

Generalized Parton Distributions from the Lattice



施羅斯

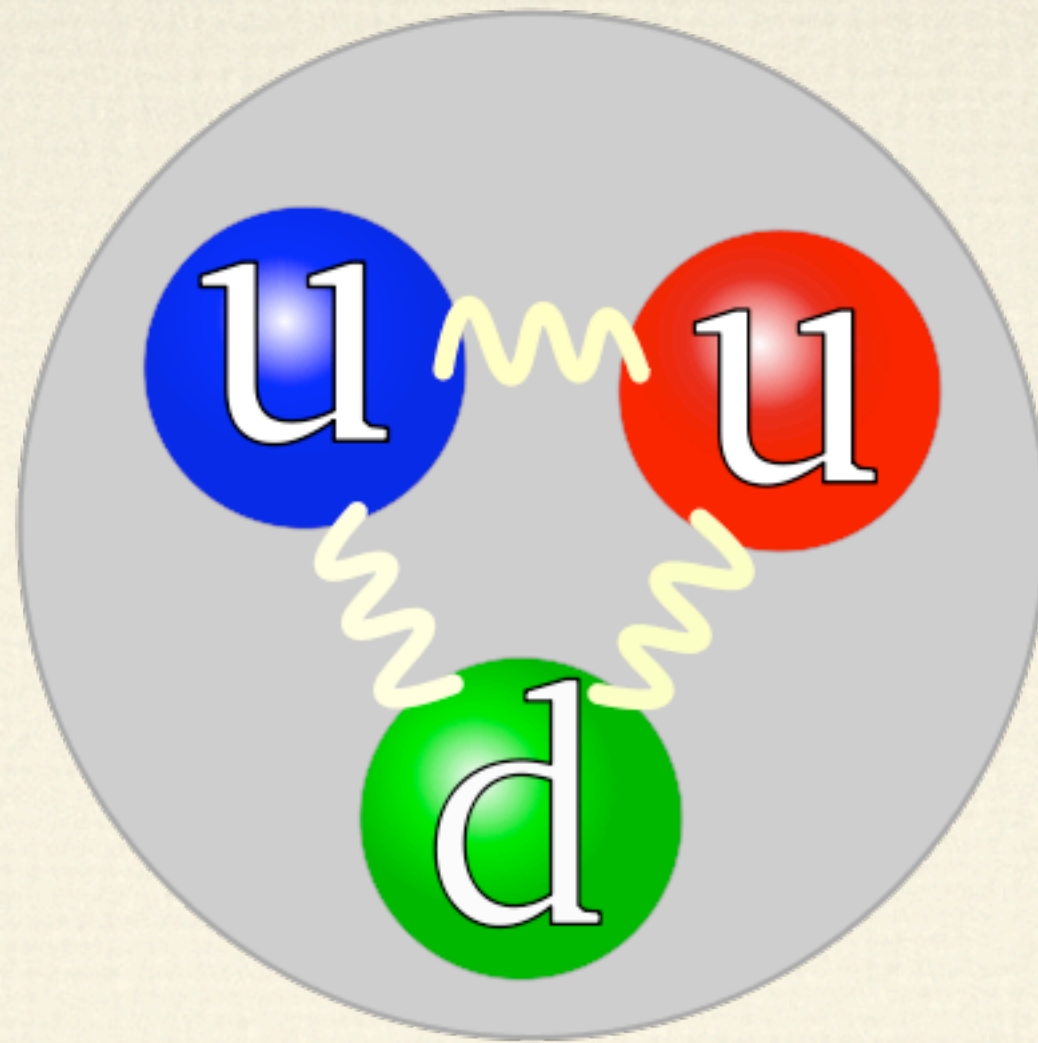
Wolfram Schroers

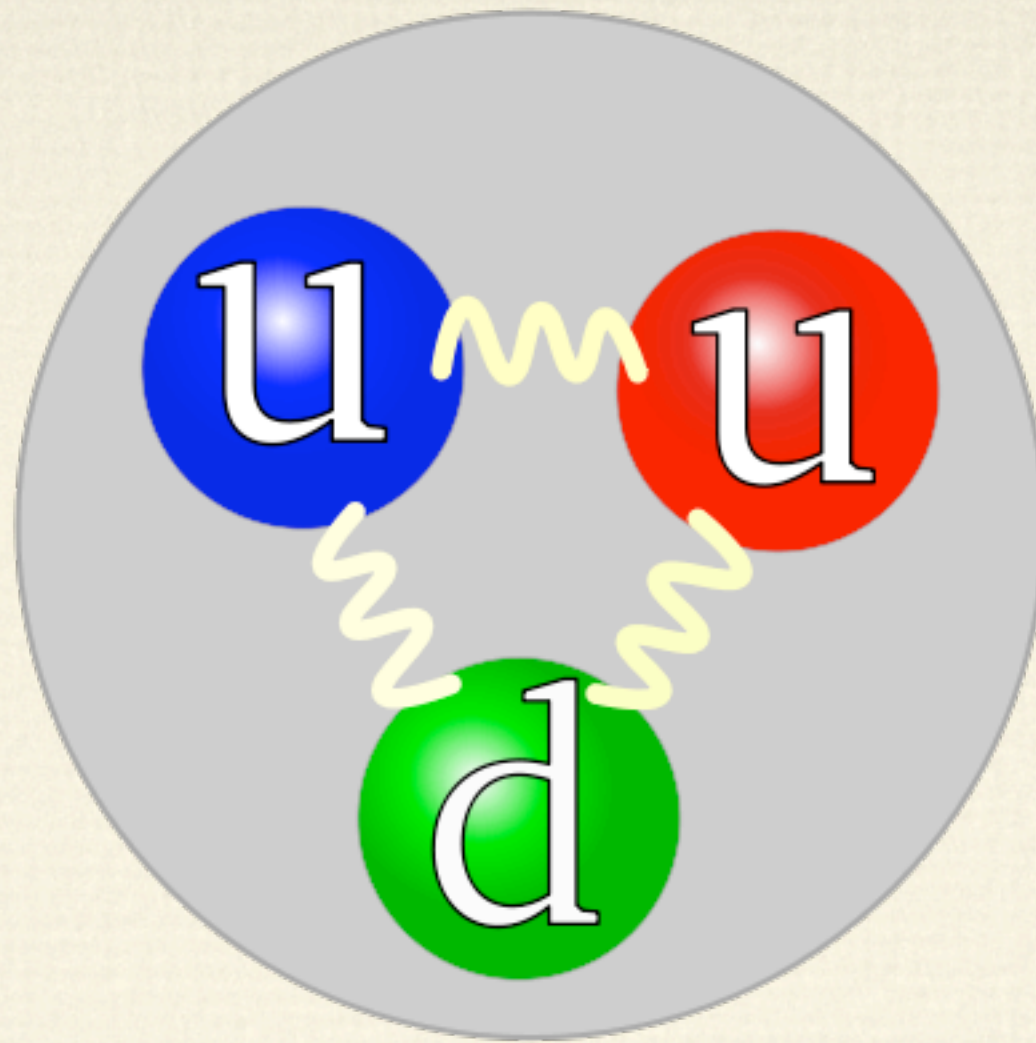


Outline

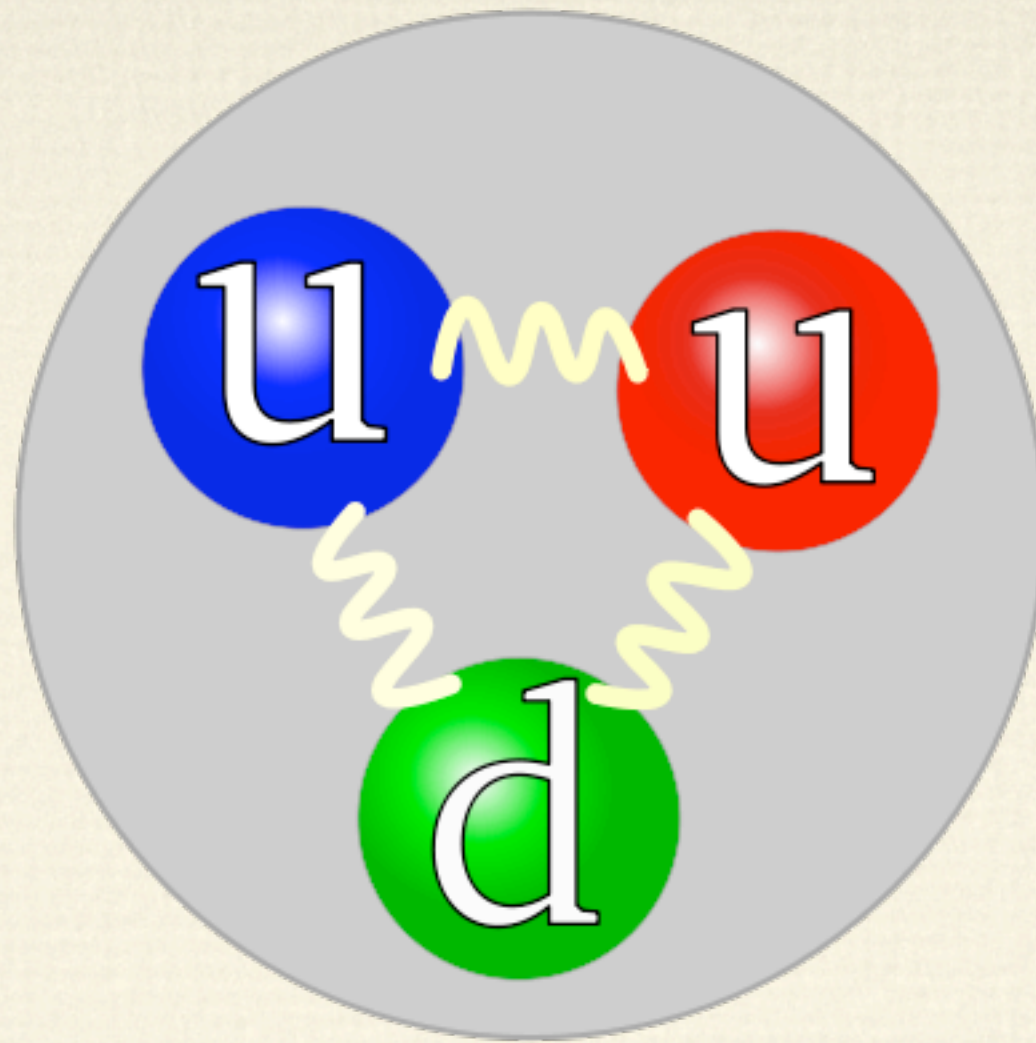
- ❖ Nucleon structure
- ❖ Lattice simulations & their challenges
- ❖ Achievements: Three key calculations
- ❖ Summary
- ❖ Outlook

Nucleon structure





How do we know?

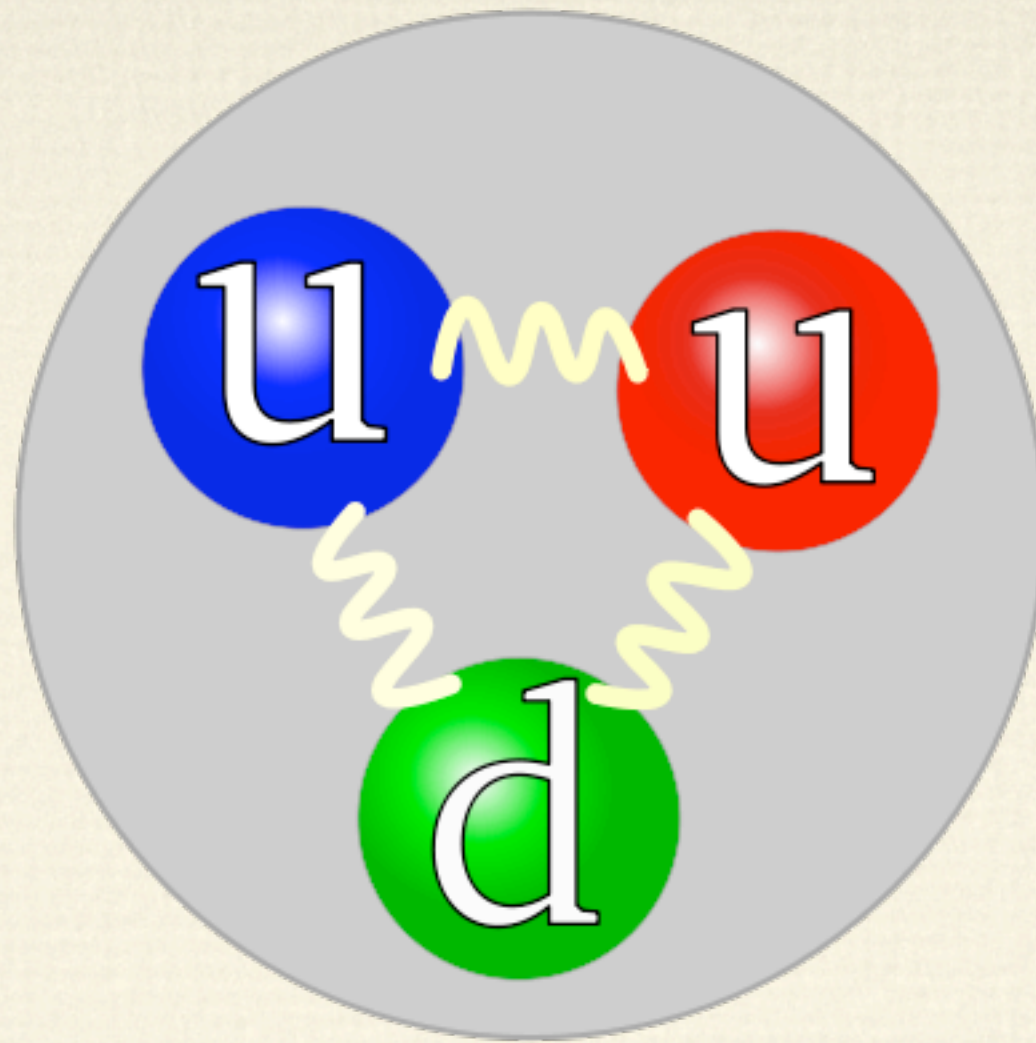


How do we know?

Scattering experiments:

Exclusive

Inclusive



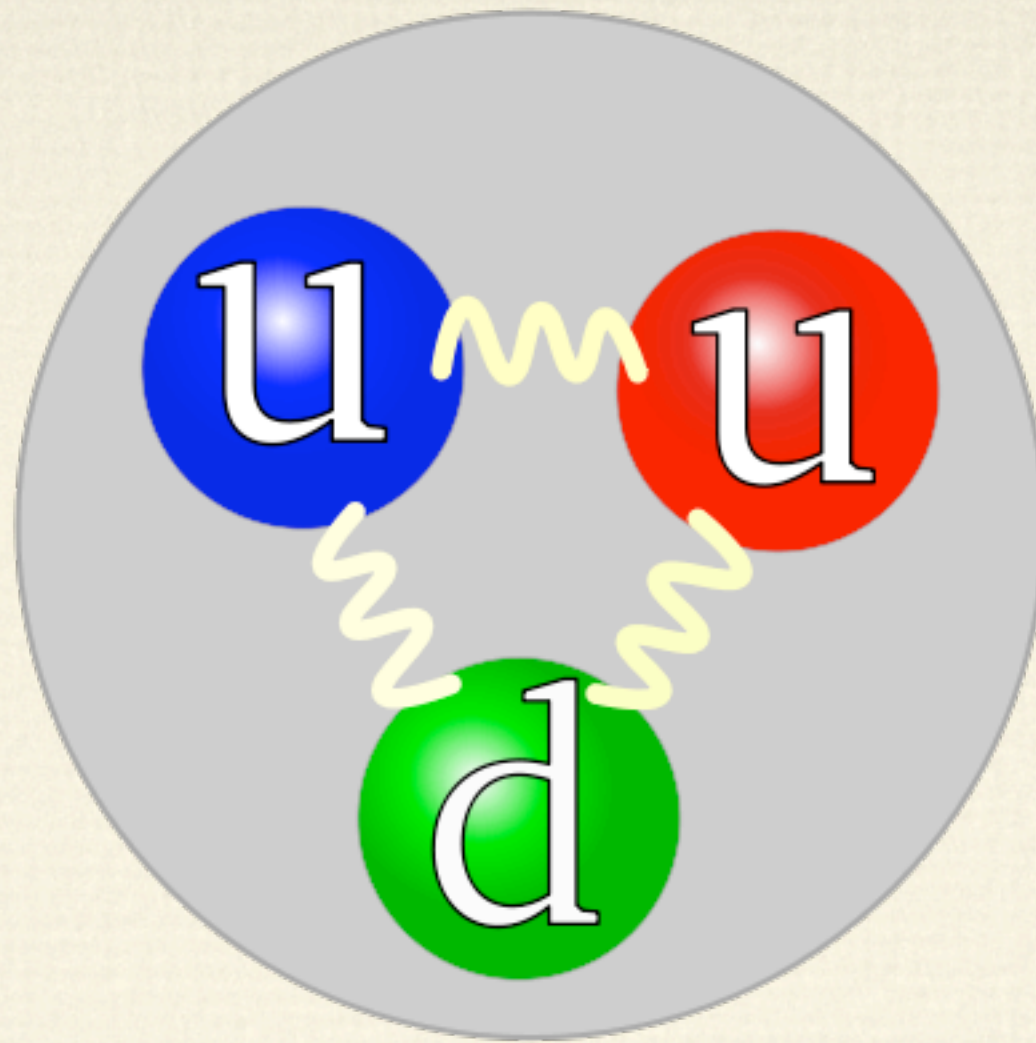
How do we know?

Scattering experiments:

e.g. Form factors

Exclusive
Inclusive





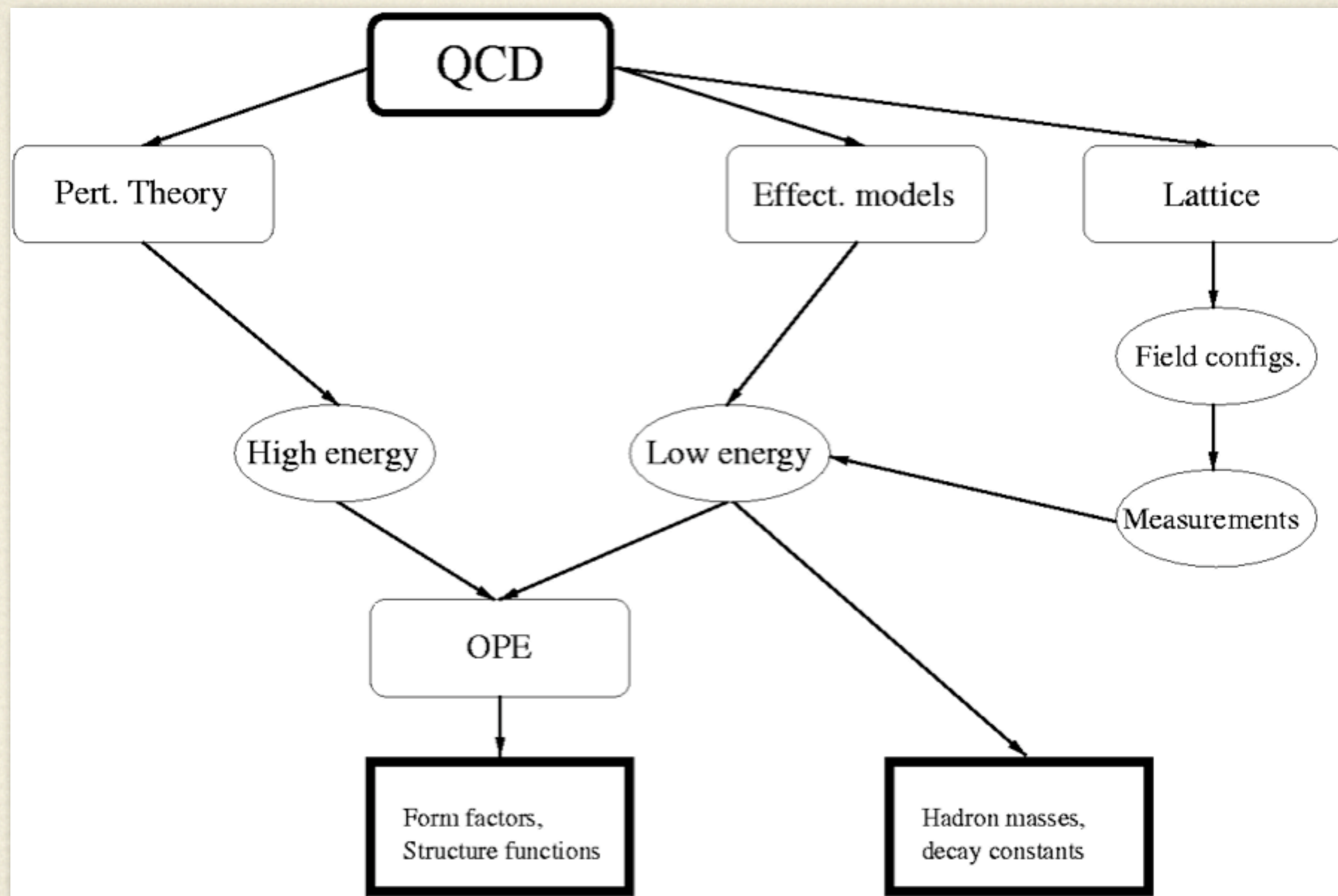
How do we know?

Scattering experiments:

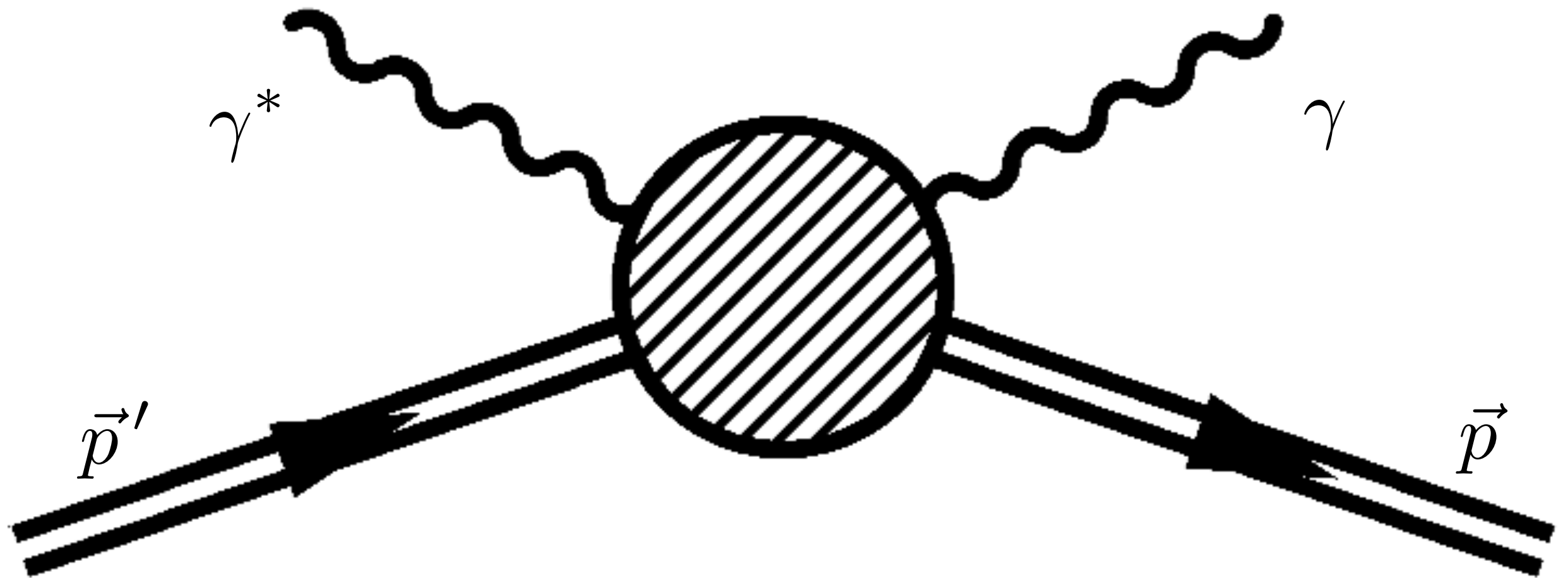
e.g. Form factors

Exclusive
Inclusive

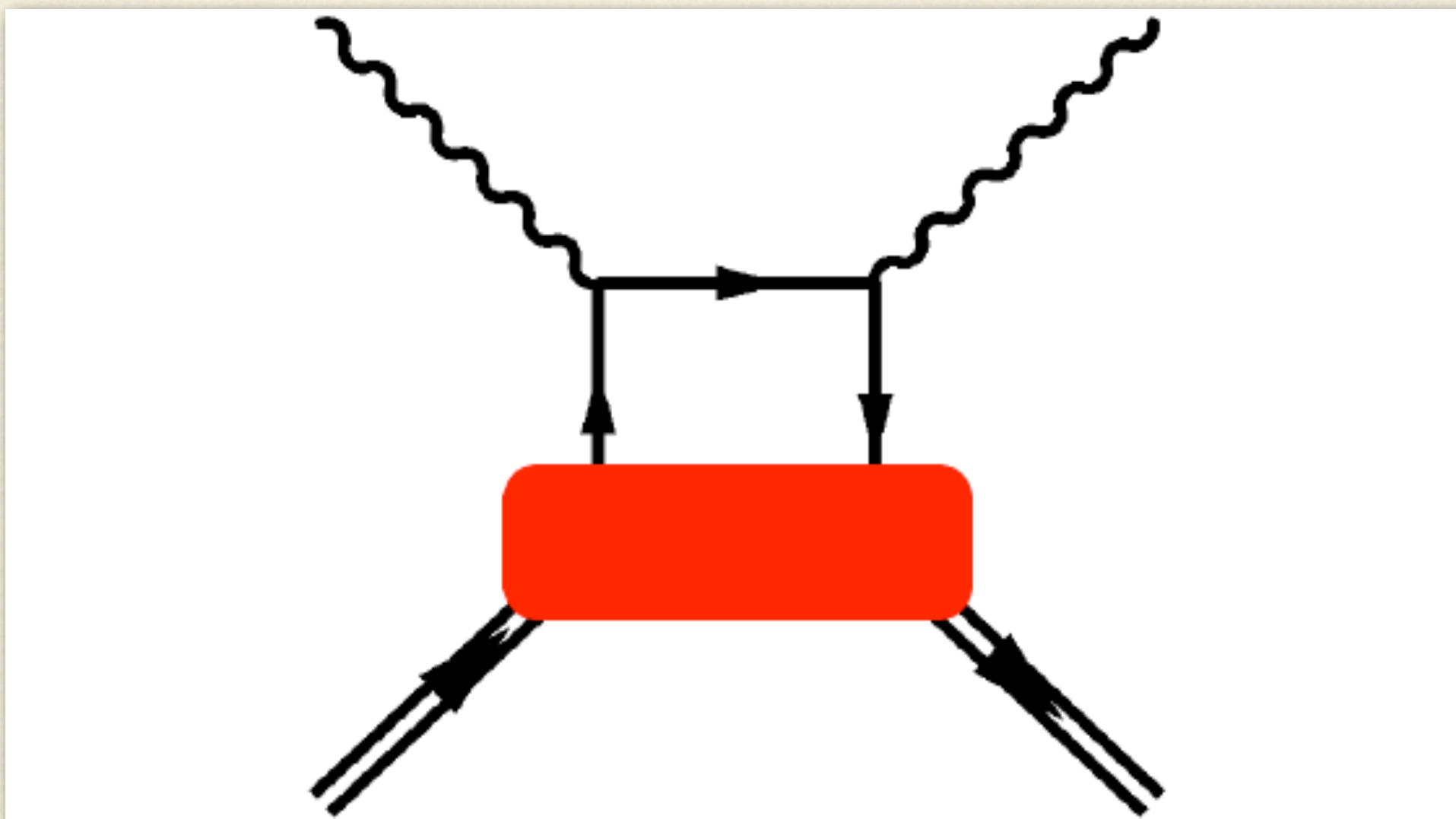
e.g. parton
distributions



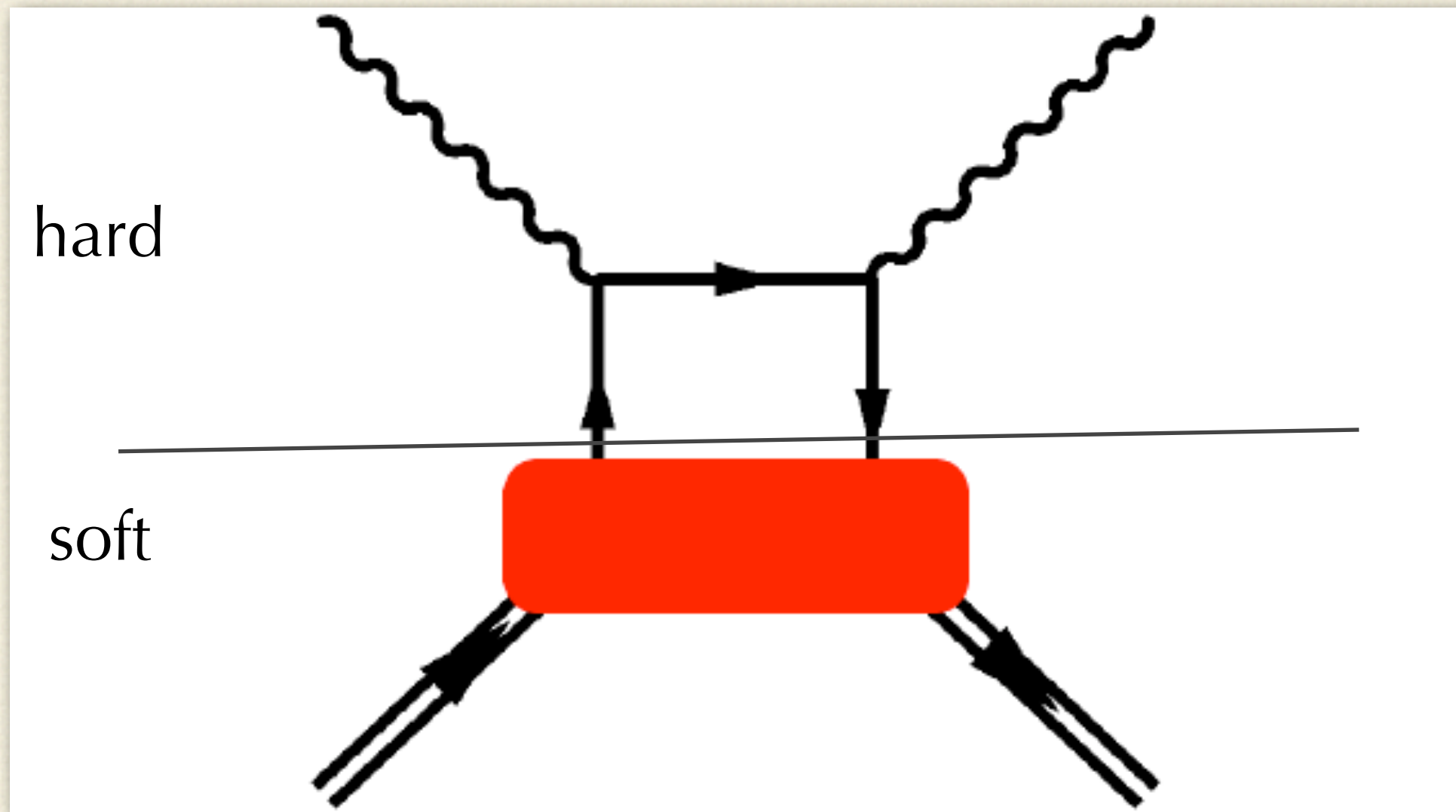
DVCS



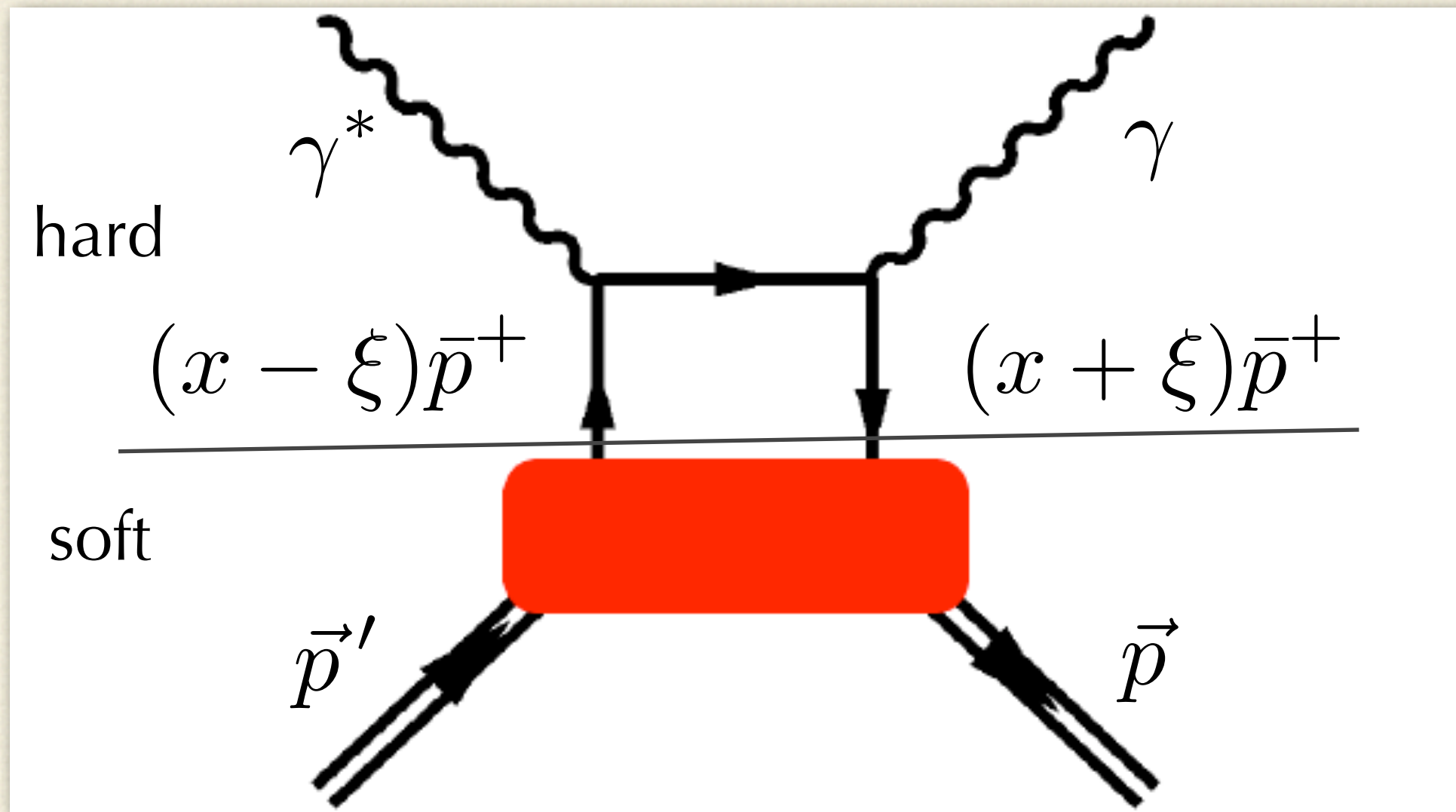
Signature: $e + p \rightarrow e + p + \gamma$

[illegible]

