Measuring the Trilinear Higgs Coupling at the LHC

Chung Kao [高 鐘]
University of Oklahoma

†Presented at the Chung Yuan Christian University University, March 27, 2014.

Measuring the Trilinear Higgs Coupling at the LHC

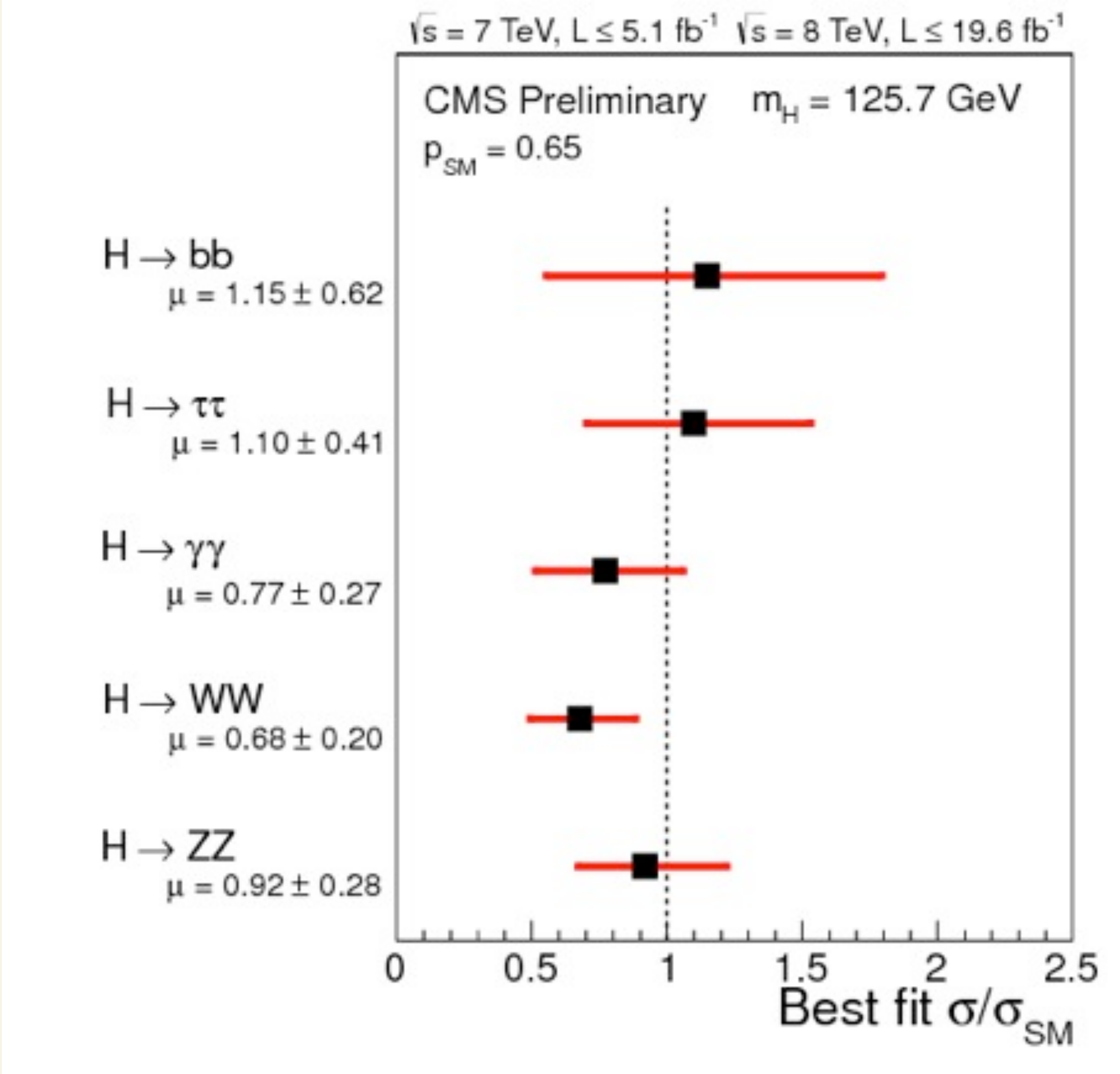
Dicus and Kao (2004)

- Introduction
- Higgs Pair Production from Gluon Fusion
- Higgs Pair Production via Bottom Quark Fusion
- Loop Integrals and Effective Lagrangian
- The Trilinear Higgs Coupling(s)
- The Discovery Potential of Higgs Pairs at the LHC
- Conclusions

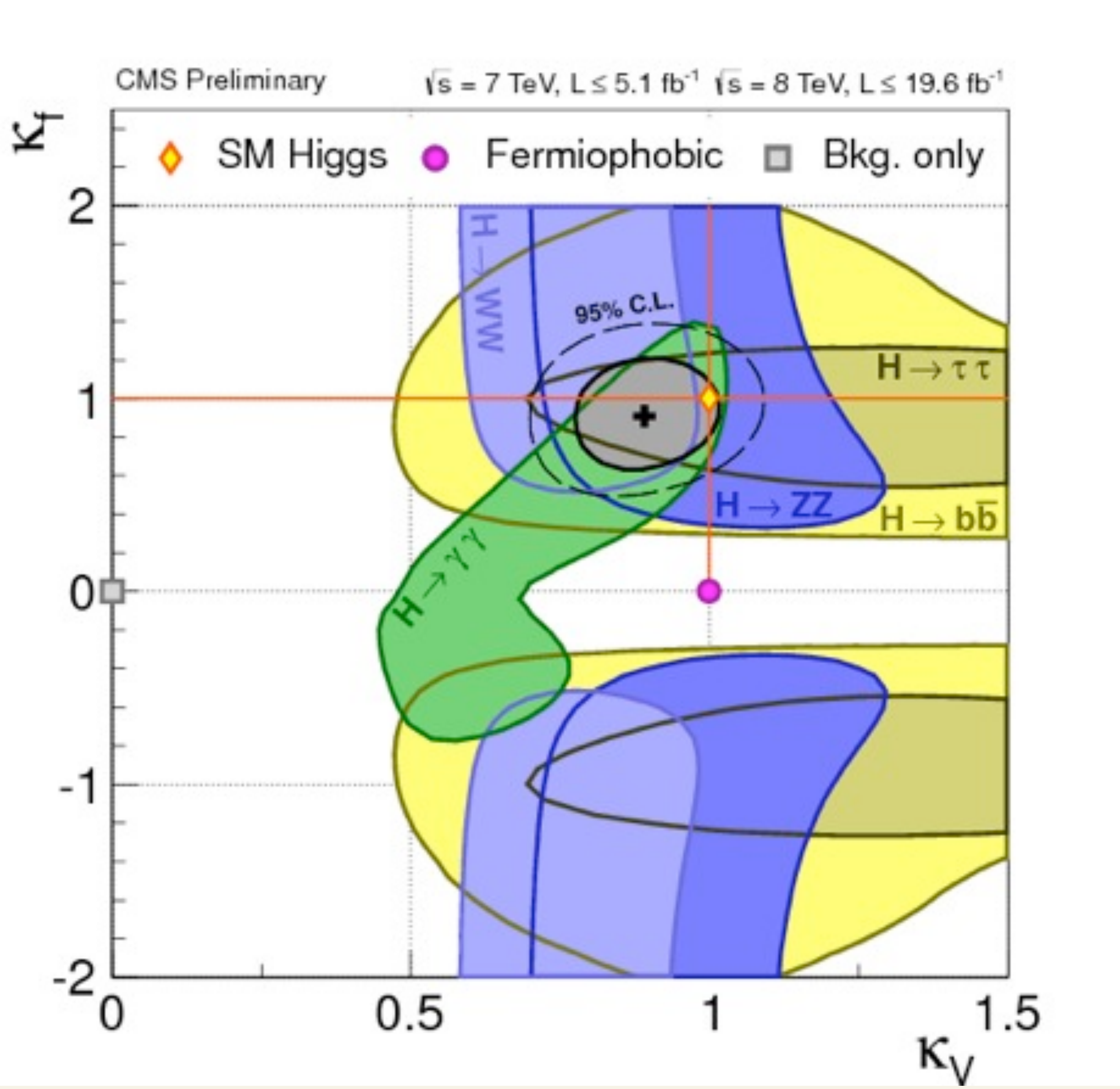
Introduction

- Thus far the results from the LHC indicate that the couplings of the Higgs boson to other particles are consistent with the Standard Model.
- But the ultimate test as to whether this particle is the SM Higgs boson will be the trilinear Higgs coupling that appears in Higgs pair production.
- There are uncertainties in the factorization and renormalization scales as well as variations in the parton distribution functions.
- I will discuss how accurately this three Higgs coupling can be determined theoretically.

