The IR obstruction to UV completion for Dante's Inferno Model with Higher-Dimensional Gauge Theory Origin

Yoji Koyama (NCTS)
and Kazuyuki Furuuchi (MCNS, India)

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Plan of talk

1. Introduction and Motivation
   1-1. our previous study, 1-2. Dante’s Inferno

2. Dante’s Inferno model with 5D gauge theory origin-1

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6. Connection to DBI action (if there will be time)

7. Summary

1. Introduction and Motivation

- Recent observation for the tensor to scalar ratio:

$$(r \simeq 0.16 \to) \quad r < 0.12$$

[BICEP2, Planck collaboration, 15]

More recently, $r < 0.09$. [BICEP/Keck Array,15]

There is still a possibility of large field inflation (Lyth bound)

$$\Delta\phi > M_P \ (r \gtrsim 10^{-3})$$

- We are interested in large field inflation model based on higher-dimensional (5D) gauge theories as a solution to the fine-tuning problem in inflation which I will explain later.

- In extra natural inflation, the weak gravity conjecture (WGC) restricts axion field range to be sub-Planckian.

[Arkani-Hamed et al, 06]

- Taking into account of WGC, previously, we studied the models that realize the super-Plankian field excursion effectively from the sub-Planckian field excursion of the original fields.

[Furuuchi & YK, 14]

- We found that Dante's Inferno (DI) model is the most preferred model in point of view of the naturalness of the 5D massive gauge theory parameters.

- However DI model for a simple chaotic inflation $V(\phi) = m^2\phi^2$ is (modestly) disfavored by the current observational bound $r < 0.12$ since $V(\phi) = m^2\phi^2 \to r \simeq 0.16$.

What we have done in this work

- We consider $V = m^2\phi^2 - \lambda\phi^4$ in order to accommodate DI model with 5D massive gauge theory origin to the updated upper bound on r .

- We examine a criterion for effective field theories to be embedded in a consistent UV theory:

  The IR obstruction to UV completion [Adams et al, 06]

- Connection to DBI action

- **Fine-tuning problem in inflation (like in the Higgs potential)**

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \; |\eta| = \frac{M_P^2}{2}\left|\frac{V''}{V}\right| \ll 1 \qquad r = P_\zeta / P_h < 0.12$$

$$\mathcal{P}_\zeta \sim V(\phi)/M_p^4 \epsilon \simeq 10^{-9} \qquad n_s = 1 - 6\epsilon_* + 2\eta_* \quad n_s \simeq 0.96$$

$$N \equiv \int_{t_i}^{t_f} H dt \simeq \frac{1}{M_P^2}\left|\int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi\right| \simeq 50 - 60$$

Inflation parameters

- Chaotic inflation (Large field inflation) $\lambda_n \phi^n$

$$m^2 \sim 10^{13}\mathrm{GeV} \ll M_P^2 \; (n=2) \qquad \lambda \simeq 10^{-12} \; (n=4)$$

Including quantum effects cause fine-tuning in the parameters, and serious divergent terms may appear.

$$V(\phi) = \underline{m_{\rm eff}^2 \phi^2} + \underline{\lambda_{\rm eff}\phi^4} + \underline{c_6 \frac{\phi^6}{M_P^2}} + \underline{c_8 \frac{\phi^8}{M_P^4}} + \cdots . \qquad (\Lambda_{UV} = M_P)$$

- In this situation, natural inflation model was constructed.

[Freese etal,90]

$$\Lambda^4(1-\cos(\phi/f))$$

- Naturalness in the sense of 't Hooft ['tHooft,79]

  - Physical parameters are allowed to be very small only if the replacement them by 0 would increase the symmetry of the system

  $\Lambda/f \ll 1$ is natural because of approximate shift symmetry

But,

$f > M_P$ ➡ SSB scale of PQ symmetry is above M_P

Quantum gravity correction can not be controllable

Extranatural inflation (an improvement of natural inflation)

[Arkani-Hamed et al, 03]

- 5D $U(1)$ gauge theory with massless matters on $M_4 \times S^1$
  All fields satisfy periodic boundary condition.