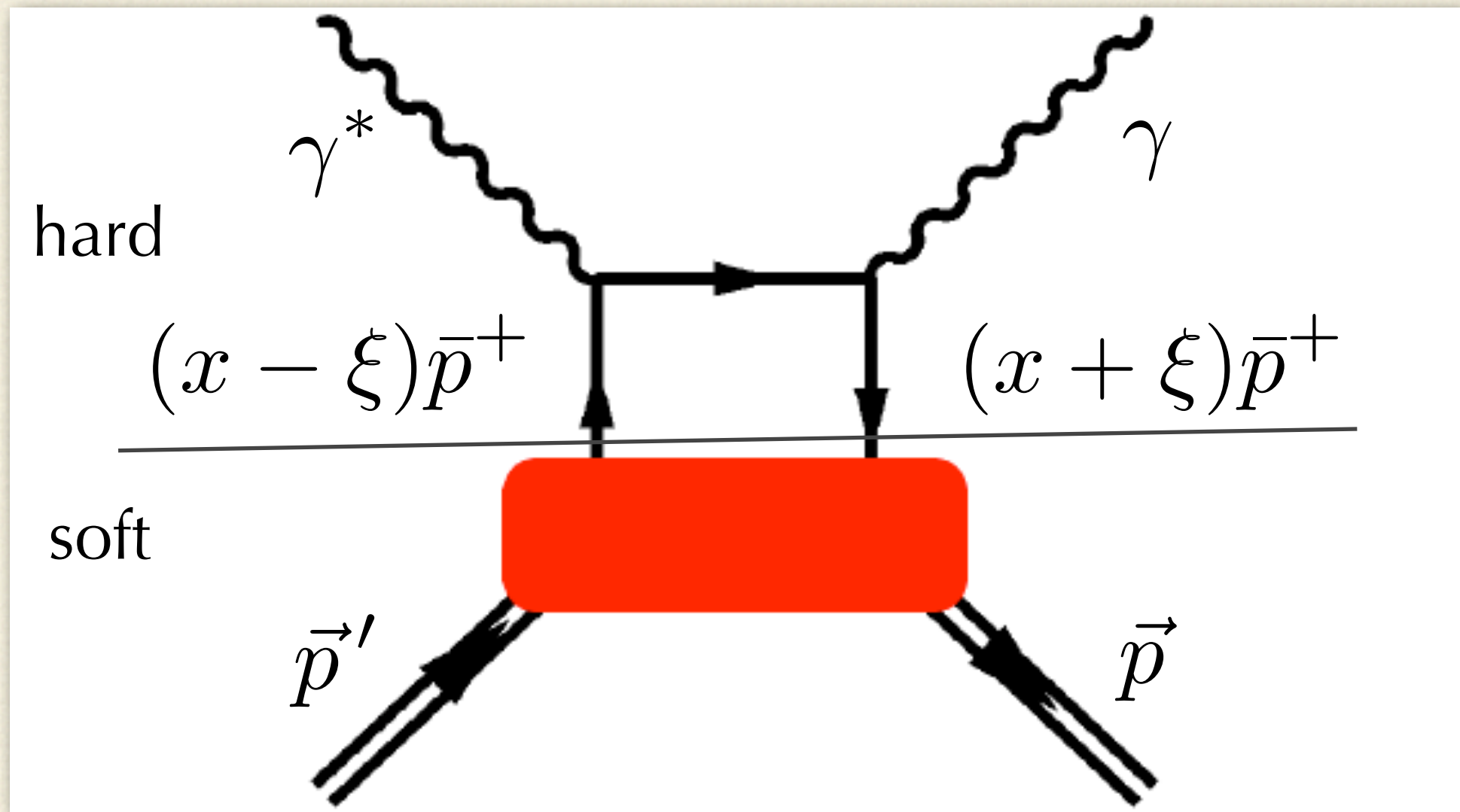
Factorization ansatz



Factorization ansatz



Factorization ansatz



x

ξ

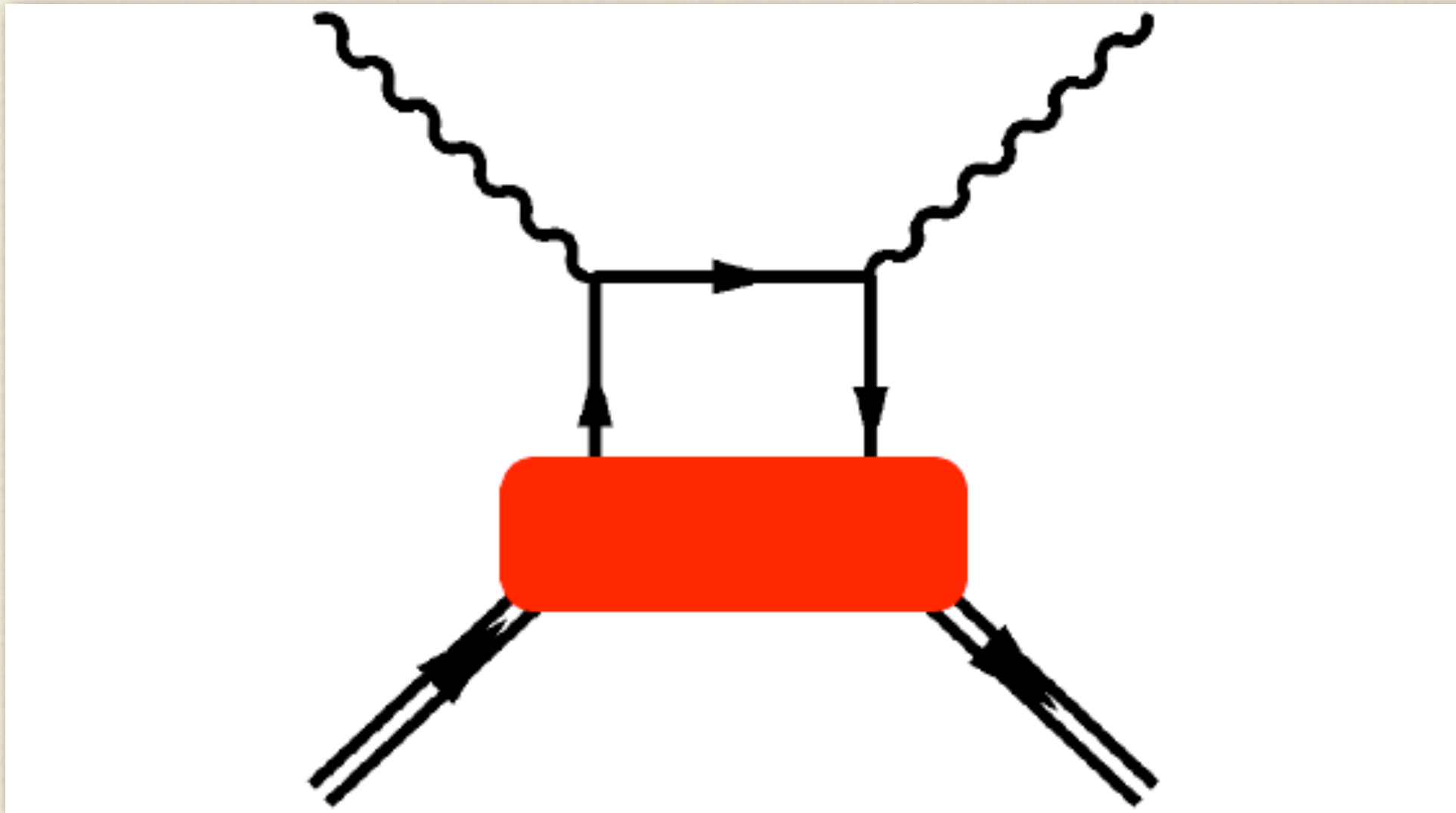
average long. mom

long. mom transfer

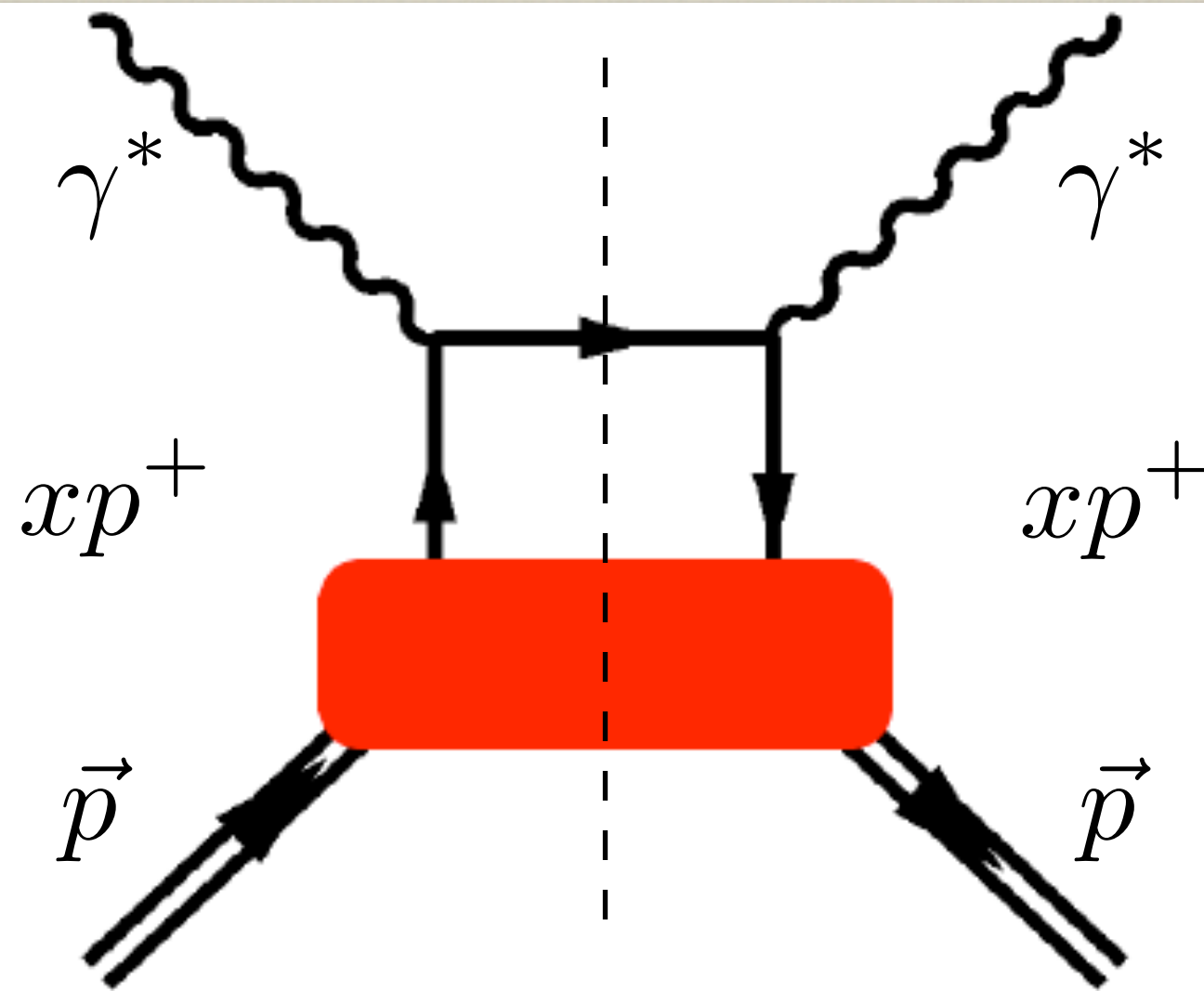
$$t = \Delta^2 = (p' - p)^2$$

virtuality

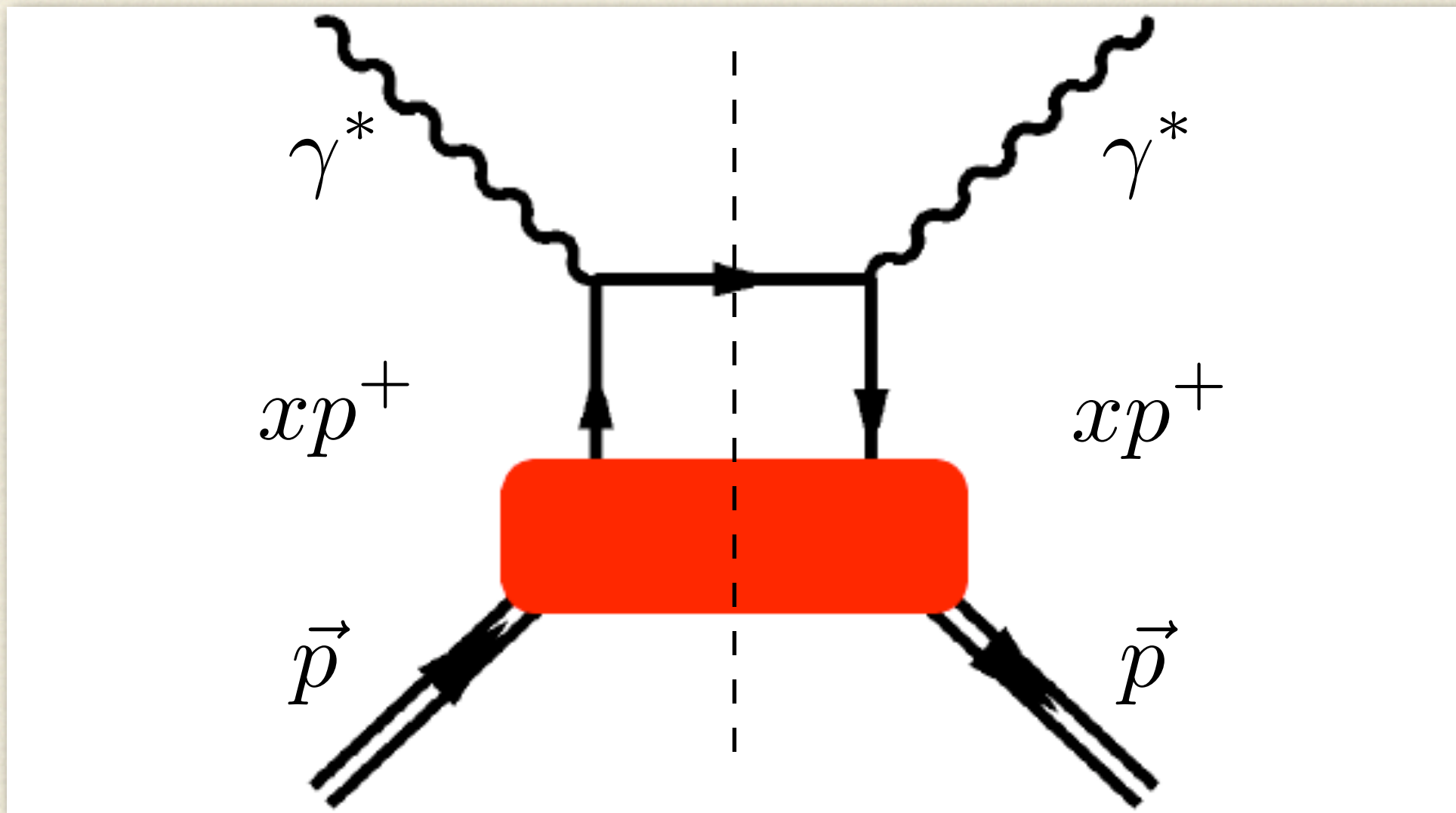
Forward limit



Forward limit



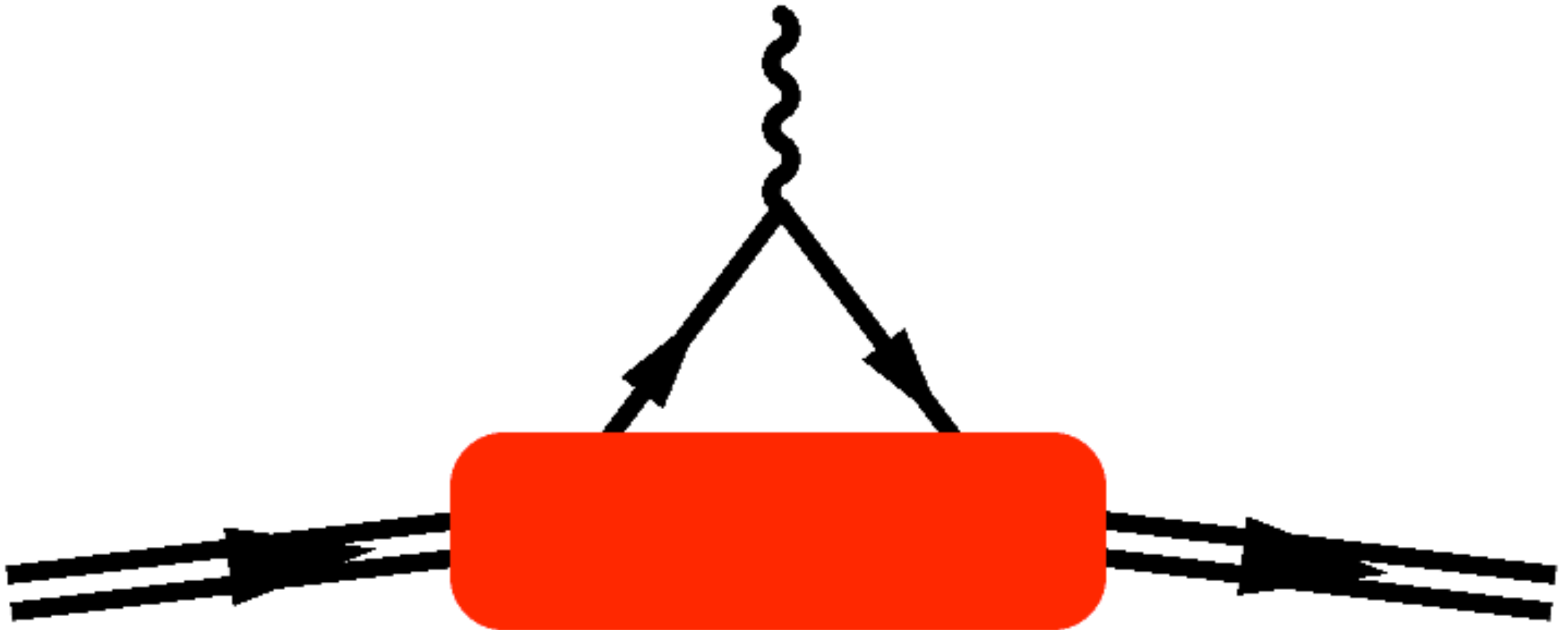
Forward limit



⇒ Recover forward parton distributions

Local limit

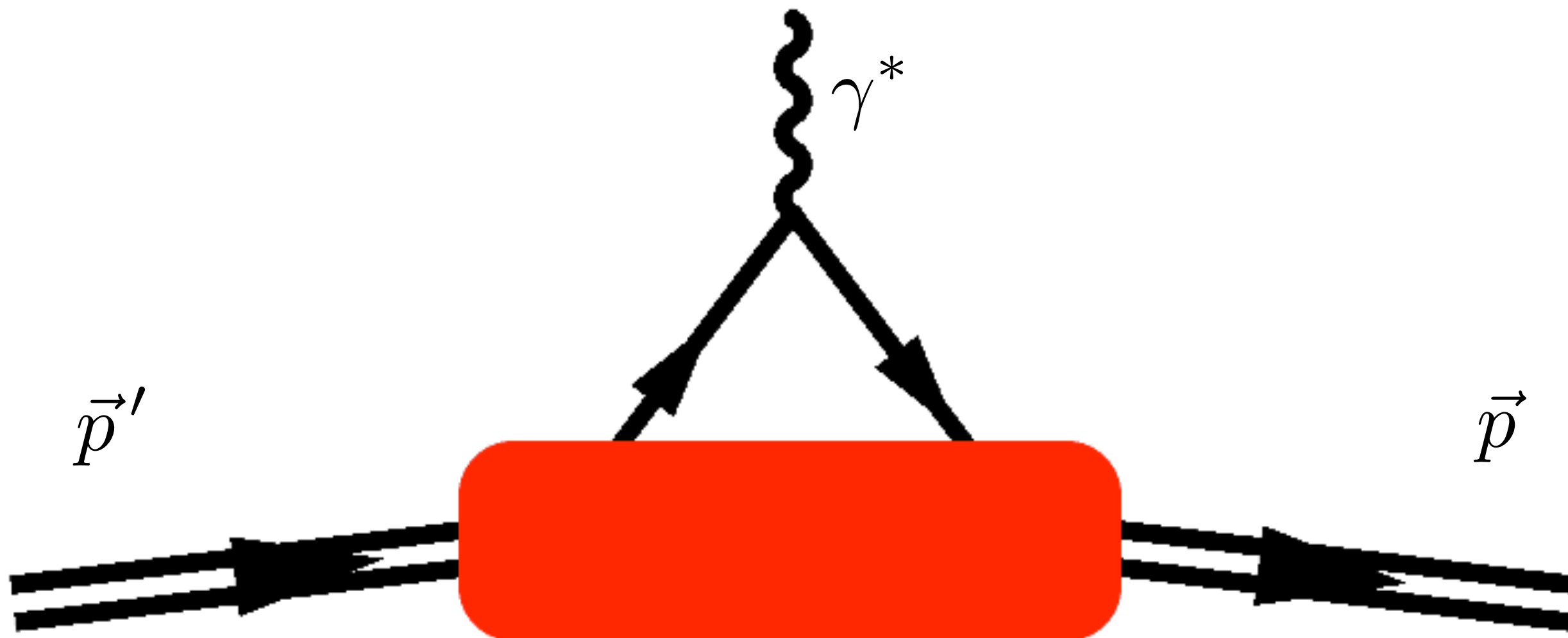
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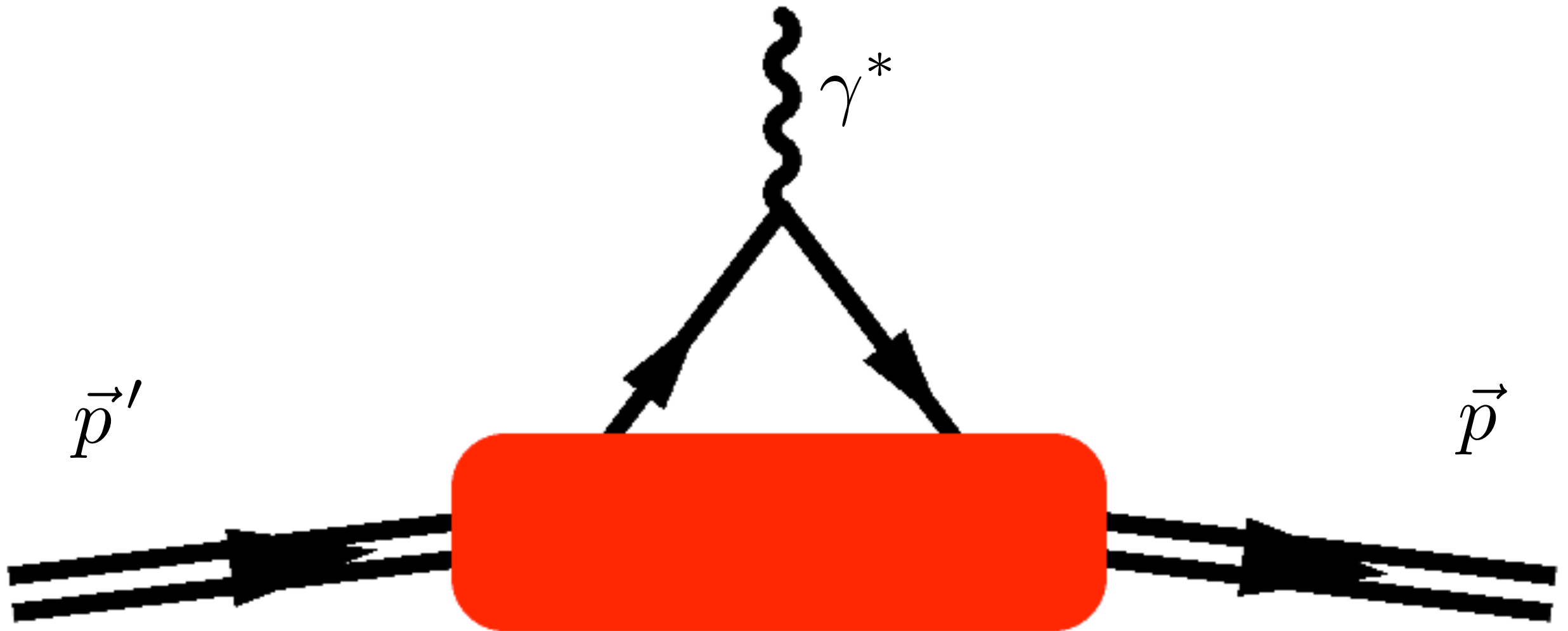
# Local limit

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# Local limit



$\Rightarrow$  Recover form factors



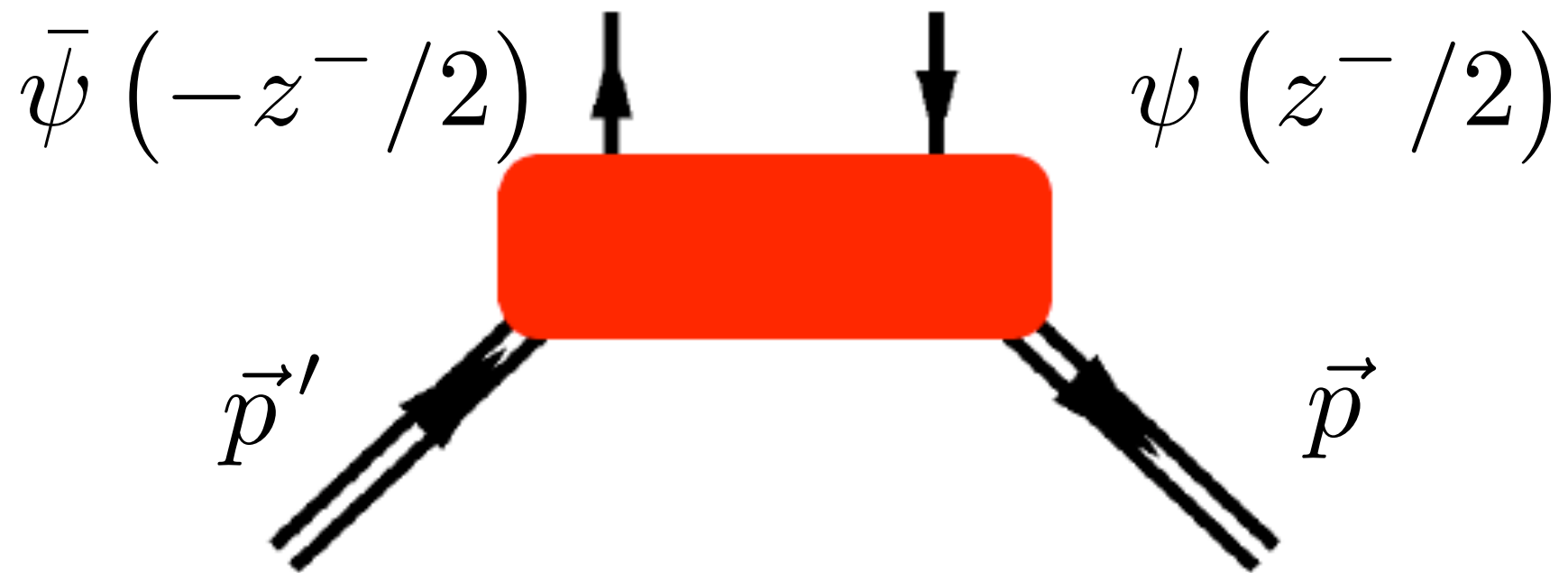
# Generalized parton distributions

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- ❖ Describe both exclusive and inclusive processes
- ❖ Pioneering paper:  
D.Müller et.al., Fortschr.Phys. 42, 101 (1994)
- ❖ Became really popular with  
Phys.Rev.Lett. 78, 610 (1997)  
Phys.Rev. D56, 5524 (1997)



# QCD matrix element

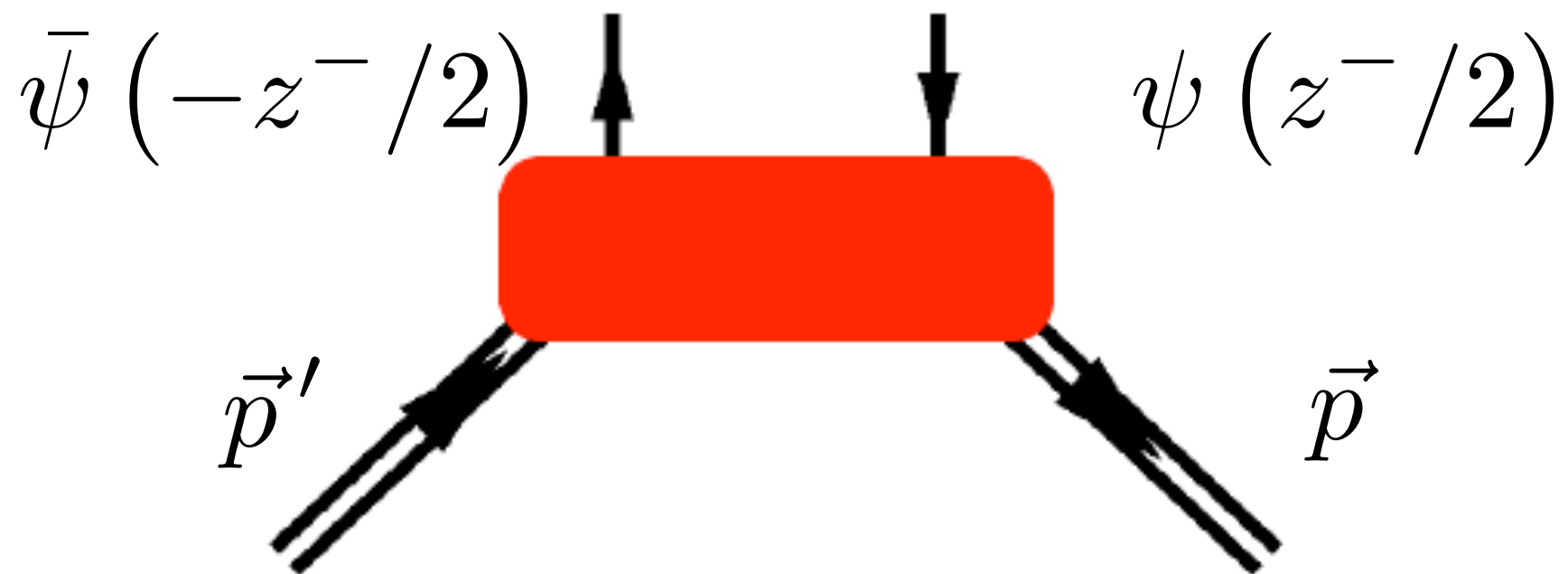


$$\bar{p}^+ \int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle p' | \bar{\psi}(-\frac{1}{2}z^-) \mathcal{A} \gamma^+ \psi(\frac{1}{2}z^-) | p \rangle$$

$$= H(x, \xi, t) \langle \langle \gamma^+ \rangle \rangle + E(x, \xi, t) \frac{i}{2m} \langle \langle \sigma^{+\alpha} \Delta_\alpha \rangle \rangle$$



# QCD matrix element

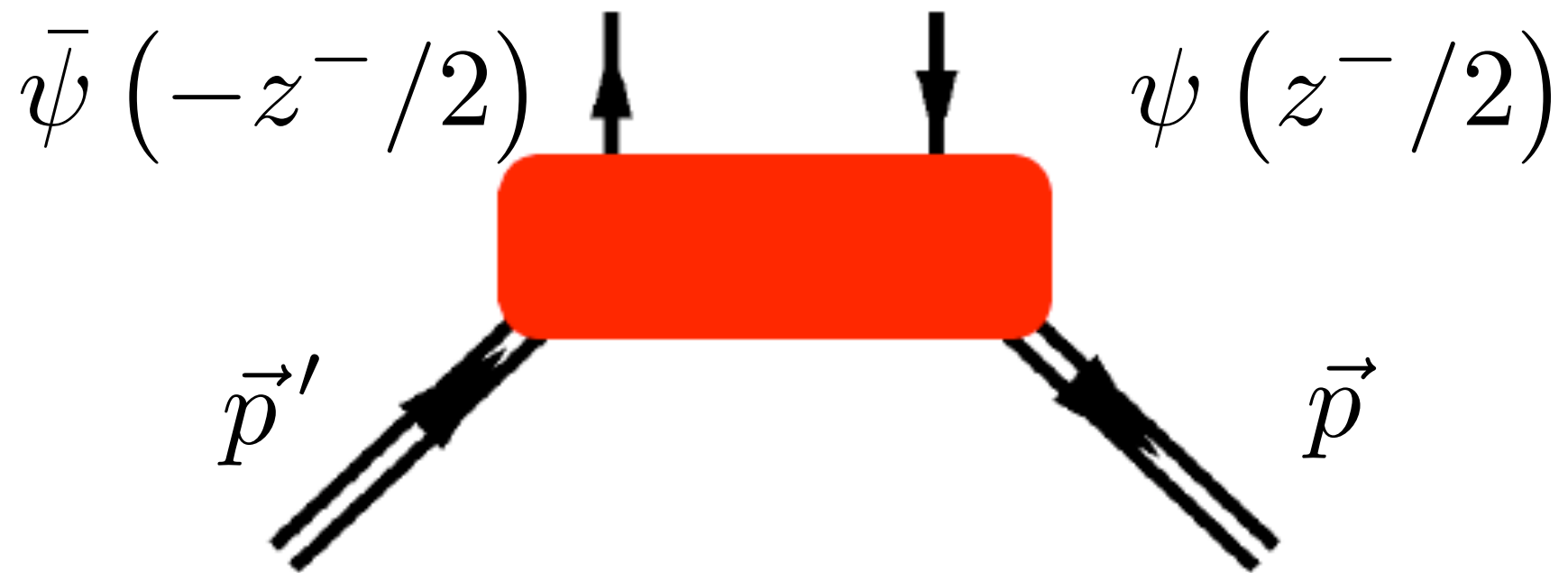


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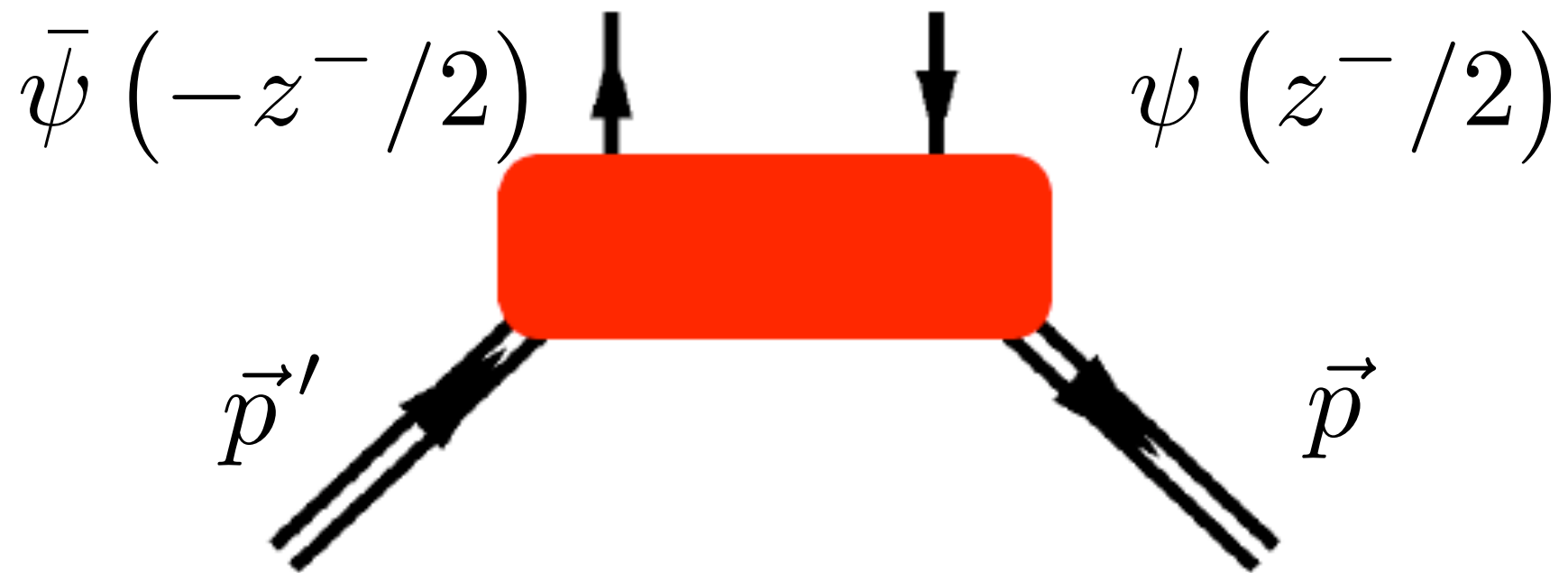


$$\bar{p}^+ \int \frac{dz^-}{2\pi} e^{i\bar{p}^+ z^-} \langle p' | \bar{\psi}(-z^-/2) \gamma_5 \gamma^+ \psi(z^-/2) | p \rangle$$

$$= \tilde{H}(x, \xi, t) \langle\langle \gamma_5 \gamma^+ \rangle\rangle - \tilde{E}(x, \xi, t) \frac{\Delta^+}{2m} \langle\langle \gamma_5 \rangle\rangle$$



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# Collection of GPDs

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$$\langle p' | \bar{\psi} \gamma^\mu \psi | p \rangle \Rightarrow H(x, \xi, t) \text{ \& } E(x, \xi, t)$$

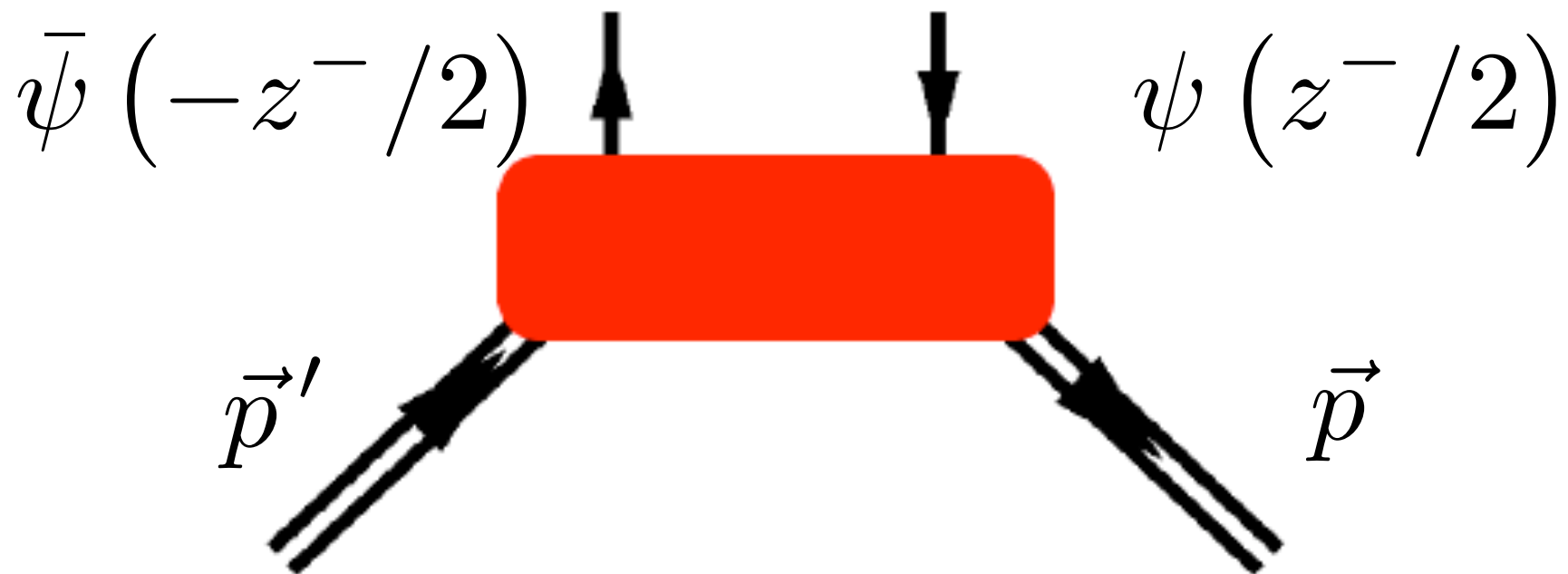
$$\langle p' | \bar{\psi} \gamma_5 \gamma^\mu \psi | p \rangle \Rightarrow \tilde{H}(x, \xi, t) \text{ \& } \tilde{E}(x, \xi, t)$$

+ four more fermion GPDs for  
transversity

+ eight more gluon GPDs



# Interpreting GPDs



- ❖ Quark emitted and absorbed with l.m.f.  $(x+\xi)$  and  $(x-\xi)$
- ❖ Quark/antiquark pair emitted with l.m.f.  $(\xi+x)$  and  $(\xi-x)$



# Phenomenology

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- ❖ Deeply virtual-wide-angle Compton scattering
- ❖ Exclusive meson production
- ❖ Form factors, Deep-inelastic scattering
- ❖ Three-dim. hadron structure
- ❖ Angular momentum sum rule

Recent review: Phys.Rept. 388:41-277 (2003)



# Phenomenology

Where has the *lattice* contributed?