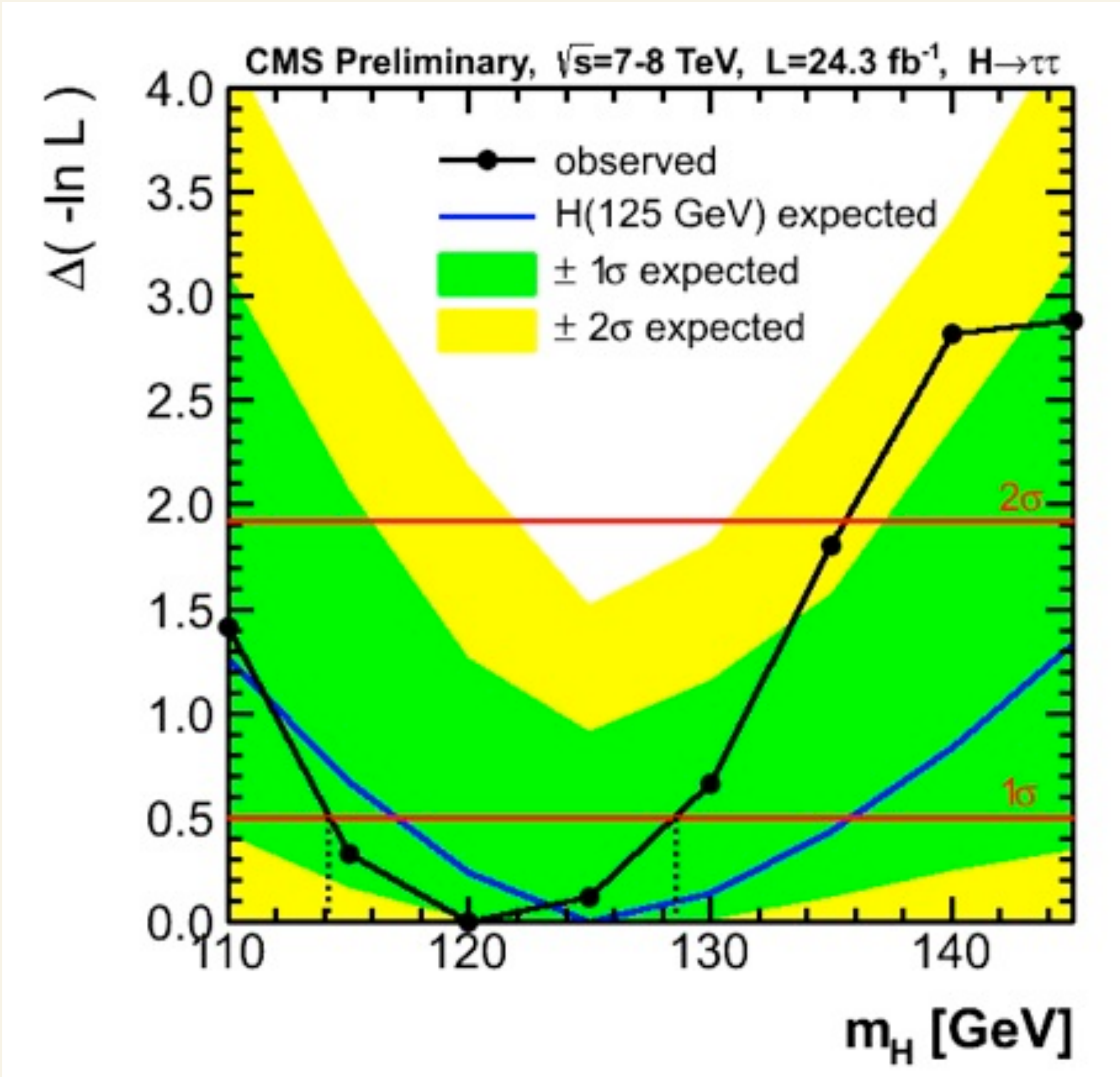
Recent CMS Higgs Results I



Recent CMS Higgs Results II



CMS Invariant Mass of Tau Pairs

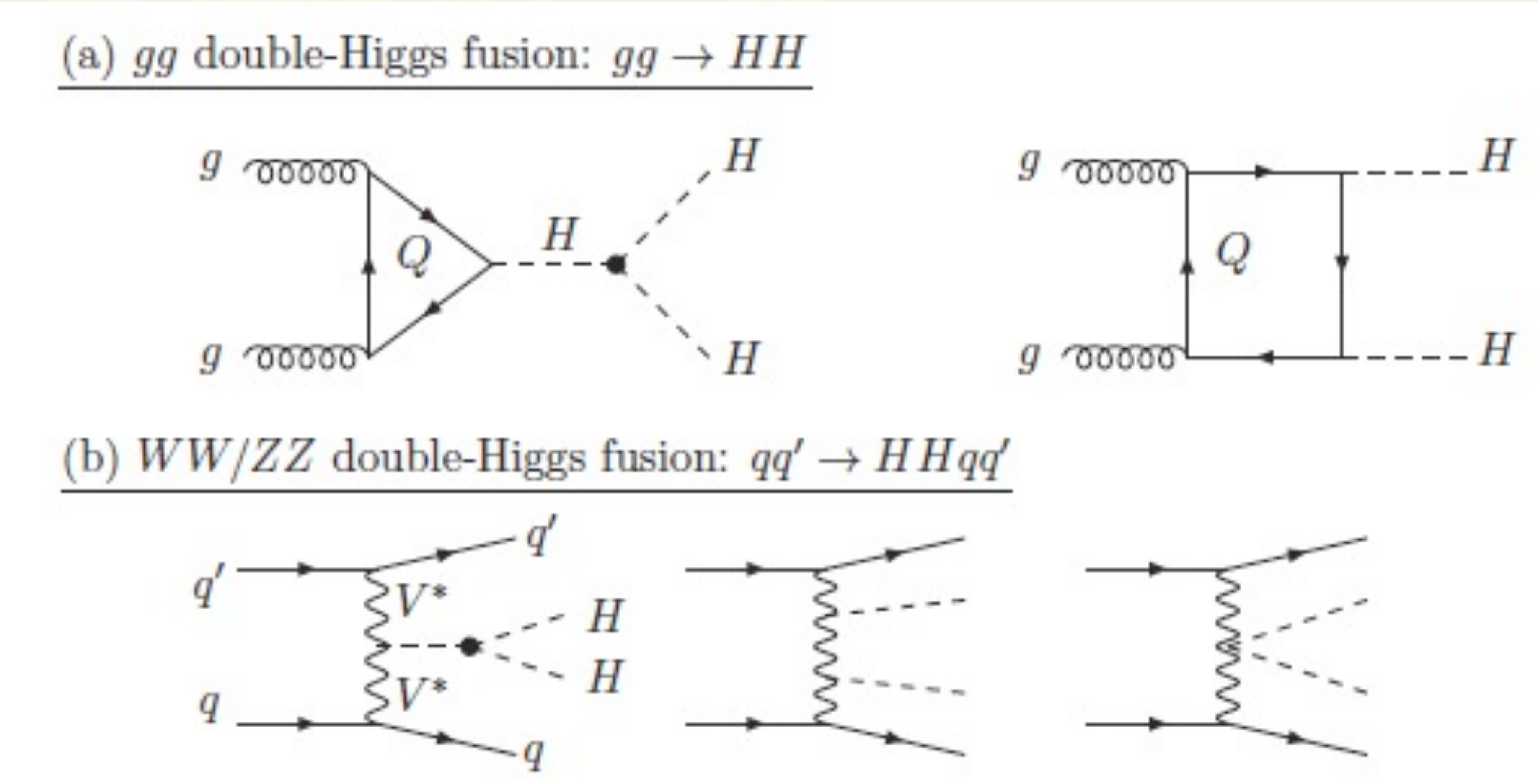


Higgs Pairs Production from Gluon Fusion

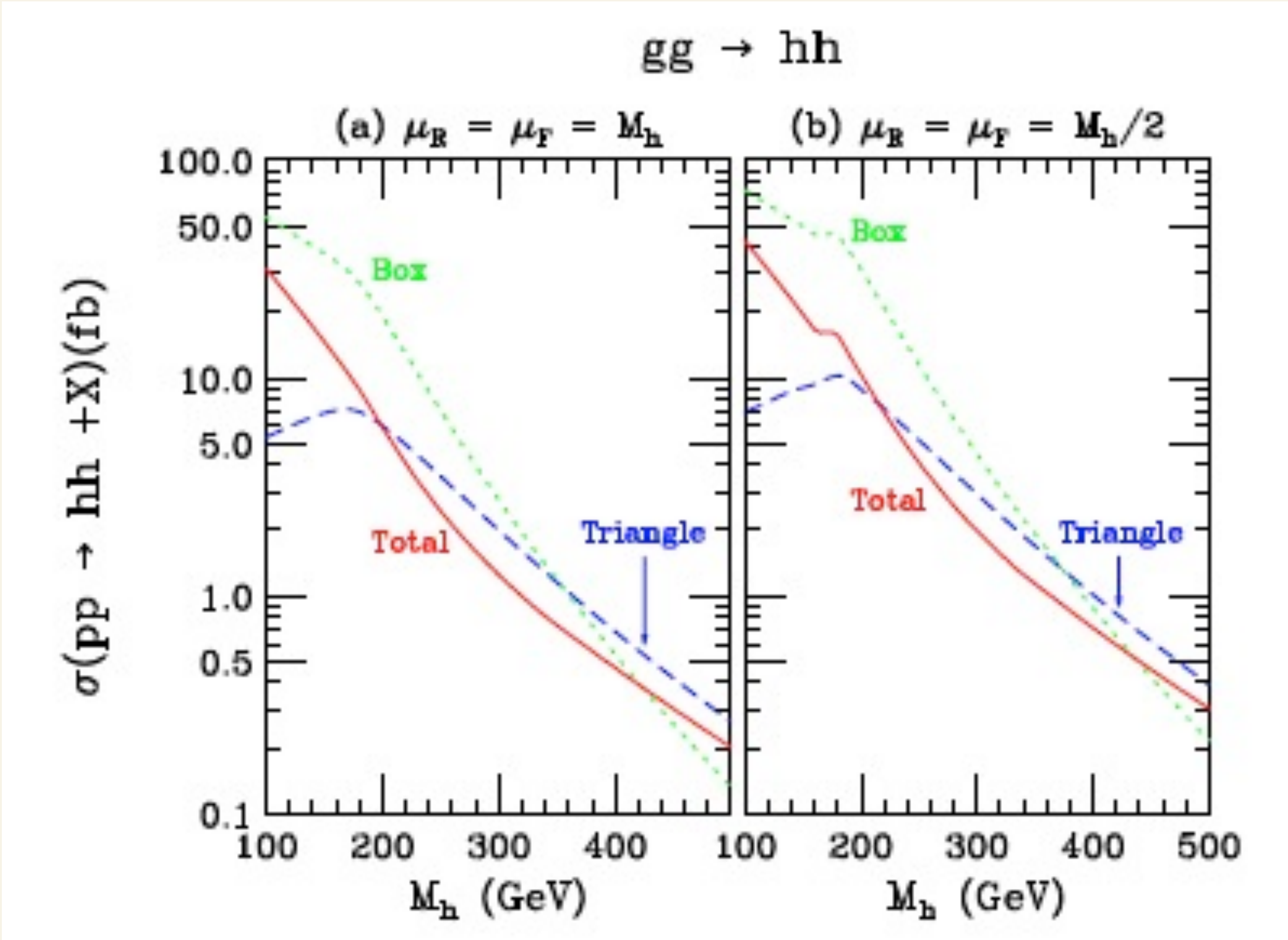
Dicus, Kao, and Willenbrock, Phys. Lett. **B203** (1988) 457;
Glover and van der Bij, Nucl. Phys. **B309** (1988) 282.

- For a light Higgs boson with $M_H < 500$ GeV, the dominant source of Higgs boson pair production is gluon fusion through both triangle and box diagrams.
- The triangle diagram involves the Higgs self-coupling while the box diagrams don't.
- For a heavy Higgs boson with $M_H \sim 1$ TeV, vector boson can become significant.

Higgs Pairs Production from Gluon Fusion

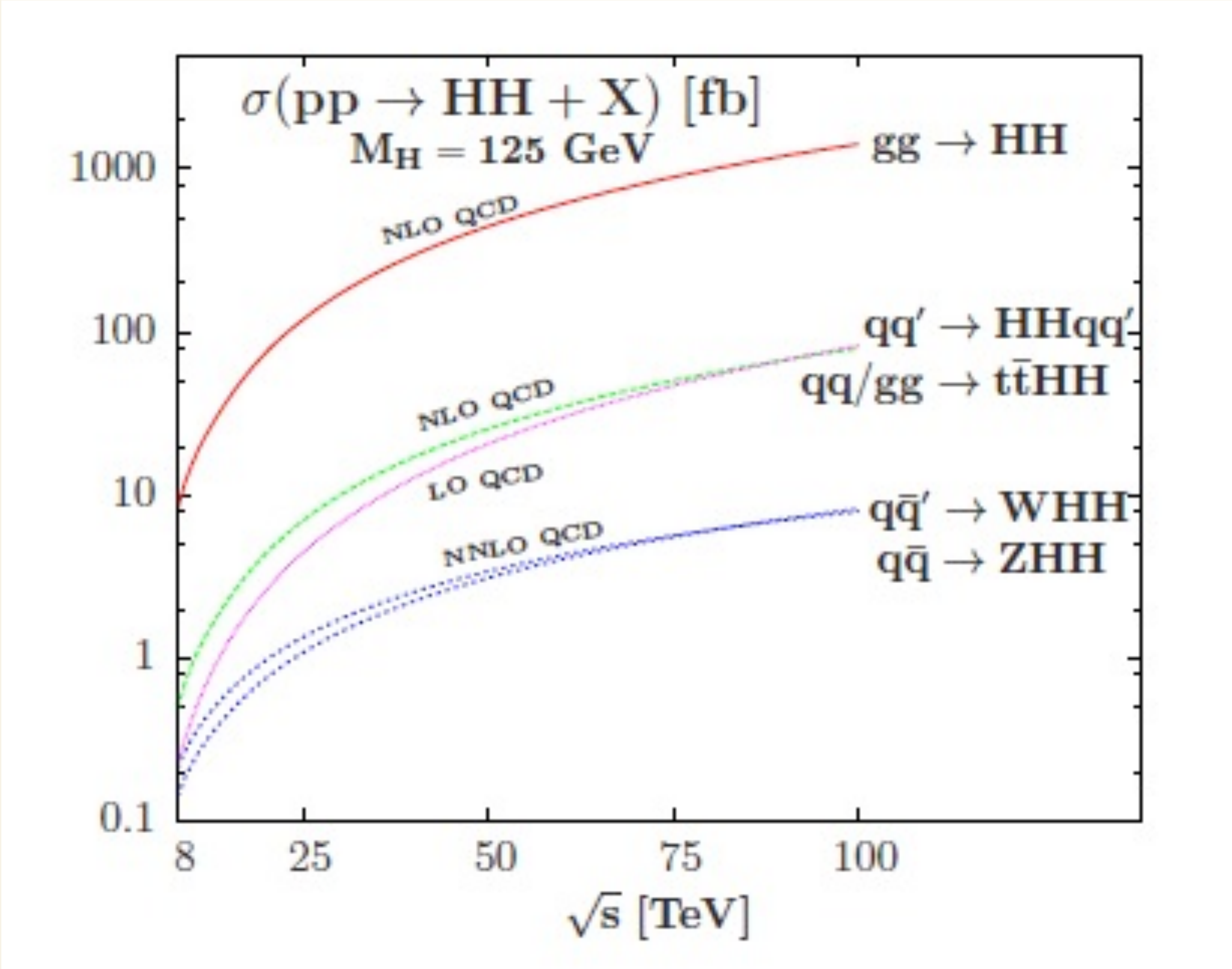


Higgs Pairs Production from Gluon Fusion



Higgs Pair Production in Hadron Collisions

Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira, JHEP **1304** (2013) 151.



