Parton Distribution Functions for Collider Physics

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CTEQ-Tung Et Al.: recent activities

- Uncertainty induced by α_s in the CTEQ-TEA PDF analysis *(PRD, arXiv:1004.4624)*
- New PDFs for collider physics
  - CTEQ6.6 set (published in 2008)$\longrightarrow$ CT09 $\longrightarrow$ CT10 *(PRD, arXiv:1007.2241)*
  - new experimental data, statistical methods, and parametrization forms
- PDFs for Event Generators *(JHEP, arXiv:0910.4183)*
- PDFs at NNLO with General Mass treatment *(in preparation)*

Uncertainty induced by α_s in the PDF analyses

- *Questions addressed:*
  - Two leading theoretical uncertainties in LHC processes are due to α_s and the PDFs;
  - These are not independent uncertainties; how can one quantify their correlation?
  - Which central $\alpha_s(M_Z)$ and which error on $\alpha_s(M_Z)$ are to be used with the existing PDFs?
  - What are the consequences for key LHC processes ($gg \to H^0$, etc.)?

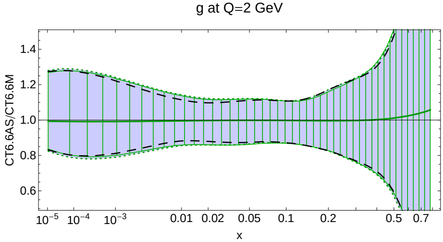
Uncertainty induced by α_s in the PDF analyses

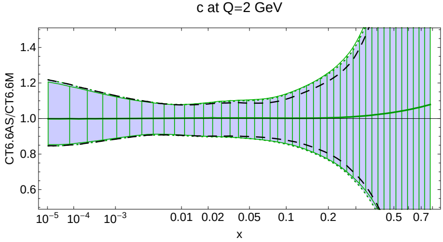
Recent activity to examine these questions, e.g.:

- **MSTW** *(arXiv:0905.3531)*
  - $\alpha_s(M_Z)$ is an **output** of the global fit (constrained by the hadronic scattering only)
  - several sets of error PDFs, each with its own $\alpha_s(M_Z)$ value $\Rightarrow$ lengthier calculations
  - The α_s uncertainty and PDF uncertainty are inseparable
- **NNPDF** *(in 2009 Les Houches Proceedings, arXiv:1004.0962)*:
  - $\alpha_s(M_Z) = 0.119 \pm 0.002$ is taken as an **input**
  - α_s-PDF correlation is examined with ~ 1000 PDF replicas and found to be small
- **H1+ZEUS** *(arXiv:0911.0884)*: sensitivity of the HERAPDF set to $\delta\alpha_s(M_Z) = \pm 0.002$ is explored

Our findings

Total PDF+α_s errors ΔX are the **same** when found (a) from a full fit with floating α_s, or (b) by adding ΔX_{PDF} and ΔX_{α_s} in quadrature



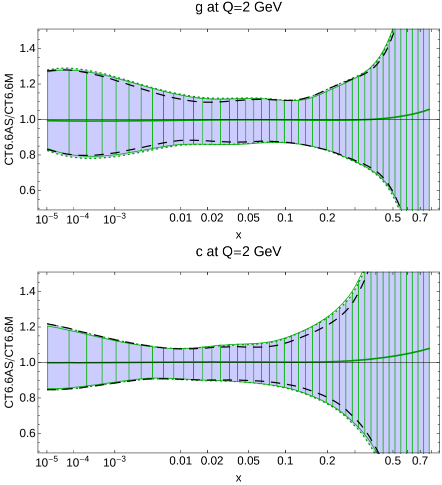


- black – CTEQ6.6 PDF uncertainty
- Blue filled – PDF+α_s uncertainty of the fit with floating $\alpha_s(M_Z)$
- Green hatched – PDF+α_s uncertainty added in quadrature

Also, agreement in cross section predictions $\Rightarrow$ backup slides

Our findings

Total PDF+α_s errors ΔX are the same if found either from a full fit with floating α_s, or by adding ΔX_{PDF} and ΔX_{α_s} in quadrature