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Phys.Rev.Lett.96:052001(2006)  
PoS LAT2007:161(2007)  
arXiv:0709.3370 (Baryons07)



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# Phenomenology

Where has the *lattice* contributed?

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Phys.Rev.Lett.92:042002(2004)  
Phys.Rev.D77:094502(2008)

❖ Three-dim. hadron structure

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# Extracting GPDs

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- ❖  $x$ -dependence: as Mellin-moments
- ❖  $\xi$ -dependence: analytically
- ❖  $t$ -dependence: via external momenta
- ❖ On the lattice: model-independent results
- ❖ Experimentally: difficult to extract functions of three variables  
⇒ Combine lattice, models, and experiment



# Lattice simulations & their challenges



# Lattice QCD for predictions

---

- ❖ **Goal:** Qualitative & quantitative results from first principles
  - $\Rightarrow$  Comparison of theory  $\Leftrightarrow$  experiment
  - $\Rightarrow$  Credibility for predictions
- ❖ Vary parameters, e.g.  $m_q$ ,  $N_c$ ,  $N_f$
- ❖ Test models of QCD
  - $\Rightarrow$  Insight into how QCD works



# Regimes of quark masses

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- ❖ **Heavy quark regime:**

confinement, flux tubes, adiabatic potential

- ❖ **Light quark regime:**

chiral symmetry breaking, instantons, chiral perturbation theory



# Regimes of quark masses

---

Major source of  
uncertainty:

Quark masses!

atic potential

antons, chiral



# Fermion discretizations

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- ❖ (Improved) Wilson fermions
- ❖ (Improved) staggered fermions
- ❖ Twisted mass Wilson fermions
- ❖ Ginsparg-Wilson fermions
  - ❖ Domain-wall
  - ❖ Overlap



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*Practically **very** important question!*



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# Fermion discretizations

Simple,  
well understood



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Cheap



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Small mps  
problematic



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Cheap

Small mps  
problematic

Chiral,  $O(a^2)$ ,  
**but** expensive



# The challenge

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- ❖ Light valence fermions possible today
- ❖ Light sea quarks remain major issue

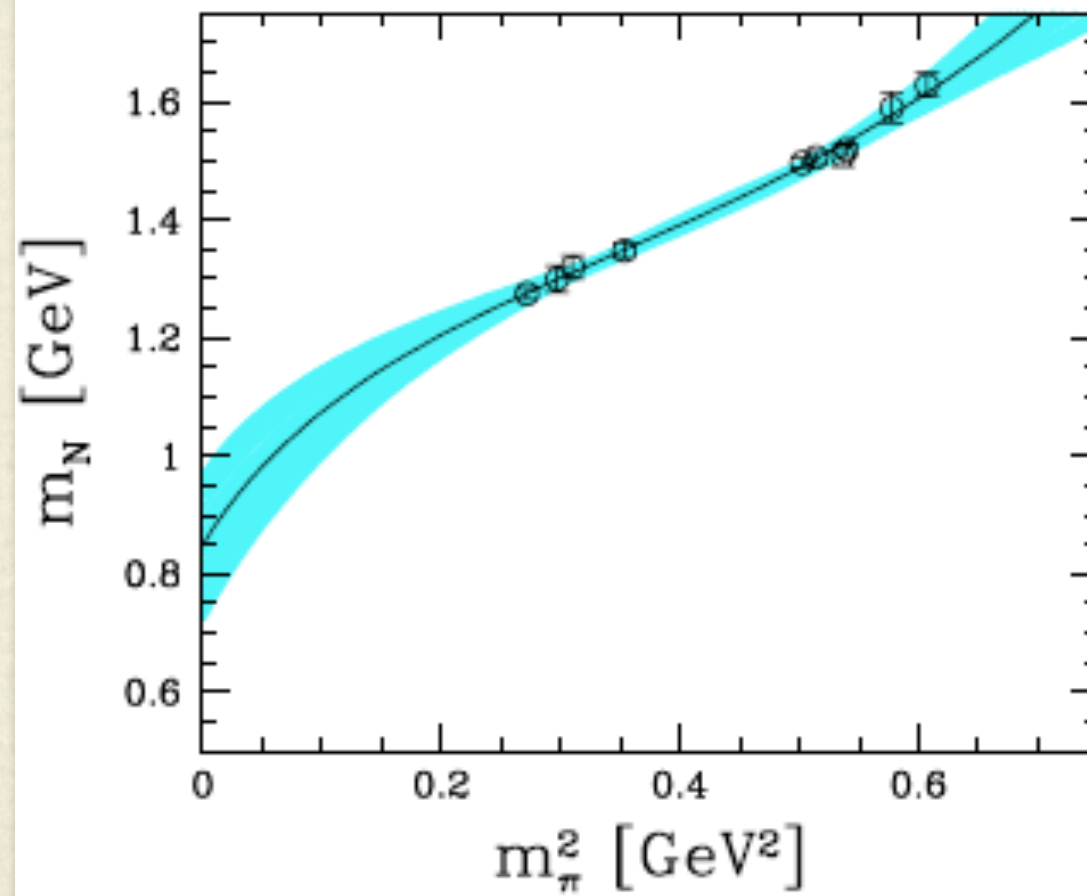
## Possible solutions:

- ❖ Hybrid calculations
- ❖ Full GW: Either full DWF or Overlap
- ❖ Full Wilson-type fermions  
(Clover/Twisted mass)

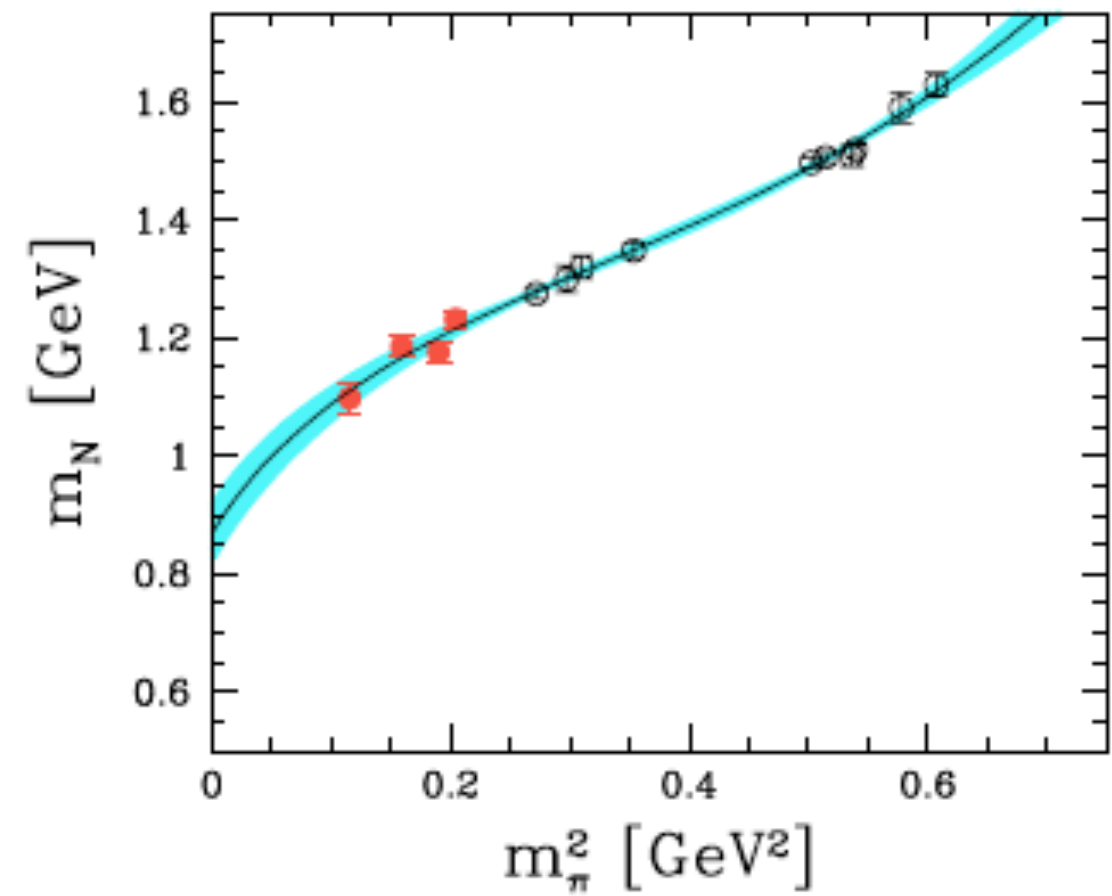


# Nucleon mass (QCDSF)

< 2006



2007





# GPDs on the lattice

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Original definition: *non-local* matrix elements

On the lattice: we can only do *local*  
matrix elements

Similar to PDs use the light-cone OPE

$$\begin{aligned} \langle p | \mathcal{O}_q^{\{\mu_1 \dots \mu_n\}} | p \rangle \\ \mathcal{O}_q^{\{\mu_1 \dots \mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} i \mathcal{D}^{\mu_2} \dots i \mathcal{D}^{\mu_n\}} \psi_q \end{aligned}$$



# Generalized form factors

Expand generalized local currents  
in

Lorentz-invariant functions

$$\langle p' | \mathcal{O}_q^{\{\mu_1 \dots \mu_n\}} | p \rangle$$

$\Rightarrow$  Generalized form factors

$$\begin{aligned} \text{e.g.: } \langle p' | \mathcal{O}^{\{\mu\nu\}} | p \rangle &= A_{20}(t) \bar{p}^{\{\mu} \bar{u}(p') \gamma^{\nu\}} u(p) \\ &+ B_{20}(t) \frac{i}{2m} \bar{p}^{\{\mu} \bar{u}(p') \sigma^{\nu\}}{}^{\alpha} u(p) \Delta_{\alpha} \\ &+ C_2(t) \frac{1}{m} \Delta^{\{\mu} \Delta^{\nu\}} \end{aligned}$$



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# GFFs as Mellin transforms

---

Explicitly for  $\xi=0$ :

$$\int dx x^{n-1} H(x, 0, t) = A_{n0}(t)$$
$$\int dx x^{n-1} E(x, 0, t) = B_{n0}(t)$$

Higher moments are always  
polynomials in  $(2\xi)^2$ :

$$\int dx x H(x, \xi, t) = (2\xi)^2 C_2(t) + A_{20}(t)$$
$$\int dx x E(x, \xi, t) = (2\xi)^2 C_2(t) + B_{20}(t)$$



# LHPC: Hybrid calculations

---

- ❖ Hybrid approach: unitarity & square root?  
Lattice artifacts?
- ❖ Achievement: 5 quark masses, full QCD down to  $m_\pi=354$  MeV
- ❖ Lattice sizes  $(2.5\text{fm})^3$  and  $(3.5\text{fm})^3$



# Achievements



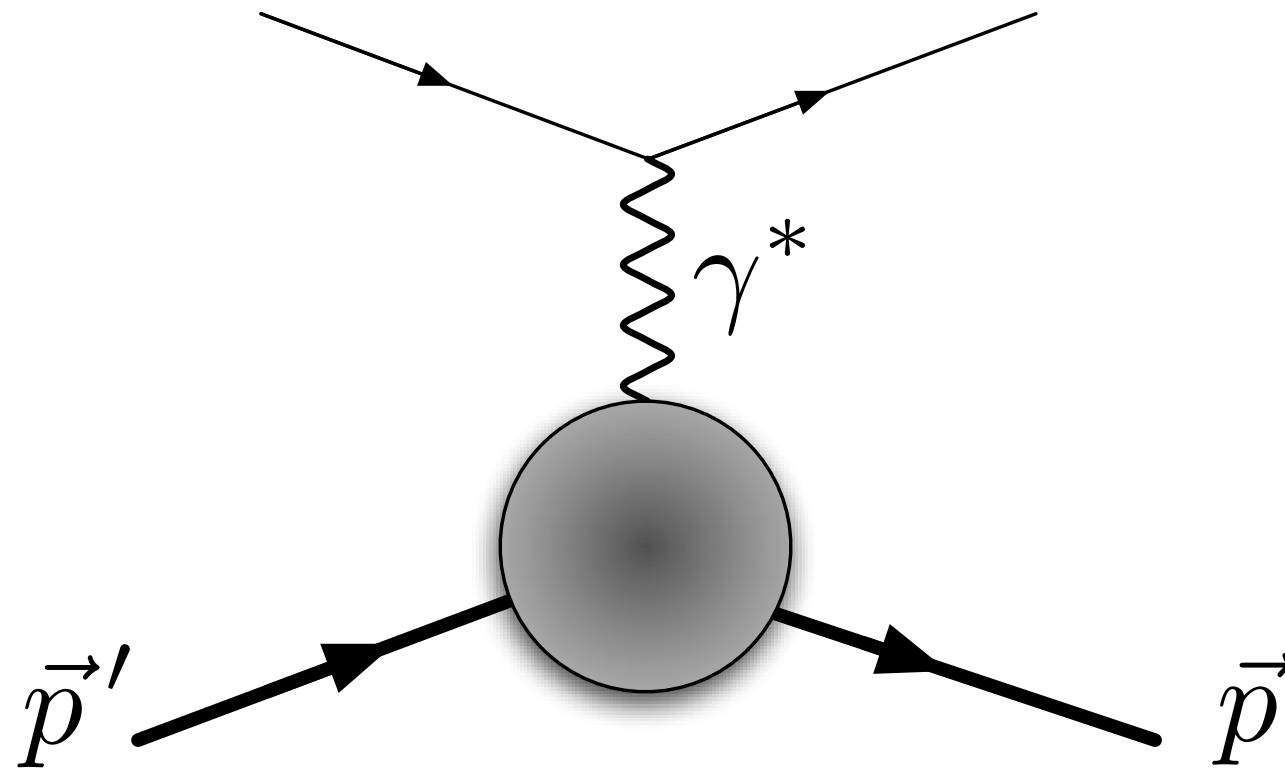
# Key calculations

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- ❖ **Three key calculations** from the lattice:
- ❖  $N \rightarrow N$  &  $N \rightarrow \Delta$  transition form factors
- ❖ GPDs - The nucleon spin & transverse structure
- ❖ Nucleon axial coupling  $g_A$  :  
First quantitative result!



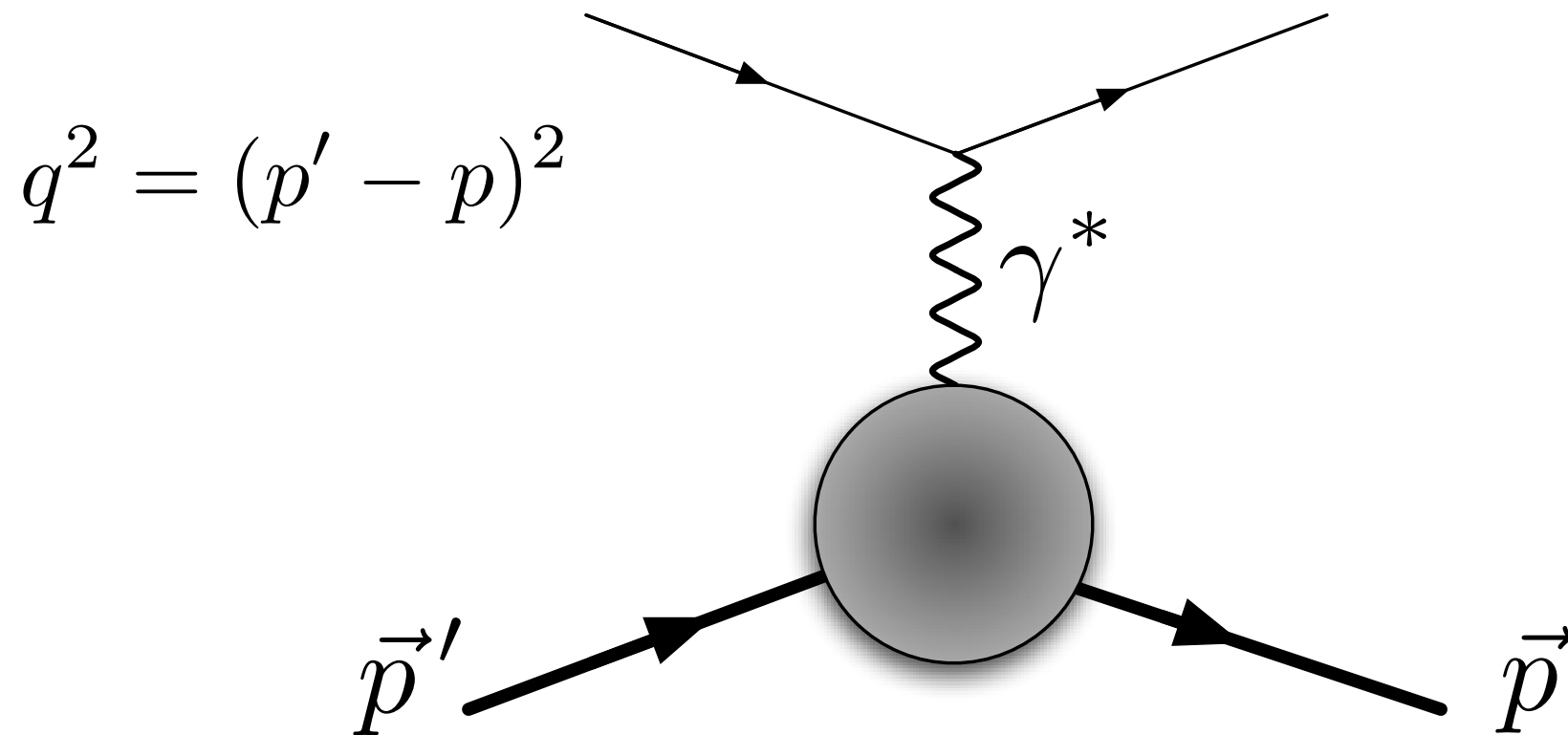
# Electromagnetic form factors



$$\langle p' | J_\mu | p \rangle = \bar{u}(p') \left[ \gamma_\mu F_1(q^2) + \frac{i q^\alpha}{2m_N} \sigma_{\alpha\mu} F_2(q^2) \right] u(p)$$



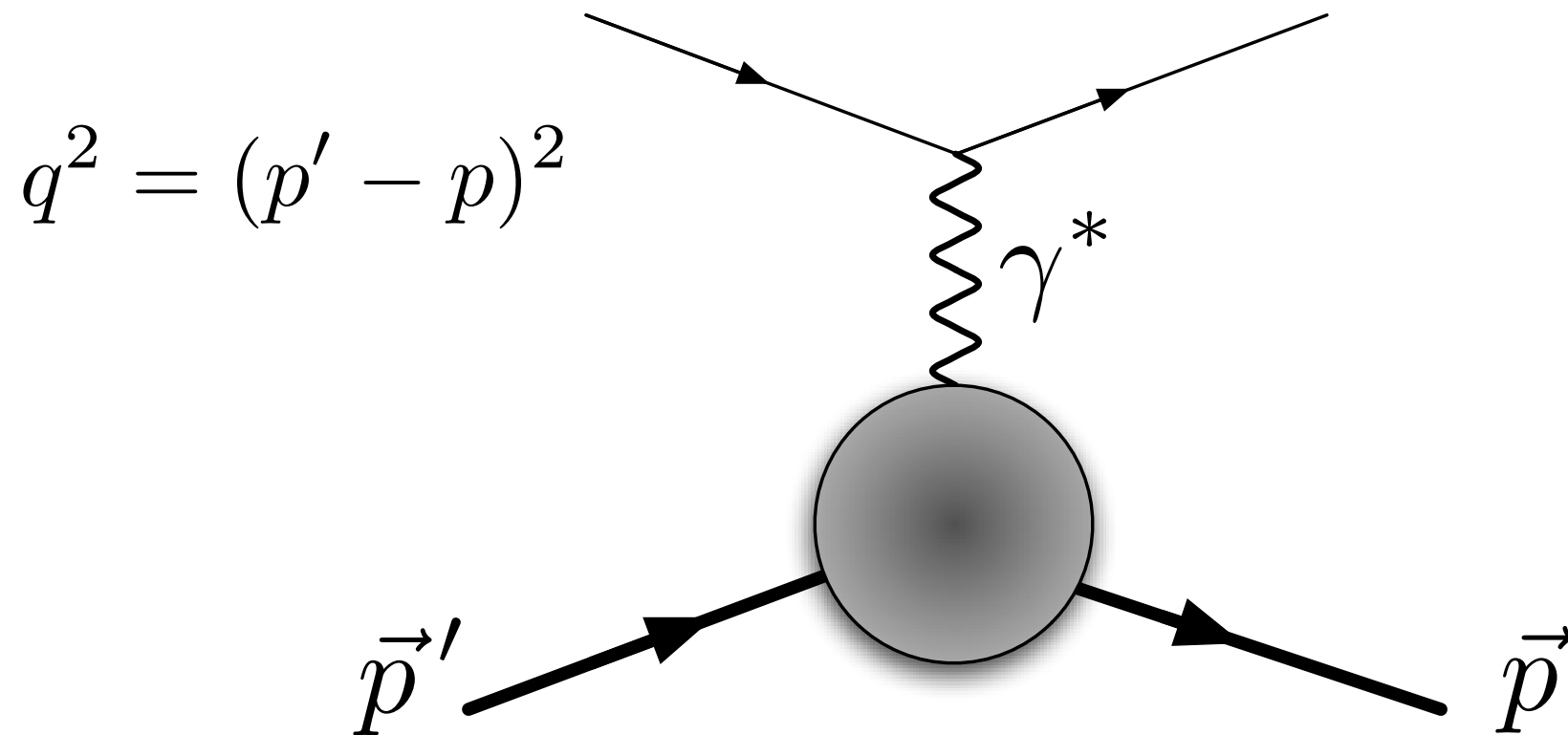
# Electromagnetic form factors



$$\begin{aligned} \langle p' | J_\mu | p \rangle &= \bar{u}(p') \left[ \gamma_\mu F_1(q^2) \right. \\ &\quad \left. + \frac{i q^\alpha}{2m_N} \sigma_{\alpha\mu} F_2(q^2) \right] u(p) \end{aligned}$$



# Electromagnetic form factors



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# Experiment

---

- ❖ Recent reviews: Phys.Rev. C71:055202 (2005)  
J.Phys. G34:S23-S25(2007)
- ❖ Two techniques:
  - ❖ Rosenbluth: elastic scattering  $\sigma(e + p \rightarrow e + p)$
  - ❖ Spin-transfer:  $\vec{e} + \vec{p} \rightarrow e + p$   
 $\vec{e} + p \rightarrow e + \vec{p}$
- ❖ Around for several decades, but  
*surprising new insights* recently