Thursday, March 27, 14

NNLO Higgs Pair Production at Hadron Colliders

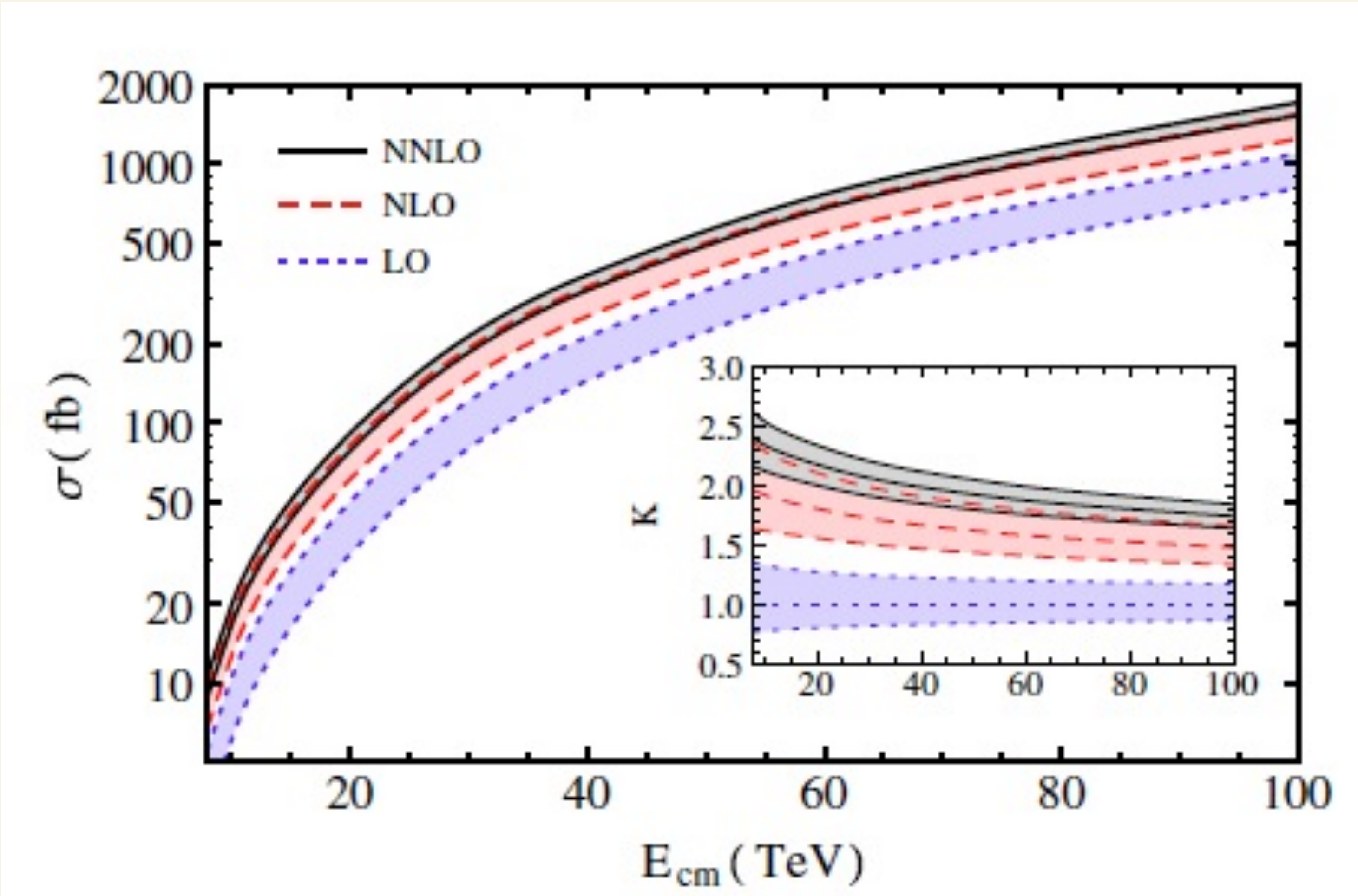
de Florian and Mazzitelli, Phys. Rev. Lett. **111** (2013) 201801.

$$\sigma_{\mathrm{LO}} = 17.8^{+5.3}_{-3.8}\ \mathrm{fb}, \quad \sigma_{\mathrm{NLO}} = 33.2^{+5.9}_{-4.9}\ \mathrm{fb},$$
$$\sigma_{\mathrm{NNLO}} = 40.2^{+3.2}_{-3.5}\ \mathrm{fb}, \tag{18}$$

$E_{\mathrm{c.m.}}$	8 TeV	14 TeV	33 TeV	100 TeV
σ_{NNLO}	9.76 fb	40.2 fb	243 fb	1638 fb
Scale [%]	$+9.0 - 9.8$	$+8.0 - 8.7$	$+7.0 - 7.4$	$+5.9 - 5.8$
PDF [%]	$+6.0 - 6.1$	$+4.0 - 4.0$	$+2.5 - 2.6$	$+2.3 - 2.6$
PDF + α_S [%]	$+9.3 - 8.8$	$+7.2 - 7.1$	$+6.0 - 6.0$	$+5.8 - 6.0$

NNLO Higgs Pair Production at Hadron Colliders

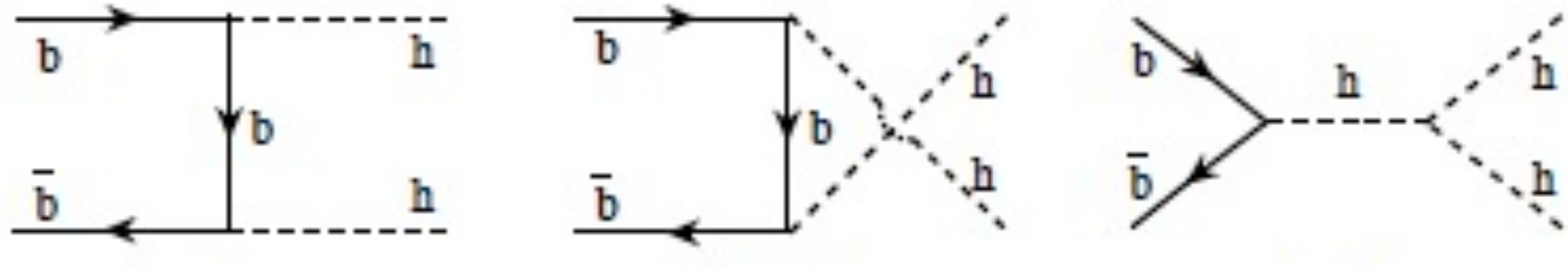
de Florian and Mazzitelli, Phys. Rev. Lett. **111** (2013) 201801.



Higgs Pair Production via Bottom Quark Fusion

- In the Standard Model, bottom quark fusion is almost negligible for Higgs pair production.
- In two Higgs doublet models with Type II Yukawa interactions, the Hbb coupling is enhanced by a large value of tanβ. Thus for $\tan\beta > 5$, bottom quark fusion makes dominant contribution.
- The physical process is gg to bbHH.
- However, it is a good approximation to calculate bb to HH if no associate b quarks are tagged.

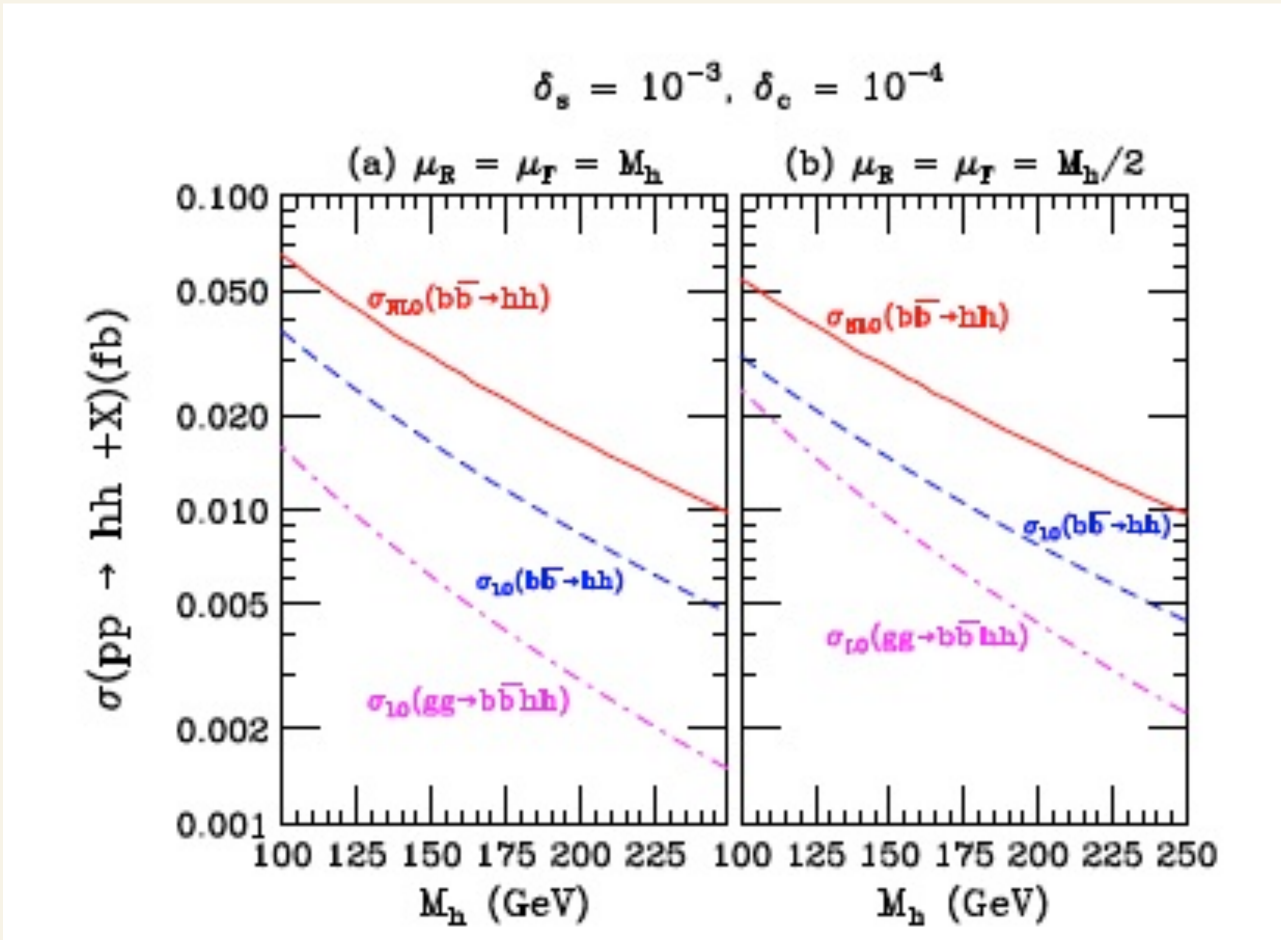
Higgs Pair Production via Bottom Quark Fusion



(1) (2) (3)

Higgs Pair Production via Bottom Quark Fusion

Dawson, Kao, Wang and Williams, Phys. Rev. **D75** (2007) 013007.