This agreement is **a rigorous consequence** of the quadratic approximation

Details of the CTEQ6.6FAS analysis

- Take the "world-average" $\alpha_s(M_Z) = 0.118 \pm 0.002$ as an **input**:

$$\alpha_s(M_Z)|_{\text{in}} = 0.118 \pm 0.002 \text{ at } 90\% \text{ C.L.}$$

- Find the theory parameter $\alpha_s(M_Z)$ as an **output** of a global fit (CTEQ6.6FAS):

$$\alpha_s(M_Z)|_{\text{out}} = 0.118 \pm 0.0019 \text{ at } 90\% \text{ C.L.}$$

- The combined PDF+α_s uncertainty is estimated as

$$\Delta X = \frac{1}{2}\sqrt{\sum_{i=1}^{22+1}\left(X_i^{(+)} - X_i^{(-)}\right)^2}$$

- **Problem:** each PDF set comes with its own $\alpha_s \Rightarrow$ cumbersome
- **A simple workaround exists!**

A quadrature sum reproduces the full α_s-PDF uncertainty

Theorem

In the quadratic approximation, the total α_s+PDF uncertainty $\Delta\sigma$ of the CTEQ6.6FAS set, with all correlation, reduces to

$$\Delta X = \sqrt{\Delta X^2_{CTEQ6.6} + \Delta X^2_{\alpha_s}},$$

where

- $\Delta X_{CTEQ6.6}$ *is the CTEQ6.6 PDF uncertainty from 44 PDFs with the same* $\alpha_s(M_Z) = 0.118$
- $\Delta X_{\alpha_s} = (X_{high} - X_{low})/2$ *is the* α_s *uncertainty computed with upper/lower* α_s *PDFs, e.g. CTEQ6.6AS PDFs for* $\alpha_s(M_Z) = 0.120$ *and* 0.116

Main Idea

Based on the Hessian method, we can write the variation from the global χ^2minimum approximately as

$$\delta\chi^2 = A^\dagger H A = A^\dagger V^\dagger D V A = Y^\dagger Y,$$

where $Y = D^{1/2} V A$

We can further rotate according to "Data Set Diagonalization" on a function of α_s variation, e.g.

$$g = (\frac{\alpha_S(M_Z) - \alpha_{S0}(M_Z)}{\triangle\alpha_{S0}})^2,$$

when expanded in the y space around the minimum:

$$\delta g \quad = \quad Y^\dagger G Y = Y^\dagger U^\dagger \Lambda U Y$$

It can be proved that Λ, the diagonal matrix, has only one non-zero eigenvalue.

Main Idea

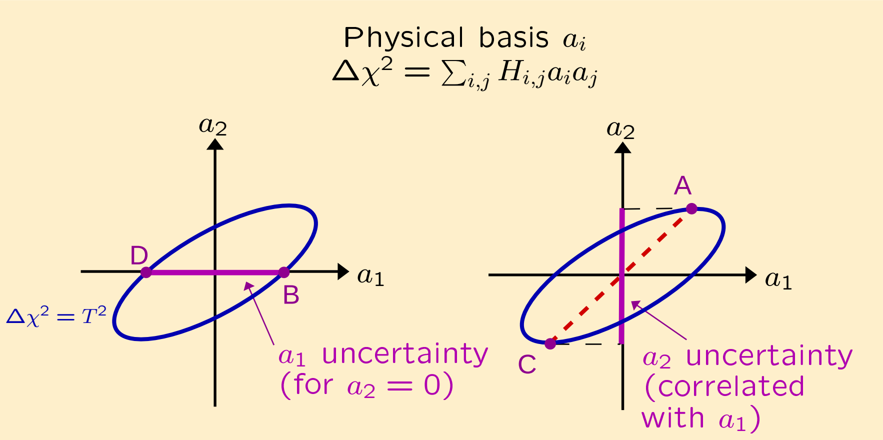
Now, we can insert the unitary matrix U back to the global χ^2,

$$\delta\chi^2 = Y^\dagger Y = Y^\dagger U^\dagger U Y = Z^\dagger Z.$$

We have only $z_1$that is fully correlated to G, i.e. α_S, whereas all the other directions are uncorrelated to α_S.
The space that spanned by $\{z_2, ..z_n\}$ would be $(n-1)$ dimensions with no correlation with α_S, which would be the same space spanned by $\{a_i; i \neq \alpha\}$ with α_S fixed at the best fitted value.
Therefore, the total uncertainty can be added up in quadrature for those due to PDFs with fixed α_S (corresponding to $\{z_2, ..z_n\}$) and the best fitted PDFs ofα_S variation (corresponding to z_1).