Classical $\quad V(A_5^{(0)}) = 0\,,\ m_{A_5^{(0)}} = 0$

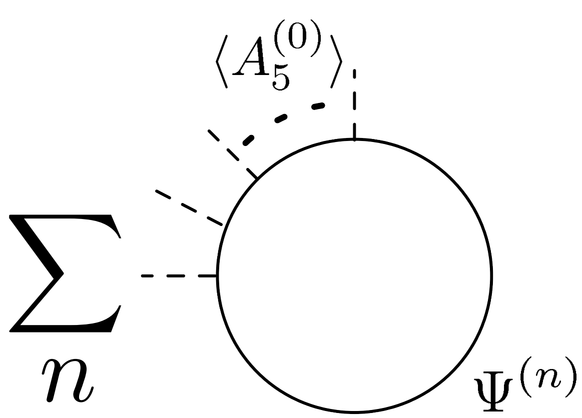
Quantum(1-loop) $\quad \langle A_5^{(0)} \rangle = \dfrac{\theta}{g(2\pi R)},$

g : 4D gauge coupling

L : S^1 radius

$$\theta = g \int_0^{2\pi L} dy \langle A_5^{(0)} \rangle$$: Wilson line phase

1-loop diagram



All KK modes should be taken into account.

- Although the theory is unrenormalizable, the effective potential of $A_5^{(0)}$ is obtained as a finite quantity

$$V(\phi)_{1-\mathrm{loop}} = \frac{3c}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos\left[n \left(\frac{\phi}{f} \right) \right]$$

$$\boxed{\langle A_5^{(0)} \rangle \sim \phi}$$

$$f \equiv 1/(2\pi g R)$$
$$L = 2\pi R$$

- Consequences of the extrantural inflation:

$f > M_P$ ➡ Large field inflation

Validity of the Effective Field Theory, and slow-roll inflation imply

$R^{-1} < M_P \;, \quad m_\phi \sim g^2 f \ll H_*$ ➡ $g \lesssim 10^{-3}$

- We can realize a large field inflation scenario within the 4D effective died theory but a few questions remain.
  - — Tiny gauge coupling constant seems unnatural.
  - — Potential value is sub-Planckian, but the field excursion of the field is still super-Planckian.

- Although extranatural inflation is a good realization of large field inflation, it is difficult to embed it to UV completion theory (String theory) due to tiny gauge coupling: it may cause an obstacle for coupling EFT to gravity
  - — Weak gravity conjecture [Arkani-Hamed et al, 06]

Weak gravity conjecture (WGC) [Arkani-Hamed et al, 06]

- WGC asserts the existence of a state with charge q and mass satisfying

$$\frac{gq}{\sqrt{4\pi}} \geq \sqrt{G_N} m = \frac{m}{\sqrt{8\pi} M_P} \qquad g : U(1) \text{ coupling}$$

- Monopole with unit magnetic charge

$$q_m = \frac{4\pi}{g}, \quad m_m \simeq \frac{4\pi \Lambda_{UV}}{g^2}$$

WGC reads

$$\Lambda_{UV} \lesssim \sqrt{2} g M_P$$

Usage of higher-dim. gauge theory is justified if $L^{-1} < \Lambda_{UV}$

$$\Rightarrow \boxed{2\pi f \lesssim M_P \qquad f = 1/(2\pi g L)}$$

X
- Extremal black holes can loose their charge by emitting such particles. If no such particles, there will be infinite number of BH remnants.

1-1. Our previous study

- Previously we have studied large field inflation models from higher-dim. gauge theories.