QCD CORRECTIONS TO bb → h h

Dawson, Kao, Wang and Williams, Phys. Rev. **D75** (2007) 013007.

- **Next-to-Leading Order Corrections**
  - **α_s Corrections: Real Emission, bb → hhg**
  - **α_s Corrections: Virtual Correction**
  - **1/Λ Corrections: bg →bhh [Λ = ln (m_h/m_b)]**
  - **gg→bbhh Cross Section (1/Λ^2)**

Running Coupling

Renormalization group equation $\alpha_s(\mu)$:

$$\mu \frac{\partial}{\partial \mu} \alpha_s(\mu) \equiv \beta(\alpha_s) = b_0 \alpha_s^2(\mu) + b_1 \alpha_s^3(\mu) + b_2 \alpha_s^4(\mu) + \cdots$$

Here b_0 and b_1 are universal. (process independence)

$$b_0 = -\frac{1}{2\pi}\left(11 - \frac{2N_F}{3}\right) \qquad b_1 = -\frac{1}{4\pi^2}\left(51 - \frac{19N_F}{3}\right)$$

Solution to next-to-leading order:

$$\alpha_s(\mu) = \frac{-2}{b_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right)} \left[1 + \frac{2b_1}{b_0^2} \frac{\ln\left(\ln\left(\frac{\mu^2}{\Lambda^2}\right)\right)}{\ln\left(\frac{\mu^2}{\Lambda^2}\right)} \right]$$

Λ is a parameter: ~ 200 – 300 MeV

Running Mass

As a consequence of renormalization, just like the coupling constant, quark masses also depend on the momentum exchange and renormalization scheme

$$\overline{m}(\mu) = \overline{m}(\mu_o)\left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_o)}\right)^{\gamma_0/\beta_0} \frac{1 + a_1 \frac{\alpha_S(\mu)}{\pi}}{1 + a_1 \frac{\alpha_S(\mu_0)}{\pi}}$$

$$\gamma_0 = 1$$

$$\gamma_1 = \frac{1}{16}\left(\frac{202}{3} - \frac{20}{9} N_f\right)$$

$$a_1 = -\frac{b_1\gamma_0}{b_0^2} + \frac{\gamma_1}{b_0}$$

Pole mass: $M_b = \overline{m}(M_b)\left(1 + C_F \frac{\alpha_S(M_b)}{\pi}\right)$

Factorization or Hadronization

PQCD does not work for physics at hadronic scale, 1/fm

Factorization is necessary for calculating cross sections involving hadrons

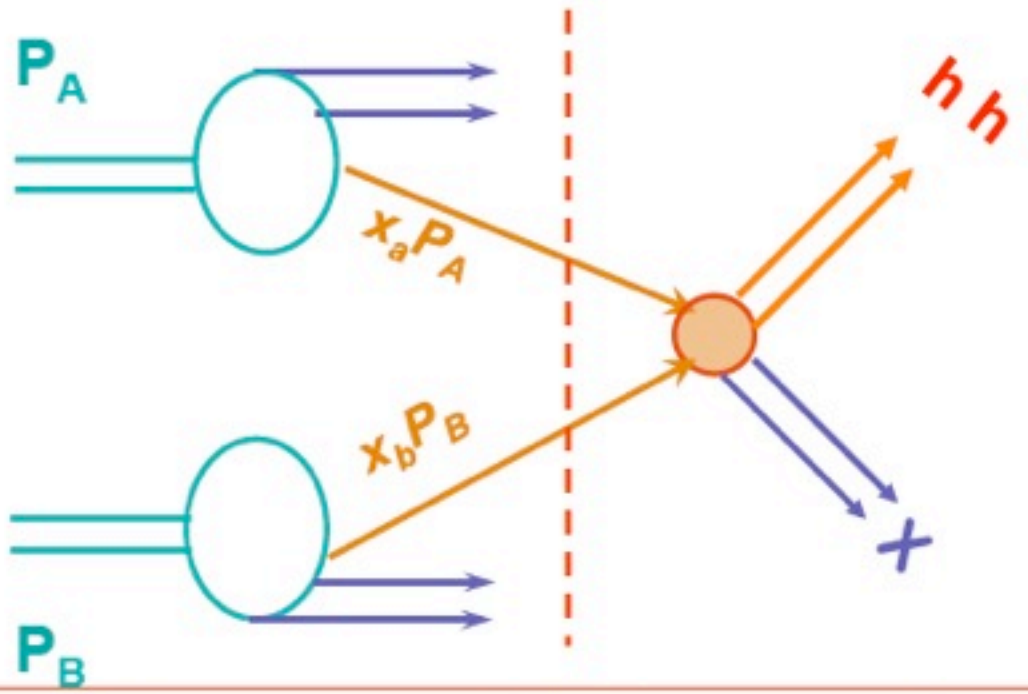
Factorization:

To factorize physical quantities into a short distance component (infrared safe, calculable, process dependent) and a long distance component (not perturbatively calculable, but process independent)

Predictive power:

Compare observables with different short distance interactions but same long distance physics

Parton Model



interference between different momentum scales are power suppressed

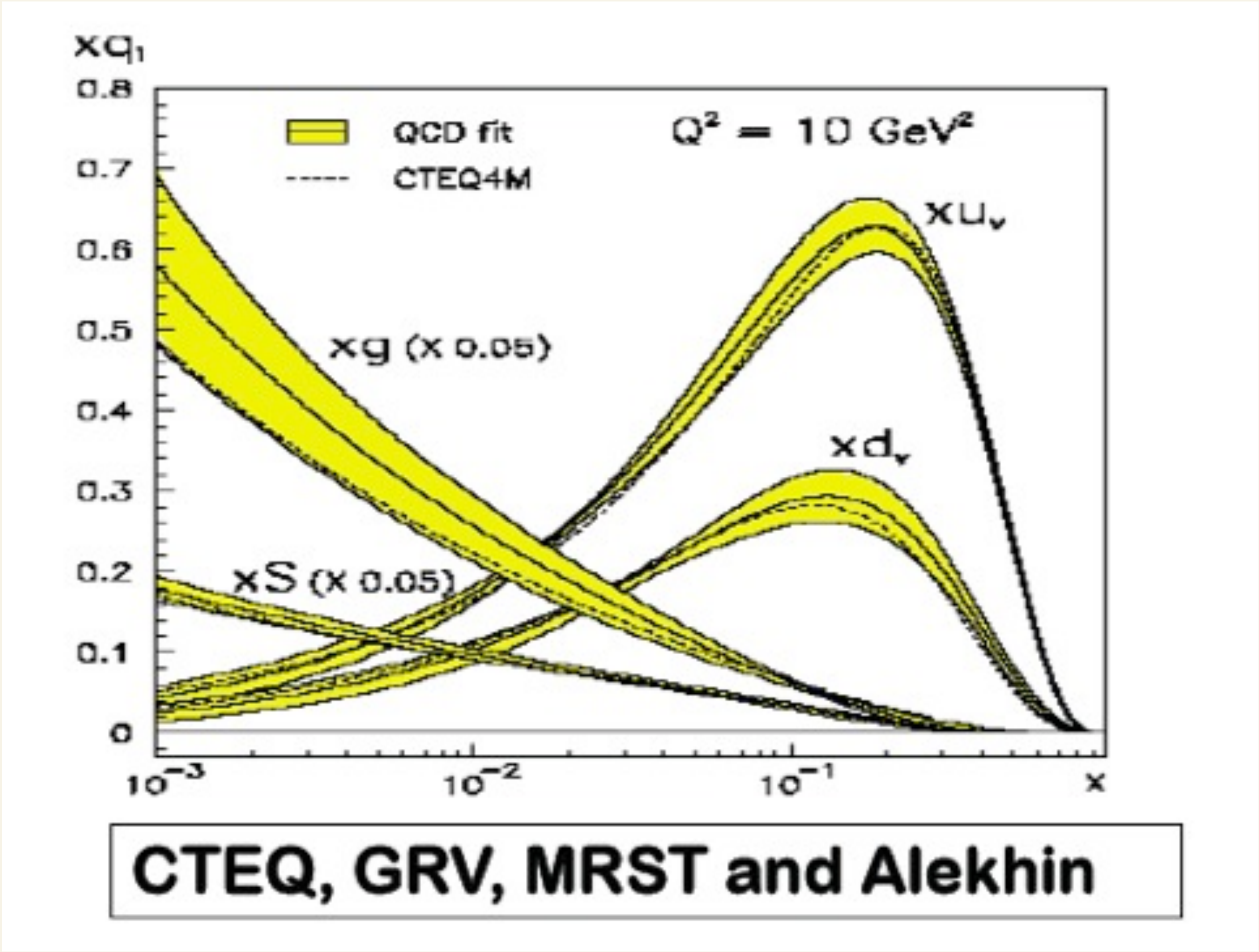
Parton distributions donot interfere with hard interaction. They are universal

$$\sigma = \sum_f \int dx_1 \varphi_{f/A}(x_1)\; dx_2 \varphi_{\bar{f}/B}(x_2)\; \hat{\sigma}(b\bar{b} \to hh)$$

Probability of finding a parton of flavor a in hardon A

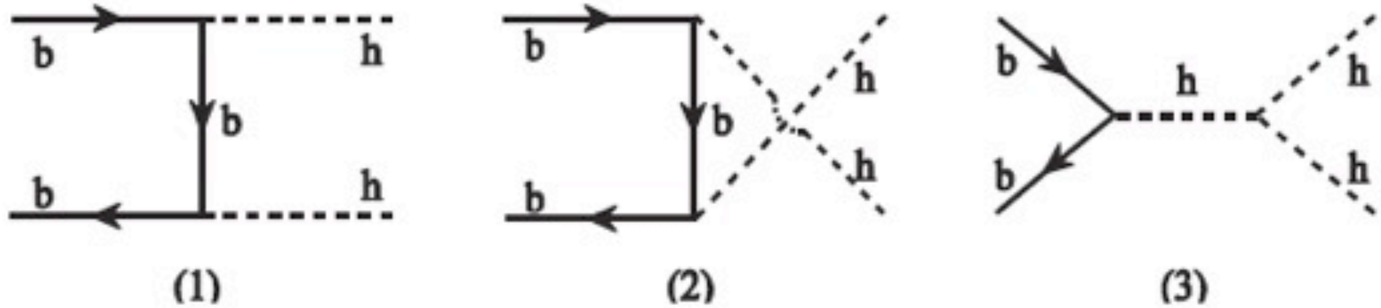
Parton Distribution Functions

$x = p_{q,g}/p_p$ = momentum fraction



Leading Order Cross Section

lowest order cross section for $b\,\bar{b} \rightarrow h\,h$:



$$b(p_1)\bar{b}(p_2) \longrightarrow h(p_3)h(p_4)$$

$$\hat{\sigma}_{b\bar{b}} = \frac{1}{2}\frac{1}{2\hat{s}}\int \frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}$$

$$(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\,|\,\overline{M}_0\,|^2$$

Final state identical

$$|\,\overline{M}_0\,|^2 = \left(\frac{1}{3}\cdot\frac{1}{3}\right)\left(\frac{1}{2}\cdot\frac{1}{2}\right)\sum_{\substack{\text{spin}\\\text{color}}}|\,M_0\,|^2$$

Next-to-Leading Order Cross Section

- α_s Corrections from $b\bar{b} \to hhg$
  - Corrections from virtual gluons.
    Infrared singularity: $p_g \to 0$,
    ultra-violet singularity: $p_g \to \infty$
  - Corrections from real gluon emission
    Infrared singularity: $p_g \to 0$
    collinear singularity: p_g parallels to one of initial b or $\bar{b}$ momentums.
- $1/\Lambda$ Corrections from $bg \to bhh$
  only collinear singularities — gluon splits into a pair of collinear b

α_s Corrections: Real Emission

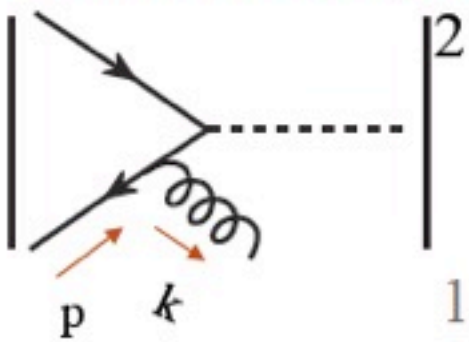
- **Relevant Lagrangian:** g = gauge coupling, T = SU(3) matrices

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - g A\!\!\!/ \cdot T - m)\psi - \frac{1}{4}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} - \frac{m_\Psi}{v}H\overline{\Psi}\Psi - 3\frac{m_h^2}{v}HHH$$

Fields: Quark, ψ, gluon and Higgs, H.

