Unified Models of Inflation and Dark Energy

Chung-Chi Lee

National Center for Theoretical Sciences
(NCTS)

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Chung-Yuan Christian University

Outline

Motivation

$f(R)$ Dark Energy

$f(R)$ Inflation

Observational Constraints on $f(R)$ Gravity

Quintessential Inflation

Observational Constraints on Quintessential Inflation

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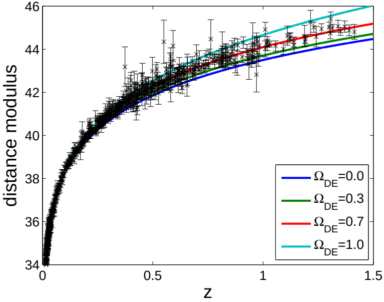
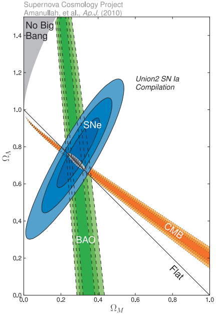
Quintessential Inflation

Observational Constraints on Quintessential Inflation

Dark Energy Problem

- Dark Energy Problem: late-time accelerating phenomenon.
  - Type-Ia supernovae (SNIa).
  - Cosmic microwave background radiation (CMB).
  - Baryon acoustic oscillations (BAO).





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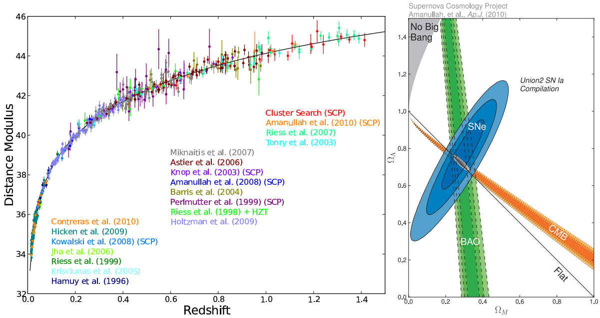


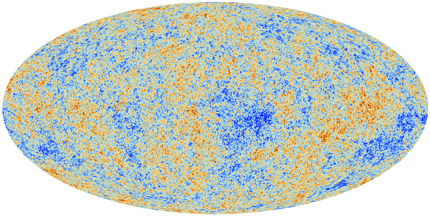
Dark Energy Problem

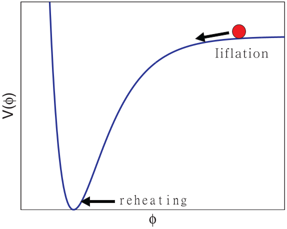
$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Modify the Energy-Momentum tensor.
  - ΛCDM model
  - Quintessence/Phantom model
  - etc.
- Modify the Gravity
  - $f(R)$ gravity
  - $f(T)$ gravity (Modified Teleparallel gravity)
  - etc.

Inflation Problem

- Inflation: early-time accelerating phase.
  - Horizon problem.
  - Flatness problem.





Inflation Problem

- Unify inflation and dark energy:
  - $f(R)$ gravity: combines the early and late-time potential.
  - Quintessential inflation: matter non-minimally coupled to quintessence field.

Massive Neutrinos

- Neutrino Flavour Oscillation.
  $|\nu_e\rangle = U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle$.
- Neutrino Mass Difference.
  $\Delta m^2_{21} \sim 7.54 \times 10^{-5} eV^2$ and $\Delta m^2_{13} \sim 2.43 \times 10^{-3} eV^2$.
- Neutrono mass from cosmology:

$$\Sigma m_\nu < 0.23 eV\,. \qquad (95\%; \mathrm{Planck} + \mathrm{BAO} + \mathrm{JLA} + \mathrm{H}_0)$$

Outline

$f(R)$ Gravity

- $f(R)$ gravity:
  One of the simplest modified gravity model, which extends Einstein-Hilbert action to higher order terms.
- The action of $f(R)$ gravity:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m \,,$$

where $\kappa^2 = 8\pi G$ and $f(R)$ is an arbitrary function.

$f(R)$ Gravity

- The field (modified Einstein) equation is obtained by using the variation principle:
  - Palatini formalism:
    Dynamical variable: metric $g_{\mu\nu}$ and connection $\Gamma^{\rho}_{\mu\nu}$
  - Metric formalism:
    Dynamical variable: metric $g_{\mu\nu}$ only
    Levi-Civita connection: $\Gamma^{\rho}_{\mu\nu} = \frac{g^{\rho\lambda}}{2}\left(g_{\lambda\mu,\nu} + g_{\lambda\nu,\mu} - g_{\mu\nu,\lambda}\right)$
- Under the metric formalism, modified Einstein equation:

$$f_R R_{\mu\nu} - \frac{f}{2} g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) f_R = \kappa^2 T_{\mu\nu} \,,$$

where the subscript R denotes d/dR and $T_{\mu\nu}$ is the energy-momentum tensor.