- Important theoretical ingredients in our EFT approach were
  - naturalness of gauge theory parameters
  - weak gravity conjecture(WGC) $2\pi f \lesssim M_P \quad f = 1/(2\pi g L)$
    $((M_P L)^{-1} \lesssim g)$

- The models we studied were those in which the defining theories are sub-Planckian but inflation effectively travels trans-Planckian field range. Good inflaton potentials had already been proposed.
  - Single axion monodromy
  - Dante's Inferno
  - Axion alignment
  - Axion hierarchy

(Typical models)

Expected parameter range

Gauge couplings	Compactification radius	Charges
$-\log_{10}[(LM_P)^2] \lesssim \log_{10}[g^2] \lesssim 0$	$\log_{10}[1/(L\,\mathrm{GeV})] \sim 3-17$	$n \sim \mathcal{O}(1)$

Expected parameter ranges from higher dimensional gauge theory.

- The lower bound in g is imposed by WGC, while the upper bound comes from applicability of perturbation theory.
- The expected value of charges is in unit of the minimal charge in the model. (Extraordinary large charge is unlikely or rare in nature)

Resultant parameter range in the previous study

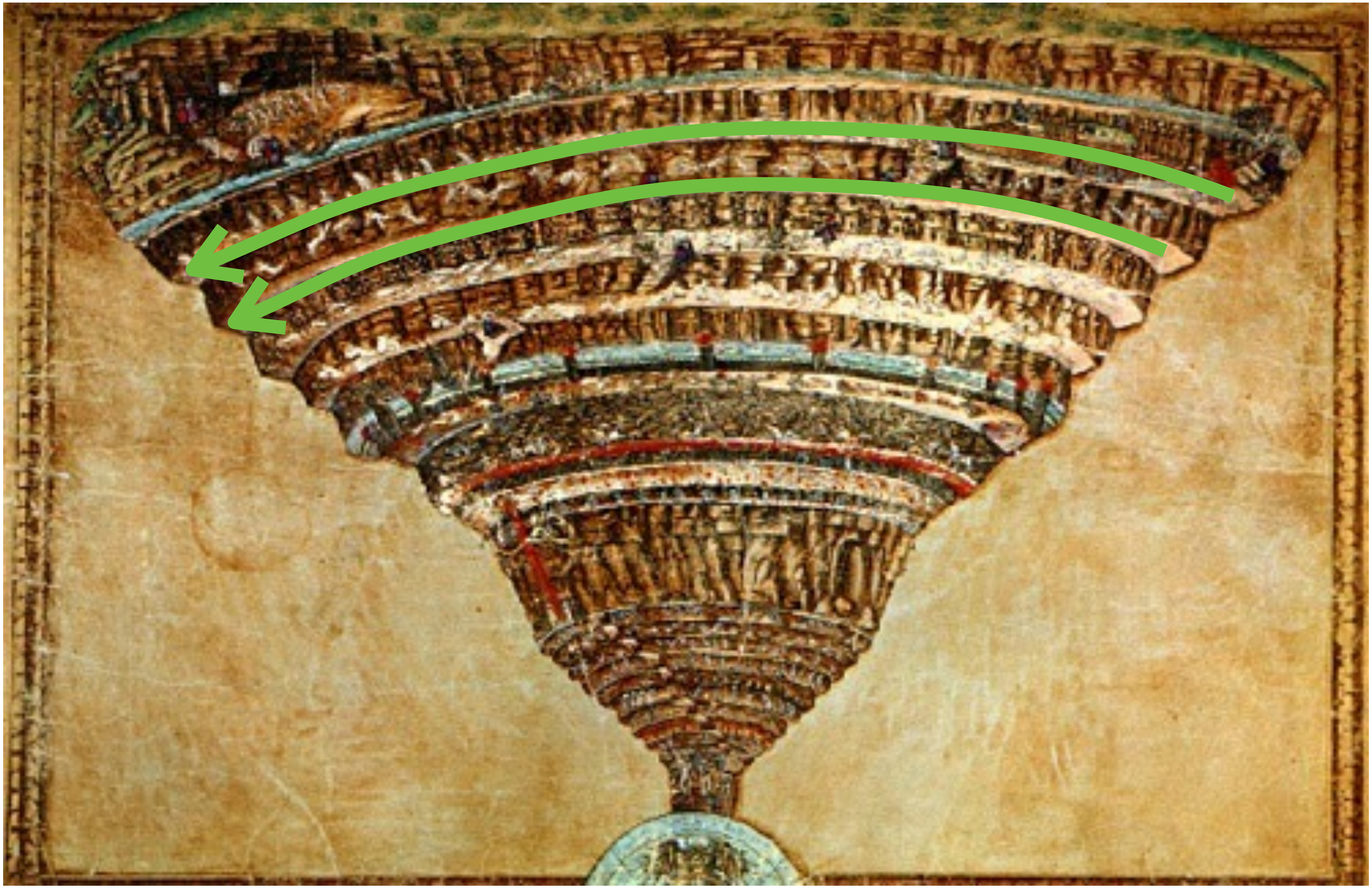
Model	Gauge coupling(s)	Compactification radius
AM	$-8 \lesssim \log_{10}[g^2] \lesssim 0$	$\log_{10}[1/(L\,\mathrm{GeV})] \sim 14-16$
DI	$-1 \lesssim \log_{10}[g_A^2] \lesssim 0$, $-3 \lesssim \log_{10}[g_B^2] \lesssim -2$	$\log_{10}[1/(L\,\mathrm{GeV})] \sim 17$
AA	$-10 \lesssim \log_{10}[g_A^2], \log_{10}[g_B^2] \lesssim -4$	$\log_{10}[1/(L\,\mathrm{GeV})] \sim 14-17$
AH	$-10 \lesssim \log_{10}[g_A^2] \lesssim -4$, $-10 \lesssim \log_{10}[g_B^2] \lesssim 0$	$\log_{10}[1/(L\,\mathrm{GeV})] \sim 14-17$