- **Problems arise from parton level interactions**

Infrared (IR) and collinear (CO) singularities



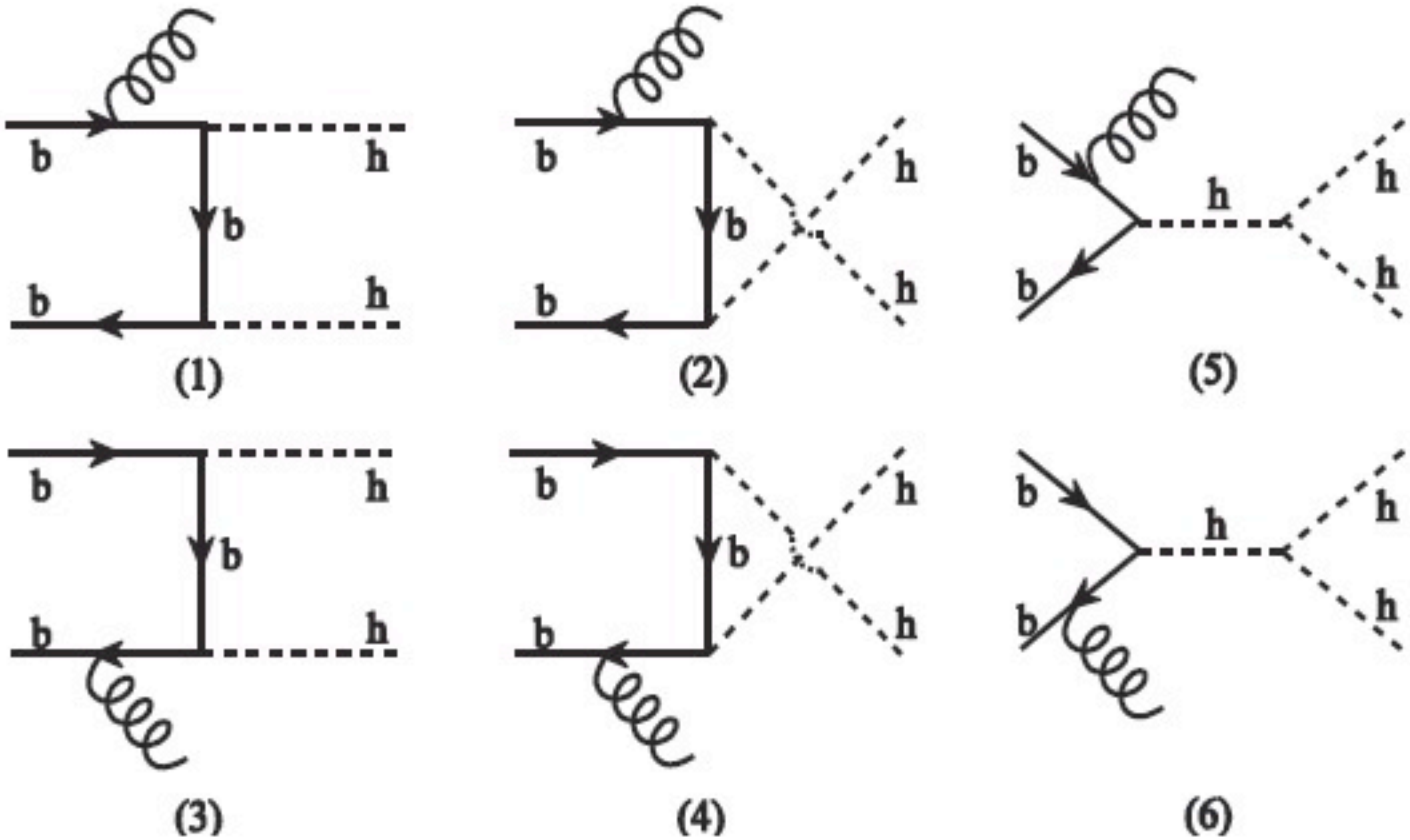
$$\left| \cdots \right|^2 \to \infty \quad as \quad \begin{cases} k^\mu \to 0 & \text{Infrared divergence} \\ k^\mu \parallel p^\mu & \text{Collinear divergence} \end{cases}$$

m = 0

$$\frac{1}{(p-k)^2 - m^2} = \frac{1}{-2p\cdot k}$$

Real Gluon Emission

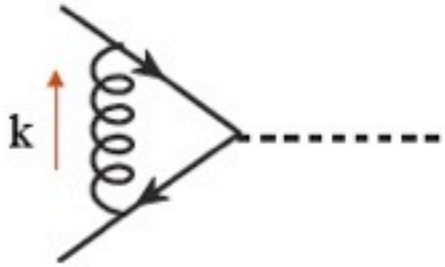
Corrections from real gluon emission



there is infrared and collinear singularities (m_b~0)

α_s Corrections: Virtual Correction

Ultra-violet singularity



$$\sim \int^{\infty} d^4k \frac{k^\mu k^\nu}{k^2 k^2 k^2} \to \infty \text{ as } |k| \to \infty$$

➢ **Vertex with Yukawa coupling must be renormalized.**

Renormalization introduces a renormalization scale μ_R

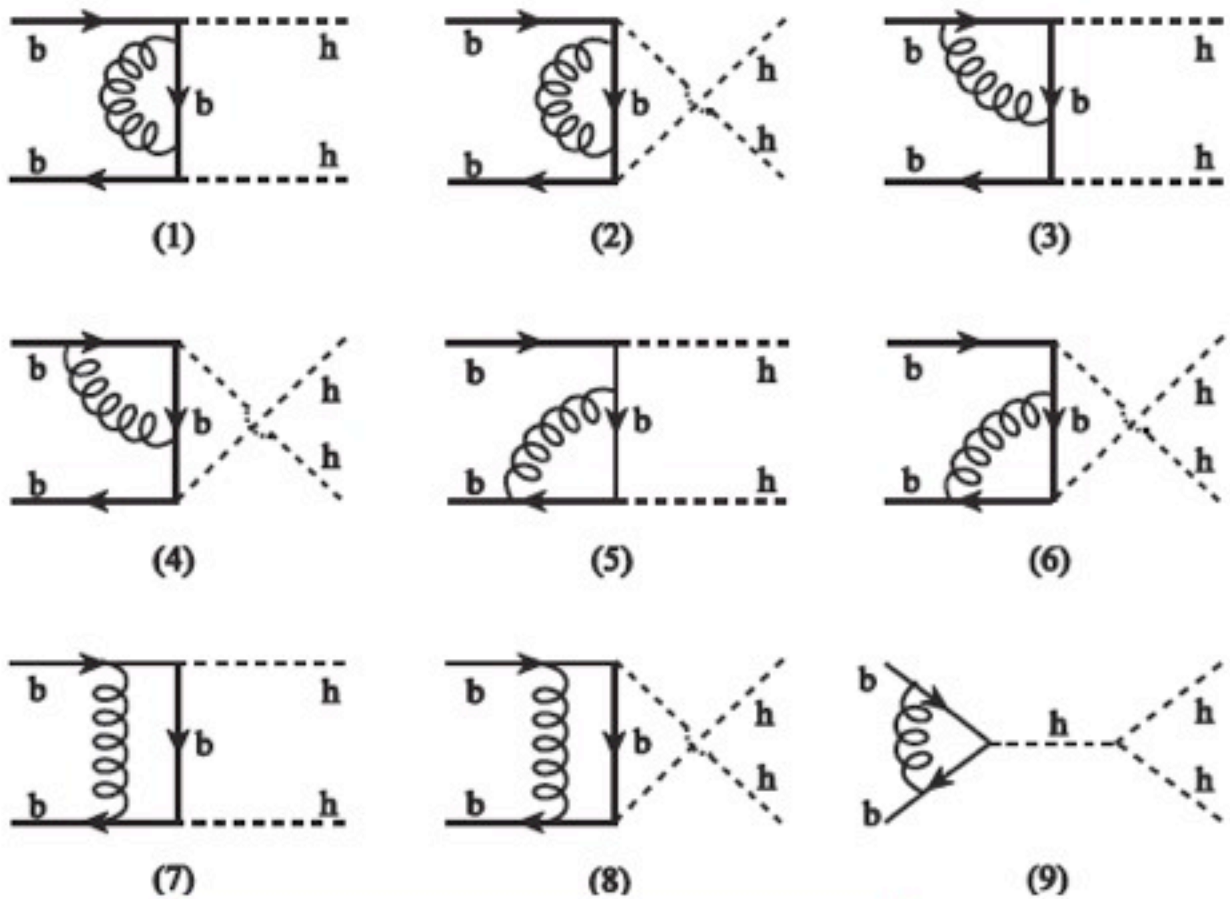
In principle, μ_R is arbitrary

In practice, μ_R is chosen to be a physical scale Q or $\sqrt{\hat{s}}$

interaction at distance « 1/μ_R or momentum scale » μ_R are integrated out.

Ultra-violet divergences are hidden into quantities which can be measured experimentally: mass, coupling

Virtual Corrections



$$M_i = g_s^2 (T^a T^a)_{ji} \hat{M}_d^0 X_i$$

Virtual Corrections with N = 4-2ε (Dimensional Regularization)

Matrix element squared

$$|M_v|^2 = 2\mathrm{Re}(M_{loop}M_0^*) + |M_{CT}|^2$$
$$= A\frac{64\pi\alpha_s}{3}\{[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\ln(\hat{s}) - \frac{3}{2\varepsilon}]|M_0|^2 - |M_D|^2\}$$

IR and UV divergences **finite terms**

$|M_D|^2$ includes all remaining finite terms.