Viability Conditions

- $\frac{df(R)}{dR}$, $\frac{d^2f(R)}{dR^2} > 0$ for $R > R_0$, where R_0 is the background curvature.
  - The effective Newtonian constant: $G_{\rm eff} = \frac{G}{f_R}$.
  - The scalar mode graviton (scalaron) mass: $m_s^2 = \frac{1}{3}\left(\frac{f_R}{f_{RR}} - R\right)_{R=R_0}$.
    - The trace of modified Einstein equation:

$$Rf_R - 2f + 3\Box f_R = \kappa^2 T\,,$$

where $T = g^{\mu\nu}T_{\mu\nu}$ is the trace of energy-momentum tensor.

- The metric perturbation: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h^T_{\mu\nu} - \bar{g}_{\mu\nu}h_f$
- The trace equation yields:

$$3\Box\delta f_R + R_0\delta f_R + f_R\delta R - 2\delta f = 0\,,$$

$$\Rightarrow \Box h_f = \frac{1}{3}\left(\frac{f_R}{f_{RR}} - R\right)_{R=R_0} h_f\,,$$

where $h_f \equiv \delta f_R / f_R$.

Viability Conditions

- Passing local gravity constraints.
  From chameleon mechanism, the maximum deviation leads to

$$|f_R - 1| \lesssim 10^{-15}\,,$$

  in solar system. Note that the inner galaxy and critical energy density are $\rho_{\rm in} \sim 10^{-24}{\rm g/cm^3}$ and $\rho_c \simeq 10^{-29}{\rm g/cm^3}$.
- Having a stable late-time de-Sitter point:

$$(2f - Rf_R)_{R=R_d} = 0\,,$$

  where R_d is the de Sitter curvature.
- Having a ΛCDM limit in the large curvature regime $(R \gg R_0)$ $f(R) \to R - 2\Lambda$.

$f(R)$ Dark Energy

- Hu-Sawicki model: (R_{ch} is a constant curvature for each model)

$$f(R) = R - R_{ch}^{(HS)} \frac{c_1 \left(R/R_{ch}^{(HS)}\right)^p}{c_2 \left(R/R_{ch}^{(HS)}\right)^p + 1}.$$

- Starobinsky model:

$$f(R) = R - \lambda R_{ch}^{(S)} \left[1 - \left(1 + \frac{R^2}{R_{ch}^{(S)2}}\right)^{-n}\right].$$

- Tsujikawa model:

$$f(R) = R - \mu R_{ch}^{(T)} \tanh\left(\frac{R}{R_{ch}^{(T)}}\right).$$

- Exponential gravity model:

$$f(R) = R - \beta R_{ch}^{(E)} \left(1 - e^{-R/R_{ch}^{(E)}}\right).$$

- Appleby-Battye model:

$$(1-g)R + g R_{ch}^{(AB)} \ln\left[\frac{\cosh\left(R/R_{ch}^{(AB)} - b\right)}{\cosh\, b}\right].$$

$f(R)$ Dark Energy

- $R_{ch} \sim \Lambda$: the same order of cosmological constant.
- In high redshift regime $(R \gg R_0)$, these viable models reduce to:
  - Hu-Sawicki, Starobinsky model $\Rightarrow f(R) \simeq R - \lambda R_{ch}\left(1 - \left(\frac{R_{ch}}{R}\right)^{2n}\right)$.
  - Tsujikawa, Appleby-Battye model
    $\Rightarrow f(R) = R - \beta R_{ch}\left(1 - e^{-R/R_{ch}}\right)$.

$f(R)$ Dark Energy

- Considering the Friedmann-Robertson-Walker (FRW) metric in flat space-time,

$$ds^2 = -dt^2 + a^2(t)\left(x^2 + y^2 + z^2\right) .$$

- The modified Einstein equation leads to the Friedmann equations,

$$\begin{aligned} 3H^2 &= \kappa^2 \rho_{\mathrm{M}} + \frac{1}{2}\left(Rf_R - f\right) - 3\left(\left(f_R - 1\right)H^2 + H\dot{f}_R\right) , \\ -2\dot{H} &= \kappa^2\left(\rho_{\mathrm{M}} + P_{\mathrm{M}}\right) + \ddot{f}_R - H\dot{f}_R + 2\left(f_R - 1\right)\dot{H} , \end{aligned}$$

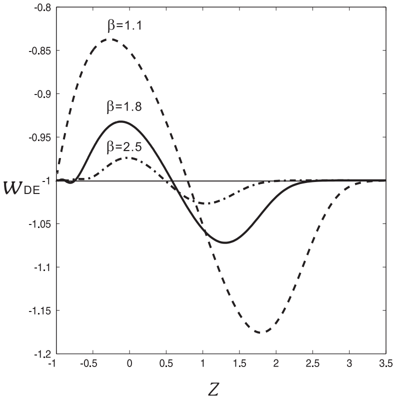
where $H \equiv \dot{a}/a$ is Hubble parameter; ρ_M and P_M are total matter energy density and pressure.

