_{4f} = G_{q'} (\bar{q}'_{Li} q'_{Rj}) (\bar{q}'_{Rj} q'_{Li})$$



auxiliary field: $\Phi_{ij} \sim \bar{q}'_{Rj} q'_{Li}$

$$\mathcal{L}'_{4f} = -y_0 (\bar{q}'_{Li} \Phi_{ij} q'_{Rj} + h.c.) - m_{\Phi 0}^2 \text{tr}(\Phi^\dagger \Phi) \quad G_{q'} = y_0^2 / m_{\Phi 0}^2$$



$$\mu \lesssim \Lambda_{4f}$$

$$\begin{aligned} \mathcal{L}''_{4f} = & -y_0 (\bar{q}'_{Li} \Phi_{ij} q'_{Rj} + h.c.) + \underline{Z_\Phi \text{tr}(\partial^\mu \Phi^\dagger \partial_\mu \Phi)} - m_{\Phi,0}^2 \text{tr}(\Phi^\dagger \Phi) \\ & - \underline{\frac{\lambda_{1,0}}{2} [\text{tr}(\Phi^\dagger \Phi)]^2} - \underline{\frac{\lambda_{2,0}}{2} \text{tr}(\Phi^\dagger \Phi)^2} \end{aligned}$$

When, $\mu \rightarrow \Lambda_{4f}, \quad Z_\Phi \rightarrow 0 \quad \lambda_{1,0} \rightarrow 0 \quad \lambda_{2,0} \rightarrow 0$

◆ by rescaling the field $\Phi \rightarrow \Phi/\sqrt{Z_\Phi}$

$$\mu \ll \Lambda_{4f}$$

$$\mathcal{L} = \bar{q}' i \gamma^\mu \partial_\mu q' - y(\bar{q}'_L \Phi q'_R + h.c.) + \underline{tr(\partial_\mu \Phi^\dagger \partial^\mu \Phi)} - m_\Phi^2 tr(\Phi^\dagger \Phi) \\ - \frac{\lambda_1}{2} [tr(\Phi^\dagger \Phi)]^2 - \frac{\lambda_2}{2} tr(\Phi^\dagger \Phi)^2$$