IR divergences will be canceled by the IR divergences from real gluon emission diagrams

Infrared Safety: KLN Theorem

Virtual diagrams plus soft contribution of real diagrams

$$|M_v|^2 + |M'_{soft}|^2$$

$$= -A\frac{64\pi\alpha_s}{3}\left(-\frac{1}{\varepsilon}\right)\left[\ln(\delta_s^2) + \frac{3}{2}\right]|M_0|^2$$

Collinear singularity from soft region, will be absorbed into PDF

$$+ A\frac{64\pi\alpha_s}{3}\left[\frac{1}{2}\ln^2(s\delta_s^2) - \frac{\pi^2}{3}\right]|M_0|^2$$

Finite contributions from soft region

$$- A\frac{64\pi\alpha_s}{3}|M_D|^2$$

Finite virtual contributions

Cancellation of Collinear Divergence

Replace b(x) by b(x, μ_f) and drop terms high order than α_s

Extra terms in LO contributions.

$$\sigma_{LO} = \int dx_1 dx_2 b(x_1,\mu)\bar{b}(x_2,\mu)\hat{\sigma}_{LO}$$

$$+\int dx_1 dx_2 b(x_1,\mu)\bar{b}(x_2,\mu)\hat{\sigma}_{LO} \frac{4\alpha_S}{3\pi}(4\pi)^{\varepsilon}\Gamma(1+\varepsilon)\left(\frac{1}{\varepsilon}\right)\left[\ln(\delta_s^2)+\frac{3}{2}\right]$$

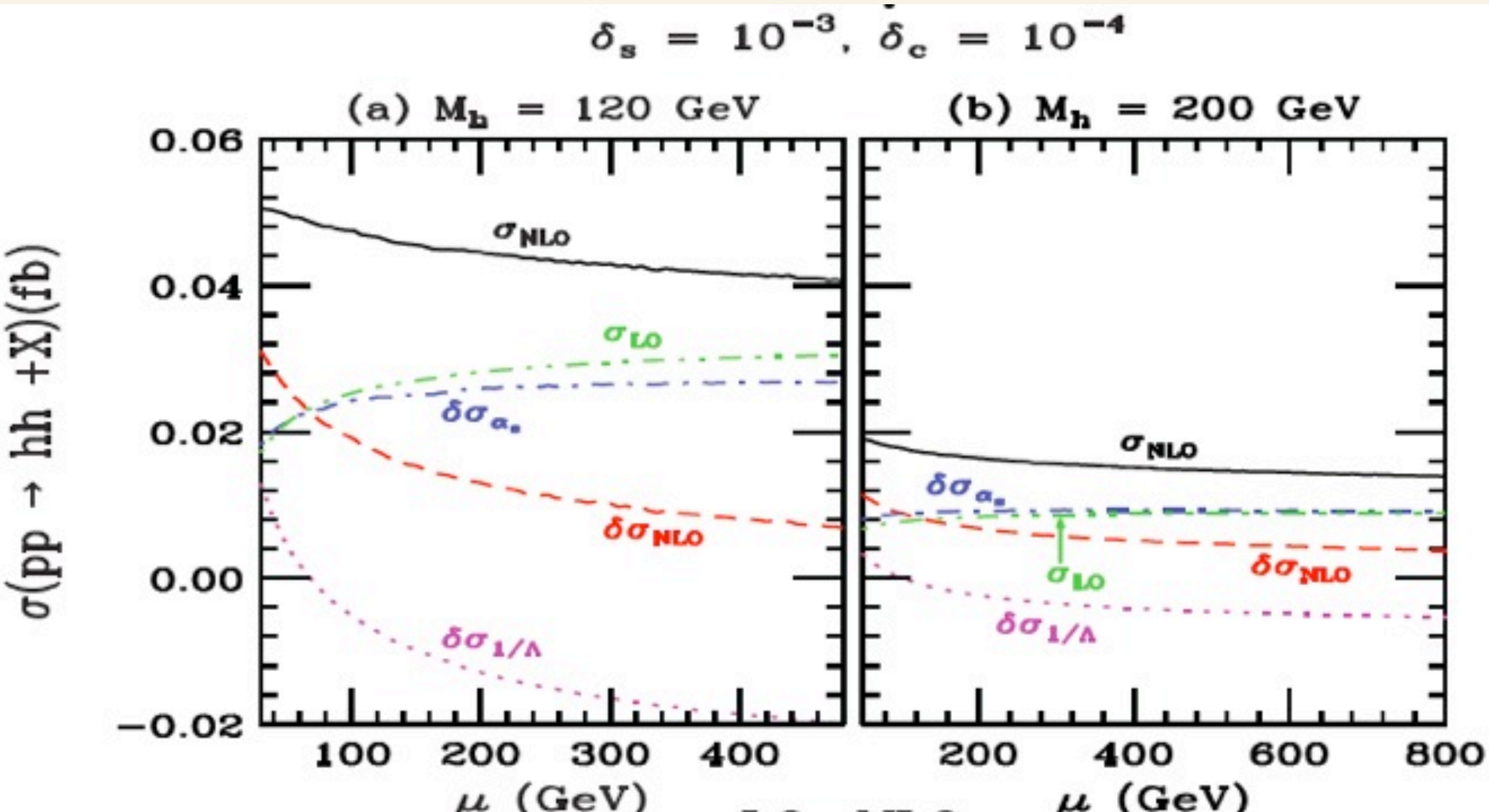
$$+\int dx_1 dx_2 \bar{b}(x_2,\mu)\hat{\sigma}_{LO}\frac{\alpha_S}{2\pi}(4\pi)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{1}{\varepsilon}\right)\int_{x_1}^{1-\delta_S} P_{bb}(z,\varepsilon)\frac{dz}{z}b(x_1/z,\mu)$$

To cancel the collinear singularity in soft region

To cancel the collinear singularity in hard collinear region

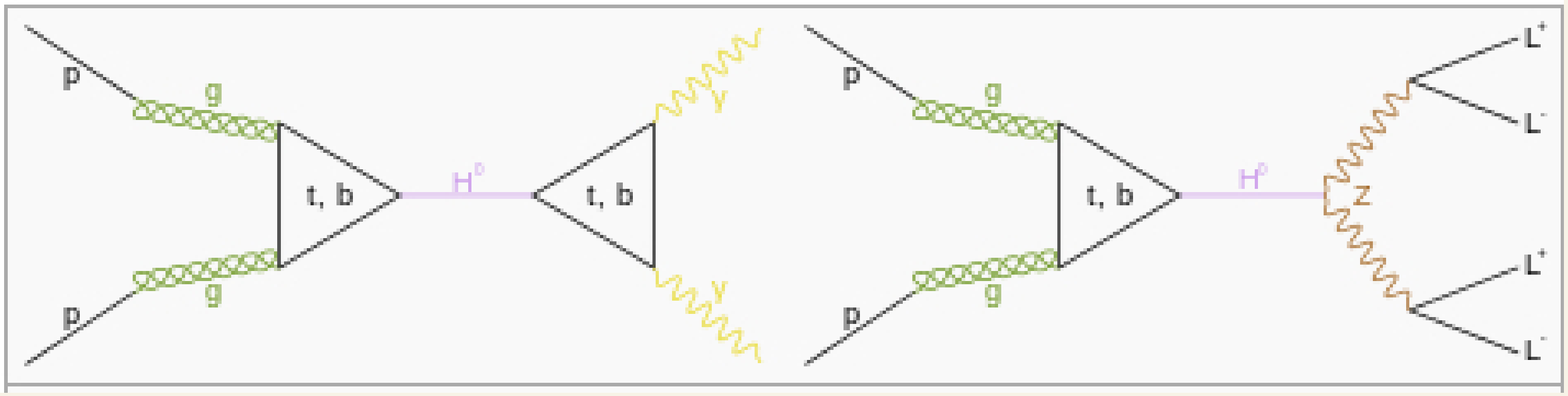
For simplification, we use $\mu_R = \mu_f = \mu$

NLO Corrections to bb → h h



$$K = \frac{\mathrm{LO} + \mathrm{NLO}}{\mathrm{LO}} \sim 1.5$$

Higgs Production from Gluon Fusion



Loop Integrals

't Hooft and Veltman

$$\int \frac{d^N l}{i\pi^{N/2}} \frac{1}{(l^2 + 2l \cdot p + M^2)^A} = \frac{\Gamma(A - N/2)}{\Gamma(A)} \frac{1}{(M^2 - p^2)^{A-N/2}}$$

For $(q^2/M^2)^2 < 1$, we can expand the propagator* as

$$\begin{aligned}
[(l+q)^2 + M^2]^{-1} &= [l^2 + M^2 + 2l \cdot q + q^2]^{-1} \\
&= [l^2 + M^2]^{-1}[1 + \frac{2l \cdot q + q^2}{l^2 + M^2}]^{-1} \\
&= [l^2 + M^2]^{-1}[1 - \frac{2l \cdot q + q^2}{l^2 + M^2} + \frac{(2l \cdot q + q^2)^2}{(l^2 + M^2)^2} + O(M^{-6})].
\end{aligned}$$

The Decoupling Theorem

Appelquist and Carazzon, Phys. Rev. **D11** (1975) 2856.

- Sakurai Prize 1997 awareded to Thomas Appelquist: "For his pioneering work on charmonium and on the de-coupling of heavy particles".
- However, there are non-decoupling interactions between heavy fermions and Higgs bosons.

Loop Integrals and Effective Lagrangian

- Within the large top-mass approximation, the effective single and double-Higgs coupling to gluons is given by the following Lagrangian where $C_H = \alpha_S/(3\pi)$ and $v = 246$ GeV:

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} \left(C_H \frac{H}{v} - C_{HH} \frac{H^2}{v^2} \right) .$$

Heavy Top Quark Limit

At the parton level, the cross section of gg to H is

$$\sigma(gg \to H^0) = \frac{1}{64}\left(\frac{\alpha_s^2 \alpha_W}{M_W^2}\right)(s)|F(\rho)|^2 \delta(s - M_H^{0\,2})$$

$$I(\rho) = +\int_0^1 \frac{dy}{y}\left\{\ln\left[1 - \frac{y(1-y)}{\rho - i\epsilon}\right]\right\}$$

$$F(\rho) = +\rho[2 + (4\rho - 1)I(\rho)]$$

In the heavy top limit with $\rho = m_t^2/m_H^2 >> 1$, we have

$$I(\rho) = -\frac{1}{2\rho} - \frac{1}{24\rho^2} + O\left(\frac{1}{\rho^3}\right),$$

$$F(\rho) = +\tfrac{1}{3} + O\left(\frac{1}{\rho}\right),$$

Loop Integrals and Effective Lagrangian

- Howard Georgi: ‘Most loop integrals are unnecessarily complicated. If we have talented and skillful experimentalists, we can just write down the effective Lagrangians and ask them to measure masses and couplings.’
- Weinberg’s guidelines for effective Lagrangians: (a) Lorentz invariance, (b) cluster decomposition, and (c) unitarity.

More Quotes from Georgi to Promote Effective Theories

- Georgi: What therefore God has put asunder, let not man joined together.
  Mark 10:9 What therefore God has joined together, let not man put asunder.
- “Therefore do not be anxious about tomorrow, for tomorrow will be anxious for itself. Let the day’s own trouble be sufficient for the day.
  Matthew 6:34 (Sermon on the Mount)

Loop Integrals and Effective Lagrangian

- Peter Lepage
  - Most relevant effective Lagrangians should be consequences from fundamental renormalizable theories with heavy particles integrated out.
  - Pauli-Villars regularization is more meaningful than dimensional regularization. If you know the right Pauli-Villars cut-off for your effective Lagrangian, you will find new physics at a 'slightly' higher energy.
  - QED is so successful because $M_W >> m_e$.