Model	Charge(s)
AM	$\mathcal{O}(1)$
DI	$\mathcal{O}(1)$
AA	$\max(\lvert m_1, m_2 \rvert) \gtrsim \mathcal{O}(100)$
AH	$m_1 \gtrsim \mathcal{O}(100)$

- Axion Monodromy (AM)
- Dante's Inferno (DI)
- Axion Alignment (AA)
- Axion Hierarchy (AH)

1-2. Dante's Inferno(DI) [Berg et al, 08]

all fundamental quantities can be sub-Planckian



[http://hellinspace.com]

Dante's Inferno [Berg et al, 08]

- Dante's Inferno model is described by the potential

$$V_{DI}(A,B) = V_A(A) + \Lambda^4 \left\{1 - \cos\left(\frac{A}{f_A} - \frac{B}{f_B}\right)\right\}$$

For an illustration, let us assume $V_A(A) = \frac{1}{2} m_A^2 A^2$ (our previous study)

- To see the inflation, it is convenient to rotate the fields as

$$\begin{pmatrix} \tilde{B} \\ \tilde{A} \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} B \\ A \end{pmatrix}, \qquad \sin\xi = \frac{f_A}{\sqrt{f_A^2 + f_B^2}}, \quad \cos\xi = \frac{f_B}{\sqrt{f_A^2 + f_B^2}}.$$