# Form factors

---

$t = Q^2$  dependence

$$\frac{tF_2(t)}{F_1(t)}$$

$\propto \text{const.}$

Naive quark  
counting rules

$$\frac{\sqrt{t}F_2(t)}{F_1(t)}$$

$\propto \text{const.}$

JLab spin transfer  
experiment

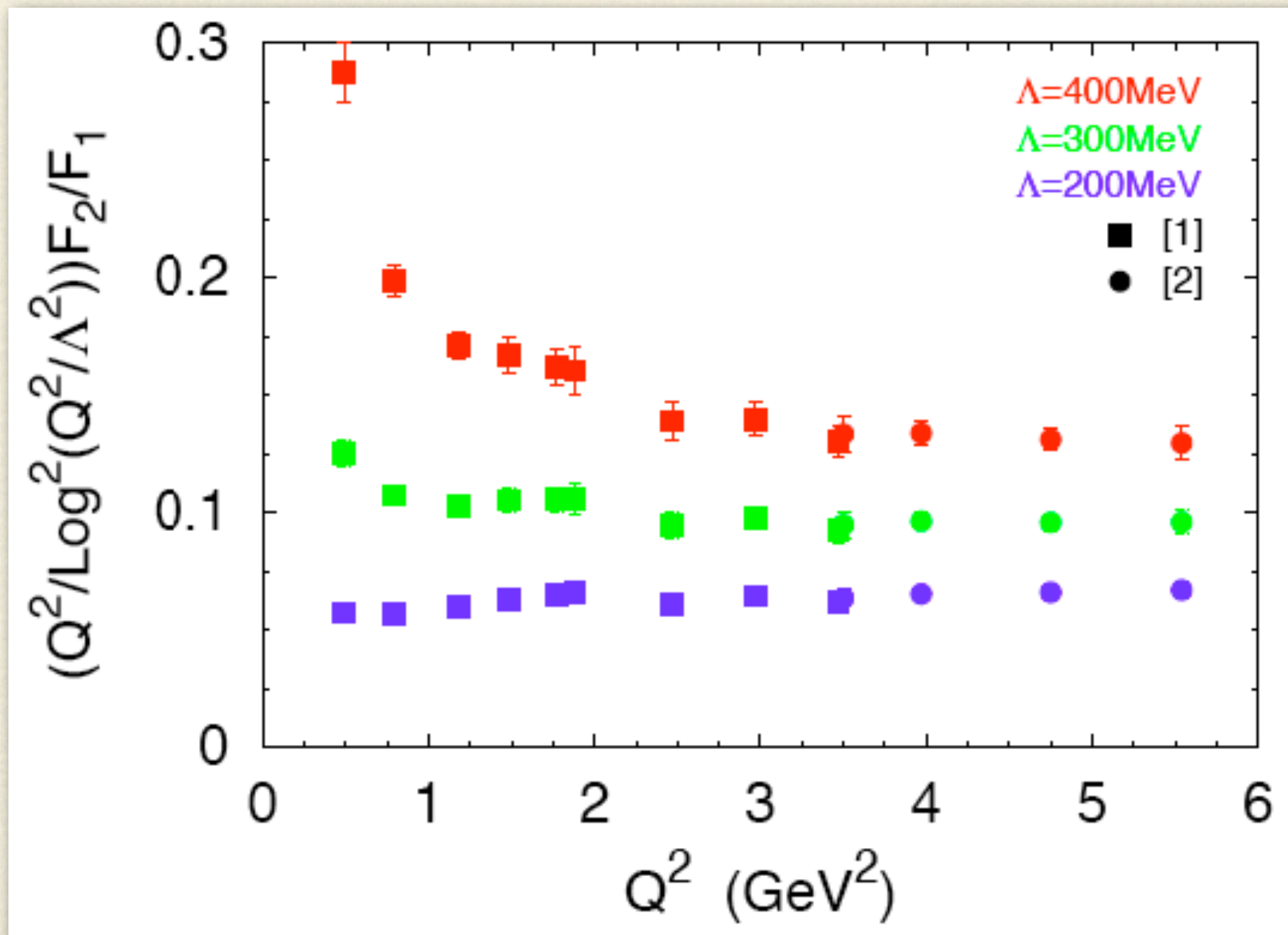
$$\frac{tF_2(t)}{\log^2(t)F_1(t)}$$

$\propto \text{const.}$

Phys.Rev.Lett.  
91:092003 (2003)

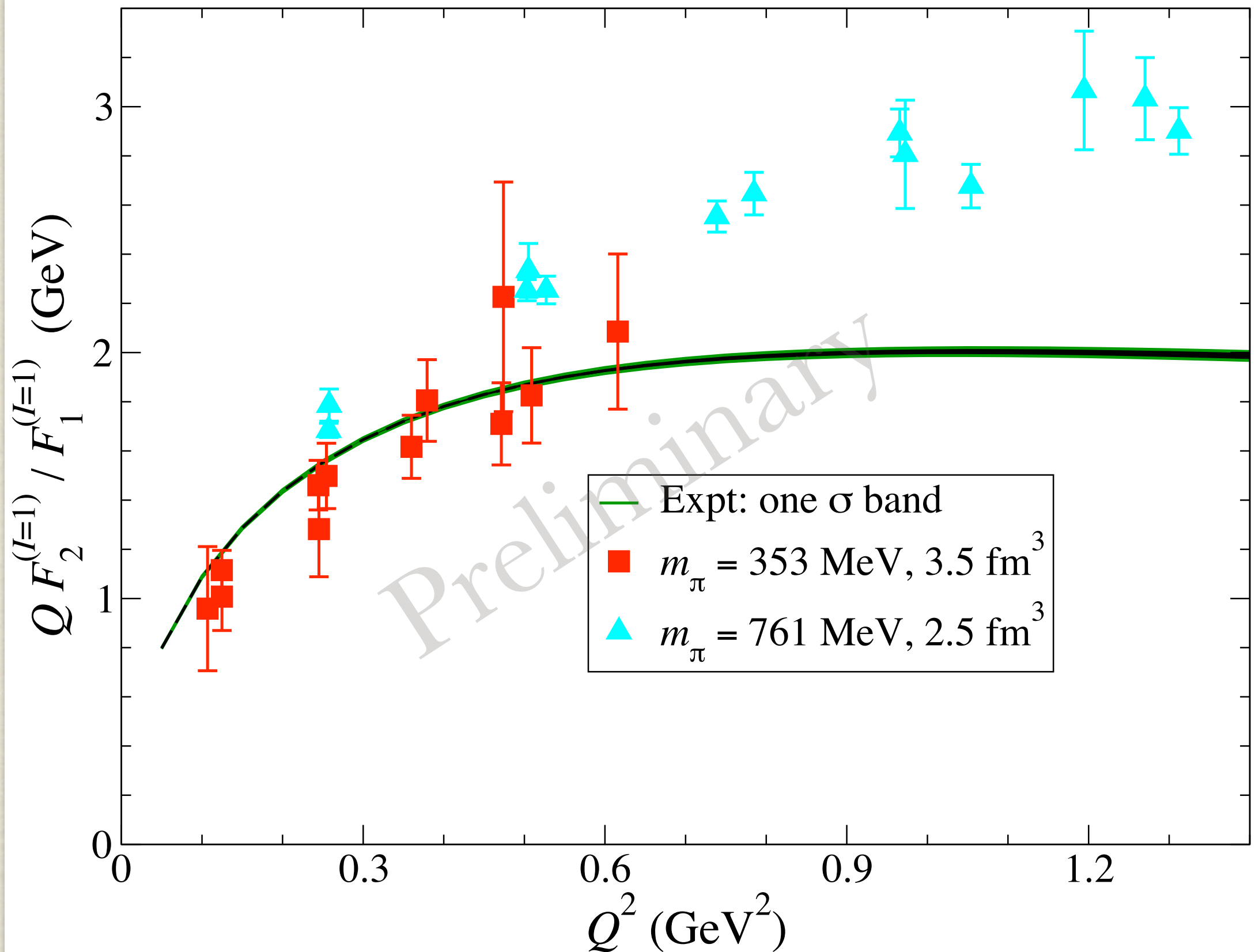


# Form factors in Nature



Phys.Rev.Lett. 91:092003 (2003)

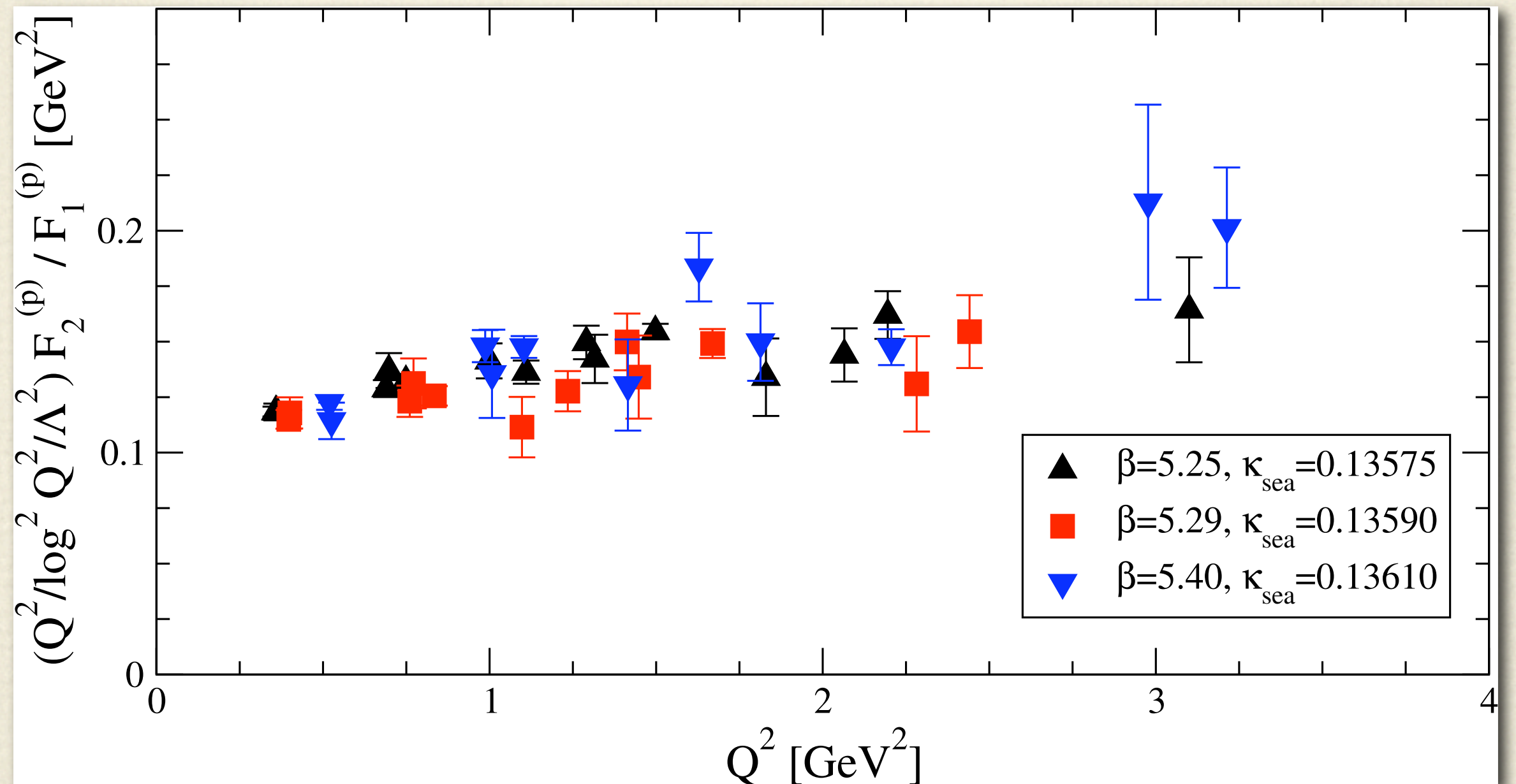




LHPC collaboration, in preparation



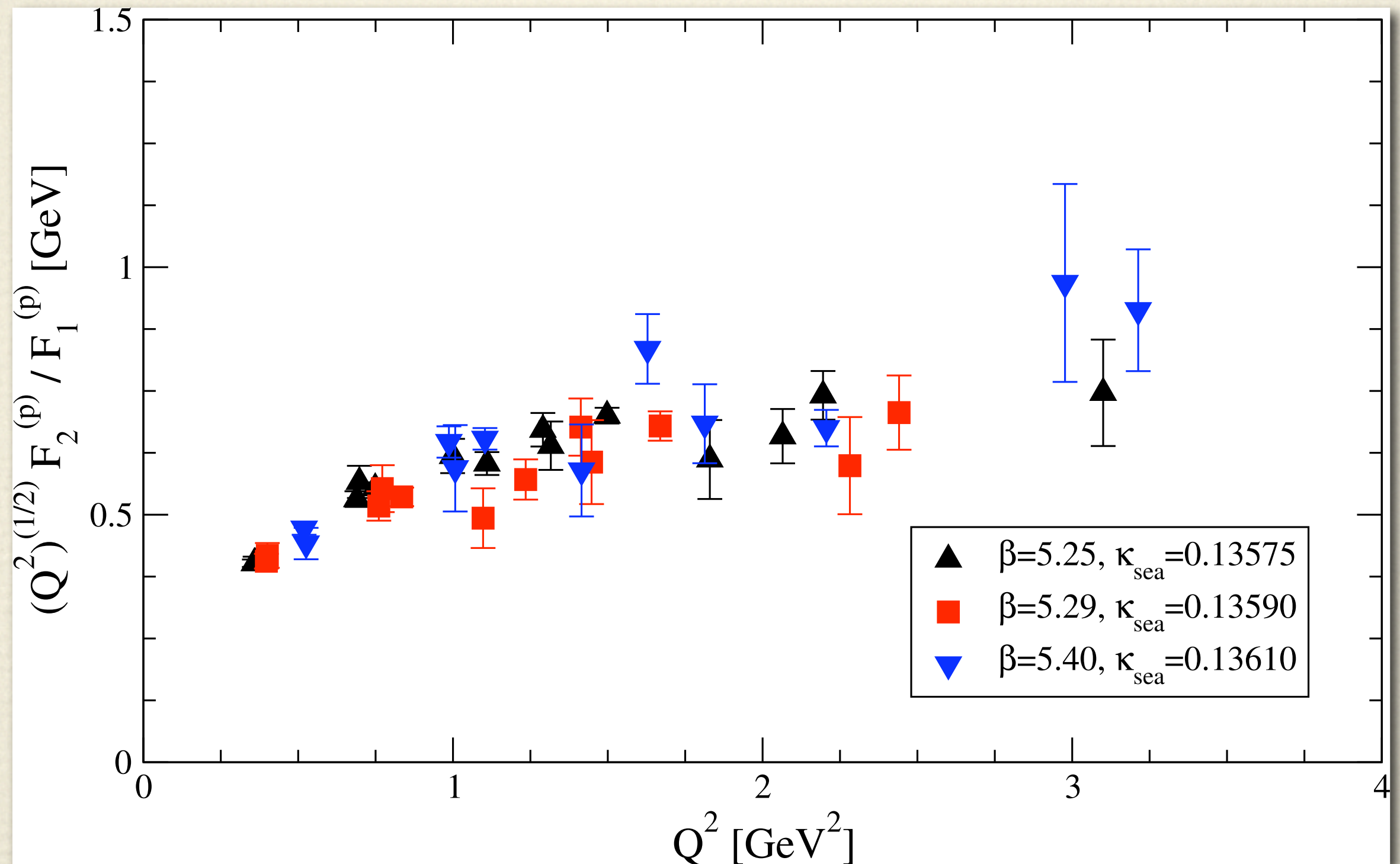
# Form factor scaling



$m_{\text{PS}} \approx 600 \text{ MeV}, a = 0.070 \dots 0.084 \text{ fm}$

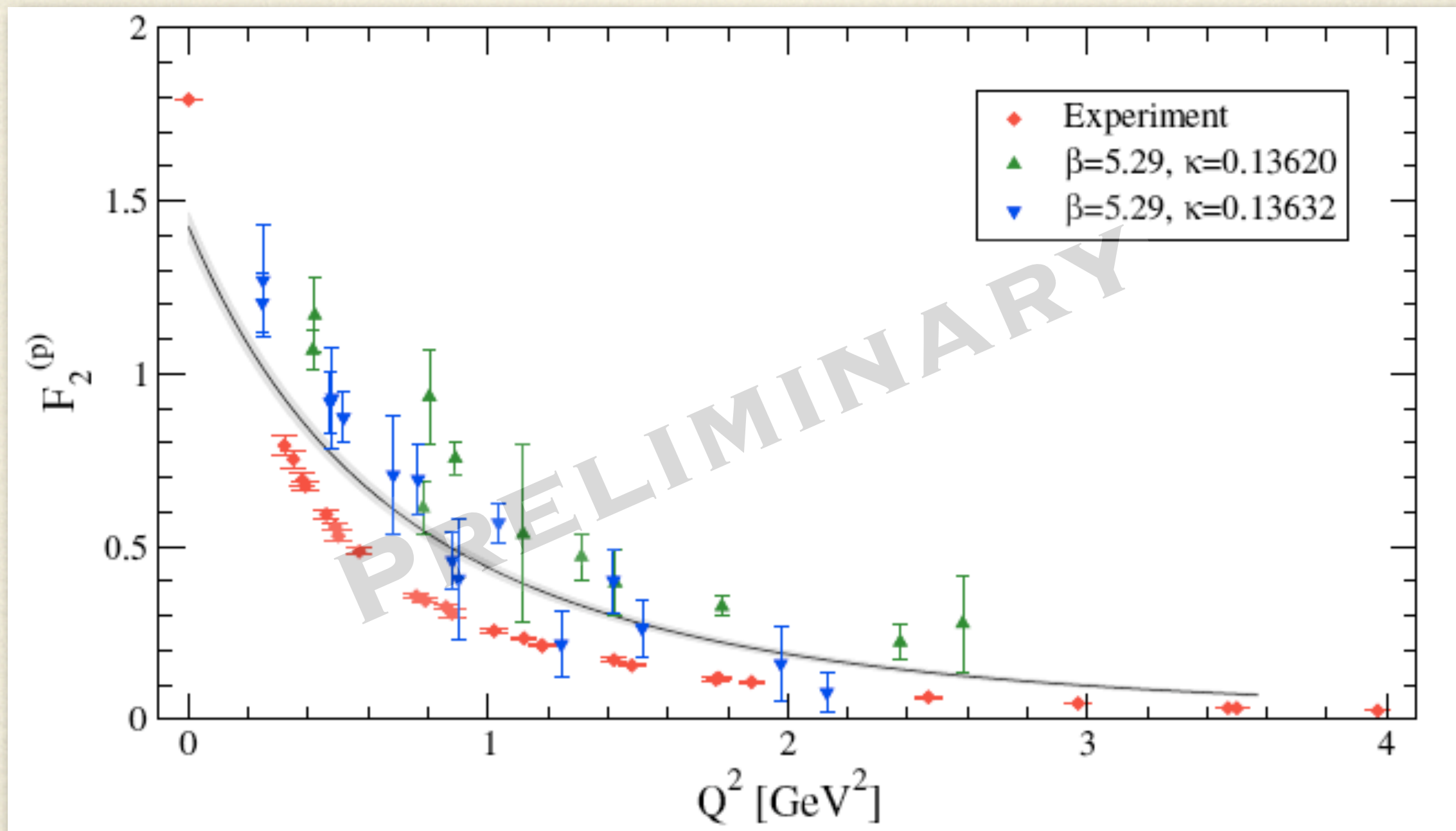


# Form factor scaling





# $F_2^p$ Lattice vs Expmt.





# Large distance behavior

---

❖ Charge radii:

$$F_i(q^2) = F_i(0) \left( 1 + \frac{1}{6} r_i^2 q^2 + \mathcal{O}(q^4) \right)$$

❖ Magnetic moment:

$$\mu \equiv F_1(0) + F_2(0) \equiv 1 + \kappa$$



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# Large distance behavior

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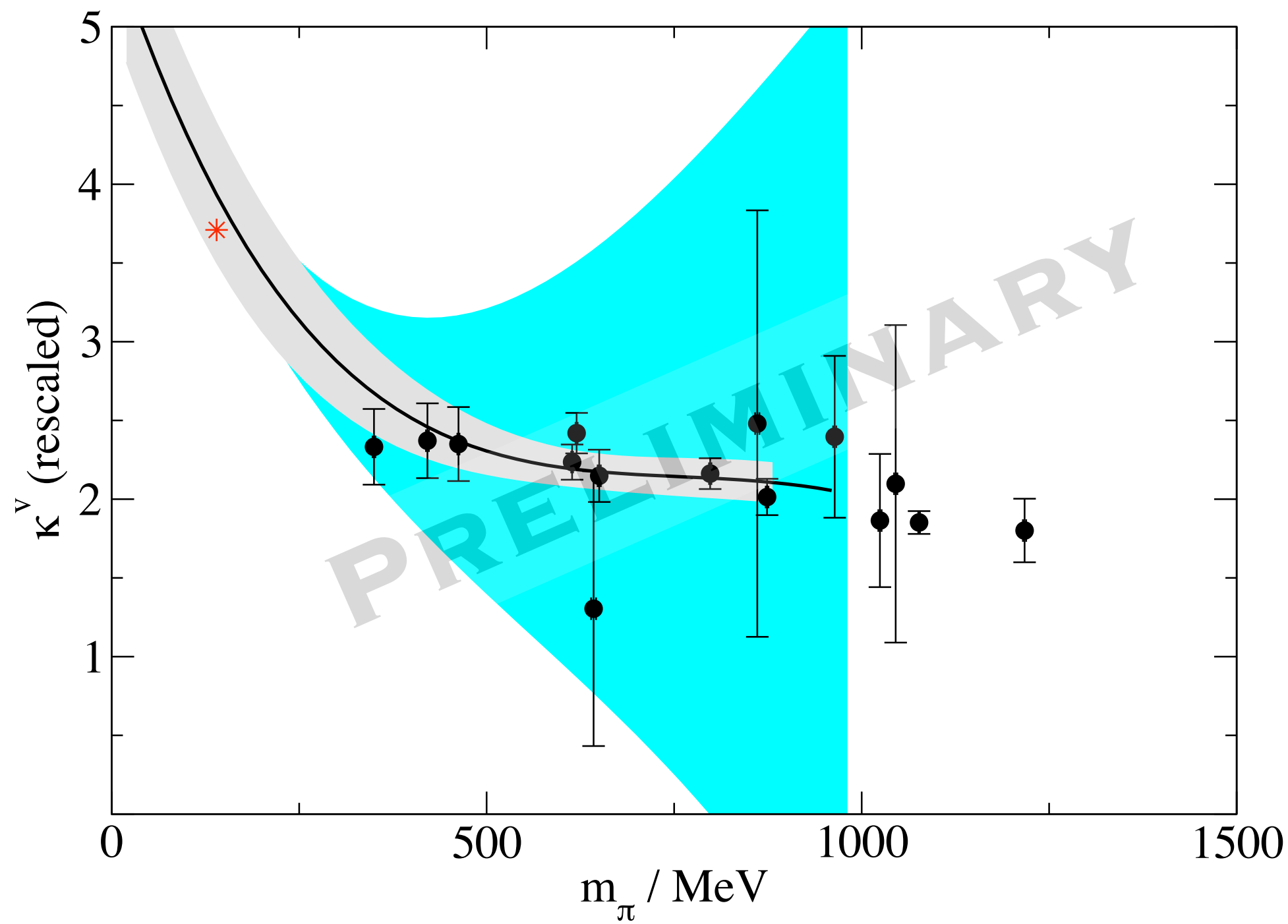
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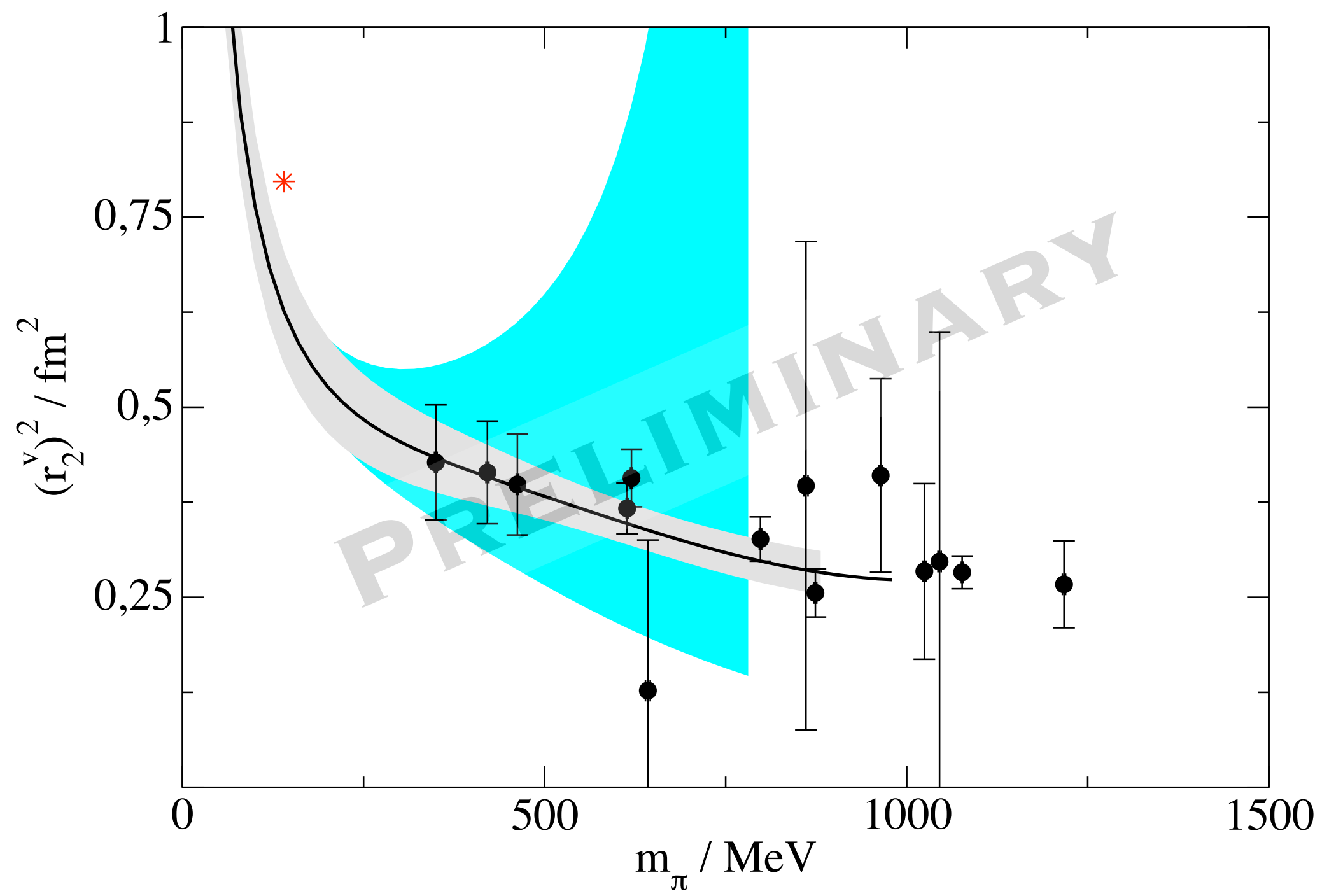
$$\mu \equiv F_1(0) + F_2(0) \equiv 1 + \kappa$$