- The effecitve dark energy is defined by rewriting the Friedmann equations

$$H^2 = \frac{\kappa^2}{3}\left(\rho_M + \rho_{DE}\right) \quad \text{and} \quad \dot{H} = -\frac{\kappa^2}{2}\left(\rho_M + P_M + \rho_{DE} + P_{DE}\right) .$$

$f(R)$ Dark Energy

- We choose exponential model to demonste the cosmological evolution of "equation of state" (EoS) $w_{DE} \equiv P_{DE}/\rho_{DE}$ as a function of the redshift z.



Outline

Single field inflation

- Slow-roll parameters:

$$\epsilon \equiv \frac{M_{\mathsf{Pl}}^2}{2}\left(\frac{V_\phi}{V}\right)^2, \ \eta \equiv M_{\mathsf{Pl}}^2 \frac{V_{\phi\phi}}{V}, \ \xi \equiv M_{Pl}^4 \frac{V_\phi V_{\phi\phi\phi}}{V^2}\,.$$

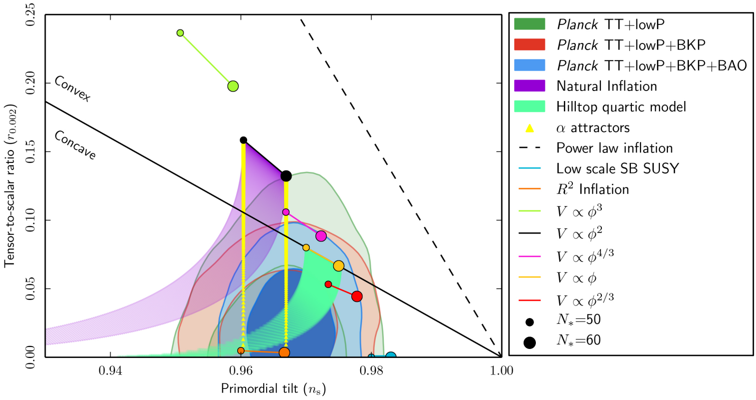
- Inflation observables: scalar and tensor spectral index (n_s, n_t), tensor-to-scalar ratio r, scalar spectral index running α_s.

$$\begin{aligned} n_s - 1 &= -6\epsilon + 2\eta\,, \\ n_t &= -2\epsilon\,, \\ r &= 16\epsilon\,, \\ \alpha_s &= 16\epsilon\eta - 24\epsilon^2 - 2\xi\,. \end{aligned}$$

$f(R)$ Inflation

- Starobinsky R^2 inflation: $f(R) = R + R^2/M^2$.

  (Planck 2015 results)



$f(R)$ Inflation

- Generalized R^{2-q} inflation.
- Conformal transformation.
  - $\tilde{g}_{\mu\nu} = f_R g_{\mu\nu}$
  - $f_R \equiv \partial f(R)/\partial R = e^{\sqrt{2/3}\ \phi/M_{\mathrm{Pl}}}$
- $f(R)$ action under conformal transformation:

$$S = \frac{M_{\mathrm{Pl}}^2}{2} \int d^4x \sqrt{-g} f(R) + S_M$$

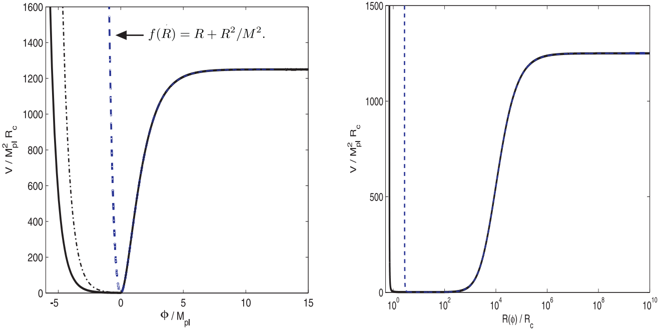
$$\Rightarrow \qquad S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\mathrm{Pl}}^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] .$$

- The potential in the Einstein frame:

$$V(\phi) = \frac{M_{\mathrm{Pl}}^2}{2} \frac{R f_R - f}{f_R^2} = V_0 e^{-2\sqrt{\frac{2}{3}} \frac{\phi}{M_{\mathrm{Pl}}}} \left(e^{\sqrt{\frac{2}{3}} \frac{\phi}{M_{\mathrm{Pl}}}} - 1 \right)^{\frac{2-q}{1-q}} .$$

$f(R)$ Inflation

- The action: $f(R) = R + R^{2-q}/M^{2-2q} + F(R)$ with $q = 0$, and $F(R)$ is responsible for the late-time dark energy model.
- The potential in the Einstein frame as function of ϕ and R.
- R oscillates around $R = 0$ in Jordan frame in R^2 model. R always positive in combined model.