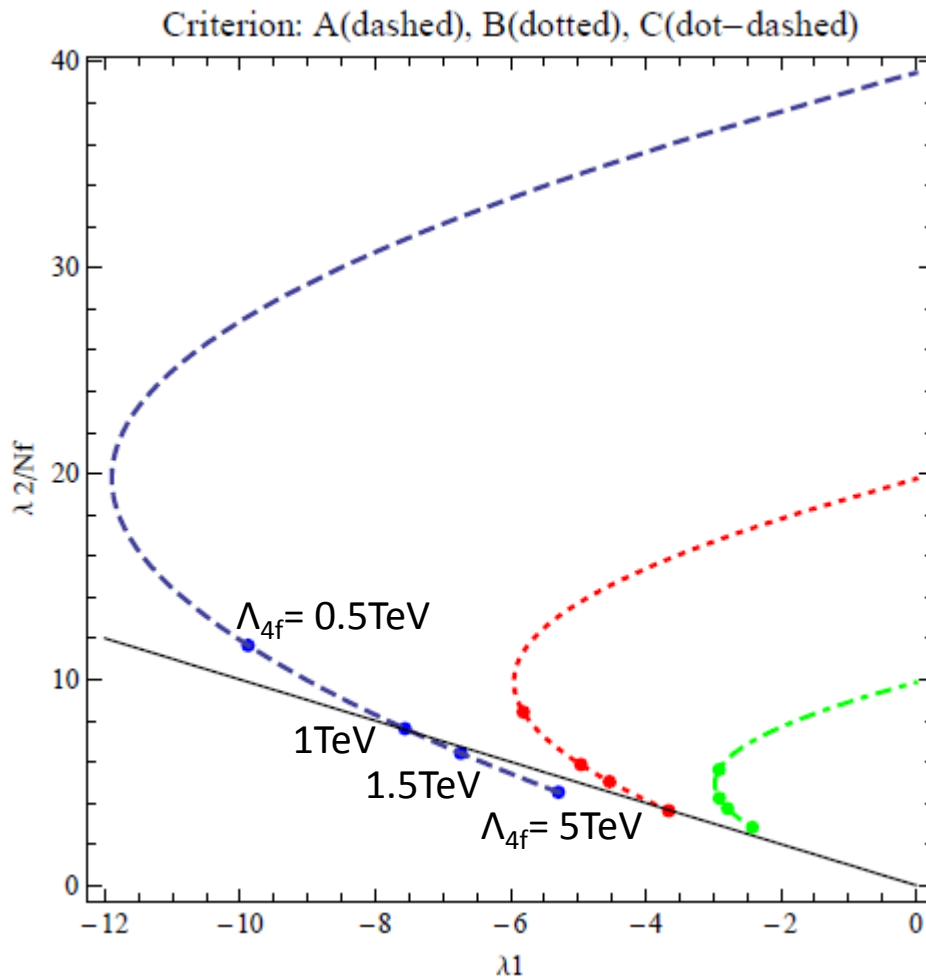
◆ compositeness condition

when $\mu \rightarrow \Lambda_{4f}$

$$\lambda_1(\mu) \rightarrow 0 \quad \lambda_2(\mu) \rightarrow \infty \quad y(\mu) \rightarrow \infty$$

Correspondence with NJL

- ◆ prediction of couplings at μ_{EW} by using one-loop RGEs



← compositeness conditions

$$\bar{\lambda}_1(\Lambda_{4f}) = 0, \quad \bar{\lambda}_2(\Lambda_{4f}) = \frac{16\pi^2}{N_f}, \quad \bar{y}(\Lambda_{4f})^2 = \frac{16\pi^2}{N_c}$$

← another initial conditions

$$\bar{\lambda}_1(\Lambda_{4f}) = 0, \quad \bar{\lambda}_2(\Lambda_{4f}) = \frac{8\pi^2}{N_f}, \quad \bar{y}(\Lambda_{4f})^2 = \frac{8\pi^2}{N_c}$$



quite sensitive to initial cond.
→ no meaningful prediction

Strategy to define cutoff Λ of effective model

➤ use one-loop RGEs:

$$\begin{aligned}\mu \frac{\partial}{\partial \mu} \bar{\lambda}_1 &= \frac{1}{8\pi^2} [(N_f^2 + 4)\bar{\lambda}_1^2 + 4N_f \bar{\lambda}_1 \bar{\lambda}_2 + 3\bar{\lambda}_2^2 + 2N_c y^2 \bar{\lambda}_1], \\ \mu \frac{\partial}{\partial \mu} \bar{\lambda}_2 &= \frac{1}{8\pi^2} (6\bar{\lambda}_1 \bar{\lambda}_2 + 2N_f \bar{\lambda}_2^2 + 2N_c y^2 \bar{\lambda}_2 - 2N_c \bar{y}^4), \\ \mu \frac{\partial}{\partial \mu} \bar{y} &= \frac{1}{16\pi^2} (N_f + N_c) \bar{y}^3\end{aligned}$$

- Starting from $(\lambda_1, \lambda_2, y)$ at $\mu=246$ GeV, define cutoff Λ as scale at which one of conditions is satisfied :

(1) vacuum instability

$$\bar{\lambda}_1(\Lambda) + \bar{\lambda}_2(\Lambda)/N_f = 0, \quad \bar{\lambda}_2(\Lambda) = 0$$

