No-go theorem of anisotropic inflations via Schwinger mechanism

Hiroyuki Kitamoto (NCTS)

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Introduction
（No-go theorem of anisotropic inflations）

- Concerning the primordial universe, we find no significant evidence for violation of rotational symmetry from the current status of cosmic microwave background observations

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{k}') P(\mathbf{k})$$

$$P(\mathbf{k}) = P_0(|\mathbf{k}|) \left\{ 1 + g_* (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}})^2 \right\}, \quad |g_*| \lesssim 10^{-2}$$

$\hat{\mathbf{n}}$: preferred direction

’13 J. Kim, E. Komatsu, ’18 Planck Collaboration

- How to understand the absence (or the smallness) of the anisotropic hair theoretically?

- From a theoretical viewpoint, an anisotropic inflation can be obtained if an U(1) gauge field has a classical value like an inflaton

$$A_i \neq 0$$

$\Rightarrow$ There may exist a special direction

- In fact, if the gauge field respects the conformal symmetry as its kinetic term is canonical, (the physical scale of) the electromagnetic field decays with the cosmic expansion

$\Rightarrow$ This model has no anisotropic hair

Introduction
（Model with a canonical kinetic term）

$$
\begin{aligned}
ds^2 &= -dt^2 + a^2(t)d\mathbf{x}^2 \\
&= a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)
\end{aligned}
\qquad H \equiv \frac{1}{a}\frac{da}{dt} \simeq \text{const.}
$$

$$S_{\text{gauge}} = \int \sqrt{-g}d^4x \left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\right] = \int d^4x \left[-\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\right]$$

conformal symmetry

$$\Rightarrow \qquad \frac{d^2}{d\tau^2}A = 0 \;\Leftrightarrow\; \frac{d}{d\tau}A = \text{const.}$$

temporal gauge: $A_0 = 0$

homogeneity: $A_i = A(\tau)\delta_i{}^1$

$$\Rightarrow \qquad E_{\text{phys}} = -a^{-2}\frac{d}{d\tau}A \propto a^{-2}$$

The electric field decays with the cosmic expasion $\Rightarrow$ Isotropic inflation

If the conformal symmetry is broken, this discussion does not hold true

Introduction
（Model with a dilatonic coupling）

'09, '10 M. Watanabe, S. Kanno, J. Soda

$$S_{\rm bg} = \int \sqrt{-g} d^4x \left[\frac{M_{\rm pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) - \frac{1}{4} \underline{f^2(\varphi)} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

Using the ansatz $f(\varphi) = \exp\left\{ \frac{2c}{M_{\rm pl}^2} \int d\varphi \ \frac{V}{\partial_\varphi V} \right\}$ and the slow-roll condition,

f is determined as a solution of the classical field eqs.:

$$f = (a^{-4} + q a^{-4c})^{\frac{1}{2}}$$

q: integration const.

- If $c > 1$ (strong coupling), $\quad f \to a^{-2}$
- If $c < 1$ (weak coupling), $\quad f \to q^{\frac{1}{2}} a^{-2c}$

Inflaton-driven electric field

Our case

- In the $c > 1$ case, $f = a^{-2}$ and then

$$E_{\rm phys} = -f a^{-1} \frac{d}{dt} A = \underline{f^{-1}} a^{-2} E$$

$$= E$$

$$E = \frac{\sqrt{3\epsilon_V (c-1)}}{c} M_{\rm pl} H$$

$$\epsilon_V \equiv \frac{1}{2} \left(\frac{M_{\rm pl} \partial_\varphi V}{V} \right)^2$$

$E_{\rm phys}$ is persistent $\Rightarrow$ Anisotropic inflation

In fact, this discussion lacks a microscopic viewpoint

- In the $c < 1$ case, $f \propto a^{-2c}$ and then

$$E_{\rm phys} \propto a^{2(c-1)}$$

$E_{\rm phys}$ decays with the cosmic expansion $\Rightarrow$ Isotropic inflation

Motivation

- We consider the case that a charged test scalar field exists

$$S_{\rm test} = \int \sqrt{-g} d^4x \ \left[-g^{\mu\nu} (\partial_\mu + ieA_\mu)\phi^* (\partial_\nu - ieA_\nu)\phi - m^2 \phi^* \phi \right]$$