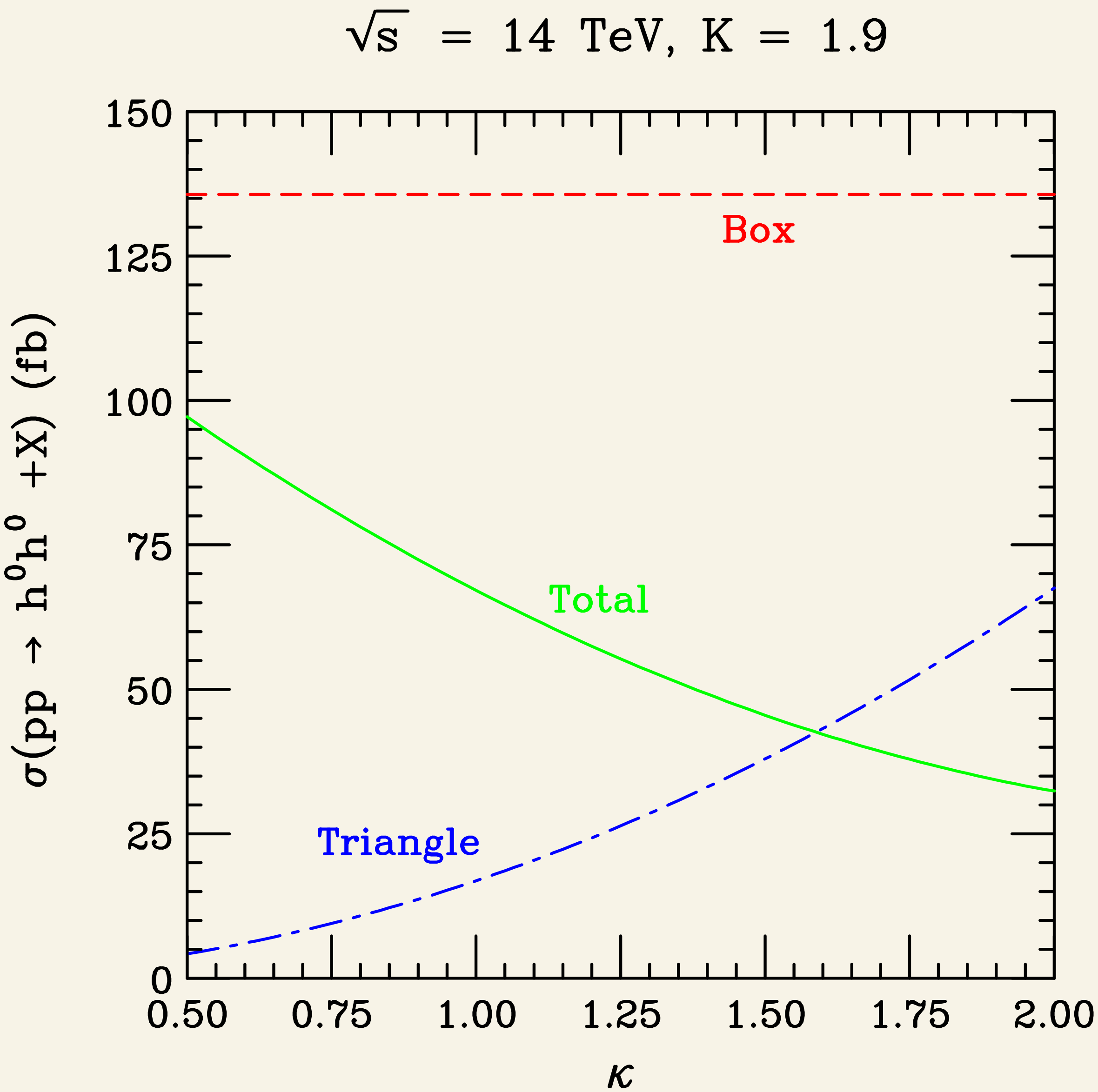
The Trilinear Higgs Coupling(s)

- Higgs pair production from gluon fusion involves ttH and HHH couplings.
- The box and triangle diagrams are separately gauge invariant so we can vary the two couplings independently by introducing parameters κ_t and κ or κ_H ,

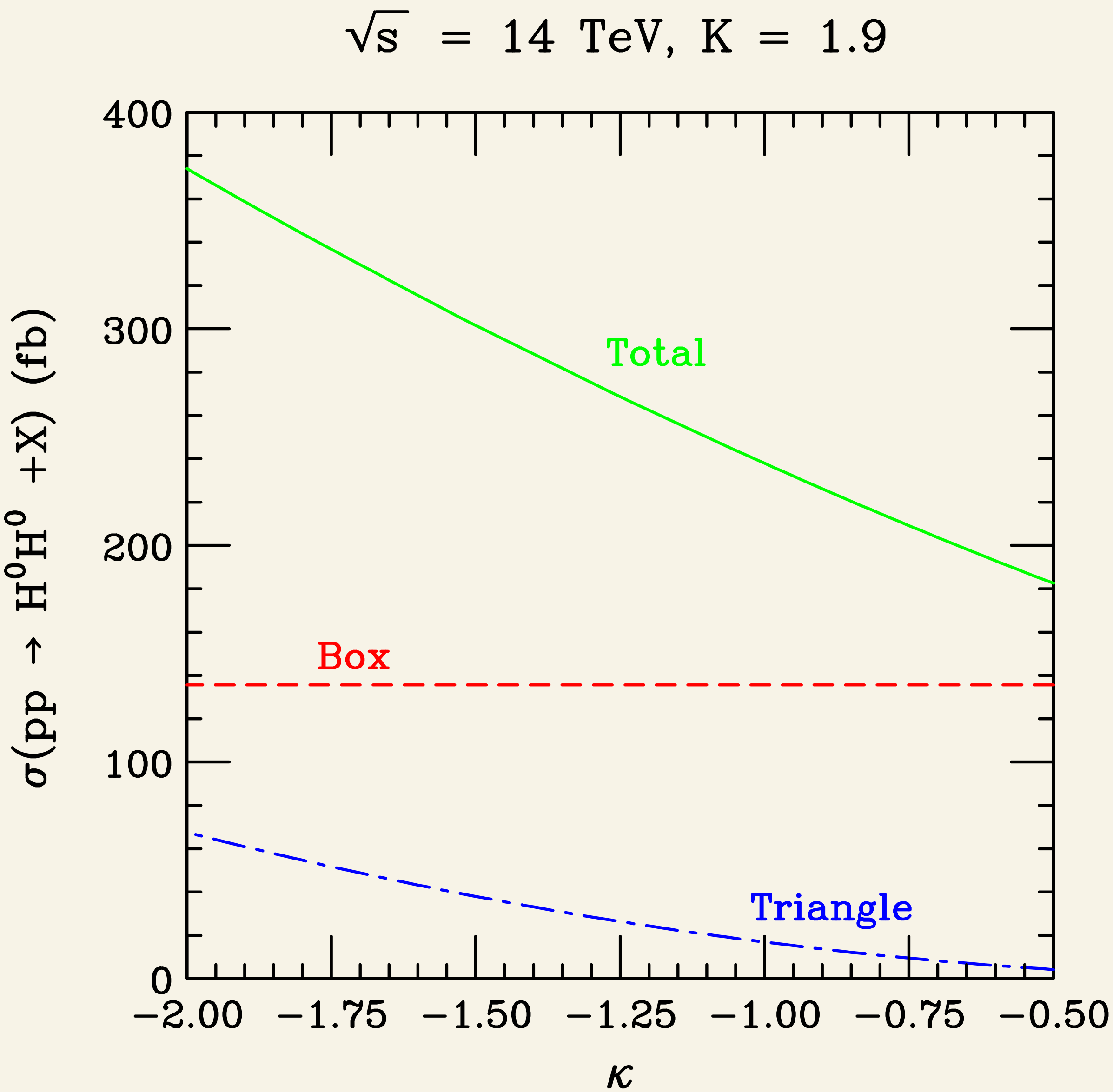
$$t\bar{t}H \; : \qquad -\frac{m_t}{v}\,k_t$$