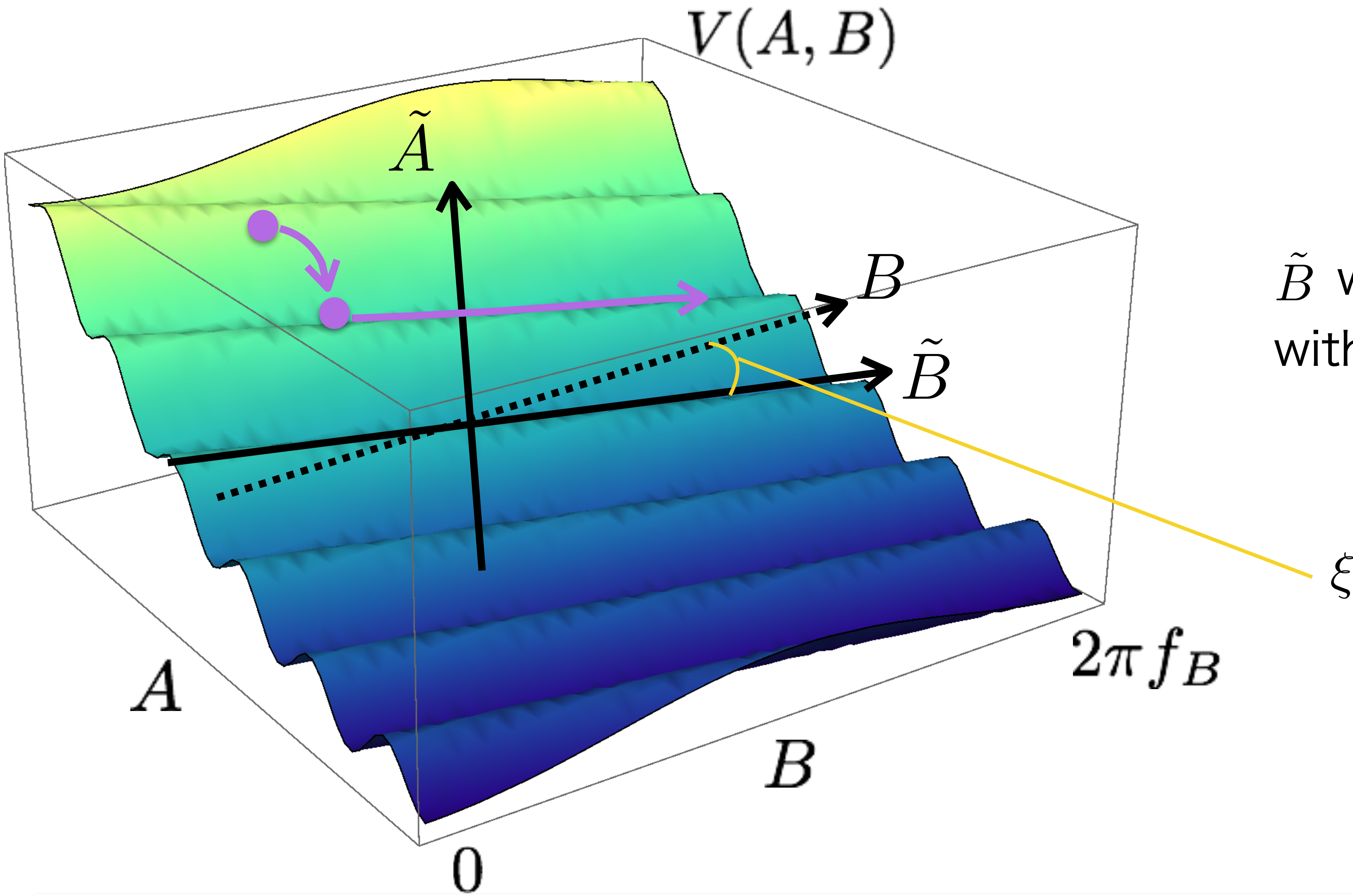
Then the potential takes the form,

$$V(\tilde{A},\tilde{B}) = \frac{m_A^2}{2}\left(\tilde{A}\cos\xi + \tilde{B}\sin\xi\right)^2 + \Lambda^4\left(1 - \cos\frac{\tilde{A}}{f}\right), \qquad f \equiv \frac{f_A f_B}{\sqrt{f_A^2 + f_B^2}}$$

In this model, the regime of interest is consistent with WGC

$$2\pi f_A \ll 2\pi f_B \lesssim M_P \qquad \cos\xi \simeq 1, \quad \sin\xi \simeq \frac{f_A}{f_B}, \quad f \simeq f_A$$

The potential of DI model: $V_{DI}(A,B) = V_A(A) + \Lambda^4 \left\{1 - \cos\left(\frac{A}{f_A} - \frac{B}{f_B}\right)\right\}$



$\tilde{B}$ will be identified with the inflaton.