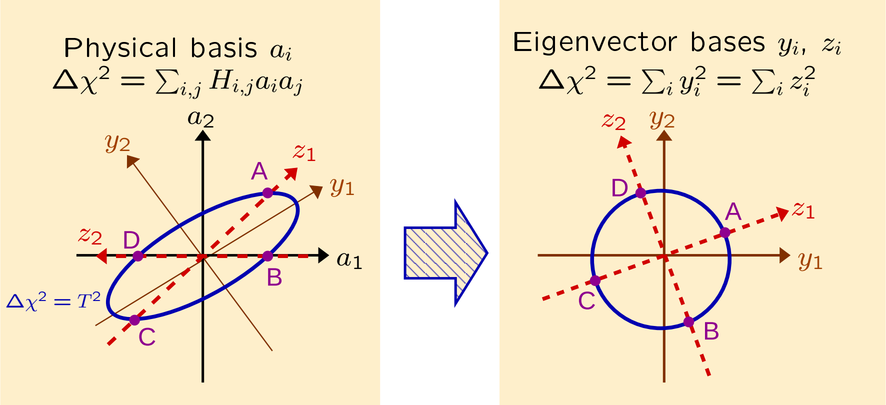
The full proof is given in the paper; the main idea is illustrated for 1 PDF parameter a_1 and α_s parameter a_2

Illustration of the theorem for 2 parameters



$\Delta X_1^2 = \frac{1}{4}\left(X(B) - X(D)\right)^2$ $\Delta X_2^2 = \frac{1}{4}\left(X(A) - X(C)\right)^2$

Illustration of the theorem for 2 parameters, cont.



$$\Delta X^2 = \frac{1}{4}\left[(X(A) - X(C))^2 + (X(B) - X(D))^2\right]$$
$$= \Delta X_1^2 + \Delta X_2^2$$

Our findings (PRD, arXiv:1004.4624)

Theorem

In the quadratic approximation, the total α_s*+PDF uncertainty* ΔX*, with all correlation, reduces to*

$$\Delta X = \sqrt{\Delta X_{PDF}^2 + \Delta X_{\alpha_s}^2},$$

where

- ΔX_{PDF} *is the PDF uncertainty with fixed* α_s*, e.g. uncertainty from 44 CTEQ6.6 PDFs with the same* $\alpha_s(M_Z) = 0.118$
- $\Delta X_{\alpha_s} = (X_{high} - X_{low})/2$ *is the* α_s *uncertainty computed with upper/lower* α_s *PDFs, e.g. CTEQ6.6AS PDFs for* $\alpha_s(M_Z) = 0.120$ *and* 0.116

The full proof is given in the paper

CTEQ-Tung Et Al.: CT10 analysis *(PRD, arXiv:1007.2241)*

- Uncertainty induced by α_s in the CTEQ-TEA PDF analysis
- New PDFs for collider physics

CT10 analysis

Experimental data

- Combined HERA-1 neutral-current and charged-current DIS data with 114 correlated systematic effects *(see Guzzi's talk)*
  - replaces 11 separate HERA-1 sets used in the CTEQ6.6 fit
- CDF Run-2 and D0 Run-2 inclusive jet production
- Tevatron Run-2 Z rapidity distributions from both CDF and D0
- W electron asymmetry from CDF II and D0 II; W muon asymmetry from D0 II (CT10W set)
- Other data sets inherited from CTEQ6.6

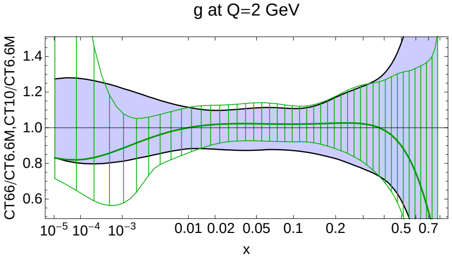
CT10 analysis

Developments in techniques

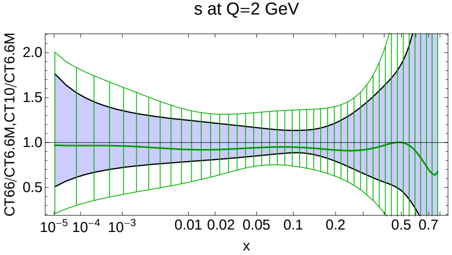
- Experimental normalizations N_i are treated on the same footing as other correlated systematic errors
- Set all data weights of 1, unless otherwise specified
- Require 90% CL for each experiment dynamically
- More flexible (i.e. less biased) parametrizations for $g(x, Q_0)$, $d(x, Q_0)$, and $s(x, Q_0)$
- Apply soft constraint on $R_s = \lim_{x\to 0} \left(s(x) + \bar{s}(x)\right) / \left(\bar{u}(x) + \bar{d}(x)\right)$ which has little information from current data