$$HHH \; : \qquad -\frac{3\,M_H^2}{v}\kappa$$

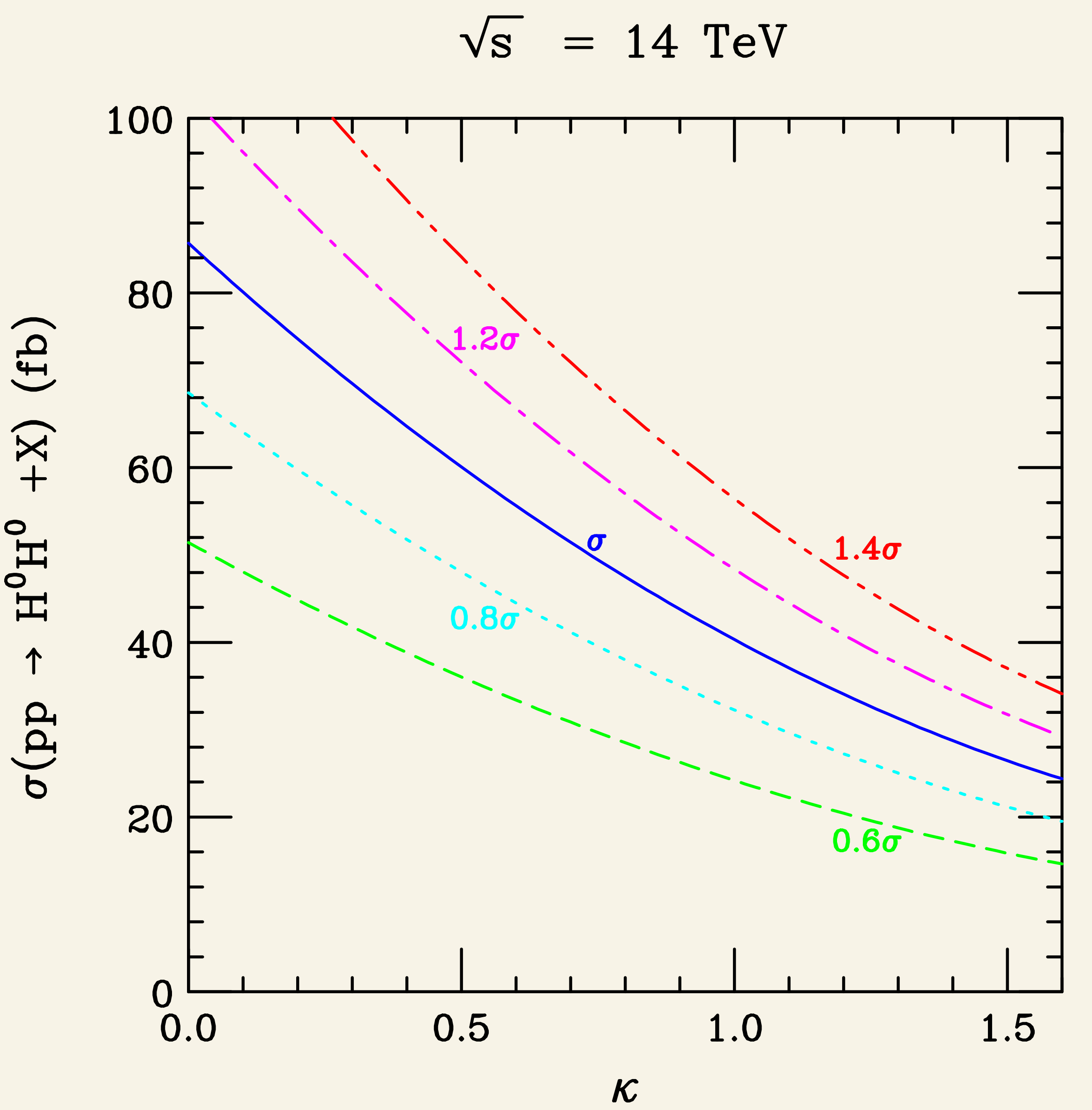
Effects of kappa with κ > 0



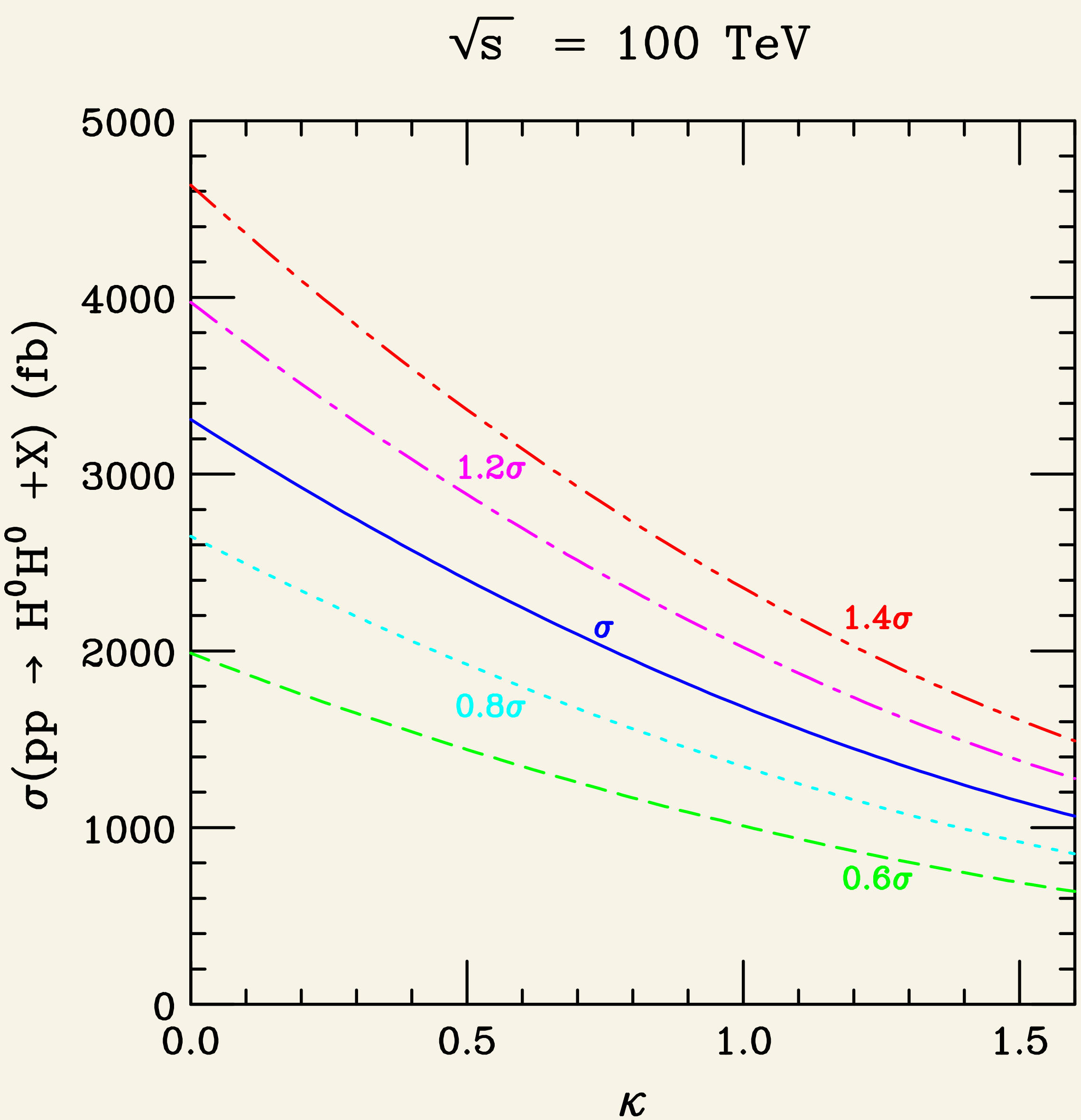
Effects of kappa with κ < 0



Uncertainties in Cross Section

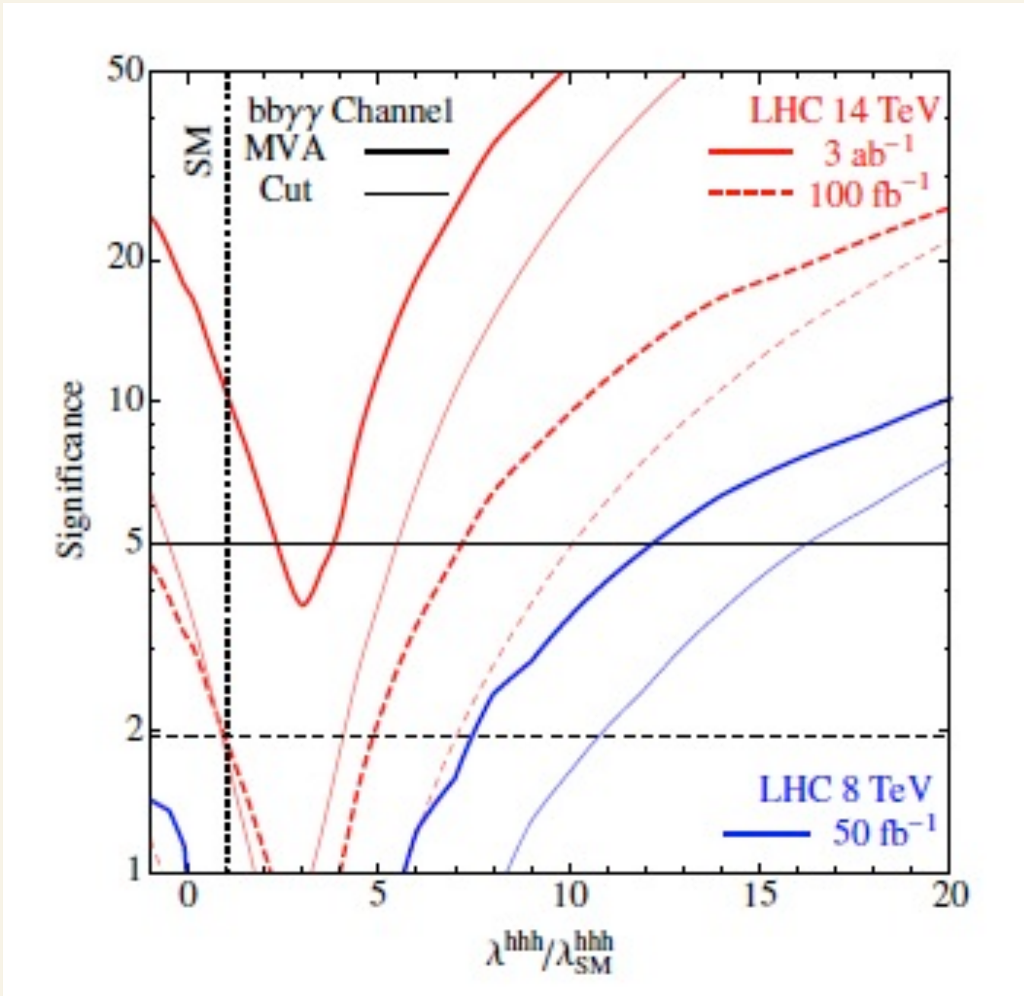


Uncertainties in Cross Section



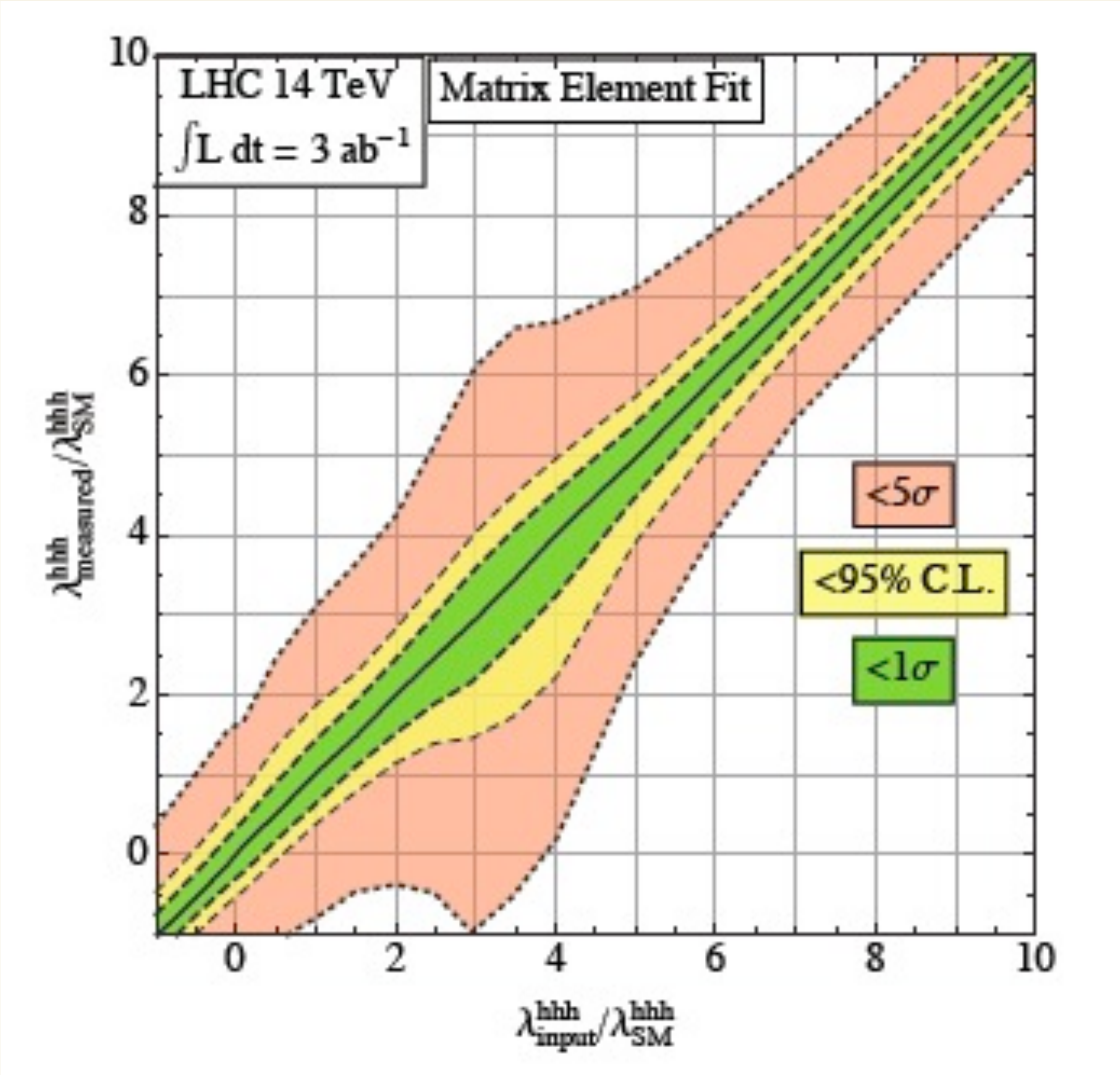
The Discovery Potential of Higgs Pairs

Barger, Everett, Jackson, Shaughnessy, Phys. Lett. B728 (2014).



Simulated Coupling Measurement

Barger, Everett, Jackson, Shaughnessy, Phys. Lett. B728 (2014) 433.



Conclusions

Barger, Everett, Jackson, Shaughnessy, Phys. Lett. B728 (2014) 433.

- The $bb\gamma\gamma$ channel is the only promising channel; reducible backgrounds swamp the signals of other channels such as $bb\tau\tau$.
- The minimum in the integrated cross section versus the trilinear coupling coincides with the minimum in the M_{hh} distribution at $2m_t$ for a hhh coupling $\kappa_h \approx 2.45$ where $\kappa_h = \lambda_{hhh}/\lambda_{hhh}^{SM}$.
- The SM amplitude of gg → hh has a zero in the Mhh distribution for $1.1 < \kappa_h < 2.45$.

Conclusions

Barger, Everett, Jackson, Shaughnessy, Phys. Lett. B728 (2014) 433.

- Multivariate analysis gives a substantially better reach on λ_{hhh} over the cut-based analysis.
- LHC data at 7-8 TeV should probe large deviations of λ_{hhh} from the SM (κ_h > 7.5 at 95% C.L.).
- At the LHC with a CM energy of 14 TeV, ATLAS and CMS will be able to measure λ_{hhh} to 25-80%.
- At LHC14 with 3 ab^{-1}, λ_{hhh} can be determined within 40% uncertainty.