Under the conditions, DI model shows the layered structure in the potential. $\frac{\Lambda^4}{f} \gg m_A^2 A_{in}\,, \quad \frac{\partial^2}{\partial \tilde{A}^2} V(\tilde{A}, \tilde{B}) > H^2$

- Model constraints for DI:

$$\frac{\Lambda^4}{f} \gg m_A^2 A_{in}\,, \quad \frac{\partial^2}{\partial \tilde{A}^2} V(\tilde{A}, \tilde{B}) > H^2$$

$\Rightarrow$ $\tilde{A}$ is heavy and can be integrated out, then inflation occurs along the bottom of the sinusoidal potential, namely $\tilde{B}$ dependent minimum

- DI model is effectively reduced to chaotic model, identifying $\tilde{B}$ with inflaton, $\phi \equiv \tilde{B}$.

$$V_{\rm eff}(\phi) = \frac{m^2}{2}\phi^2, \quad m \equiv \frac{f_A}{f_B} m_A\,, \qquad \text{with} \quad A \sim \frac{f_B}{f_A}\phi$$

$\Delta\phi \simeq 13 M_P$ and $m \simeq 10^{13}{\rm GeV}$ can be realized thanks to the factor f_A/f_B even if $\Delta A, \Delta B < M_P$, and the original mass $m_A \gtrsim H$.

What we have done in this work (again)

- We consider $V = m^2\phi^2 - \lambda\phi^4$ in order to accommodate DI model with 5D massive gauge theory origin to the updated upper bound on r .

- We examine a criterion for effective field theories to be embedded in a consistent UV theory:

  The IR obstruction to UV completion [Adams et al, 06]

- Connection to DBI action

2. Dante's Inferno model with 5D gauge theory origin-1

- Dante's Inferno model: $V_{DI}(A,B) = V_A(A) + \Lambda^4 \left\{ 1 - \cos\left(\frac{A}{f_A} - \frac{B}{f_B} \right) \right\}$

- This potential is derived from a 5D $U(1)$ gauge theory on $M^4 \times S^1$

$$S = \int d^5x \Big[-\frac{1}{4} F^{(A)}_{MN} F^{(A)MN} - \underline{V_A(\mathcal{A}_M)} - \frac{1}{4} F^{(B)}_{MN} F^{(B)MN}$$

$$- i\bar{\psi}\gamma^M \left(\partial_M + i g_{A5} A_M - i g_{B5} B_M \right) \psi \Big] \quad (M, N = 0, 1, 2, 3, 5)$$

where

$$\mathcal{A}_M = A_M - g_{A5} \partial_M \theta \qquad \theta \text{ : Stueckelberg field}$$

and a matter has two kinds of charge belonging to $U_A(1)$ and $U_B(1)$.

- Two scalar fields A, B are identified with $A_5^{(0)}, B_5^{(0)}$, respectively.

$$A,\ B \equiv \sqrt{2\pi L_5} A_5^{(0)},\ \sqrt{2\pi L_5} B_5^{(0)}$$

- Note that the form of $V_A(A)$ in $V_{DI}(A,B)$ will have the same form as $V_A(\mathcal{A}_M)$ after the dimensional reduction.

- We consider the potential for the massive gauge field:

$$V_A(\mathcal{A}_M) = v_2 \mathcal{A}_M \mathcal{A}^M + v_4 (\mathcal{A}_M \mathcal{A}^M)^2 + v_6 (\mathcal{A}_M \mathcal{A}^M)^3 \cdots = \sum_{n=1}^{\infty} v_{2n} (\mathcal{A}_M \mathcal{A}^M)^n$$

It is here that the IR obstruction to UV completion is relevant.