More flexible parametrizations

CT10(green) vs. CTEQ6.6(blue)



$g(x, Q)$: large uncertainty at $x < 10^{-3}$, despite tighter constraints by the combined HERA data



$s(x, Q)$: wider uncertainty, covers both CTEQ6.6 and MSTW'08

Agreement between data sets

- Good overall agreement: $\chi^2/d.o.f. = 1.1$ (out of ~2800 data points)

- Noticable observations on the quality of the fit:

  - **Tevatron single-inclusive jet production:** Run-1 and Run-2 sets are moderately compatible *(arXiv:0904.2424)*

  - **Tevatron Run-2 Z rapidity:** D0 well described; CDF acceptable (higher stat.)

  - **Tevatron Run-2 W lepton asymmetry**
    - is precise; constrains $d(x)/u(x)$ at $x \to 1$
    - apparently disagrees with existing constraints on d/u, mainly provided by the NMC F_2^d/F_2^p and Run-1 W lepton asymmetry data; minor tension against BCDMS F_2^d data

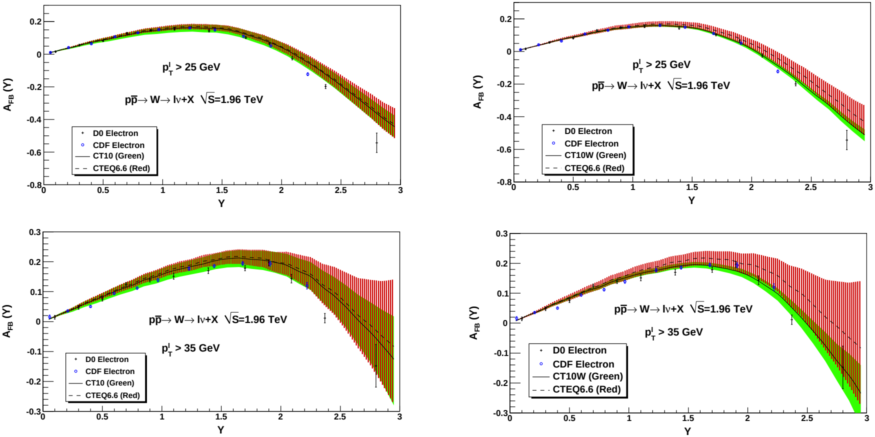
Agreement between data sets

- Reaonable fits to electron (e) asymmetry data are possible without NMC and BCDMS; and vice versa
- No acceptable fit to D0 II e asymmetry and NMC/BCDMS data can be achieved, if they are included on the same footing
- Tension between Run-2 e asymmetry and μ asymmetry
- Good agreement between Run-2 e W asymmetry data and Z y data
- With special emphasis on D0 II e asymmetry data (weight>1), it is possible to obtain a reasonable agreement for W asymmetry ($\chi^2/d.o.f. = 1 - 2$) , with some remaining tension with NMC & BCDMS data, especially at $x > 0.4$

CT10 family

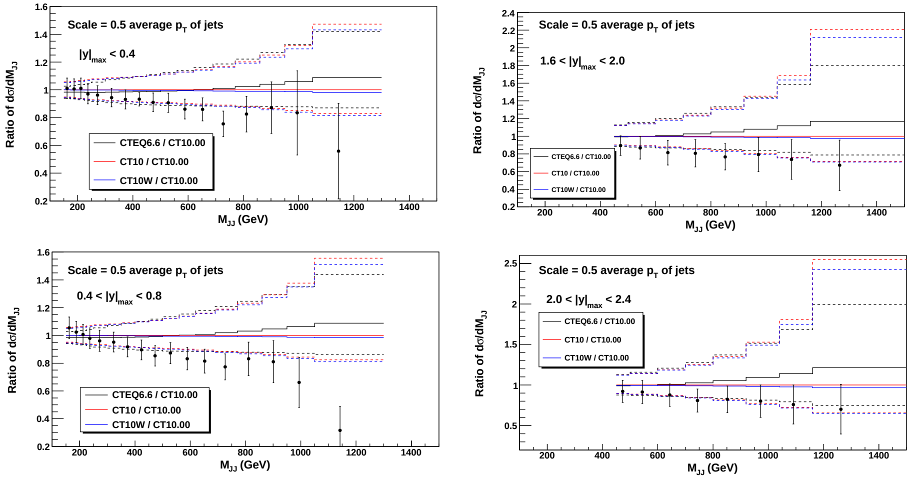
- Two series of PDFs are produced:
  - **CT10:** no D0 Run-2 W asymmetry data are included
  - **CT10W:** include D0 Run-2 W asymmetry, with an extra weight