Fitting formula from Phys.Rev. D71:034508 (2005)







# Hadron deformation

---

## ❖ How to measure?

- ❖ Quadrupole moment of ground state  
⇒ vanishes for spin-1/2 system
- ❖ Excitation spectrum of the system  
⇒ exceedingly complicated,  
broad & overlapping resonances
- ❖ Radiation of emitted de-excitation  
radiation  
⇒ viable from  $\Delta^+(1232)$



# Transition form factors

---

Electromagnetic current (local operator):

$$\langle \text{state} | \bar{\psi} \gamma^\mu \psi | \text{state} \rangle$$

Expand m.e. in terms of scalar functions  
 (“(generalized) form factors”):

$$\langle n(p') | \bar{\psi} \gamma^\mu \psi | n(p) \rangle = \mathcal{P}_1^\mu F_1(t) + \mathcal{P}_2^\mu F_2(t)$$

Transition form factors:

$$\langle \Delta^\sigma(p') | \bar{\psi} \gamma^\mu \psi | n(p) \rangle = \mathcal{A}_1^{\sigma\mu} \mathcal{G}_{M1}(t) + \mathcal{A}_2^{\sigma\mu} \mathcal{G}_{E2}(t) + \mathcal{A}_3^{\sigma\mu} \mathcal{G}_{C2}(t)$$



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$$\langle n(p') | \bar{\psi} \gamma^\mu \psi | n(p) \rangle = \mathcal{P}_1^\mu F_1(t) + \mathcal{P}_2^\mu F_2(t)$$

Transition form factors:

$$\langle \Delta^\sigma(p') | \bar{\psi} \gamma^\mu \psi | n(p) \rangle = \mathcal{A}_1^{\sigma\mu} \mathcal{G}_{M1}(t) + \mathcal{A}_2^{\sigma\mu} \mathcal{G}_{E2}(t) + \mathcal{A}_3^{\sigma\mu} \mathcal{G}_{C2}(t)$$



# Transition form factors

---

Electromagnetic current (local operator):

$$\langle \text{state} | \bar{\psi} \gamma^\mu \psi | \text{state} \rangle$$

Expand m.e. in terms of scalar functions  
 (“(generalized) form factors”):

$$\langle n(p') | \bar{\psi} \gamma^\mu \psi | n(p) \rangle = \mathcal{P}_1^\mu F_1(t) + \mathcal{P}_2^\mu F_2(t)$$

Transition form factors:

$$\langle \Delta^\sigma(p') | \bar{\psi} \gamma^\mu \psi | n(p) \rangle = \mathcal{A}_1^{\sigma\mu} \mathcal{G}_{M1}(t) + \mathcal{A}_2^{\sigma\mu} \mathcal{G}_{E2}(t) + \mathcal{A}_3^{\sigma\mu} \mathcal{G}_{C2}(t)$$



# Observables & lattice m.e.

---

❖ Matrix element

$$\langle \Delta^\sigma(p') | \bar{\psi} \gamma^\mu \psi | n(p) \rangle = \mathcal{A}_1^{\sigma\mu} \mathcal{G}_{M1}(t) + \mathcal{A}_2^{\sigma\mu} \mathcal{G}_{E2}(t) + \mathcal{A}_3^{\sigma\mu} \mathcal{G}_{C2}(t)$$

❖ Signal for deformation:

spherical  $\Rightarrow$  M1

deformed  $\Rightarrow$  M1, E2, C2

❖ Ratios:  $R_{EM} = -\frac{\mathcal{G}_{E2}(t)}{\mathcal{G}_{M1}(t)}$ ,  $R_{SM} = -\frac{|\Delta|}{2m_\Delta} \frac{\mathcal{G}_{C2}(t)}{\mathcal{G}_{M1}(t)}$



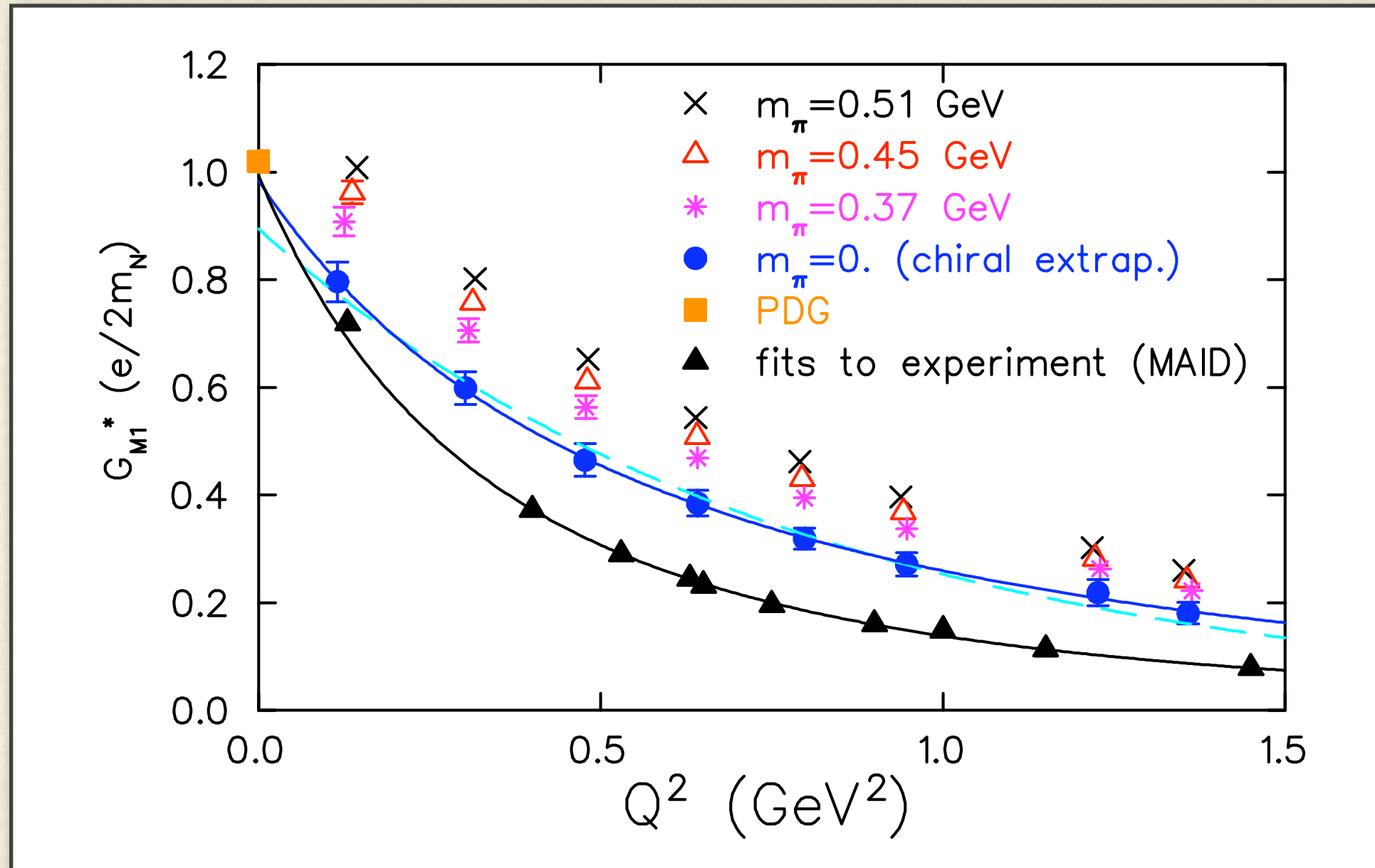
# First results

---

- ❖ M1 transition form factor: nucl-th/0012046  
quark models predict M1 30% too small
- ❖ Phys.Rev.D66:094503(2002)  $\Rightarrow R_{EM}, R_{SM}$
- ❖ Phys.Rev.Lett. 86,2963(2001)
- ❖ Phys.Rev.Lett. 88:122001(2002)
- ❖ Eur.Phys.J.A18,141(2003)  
Eur.Phys.J.A17,349(2003)



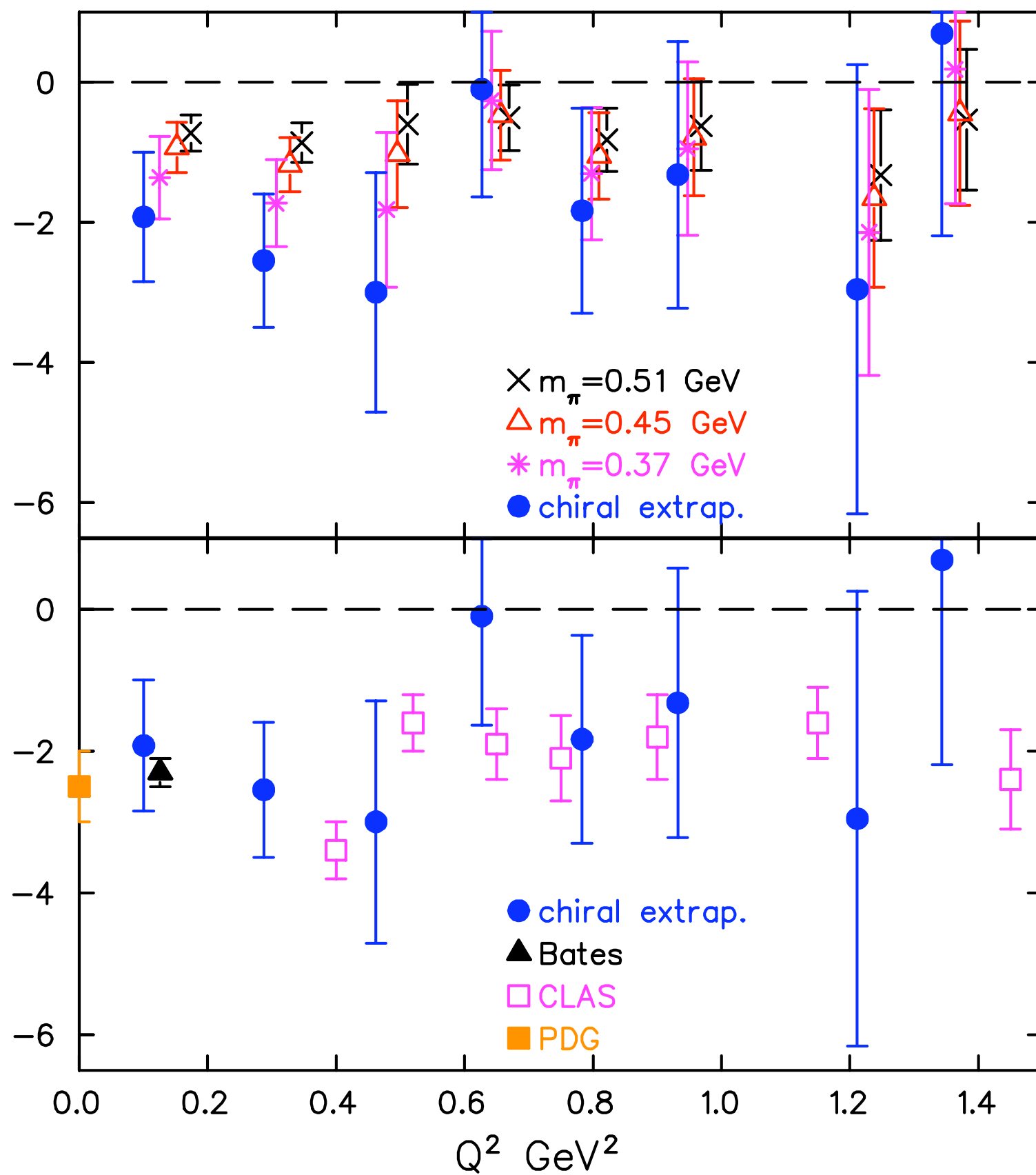
# Quenched results



Phys.Rev.Lett. 94:021601 (2005)

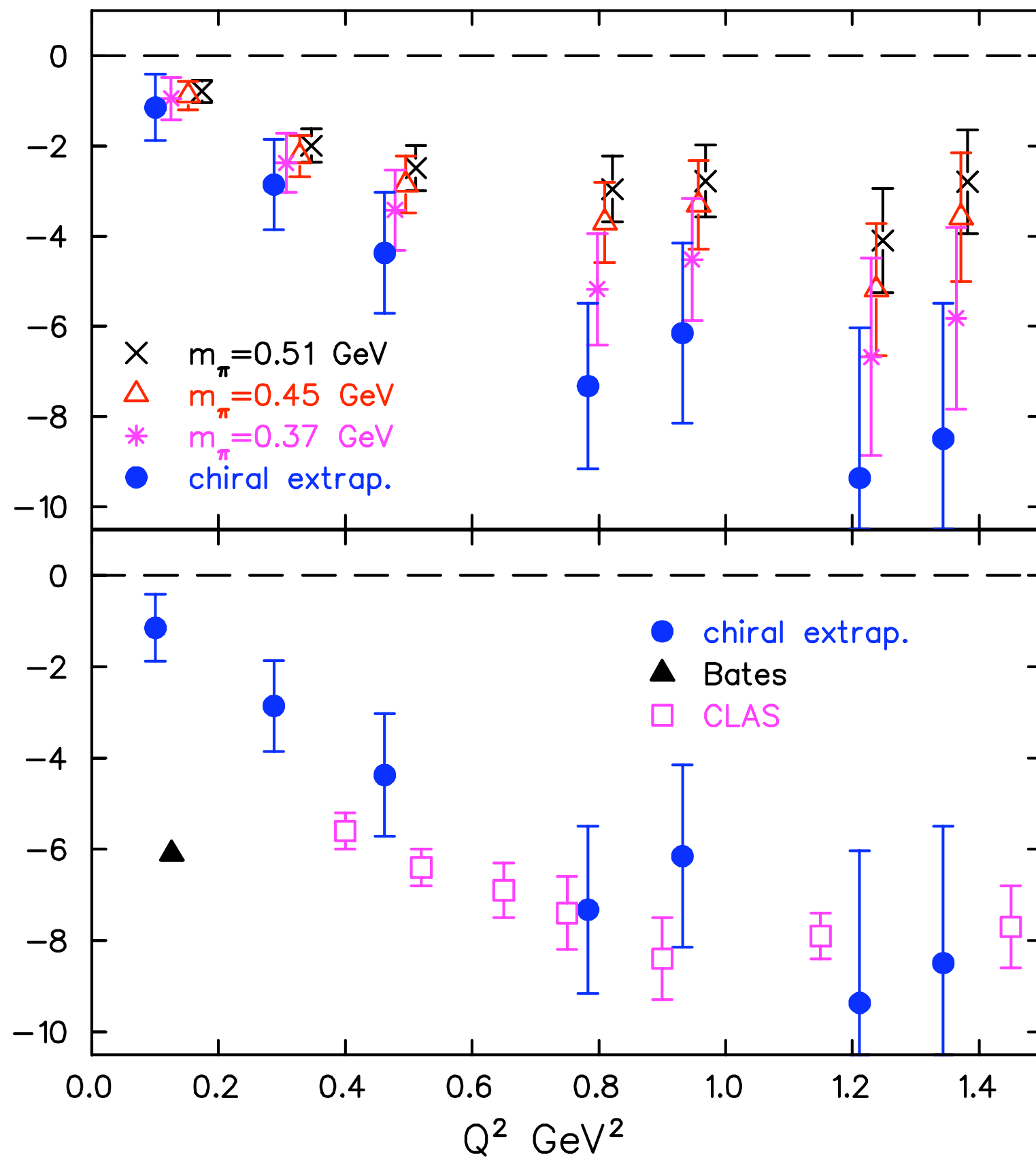


$R_{EM} (%)$



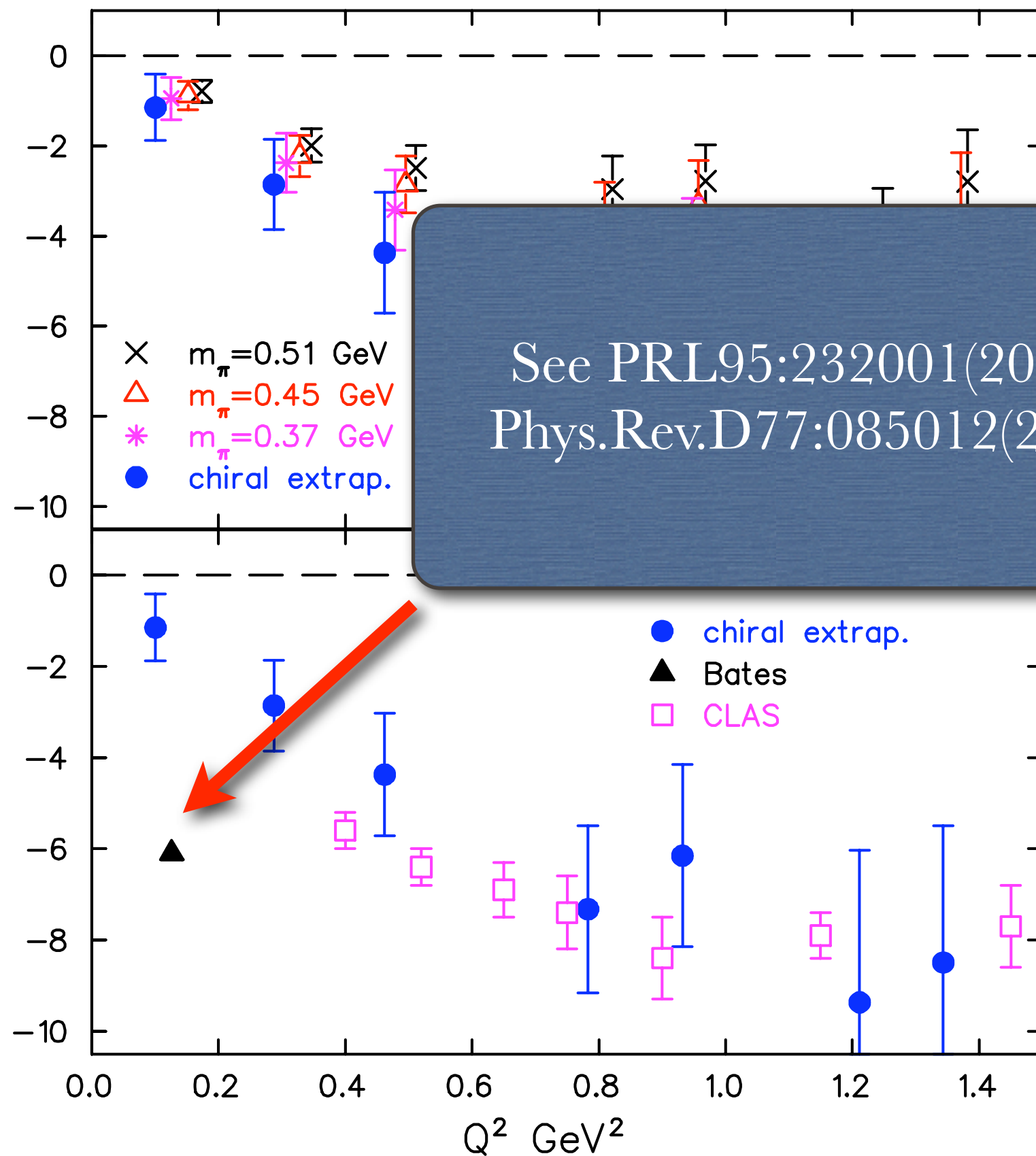


$R_{SM} \text{ (%)}$



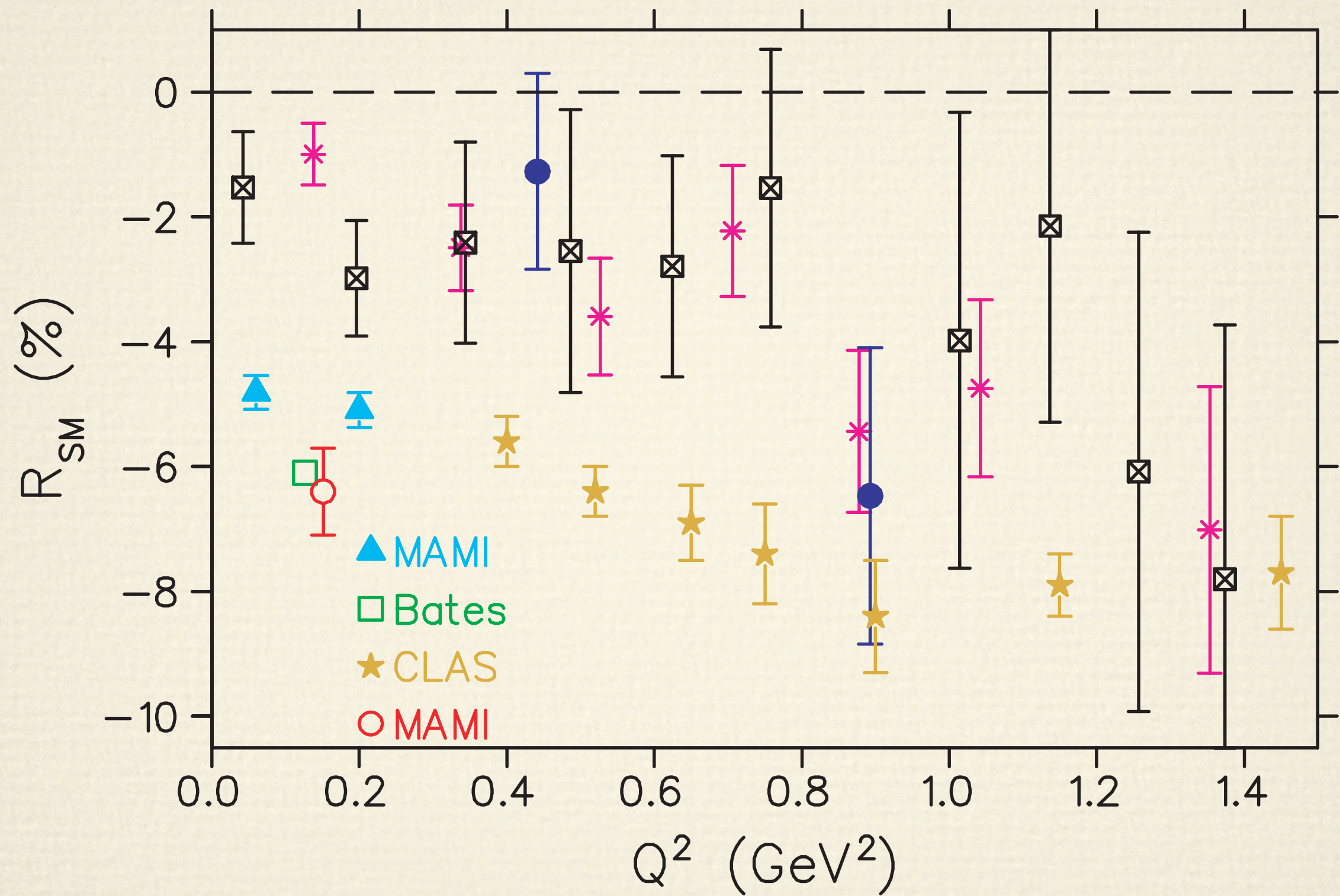


$R_{SM} (\%)$



See PRL95:232001(2005),  
Phys.Rev.D77:085012(2008)





Phys.Rev.D77:085012(2008)



# Quark spin contribution

---

Decomposition of nucleon spin:

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Sigma + L_q + J_g$$



# Quark spin contribution

---

Decomposition of nucleon spin:

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Sigma + L_q + J_g$$



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# Quark spin contribution

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Decomposition of nucleon spin:

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# Quark spin contribution

---

Decomposition of nucleon spin:

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Sigma + L_q + J_g$$

Total quark  
contribution:

$$\begin{aligned}\vec{J}_q &= \int d^3r \left[ \vec{r} \times \vec{T} \right] \\ &= \int d^3r \psi^\dagger \left[ \vec{\Sigma}/2 + \vec{r} \times (-i\vec{D}) \right] \psi\end{aligned}$$



# Energy momentum tensor

---

$$\begin{aligned}\langle p' | \bar{\psi}_q \gamma^{\{\mu} i D^{\nu\}} \psi_q | p \rangle = & A_{20}^q(t) \langle \langle \gamma^{\{\mu} \rangle \rangle \bar{p}^{\nu\}} \rangle \\ & + B_{20}^q(t) \frac{i}{2m} \langle \langle \gamma_5 \rangle \rangle \bar{p}^{\{\mu} \Delta^{\nu\}} \rangle \\ & + C_2^q(t) \frac{1}{m} \langle \langle 1 \rangle \rangle \Delta^\mu \Delta^\nu\end{aligned}$$