The point is that the following sign constraints are derived from the condition that massive gauge theory to be embedded to a UV theory with canonical analyticity property for the S-matrix:

$$v_2, v_4 < 0$$

[A. Hashimoto, 08]

* Our metric convention is $\eta_{MN} = \mathrm{diag}(+ - - - -)$

$$A_M A^M = A_\mu A^\mu - A_5^2$$

3. The IR obstruction to UV completion for massive gauge field

[Adams et al, 06] [A. Hashimoto, 08]

i) To UV completion

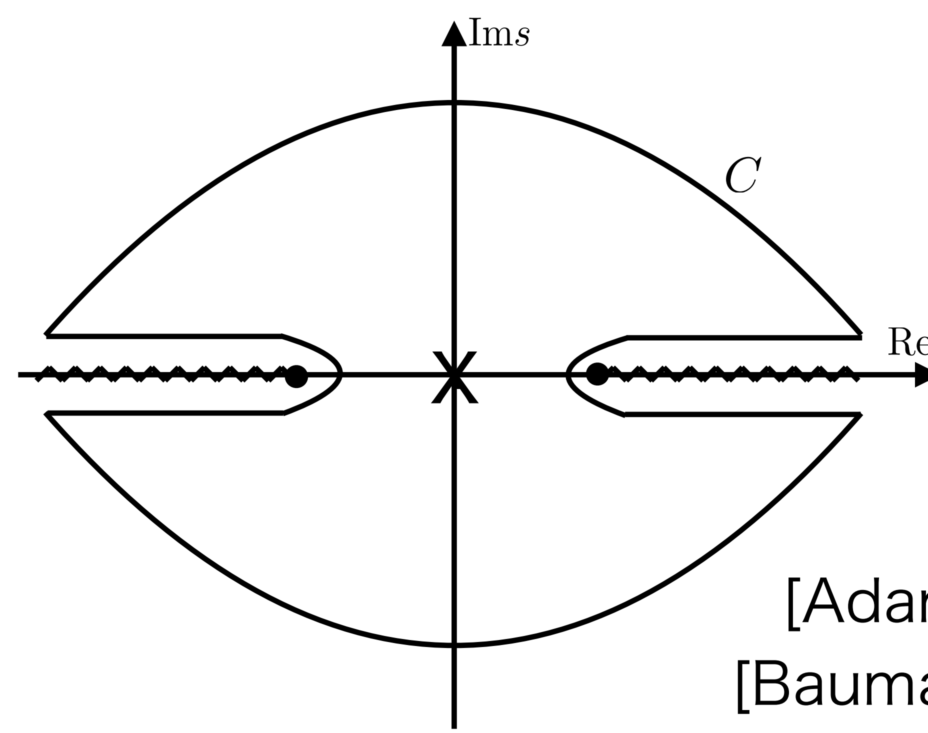
Let us focus on the forward scattering amplitude
$\mathcal{M}(s, t=0) \equiv \mathcal{A}(s)$ of 2 → 2 scattering.

The analytic property of the S-matrix: $\mathrm{Im}\,\mathcal{A}(s)$ appears as a discontinuity across the branch cut singularity on the real axis, associated with on-shell intermediate states.

$$\mathrm{Disc}[\mathcal{A}(s)] = 2i\mathrm{Im}\mathcal{A}(s)$$

Consider the Cauchy theorem

$$\frac{1}{2}\partial_s^2 \mathcal{A}(s \to 0) = \oint_C \frac{ds}{2\pi i} \frac{\mathcal{A}(s)}{s^3}$$



[Adams et al, 06]
[Baumann et al,15]

- Using the analytic property and the unitarity (optical theorem) of the S-matrix, in relativistic field theories, it holds that

$$\frac{1}{2}\partial_s^2\mathcal{A}(s\to 0)=\oint_C \frac{ds}{2\pi i}\frac{\mathcal{A}(s)}{s^3}\quad\longrightarrow\quad \partial_s^2\mathcal{A}(s\to 0)=\frac{4}{\pi}\int_0^\infty ds\frac{\sigma(s)}{s^2}$$

LHS is the IR limit of the forward scattering.
RHS, which is manifestly positive since $\sigma(s)>0$, is given by the pole structure of the UV theory.

➡ The sign of coupling constants of IR effective theory is constrained by the analytic property of the S-matrix in the UV theory

[Adams et al, 06]

ii) The IR obstruction

- Superluminal propagation in a certain background.