$f(R)$ Inflation

- The action: $f(R) = R + R^{2-q}/M^{2-2q} + F(R)$.
- The end of inflation:
  - Condition:

$$\epsilon\big|_{\phi=\phi_{end}} = 1\,.$$

  - The scalar at the end of inflation

$$\frac{\phi_{end}}{M_{\mathsf{Pl}}} = \sqrt{\frac{3}{2}}\ln\left[\frac{(2+\sqrt{3})(1-q)}{\sqrt{3}-(1+\sqrt{3})q}\right]\,.$$

  - The number of e-folding

$$N = N(\phi) \equiv \int_{\tilde{t}}^{\tilde{t}_{end}} \tilde{H} d\tilde{t} \simeq \frac{1}{M_{\mathsf{Pl}}^2}\int_{\phi_{end}}^{\phi} \frac{V}{V_\phi} d\phi\,,$$

$f(R)$ Inflation

- The scalar field $\phi = \phi(q, N)$.
- The slow-roll parameters:

$$\begin{aligned}
\epsilon &= \frac{[q f_R + 2(1-q)]^2}{3(1-q)^2(f_R-1)^2}, \\
\eta &= \frac{2[q^2 f_R^2 - (1-q)(2-5q) f_R + 4(1-q)^2]}{3(1-q)^2(f_R-1)^2}, \\
\xi &= 4[q f_R + 2(1-q)][q^3 f_R^3 + (1-q)(1-2q)(2-5q) f_R^2 \\
&\quad -(1-q)^2(10-17q) f_R + 8(1-q)^3][9(1-q)^4(f_R-1)^4]^{-1},
\end{aligned}$$

where $f_R = e^{\sqrt{2/3}\ \phi/M_{\mathrm{Pl}}}$.

$f(R)$ Inflation

- The scalar field $\phi = \phi(q, N)$.

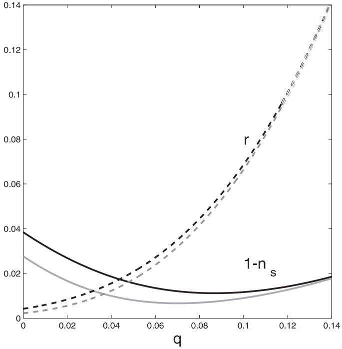


Figure: The black and gray lines correspond to $N = 50$ and 70.

$f(R)$ Inflation

- The scalar field $\phi = \phi(q, N)$.

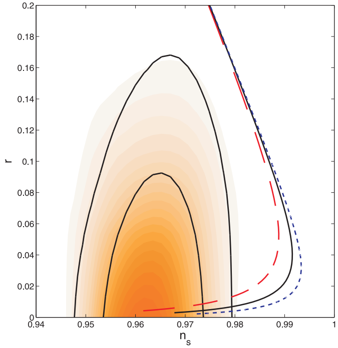


Figure: The red, black and blue denote $N = 50$, 60 and 70, and the contour plot presents the 1σ and 2σ bound from Planck 2013.

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Quintessential Inflation

Observational Constraints on Quintessential Inflation

Perturbation in $f(R)$ Gravity

- The scalar perturbed FRW metric in Newtonian gauge:

$$ds^2 = a(\tau)^2 \left[-(1+2\Psi)d\tau^2 + (1-2\Phi)dx_i dx^i\right] \ ,$$

where τ is the conformal time.

- The perturbed energy-momentum tensor is given by

$$\begin{aligned} T^0_0 &= -(\rho + \delta\rho) \ , \\ T^0_i &= (1+w)\,\rho v_i \ , \\ T^i_j &= (P + \delta P)\,\delta^i_j \ . \end{aligned}$$

Perturbation in $f(R)$ Gravity

- Inside the subhorizon limit ($k^2 \gg \mathcal{H}^2$), the perturbation equations reduce to

$$\frac{k^2}{a^2}\Psi = -4\pi G\ \mu(k,a)\ \rho\Delta\,, \qquad \frac{\Phi}{\Psi} = \gamma(k,a)\,.$$

where

$$\mu(k,a) = \frac{1}{f_R}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}{1+3\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}\,, \qquad \gamma(k,a) = \frac{1+2\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}{1+4\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}\,,$$

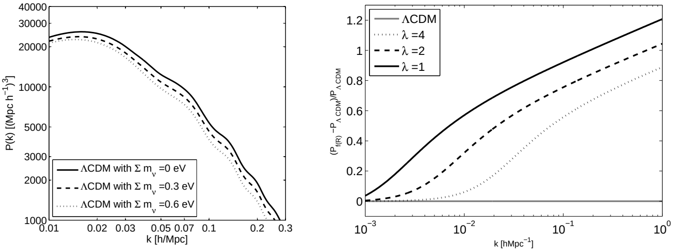
and $\Delta = \frac{\delta\rho}{\rho} + 3\frac{\mathcal{H}}{k}(1+w)$ is the gauge-invariant matter density perturbation.