After some algebra we find

$$J_q = \sum_{q=u,d} \lim_{t \rightarrow 0} (A_{20}^q(t) + B_{20}^q(t))$$

See Phys.Rev.Lett. 78:610 (1997) (X.D. Ji)



# Nucleon axial structure

---

$$\langle p' | \bar{\psi}_q \gamma^\mu \gamma_5 \psi_q | p \rangle = \tilde{A}_{10}^q(t) \langle\langle \gamma^\mu \gamma_5 \rangle\rangle + \tilde{B}_{10}^q(t) \frac{i}{2m} \langle\langle \gamma_5 \rangle\rangle \Delta^\mu$$

Now one obtains

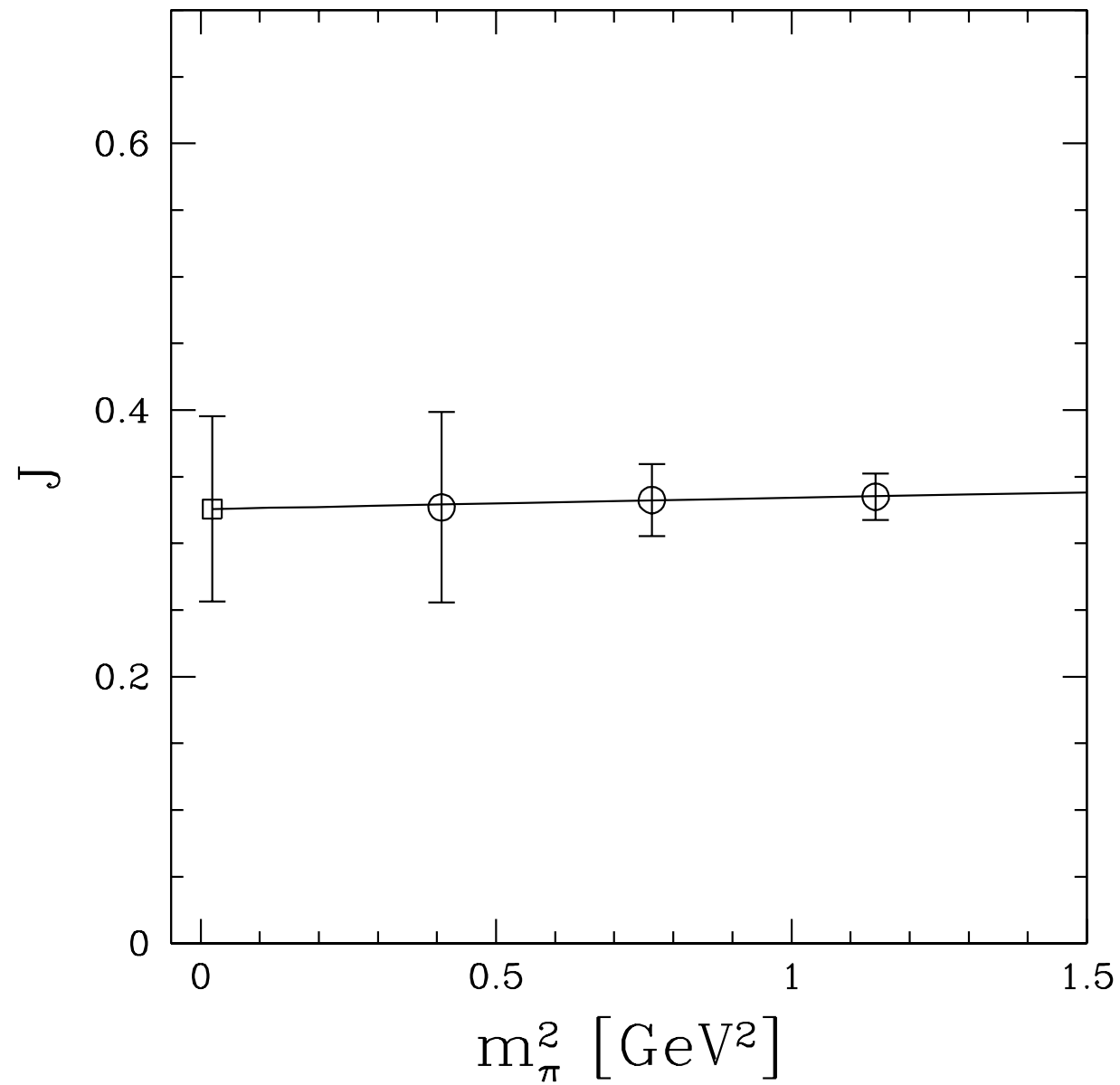
$$\frac{1}{2} \Sigma = \tilde{A}_{10}^{u+d}(0)$$

Then we can deduce

$$L_q \equiv J_{u+d} - \frac{1}{2} \Sigma$$



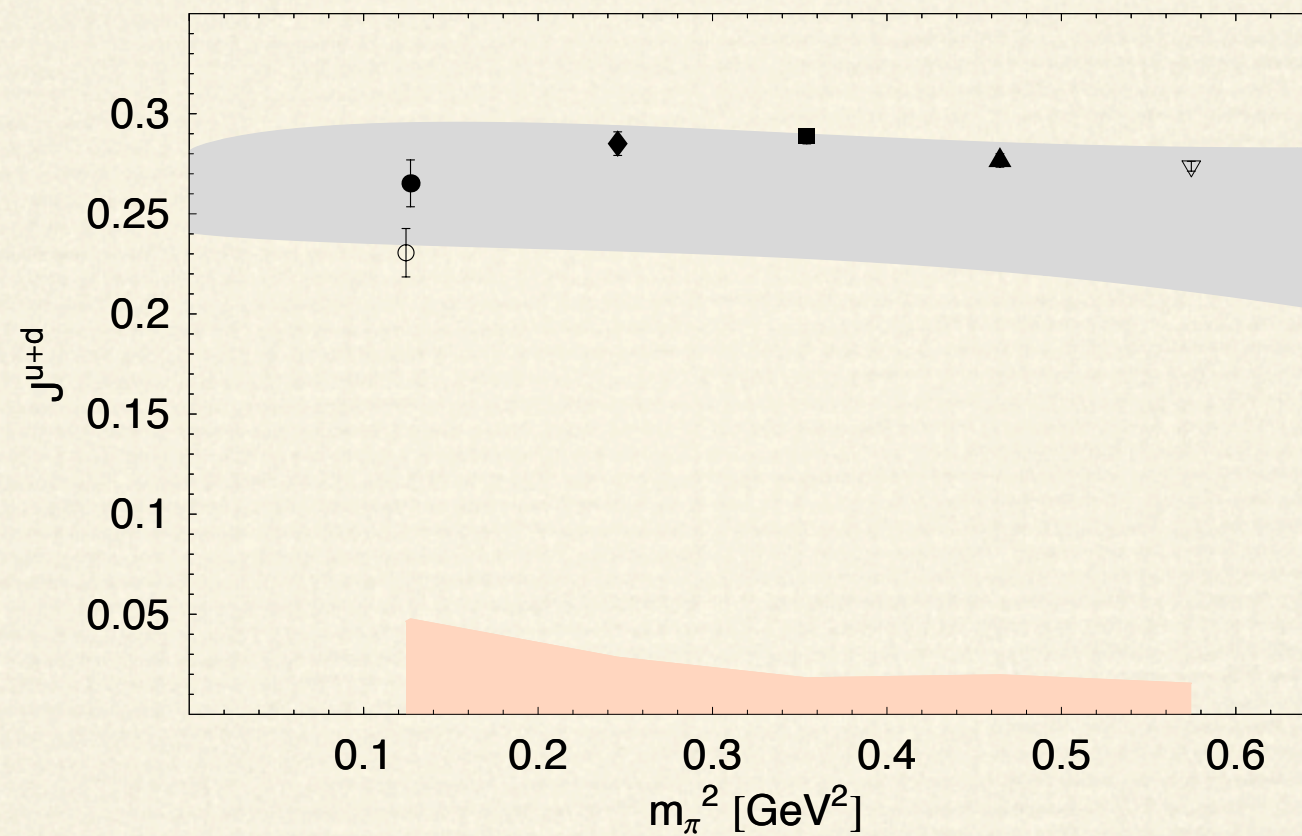
# Quark spin contribution



Phys.Rev.Lett. 92:042002 (2004)



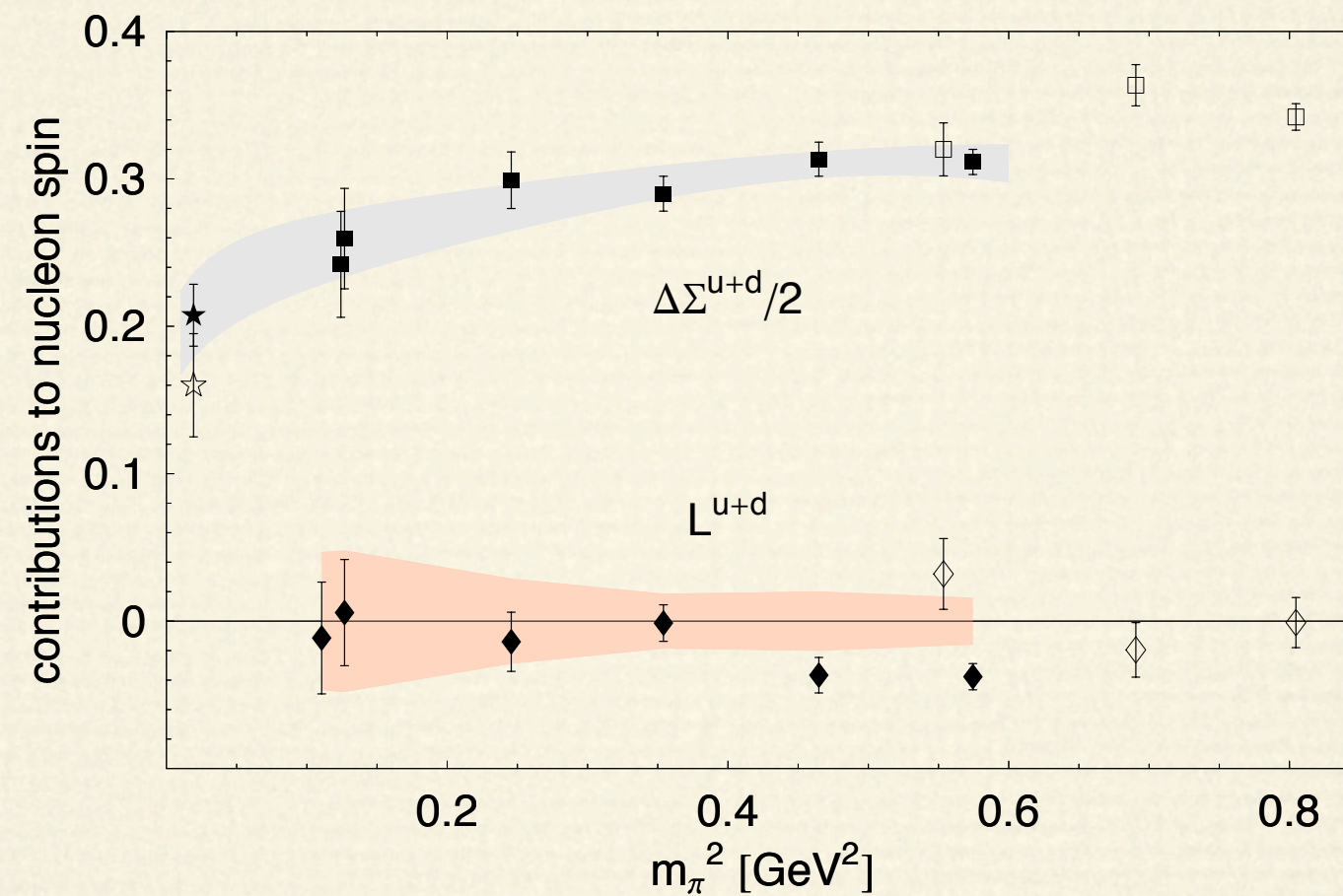
# LHPC hybrid action



Phys.Rev.D77:094502(2008)



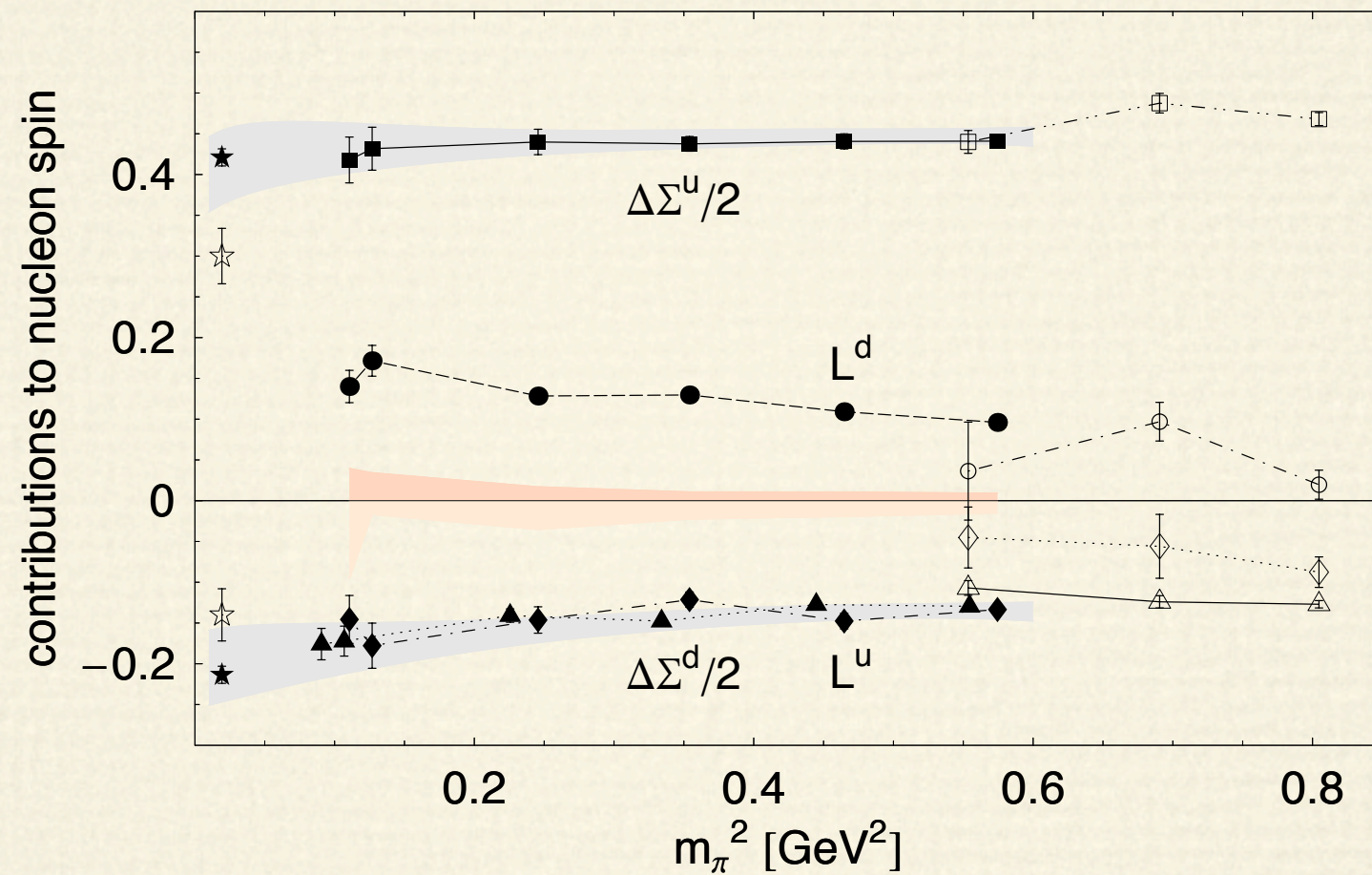
# LHPC hybrid action



Phys.Rev.D77:094502(2008)



# LHPC hybrid action



Phys.Rev.D77:094502(2008)



# Remarkable Features

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- ❖ Cancellation of OAM for  $u+d$  quarks
- ❖ Cancellation between OAM and spin contribution for down quarks
- ❖ Qualitative features over entire mass range



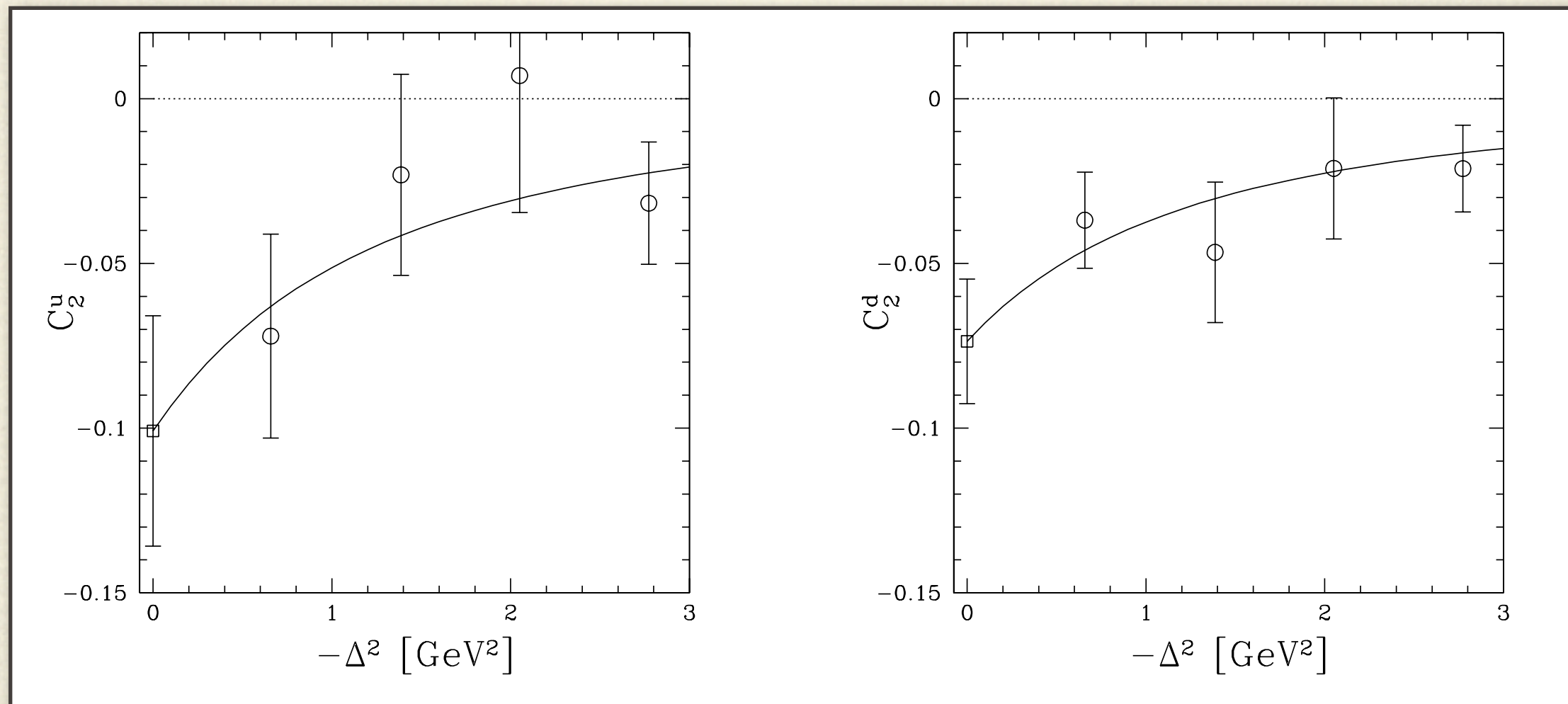
# Other Publications

---

- ❖ Similar approach: PRD62:114504 (2000)
- ❖ Alternative approach:  
Direct computation of  $\langle \vec{p} | y_j J_i(y) | \vec{p} \rangle$   
PD65:094510 (2002) (V. Gadiyak et al)  
But: see PD66:017502 (2002) (W. Wilcox)



# The “D”-term



From Phys.Rev.Lett. 92:042002 (2004)  
Compare to Phys.Rev. D63:114014 (2001)



# Hadron transverse structure

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$$H(x, \xi = 0, -\Delta_{\perp}^2)$$

measures Fourier transform of  
 $q(x, b_{\perp})$

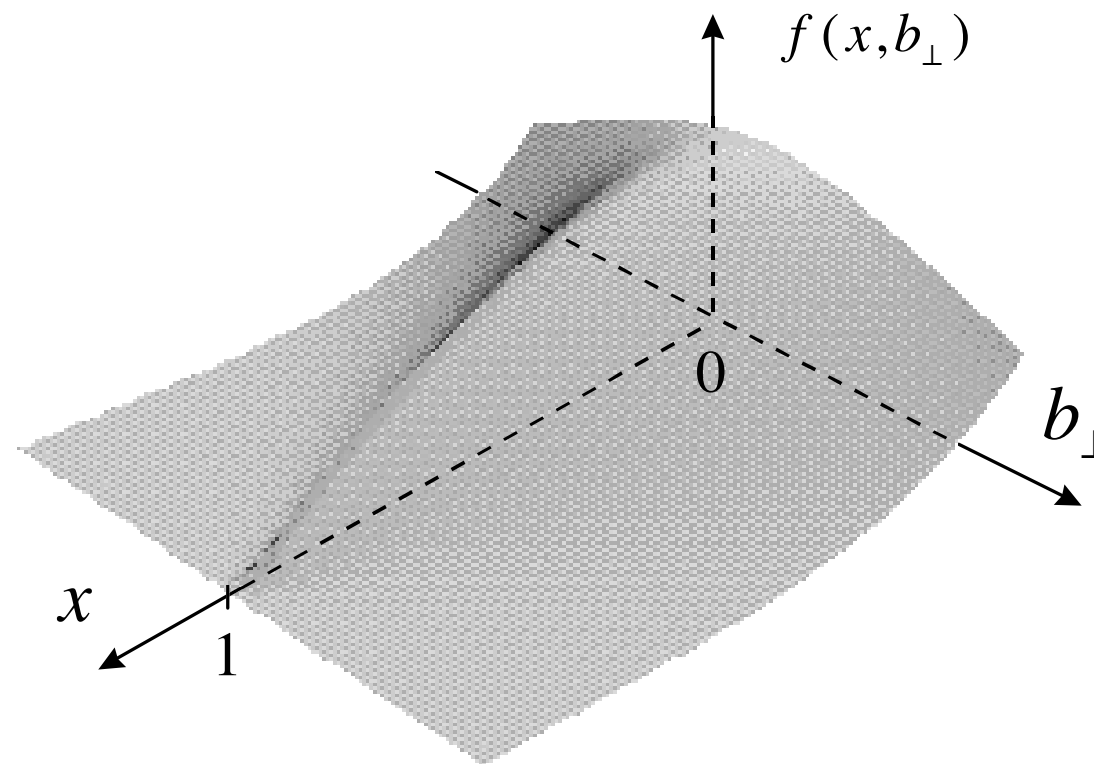
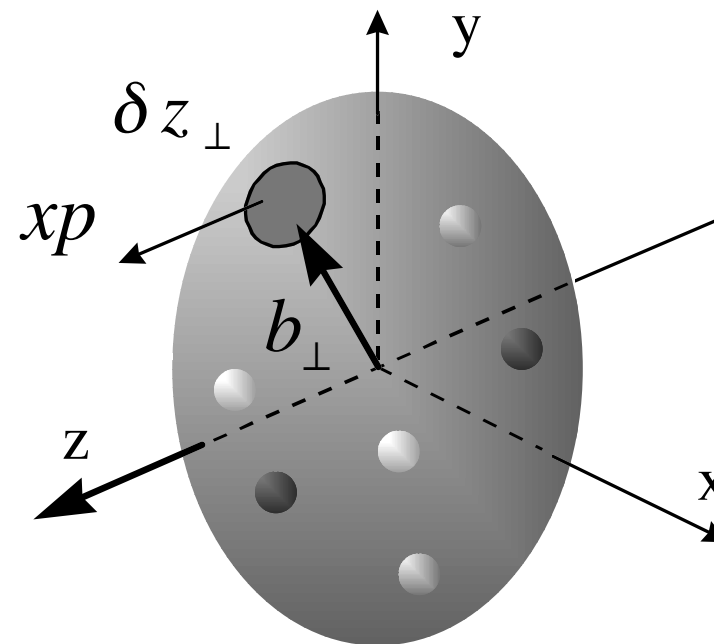
Phys.Rev. D62:071503 (2000) (M. Burkardt)

For  $\xi \neq 0$ : Phys.Rev. D66:111501 (2002) (Ralston, Pire)

Eur.Phys.J. C25:223 (2002) (M. Diehl)