- A strong electric field leads to the pair production of charged particles (Schwinger mechanism), and the pair production induces the U(1) current

$$\tilde{j} = 2e \int \frac{d^3k}{(2\pi)^3} \ v_{\mathbf{k}} n_{\mathbf{k}}$$

$n_{\mathbf{k}}$: particle number

$v_{\mathbf{k}}$: velocity of particle

- It is reasonable to conjecture that if we take into account the Schwinger mechanism, the induced current screens the inflaton-driven electric field

- Evaluating the induced current, and solving the field eqs. with it, we verify the no-anisotropic hair conjecture for inflation

Differences from other studies

The studies of Schwinger mechanism in 4-dimensional inflation can be divided into the two cases:

Our case ⊙ Introducing the dilatonic coupling, the classical field eqs. show that the electric field approaches to a constant value E

$$E_{\text{phys}} = -fa^{-1}\frac{d}{dt}A = E \quad \Rightarrow \quad A = -\frac{E}{3H}a^{1+2} \qquad \because\ f = a^{-2}$$

'17 J. J. Geng, B. F. Li, J. Soda, A. Wang, Q. Wu, T. Zhu, '18 H. Kitamoto

- Without considering the mechanism to generate a persistent electric field, the electric field is fixed at a constant value E

$$E_{\text{phys}} = -a^{-1}\frac{d}{dt}A = E \quad \Rightarrow \quad A = -\frac{E}{H}a^{1}$$

'14 T. Kobayashi, N. Afshordi, '16 T. Hayashinaka, T. Fujita, J. Yokoyama, '18 T. Hayashinaka, S. S. Xue, '18 M. Banyeres, G. Domenech, J. Garriga

In other dimensions

In D-dimension inflation theories without dilatonic couplings,

$$\frac{d}{d\tau}\left(a^{D-4}\frac{d}{d\tau}A\right) = 0 \;\Leftrightarrow\; \frac{d}{d\tau}A \propto a^{-D+4}$$

$$\Rightarrow \qquad E_{\rm phys} = -a^{-2}\frac{d}{d\tau}A \propto a^{-D+2}$$

- In D>2, the electric field decays with the cosmic expansion as long as a dilatonic coupling is absent

- In D=2, a persistent electric field can be obtained without introducing a dilatonic coupling, and then $A \propto a^1$. The pair production on the background has been investigated

’14 M. B. Frob, J. Garriga, S. Kanno, M. Sasaki, J. Soda, T. Tanaka, A. Vilenkin

Validity of WKB approximation

Klein–Gordon eq.: $$\left\{\frac{d^2}{d\tau^2}+\omega_{\mathbf{k}}^2(\tau)\right\}\tilde{\phi}_{\mathbf{k}}(x)=0$$ $\tilde{\phi}=a\phi$

$$\omega_{\mathbf{k}}^2=(k_1-eA)^2+k_2^2+k_3^2+(m^2-2H^2)a^2$$

$$A=-\frac{E}{3H}a^{1+2}$$

+2 comes from f

At $a\to 0$, the WKB approximation is trivially valid

$$\omega_{\mathbf{k}}\simeq|\mathbf{k}| \qquad \Rightarrow \qquad \omega_{\mathbf{k}}^{-4}\left(\frac{d\omega_{\mathbf{k}}}{d\tau}\right)^2\simeq 0, \quad \omega_{\mathbf{k}}^{-3}\frac{d^2\omega_{\mathbf{k}}}{d\tau^2}\simeq 0$$

At $a\to\infty$, the validity is ensured due to the presence of f

$$\omega_{\mathbf{k}}\simeq\frac{eE}{3H}a^{1+2} \qquad \Rightarrow \qquad \omega_{\mathbf{k}}^{-4}\left(\frac{d\omega_{\mathbf{k}}}{d\tau}\right)^2\simeq 9\left(\frac{eE}{3H^2}a^2\right)^{-2}, \quad \omega_{\mathbf{k}}^{-3}\frac{d^2\omega_{\mathbf{k}}}{d\tau^2}\simeq 12\left(\frac{eE}{3H^2}a^2\right)^{-2}$$

Particle number and Induced current

In the semiclassical picture,

$$n_{\mathbf{k}} = \exp\left\{ 4\ \mathrm{Im} \int^{\tau_*} d\tau'\ \omega_{\mathbf{k}}(\tau') \right\}, \quad \omega_{\mathbf{k}}(\tau_*) \equiv 0$$