- In ΛCDM limit ($\mu = \gamma = 1$):

$$\frac{k^2}{a^2}\Psi = -4\pi G\rho\Delta\,, \qquad \Psi = \Phi\,.$$

Perturbation in $f(R)$ Gravity

- Matter density perturbation:
  $\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\,\mu(k,a)\,\rho_m\delta_m = 0.$
- The matter power spectra between $f(R)$ and ΛCDM models with Starobinsky (n=2) models.



Perturbation in $f(R)$ Gravity

- CAMB & MGCAMB[1]: Synchronous gauge $ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})\, dx^i dx^j\right]$.
- The scalar perutrbation of h_{ij} in k-space,

$$h_{ij}(\vec{x},\tau) = \int d^3k e^{-i\vec{k}\vec{x}} \left[\hat{k}_i\hat{k}_j h(\vec{k},\tau) + \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right) 6\eta(\vec{k},\tau)\right] ,$$

where $h(\vec{k},\tau)$ and $\eta(\vec{k},\tau)$ are the scalar perturbation in k-space, and $h(\vec{k},\tau)$ denotes the trace of h_{ij}.

[1] A. Hojjati, G.B. Zhao, L. Pogosian and A. Silvestri, JCAP **1108** 005, http://www.sfu.ca/ aha25/MGCAMB.html

Perturbation in $f(R)$ Gravity

- Newtonian and synchronous gauge are related under the coordinate transformation $\hat{x}^\mu \to x^\mu + d^\mu$. Thus,

$$\Psi = \dot{\alpha} + \mathcal{H}\alpha\,,$$
$$\Phi = \eta - \mathcal{H}\alpha\,,$$

  where $\alpha = \left(\dot{h} + 6\dot{\eta}\right)/2k^2$.

- The gauge invariant equations:

$$\frac{k^2}{a^2}\Psi = -4\pi G\ \mu(k,a)\ \rho\Delta\,, \qquad \frac{\Phi}{\Psi} = \gamma(k,a)\,.$$

Perturbation in $f(R)$ Gravity

- The tensor perturbation:

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + D_{ij})dx^i dx^j\right] ,$$

where D_{ij} is a traceless divergence free tensor field.

- The tensor mode evolution:

$$D''_{ij} + \left(2\mathcal{H} + \frac{F'_R}{F_R}\right) D'_{ij} + k^2 D_{ij} = \frac{a^2 \pi^T_{ij}}{M^2_{\mathsf{Pl}} F_R} ,$$

where $\mathcal{H} = a^{-1} da/d\tau$, the prime denotes the derivative with respect to the conformal time, and π^T_{ij} is the tensor perturbation of T_{ij}.

Observational Constraints

- Combined $f(R)$ models: $f(R) = R + R^{2-q}/M^{2-2q} + F(R)$
  - Starobinsky model:

$$F(R) = -\lambda R_{\mathrm{S}} \left[1 - \left(1 + \frac{R^2}{R_{\mathrm{S}}^2} \right)^{-n} \right] .$$

  - Exponential model:

$$F(R) = -\beta R_s \left(1 - e^{-R/R_s} \right) .$$

- CosmoMC program with MGCAMB.
- Dataset:
  - CMB: Planck ($l < 50$ and $50 < l < 2500$) and WMAP (low-l).
  - BAO: BOSS (Baryon Oscillation Spectroscopic Survey).
  - SNIa: SNLS (Supernova Legacy Survey).

Observational Constraints

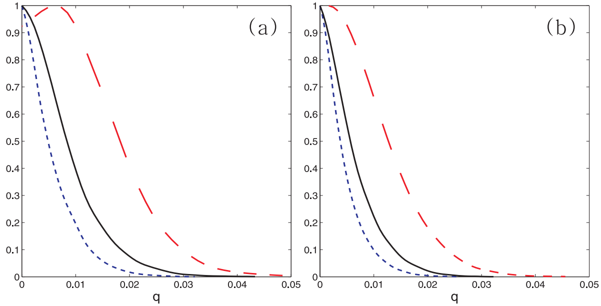
- The primordial scalar and tensor perturbations

$$\ln \mathcal{P}_s(k) = \ln A_s + (n_s - 1) \ln \left(\frac{k}{k_s}\right) + \alpha_s \left[\ln \left(\frac{k}{k_s}\right)\right]^2 .$$

$$\ln \mathcal{P}_t(k) = \ln A_t + n_t \ln \left(\frac{k}{k_s}\right) .$$

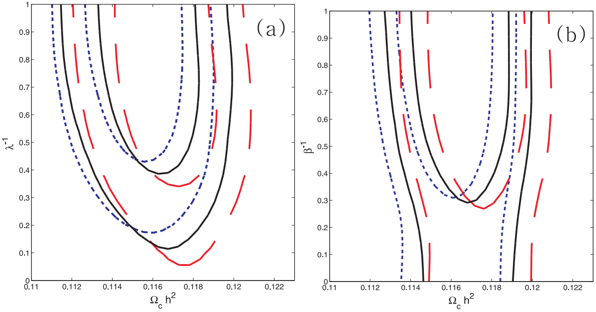
Observational Constraints

- Marginalized probability for the inflation power parameter q in (a) Starobinsky and (b) exponential gravity models, where the long-dashed, solid and dashed lines correspond to $N = 50$, 60 and 70, respectively.