CT10 and CT10W fits with Tevatron Run-2 data



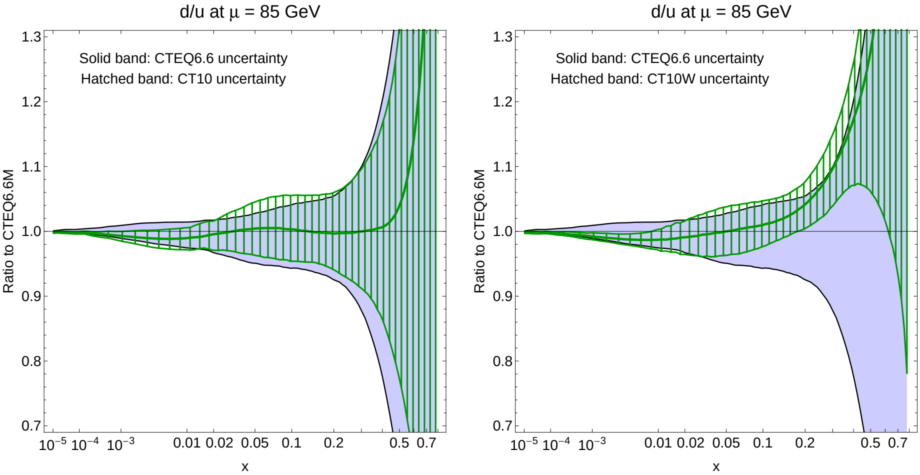
CT10W agrees better with W asy data; has smaller uncertainty than CTEQ6.6 or CT10

Tevatron Dijet invariant mass data



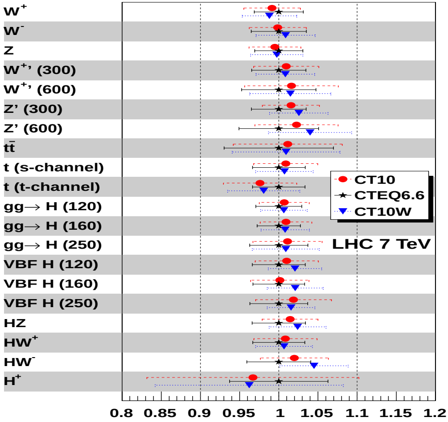
Agreement much improved with more consistent scale choice, as compared to shown in the orignal D0 paper; With Run-II inclusive jets, CT10(W) show better agreement than CTEQ6.6.

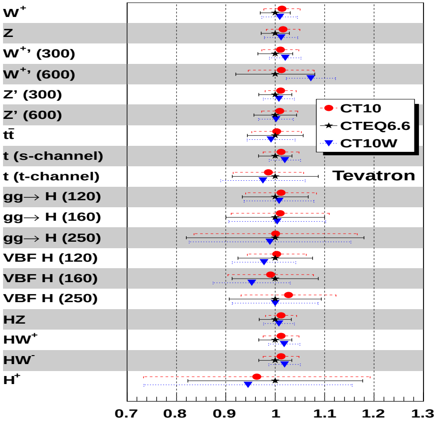
$d(x,Q)/u(x,Q)$ at $Q = 85$ GeV



CT10W prefers larger d/u, has smaller uncertainty than CTEQ6.6 or CT10

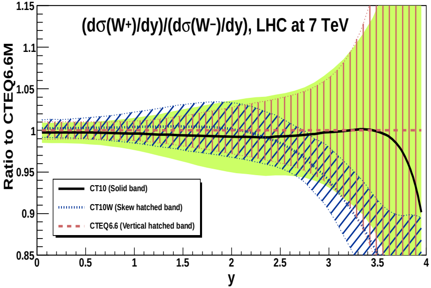
CT10 & CT10W predictions for the LHC & Tevatron





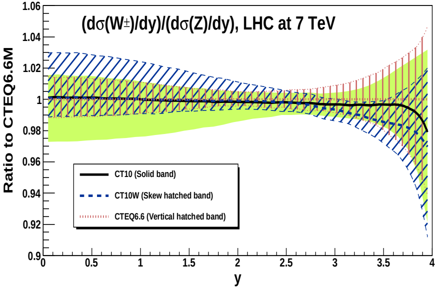
CT10 & CT10W predictions for the LHC

$\sigma(W^+)/\sigma(W^-)$ rapidity dist.