(2) Landau pole

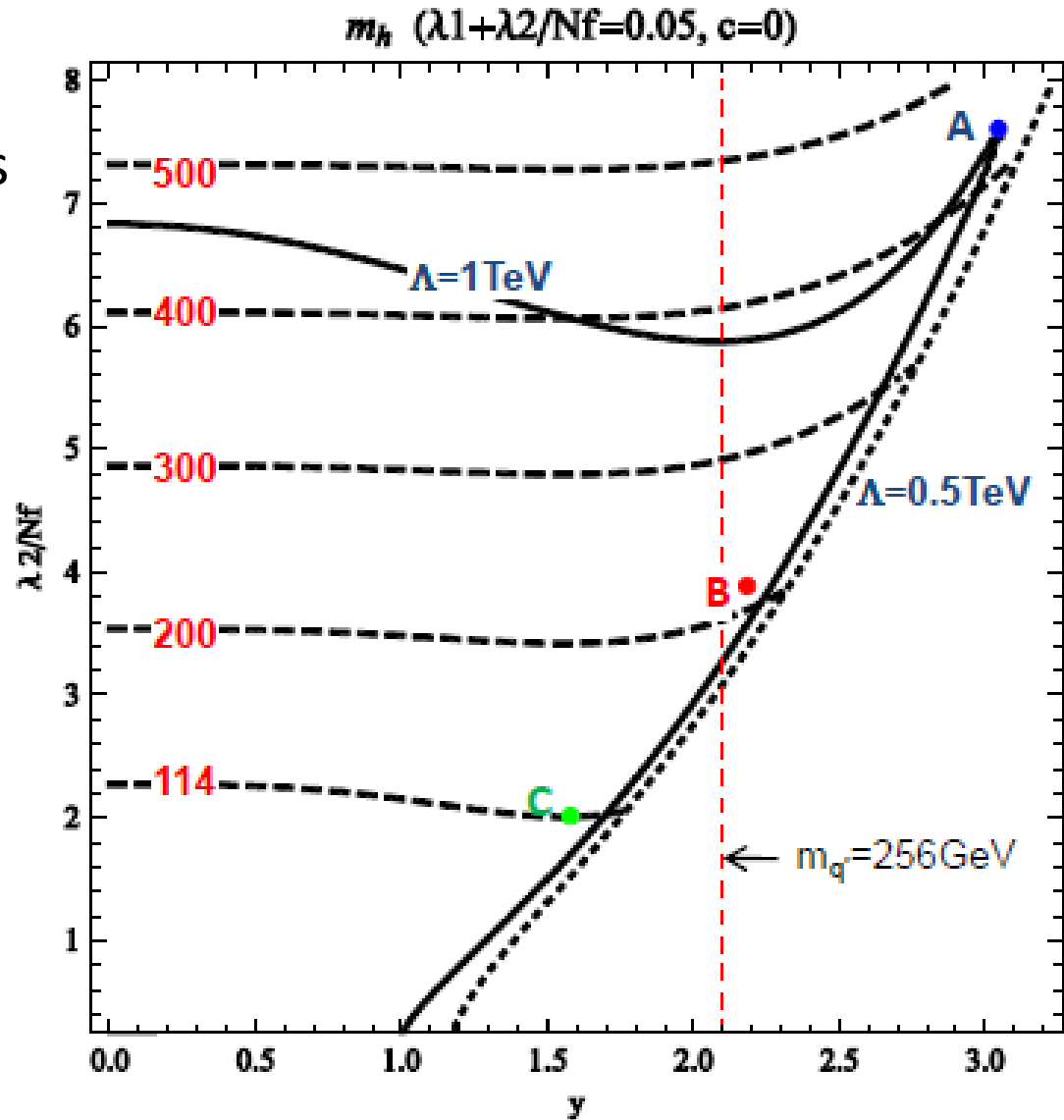
$$\bar{y}(\Lambda)^2 = \frac{16\pi^2}{N_c}, \quad \bar{\lambda}_1(\Lambda) + \bar{\lambda}_2(\Lambda)/N_f = \frac{16\pi^2}{N_f^2}, \quad \bar{\lambda}_2(\Lambda) = \frac{16\pi^2}{N_f}$$

(perturbativity bound)

one-loop Higgs mass on y - λ^2/N_f plane ($m_\eta=0$)

◆ define one-loop
(SM-like) Higgs mass as
curvature of effective
potential at VEV

$$m_h^2 \equiv \left. \frac{\partial^2 (V_0 + V_1^{(0)})}{\partial \phi^2} \right|_{\phi=\phi_0}$$



ϕc and T_c -- m_h dependence--