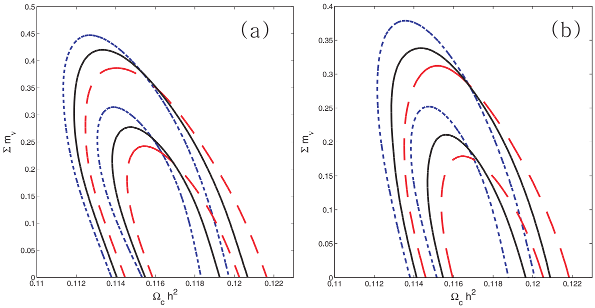
Observational Constraints

- Contour plots in the planes (a) $\lambda^{-1} - \Omega_c h^2$ for Starobinsky and (b) $\beta^{-1} - \Omega_c h^2$ for exponential gravity models, where the inner and outer curves represent $1-\sigma$ and $2-\sigma$ confidence levels, while the long-dashed, solid and dashed lines correspond to $N = 50$, 60 and 70, respectively.



Observational Constraints

- Contour plots in the planes $\Sigma m_\nu - \Omega_c h^2$ for (a) Starobinsky and (b) exponential gravity models, where the inner and outer curves represent $1-\sigma$ and $2-\sigma$ confidence levels, while the long-dashed, solid and dashed lines correspond to $N = 50$, 60 and 70, respectively.



Observational Constraints

- Cosmological parameters with 95% CL and $\xi \equiv 1 - \lambda^{-1}\left(1 - \beta^{-1}\right)$ in the Starobinsky (exponential) model with 68% CL.

Parameters	Starobinsky	Exponential	ΛCDM
$100\Omega_b h^2$	$2.25^{+0.04}_{-0.05}$	$2.24^{+0.04}_{-0.05}$	$2.20^{+0.06}_{-0.03}$
$\Omega_c h^2$	$0.118^{+0.001}_{-0.05}$	$0.118^{+0.001}_{-0.004}$	0.118 ± 0.003
$\Sigma m_\nu/\mathrm{eV}$	$0.063^{+0.266}_{-0.063}$	< 0.264	< 0.211
$100q$	< 1.86	< 1.43	–
n_s	$0.970^{+0.009}_{-0.002}$	$0.970^{+0.007}_{-0.002}$	$0.963^{+0.012}_{-0.009}$
$10^3 r$	< 6.69	< 5.63	< 125
σ_8	$1.131^{+0.038}_{-0.133}$	$0.963^{+0.021}_{-0.151}$	$0.833^{+0.024}_{-0.059}$
ξ	$0.287^{+0.111}_{-0.287}$	$0.132^{+0.331}_{-0.132}$	–
$\Delta\chi^2$	-2.61	-0.88	–

Conclusion of $f(R)$ Gravity

- The $f(R)$ dark energy is good scenario in realizing the dark energy problem.
- The observational data prefers the R^2 inflation.
- The allowed neutrino masses are enhanced.

Outline

Quintessential Inflation

- Quintessential Inflation:
  - One of the simplest scenario, which unified the inflation and dark energy problems.
  - This model alleviates the coincident problem.
- The action of quintessential Inflation:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{R}{2\kappa^2} + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + V(\phi)\right] + \mathcal{S}_m + \mathcal{S}_r + \mathcal{S}_\nu\left(\mathcal{C}^2(\phi)g_{\alpha\beta},\ \Psi_\nu\right)\ ,$$

where $V(\phi)$ is the potential and $S_{m,r,\nu}$ correspond to the actions of matter, radiation and neutrino, respectively.

Quintessential Inflation

- The Friedmann equations:

$$3H^2 = \kappa^2 \left(\frac{\dot{\phi}^2}{2} + V(\phi) + \rho_m + \rho_r + \rho_\nu \right) ,$$

$$2\dot{H} + 3H^2 = -\kappa^2 \left(\frac{\dot{\phi}^2}{2} + V(\phi) - P_m - P_r - P_\nu \right) .$$

- The scalar field equation:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV_{\rm eff}}{d\phi} = 0 \, ,$$

where $V_{\rm eff}$ is the effective potential such that $dV_{\rm eff}/d\phi = dV(\phi)/d\phi + \kappa\beta(\rho_\nu - 3P_\nu)$.