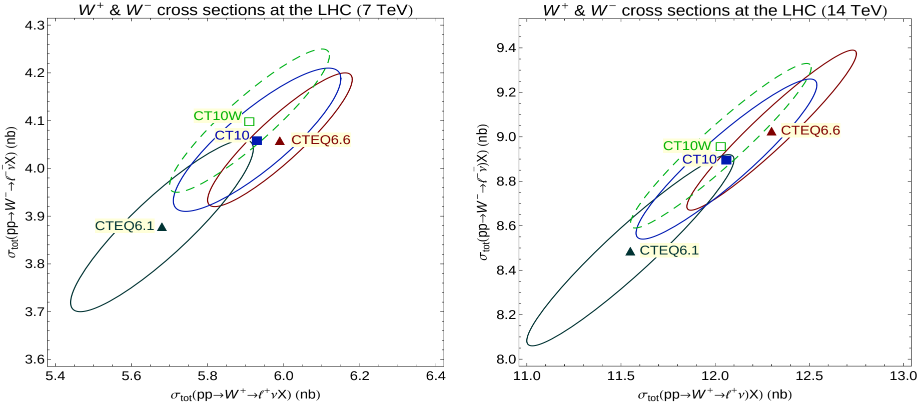
CT10W Uncertainty (blue) is clearly smaller than that of CT10 & CTEQ6.6.

$\sigma(W^{\pm})/\sigma(Z^0)$ rapidity dist.

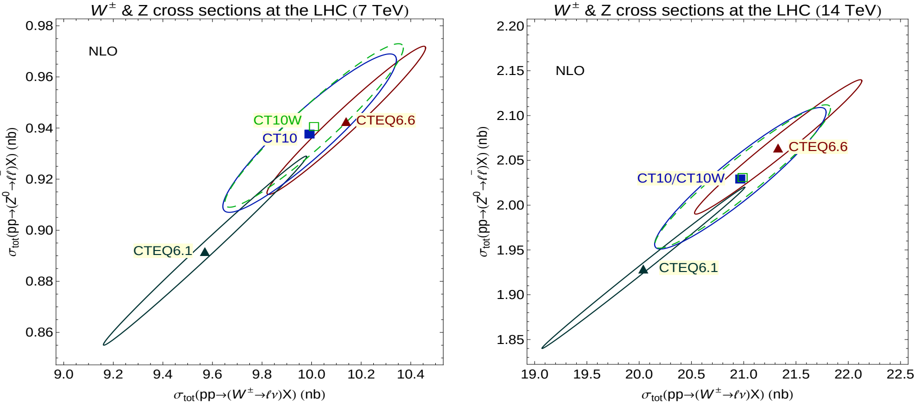


CT10 (green) & CT10W (blue) uncertainties in central y region are larger than that of CTEQ6.6 (red), mainly due to larger uncertainty on s distribution.

CT10 & CT10W predictions for the LHC



CT10 & CT10W predictions for the LHC



Heavy flavor contributions at NNLO

- General-Mass (GM) treatment of heavy quark contributions in DIS is essential for precision W, Z predictions at the LHC *(Tung et al., hep-ph/0611254)*
- Several quark mass effects are comparable to NNLO radiative contributions, therefore, must be included in a consistent way
- Simplified Aivazis-Collins-Olness-Tung (S-ACOT) scheme with GM treatment is now implemented at NNLO accuracy

S-ACOT scheme

- The default mass scheme of CTEQ6.6 and CT10 PDFs
- Based upon the proof of QCD factorization for DIS with massive quarks *(Collins, PRD, 1998)*
- Relatively simple: sets $m_Q = 0$ in ME with incoming c or b *(Collins, 1998; Kramer, Olness, Soper, PRD, 2000)*
- Reduces to the ZM $\overline{MS}$ scheme at $Q^2 \gg m_Q^2$, and to the FFN scheme at $Q^2 \approx m_Q^2$