'61 V. L. Pokrovskii, I. M. Khalatnikov

$$\tilde{j} = 2e \int \frac{d^3k}{(2\pi)^3}\ v_{\mathbf{k}} n_{\mathbf{k}}, \quad v_{\mathbf{k}} = (k_1 - eA)/\omega_{\mathbf{k}}$$

$$\tilde{j}_0 = 0,$$
$$\tilde{j}_i = \tilde{j}(t)\delta_i{}^1$$

We evaluate the late time behavior at $\frac{eE}{H^2}a^2 \gg 1$:

$$\tilde{j} \simeq \frac{e^3E^2}{4\pi^3}\frac{a^7}{7H} \exp\left\{ -\pi \frac{m^2 - 2H^2}{eEa^2} \right\}$$

At $\frac{|m^2 - 2H^2|}{eEa^2} \ll 1$, the contribution from the mass term becomes irrelevant

$$\tilde{j} \simeq \frac{e^3E^2}{4\pi^3}\frac{a^7}{7H}$$

Field eqs. with Induced current

$$\begin{cases} V = 3M_{\rm pl}^2 H^2 \\ 3H\dfrac{d}{dt}\varphi + \partial_\varphi V - f^{-1}\partial_\varphi f \cdot E_{\rm phys}^2 = 0 \\ \dfrac{d}{dt}\left(f a^2 E_{\rm phys}\right) + \underline{a^{-1}\tilde{j}} = 0 \end{cases}$$

Solving them by use of the ansatz: $f(\varphi) = \exp\left\{\frac{2c}{M_{\rm pl}^2}\int d\varphi\ \frac{V}{\partial_\varphi V}\right\}$,

$$f = a^{-2}\left\{1 - \frac{1}{1+\frac{3}{2}\frac{1}{c-1}} \cdot \frac{e^3 E}{4\pi^3}\frac{a^6}{42H^2}\right\}$$

where $c > 1$

$$E_{\rm phys} = E\left\{1 - \frac{\frac{3}{2}\frac{1}{c-1}}{1+\frac{3}{2}\frac{1}{c-1}} \cdot \frac{e^3 E}{4\pi^3}\frac{a^6}{42H^2}\right\}$$

Considering the first-order backreaction, the electric field decays with the cosmic expansion ⇒ No anisotropic hair may exist also in this model

Summary

- In the inflation theory with a dilatonic coupling between the inflaton and the U (1) gauge field, a persistent electric field (and then an anisotropic inflation) is obtained as a solution of the classical field eqs.
- We investigated the pair production of charged scalar particles in the inflaton-driven electric field. In particular, we evaluated the induced current due to the pair production
- Solving the field eqs. with the induced current, we found that the first-order backreaction screens the electric field with the cosmic expansion
- The result indicates that as long as charged particles exist, the no-go theorem of anisotropic inflations holds true regardless of whether the dilatonic coupling is present or not

Future directions (i)

- In order to prove the no-go theorem of anisotropic inflations completely, the whole time evolution of the electric field should be investigated

- For the investigation, we need to evaluate the induced current on general backgrounds $E_{\rm phys},\ f$

Using the WKB approximation,

$$\tilde{j} \simeq \frac{e^3}{4\pi^3}\int_{t_0}^{t} dt'\ a^3(t')E_{\rm phys}^2(t')f^{-2}(t')\exp\left\{-\pi\frac{m^2-2H^2}{eE_{\rm phys}(t')f^{-1}(t')}\right\}$$

$\tilde{j}^{(1)}$ can be derived for $E_{\rm phys}^{(0)} = E,\ f^{(0)} = a^{-2}$

- Substituting the general expression of the induced current into the field eqs., we obtain self-consistent eqs. for $E_{\rm phys},\ f$

Future directions (ii)

- We have to investigate the pair production of charged fermions in the inflaton-driven electric field to cover all possible effects

$$\left\{ i\gamma^0 \frac{d}{d\tau} + i\gamma^1 (k_1 - eA) + i\gamma^2 k_2 + i\gamma^3 k_3 - ma \right\} \tilde{\psi}_{\mathbf{k}} = 0 \qquad \tilde{\psi} = a^{\frac{3}{2}} \psi$$

$$-f a^{-2} \frac{d}{d\tau} A = E_{\mathrm{phys}}$$

$$\Rightarrow \qquad \tilde{j} \simeq 2 \cdot \frac{e^3}{4\pi^3} \int_{t_0}^{t} dt' \; a^3(t') E_{\mathrm{phys}}^2(t') f^{-2}(t') \exp\left\{ -\pi \frac{m^2}{e E_{\mathrm{phys}}(t') f^{-1}(t')} \right\}$$

Factor 2 comes from the spin sum

No supercurvature modes exist

- There is no significant difference between the pair production of scalar particles and that of fermions, at least at finite-orders