Quintessential Inflation

- The neutrino matter is non-minimally coupled to the scalar,

$$\dot{\rho}_\nu + 3H(\rho_\nu + P_\nu) = \kappa\beta\dot{\phi}(\rho_\nu - 3P_\nu)\,.$$

- The neutrino mass becomes time dependent,

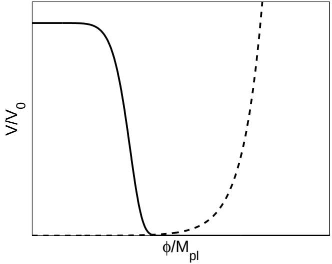
$$m_{\nu,\mathrm{eff}}(\phi) = m_{\nu,0}e^{\beta\kappa\phi}\,.$$

- The effective potential can be defined in an explicit form,

$$V_{\mathrm{eff}}(\phi) = V(\phi) + (\rho_{\nu,0} - 3P_{\nu,0})\,e^{\beta\kappa\phi}\,.$$

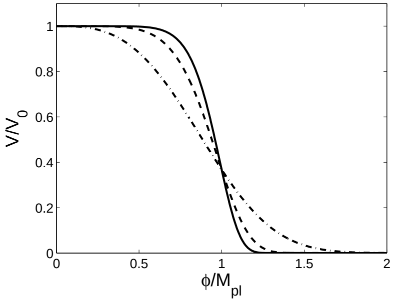
Quintessential Inflation

- Basic idea:
  - Inflation: potential is shallow in the early time.
  - Non-standerd reheating era: instant of preheating mechanism.
  - Radiation and matter dominated epoch: potential is steep and the scalar field obeys the scaling solution.
  - Dark energy dominated epoch: the massive neutrino contributes an effective potential.



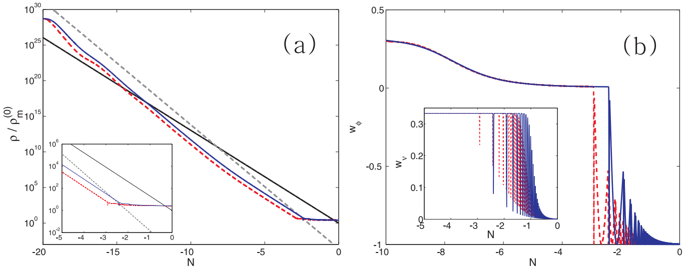
Quintessential Inflation

- Very few potentials are able to describe the inflation and dark energy.
- The generalize exponential potential $V(\phi) = V_0 e^{-\lambda(\kappa\phi)^n}$ with $\lambda = 1$, where $n = 9$ (solid line), 6 (dashed line) and 3 (dot-dashed line), respectively.



Post inflationary evolution

- Fig.a: evolution of energy densities $\rho_r/\rho_m^{(0)}$ (gray-dashed), $\rho_m/\rho_m^{(0)}$ (black-solid) and $\rho_\phi/\rho_m^{(0)}$ as functions of $N \equiv \ln a$ with $\lambda = 10^{-8}$ (blue-solid) and 10^{-6} (red-dashed).
- Fig.b: the equation-of-state w_ϕ and w_ν as functions of N with $\Sigma m_\nu = 0.45$ eV, $\Omega_m h^2 = 0.118$ and $\rho_r^{(0)}/\rho_m^{(0)} = 2.6 \times 10^{-4}$.



Quintessential Inflation

- Slow-roll parameters:

$$\epsilon \equiv \frac{M_{\mathsf{Pl}}^2}{2}\left(\frac{V_\phi}{V}\right)^2,\ \eta \equiv M_{\mathsf{Pl}}^2\frac{V_{\phi\phi}}{V},\ \xi \equiv M_{Pl}^4\frac{V_\phi V_{\phi\phi\phi}}{V^2}\,.$$

- Inflation observables: scalar and tensor spectral index (n_s, n_t), tensor-to-scalar ratio r, scalar spectral index running α_s.

$$\begin{aligned}
n_s - 1 &= -6\epsilon + 2\eta\,,\\
n_t &= -2\epsilon\,,\\
r &= 16\epsilon\,,\\
\alpha_s &= 16\epsilon\eta - 24\epsilon^2 - 2\xi\,.
\end{aligned}$$

Quintessential Inflation

- The number of the e-folding,

$$\mathcal{N} = -M_{\rm Pl}^{-2}\int_{\phi}^{\phi_{end}} \frac{V}{V_\phi} d\phi' = \frac{1}{n\lambda(n-2)}\left[\left(\frac{\phi}{M_{\rm Pl}}\right)^{2-n} - \left(\frac{2}{n^2\lambda^2}\right)^{\frac{2-n}{2n-2}}\right].$$

- The scalar at the beginning of inflation,

$$\frac{\phi}{M_{\rm Pl}} = \left[n(n-2)\lambda\mathcal{N} + \left(\frac{2}{n^2\lambda^2}\right)^{\frac{2-n}{2n-2}}\right]^{\frac{1}{2-n}}.$$