NNLO computation

- NNLO evolution for α_s and PDFs (HOPPET)
- NNLO both massless and massive Wilson coefficient functions (Moch, Vermaseren, Vogt; Smith, van Neerven, et al.)
- matching coefficients relating the PDFs in N_f and $N_f + 1$ flavors schemes
- code validated; physical results in progress; paper in preparation

Summary I

CTEQ6.6AS & CT10(W)AS PDF sets (available in the LHAPDF library):

- alternative CTEQ6.6 & CT10(W) fits for

$$\alpha_s(M_Z) = 0.116, .117, .119, .120$$

- sufficient to compute uncertainty in $\alpha_s(M_Z)$ at ≈68% and 90% C. L., including **the world-average** $\alpha_s(M_Z) = 0.118 \pm 0.002$ as an **input data point**
- **The CTEQ6.6AS** α_s uncertainty should be combined with the CTEQ6.6 PDF uncertainty as

$$\Delta X = \sqrt{\Delta X^2_{CTEQ6.6} + \Delta X^2_{CTEQ6.6AS}}$$

- The total uncertainty ΔX reproduces the full correlation between $\alpha_s(M_Z)$ and PDFs, also applicable to CT10 family and future PDFs.

Summary II

Tevatron Run-2 W asymmetry data...

...increasingly complete and precise, cannot be explained based on the d/u ratio provided by the previously existing data

- Higher-twist and nuclear corrections in the large-x BCDMS/NMC deuterium data are the usual suspects

  (Virchaux and Milsztajn; Alekhin; Accardi et al.)

- CT10 and CT10W sets of PDFs for practical applications, without and with constraints from the D0 Run-2 W asymmetry

Tevatron Run-2 Dijet invariant mass data: consistent with current analysis; still large scale uncertainty

Backup slides

Full and reduced fits with variable α_s: cross sections

Process	CTEQ6.6+CTEQ6.6AS				CTEQ6.6FAS
$t\bar{t}$ (171 GeV)	σ_0	$\Delta\sigma_{PDF}$	$\Delta\sigma_{\alpha_S}$	$\Delta\sigma$	$\sigma_0 \pm \Delta\sigma$
LHC 7 TeV	157.41	10.97	7.54	13.31	160.10 ± 13.93
LHC 10 TeV	396.50	18.75	16.10	24.71	400.48 ± 25.74
LHC 14 TeV	877.19	28.79	30.78	42.15	881.62 ± 44.27
$gg \to H$ (120 GeV)	σ_0	$\Delta\sigma_{PDF}$	$\Delta\sigma_{\alpha_S}$	$\Delta\sigma$	$\sigma_0 \pm \Delta\sigma$
Tevatron 1.96 TeV	0.63	0.042	0.032	0.053	0.64 ± 0.055
LHC 7 TeV	10.70	0.31	0.32	0.45	10.70 ± 0.48
LHC 10 TeV	20.33	0.66	0.56	0.87	20.28 ± 0.93
LHC 14 TeV	35.75	1.31	0.94	1.61	35.63 ± 1.70
$gg \to H$ (160 GeV)	σ_0	$\Delta\sigma_{PDF}$	$\Delta\sigma_{\alpha_S}$	$\Delta\sigma$	$\sigma_0 \pm \Delta\sigma$
Tevatron 1.96 TeV	0.26	0.026	0.015	0.030	0.26 ± 0.031
LHC 7 TeV	5.86	0.16	0.18	0.24	5.88 ± 0.26
LHC 10 TeV	11.73	0.33	0.33	0.47	11.72 ± 0.50
LHC 14 TeV	21.48	0.68	0.56	0.88	21.43 ± 0.94
$gg \to H$ (250 GeV)	σ_0	$\Delta\sigma_{PDF}$	$\Delta\sigma_{\alpha_S}$	$\Delta\sigma$	$\sigma_0 \pm \Delta\sigma$
Tevatron 1.96 TeV	0.055	0.0099	0.0044	0.011	0.058 ± 0.012
LHC 7 TeV	2.30	0.085	0.081	0.12	2.32 ± 0.12
LHC 10 TeV	5.08	0.14	0.15	0.21	5.10 ± 0.22
LHC 14 TeV	10.03	0.26	0.27	0.37	10.04 ± 0.41

The full (CTEQ6.6FAS) and reduced (CTEQ6.6+CTEQ6.6AS) methods perfectly agree