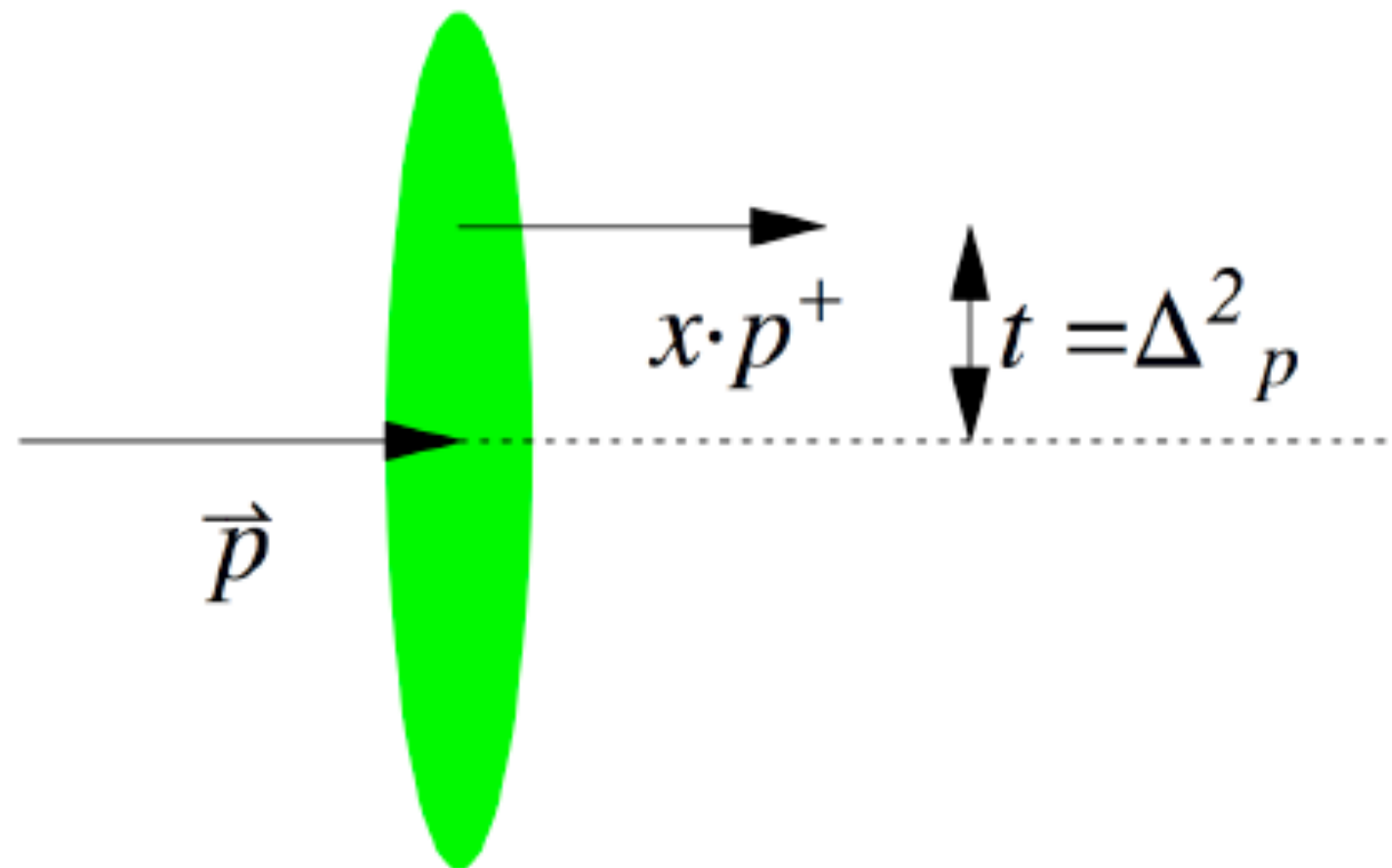


- Generalized parton distribution at  $\eta=0$





# Schematic in i.m.f.





# Moments of GPDs

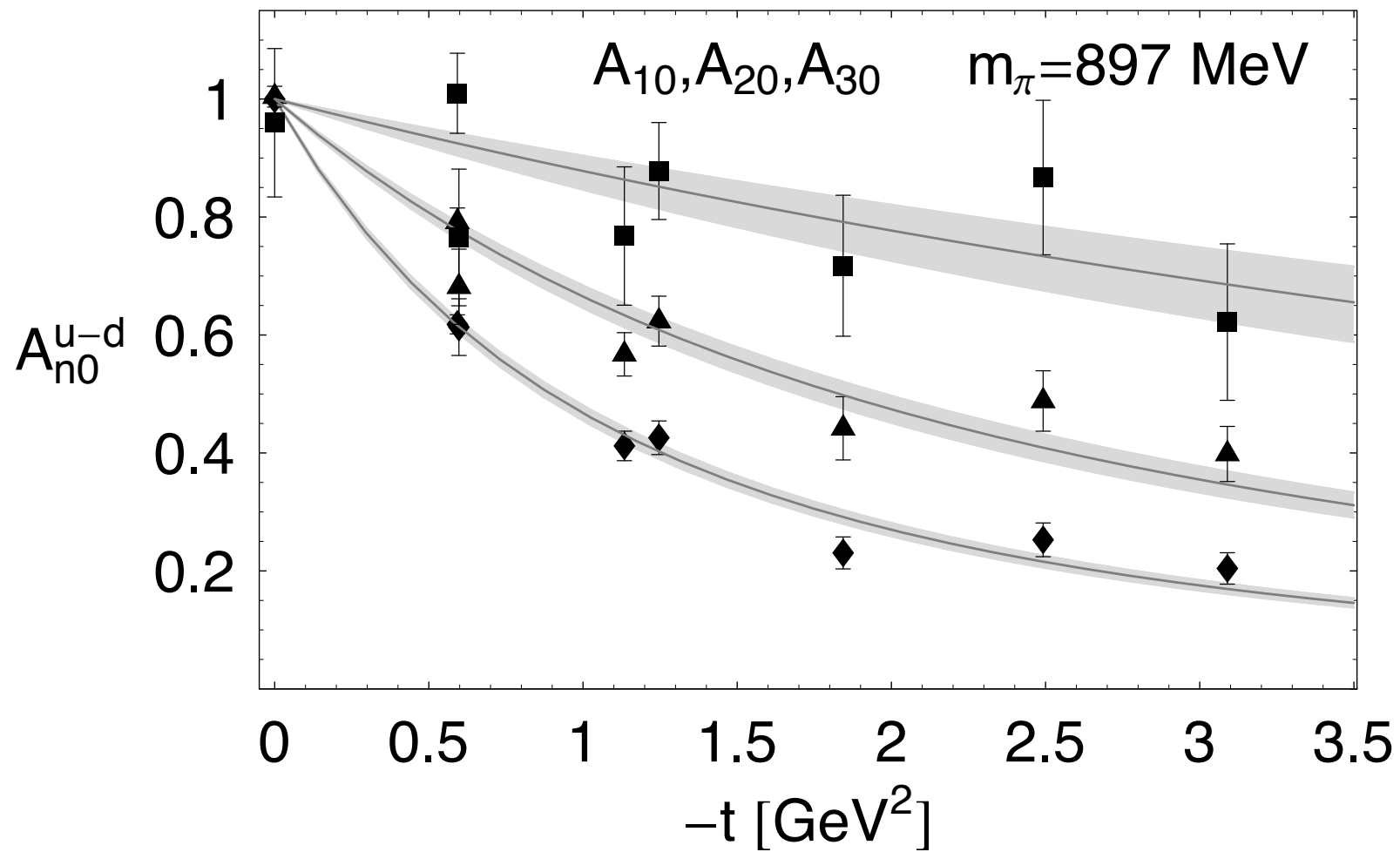
---

Do the moments depend on  $n$   
or not?

$$A_{n0}(-\Delta_{\perp}^2) \equiv \int d^2b_{\perp} dx x^{n-1} q(x, b_{\perp}) e^{i\vec{b}_{\perp} \cdot \vec{\Delta}_{\perp}}$$

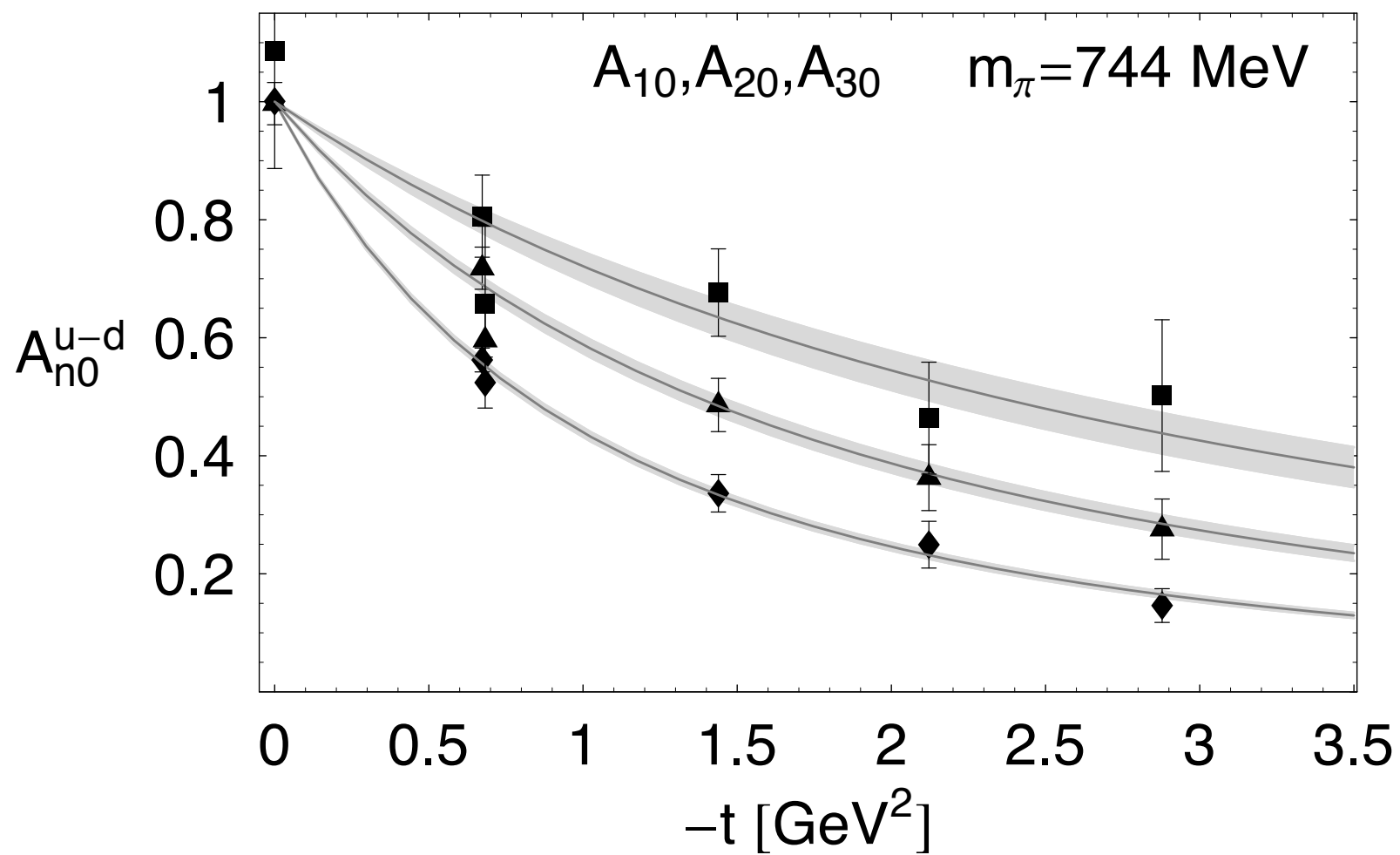


# Does $x$ -dep. factorize?

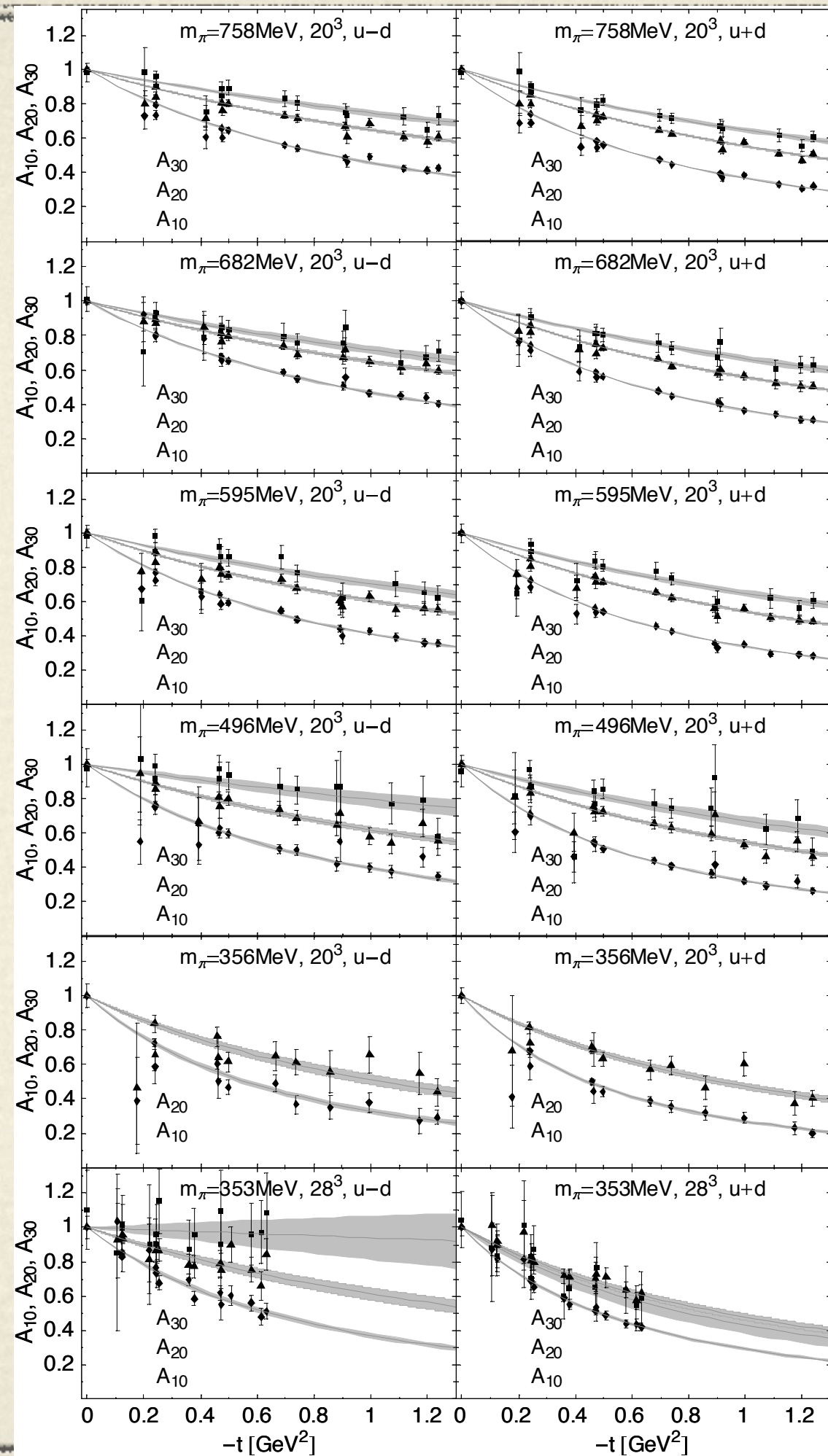




# Also for lighter quarks







LHPC,  
Phys.Rev.  
D77:094502(2008)



# The axial coupling $g_A$

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- ❖ Fundamental property of the nucleon
- ❖ Governs  $\beta$ -decay
- ❖ Quantitative measure of spont.  $\chi$ SB in hadronic physics
- ❖ Known to high accuracy experimentally from neutron  $\beta$ -decay
- ❖ Forward limit of nucleon axial form factor



# Chiral expansions

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- ❖ Phys.Rev. D70:074029 (2004) (Beane&Savage)
- ❖ Phys.Rev. D71:054510 (2005) (Detmold & Lin)
- ❖ Phys.Rev. D68:075009 (2003) (Hemmert et.al.)
- ❖ Phys.Rev. D66:054501 (2002) (Detmold et.al.)
- ❖ Phys.Rev. D74:094508 (2006) (QCDSF coll.)



# Chiral expansions

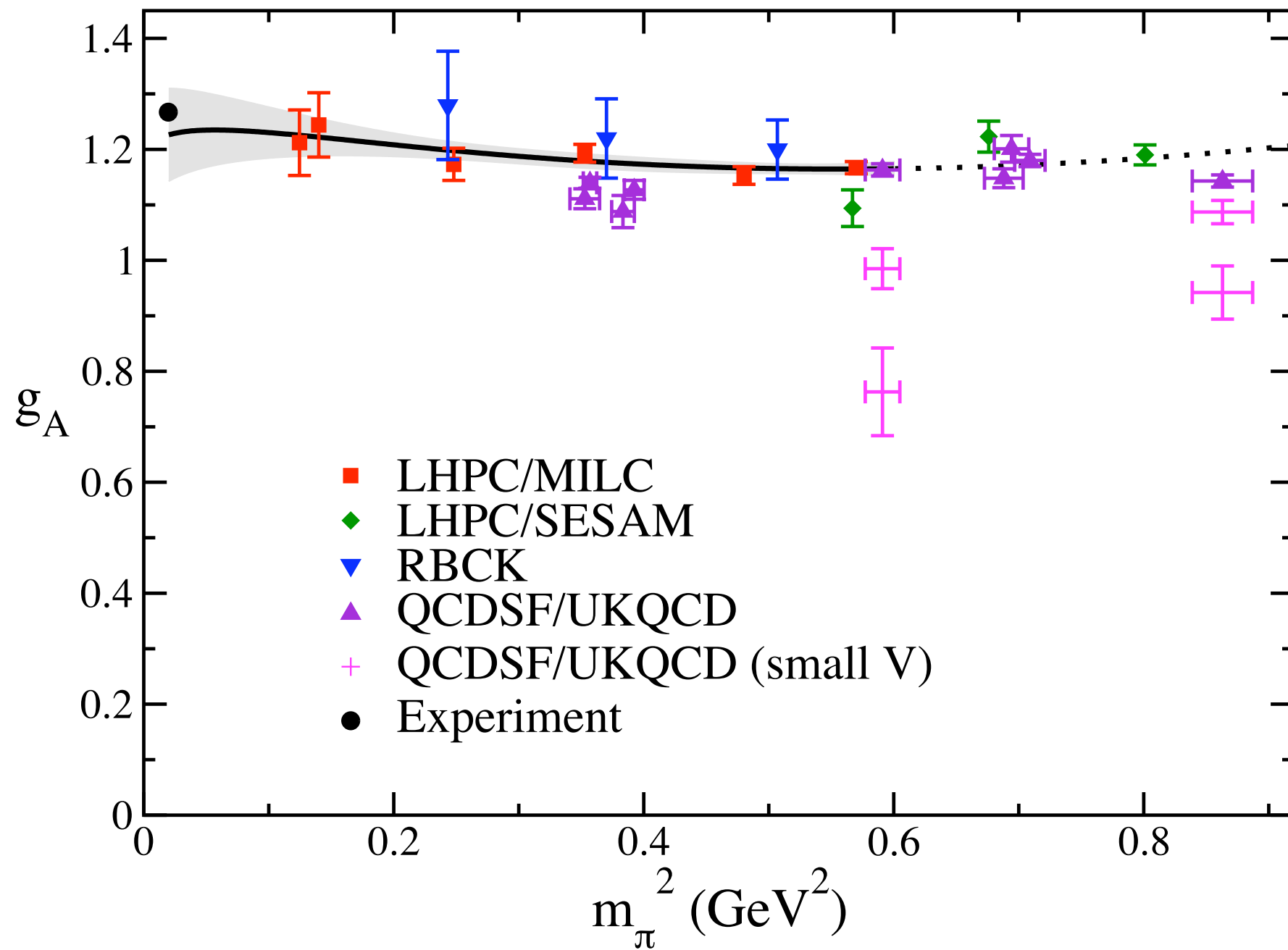
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*Our (LHPC)  
paper*



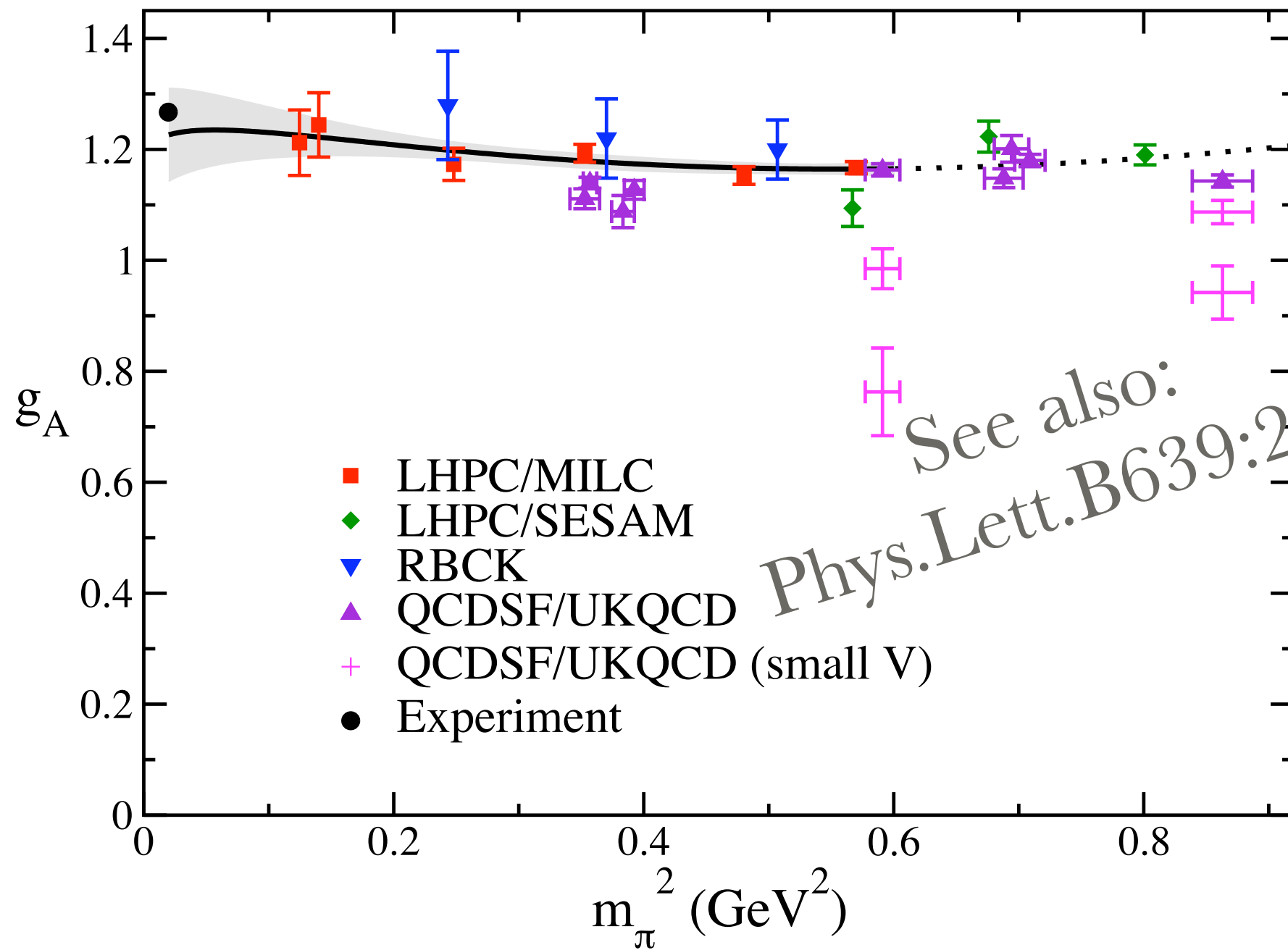
- ❖ Phys.Rev. D70:074029 (2004) (Beane&Savage)
- ❖ Phys.Rev. D71:054510 (2005) (Detmold & Lin)
- ❖ Phys.Rev. D68:075009 (2003) (Hemmert et.al.)
- ❖ Phys.Rev. D66:054501 (2002) (Detmold et.al.)
- ❖ Phys.Rev. D74:094508 (2006) (QCDSF coll.)





Phys.Rev.Lett. 96:052001 (2006)  
W. Schroers, EPJ A31:784 (2007)





See also:  
Phys.Lett.B639:278

Phys.Rev.Lett. 96:052001 (2006)  
W. Schroers, EPJ A31:784 (2007)



# Summary: $g_A$

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Experiment (neutron β decay)	$g_A = 1.2695(29)$
PRL 96:052001 (2006)	$g_A = 1.226(84)$
PR D74:094508 (2006)	$g_A = 1.31(9)(7)$

Summary & Outlook

Summary

- ❖ Three key applications
- ❖ Major progress in lattice simulations
- ❖ Qualitative insight into nucleon structure
- ❖ Quantitative results slowly becoming available
- ❖ Progress benefits from χ EFT

Outlook

- ❖ QCDSF simulations reach down to $m_{PS} \approx 350$ MeV (2006), currently running $m_{PS} \approx 250$ MeV (2009)
- ❖ Hope to reach $m_{PS} \approx 200$ MeV by the end of this decade
- ❖ LHPC focusses on full DWF, similar quark masses
- ❖ TWQCD/JLQCD: Full optimal DW + disconnected diagrams