Quintessential Inflation

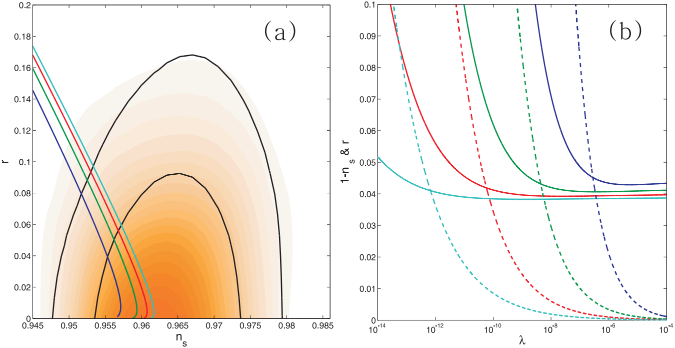
- With $\lambda \ll 1$, $n > 2$ and $\mathcal{N} \gg 1$, one can simplify the expressions $\phi/M_{\rm Pl} \simeq [n(n-2)\lambda\mathcal{N}]^{1/2-n}$.
- The inflation observables,

$$
\begin{aligned}
n_s - 1 &\simeq -\frac{2(n-1)}{(n-2)\mathcal{N}} - \frac{[n(n-2)\lambda\mathcal{N}]^{\frac{-2}{n-2}}}{(n-2)^2\mathcal{N}^2} \lesssim -\frac{2(n-1)}{(n-2)\mathcal{N}}\,,\\
n_t &\simeq -\frac{[n(n-2)\lambda\mathcal{N}]^{\frac{-2}{n-2}}}{(n-2)^2\mathcal{N}^2} \lesssim 0\,,\\
r &\simeq -8n_t \gtrsim 0\,,\\
\alpha_s &\simeq -\frac{2(n-1)}{(n-2)\mathcal{N}^2} + \frac{6(n-1)\,[n(n-2)\lambda\mathcal{N}]^{\frac{-2}{n-2}}}{(n-2)^3\mathcal{N}^3} \gtrsim -\frac{2(n-1)}{(n-2)\mathcal{N}^2}\,,
\end{aligned}
$$

corresponding to a upper bound on n_s and n_t; lower bound r and α_s.

Quintessential Inflation

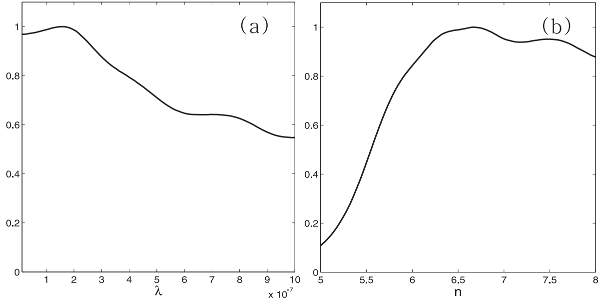
- $V(\phi) = V_0 e^{-\lambda(\kappa\phi)^n}$, $\mathcal{N} = 60$ and $\lambda \leq 10^{-4}$.
- The blue, green, red and cyan lines correspond to n =5, 6, 7 and 8, respectively.
- Contours: the 1σ and 2σ bounds in the ΛCDM model.
- $1 - n_s$ (solid line) and r (dashed line) in Fig.(b).



Outline

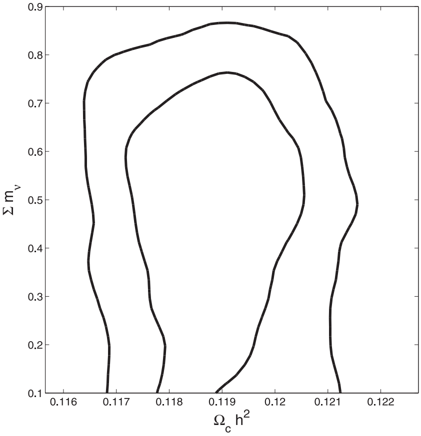
Observational Constraints

- Marginalized probabilities for the potential parameters of λ and n with $V(\phi) = V_0 e^{-\lambda(\kappa\phi)^n}$.



Observational Constraints

- Likelihood contours of the mass sum of the three neutrino species $\Sigma m_\nu^{z=0}$ in eV.



Observational Constraints

- Cosmological parameters with 95% CL and $V(\phi) = V_0 e^{-\lambda(\phi/M_{\rm Pl})^n}$.

Parameter	Quintessential Inflation	ΛCDM
Baryon density $100\Omega_b h^2$	$2.21^{+0.04}_{-0.05}$	$2.21^{+0.05}_{-0.04}$
CDM density $\Omega_c h^2$	0.119 ± 0.002	0.118 ± 0.003
Neutrino mass Σm_ν/eV	$0.473^{+0.228}_{-0.373}$	< 0.211
Optical depth τ	$0.0898^{+0.0242}_{-0.0228}$	$0.0912^{+0.0258}_{-0.0239}$
Model parameter n	$6.74^{+1.08}_{-0.59}$	–
Model parameter λ	$< 6.07 \times 10^{-7}$	–

Conclusion of Quintessential Inflation

- This model is in excellent agreement with observations and presents a successful scheme of unification of inflation and dark energy.
- The allowed neutrino masses is enhanced to 0.5 eV in this model.

Thank You!!