- i) and ii) give the same constraint on the sign of coupling constant

[Adams et al, 06]

- In the original paper of Adams etal., scalar field with shift sym. was considered as an example

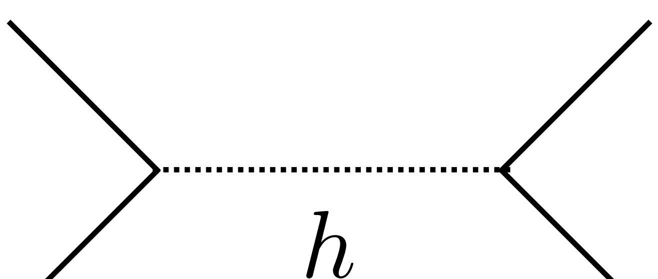
$$\mathcal{L} = \partial_\mu \pi \partial^\mu \pi + \frac{c}{\Lambda^4}(\partial_\mu \pi \partial^\mu \pi)^2 + \cdots$$

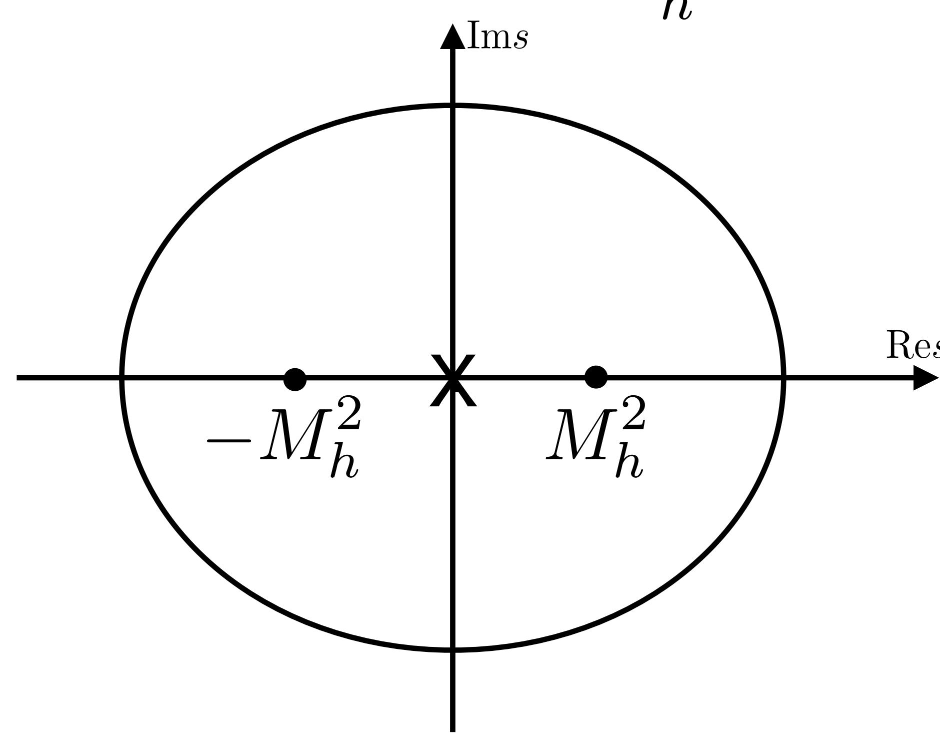
This Lagrangian can be understood as an effective theory for NG

$$\mathcal{L} = \left(1 + \frac{h}{v}\right)(\partial \pi)^2 + (\partial h)^2 - M_h^2 h^2 - \cdots$$ "UV" action

$$V(\Phi) = \lambda(|\Phi|^2 - v^2)^2 \qquad \Phi = (v+h)e^{i\pi/v}$$

Integrating out h at tree level $\mathcal{L}_{\rm eff} = \frac{\lambda}{M_h^4}(\partial\pi)^4 + \cdots \qquad \lambda > 0$





$\rightarrow \quad c > 0$

for UV completion

- Causal propagation (no superluminal propagation) $\rightarrow \; c > 0$

- In the current massive gauge theory case

i) <u>To UV completion</u>

$\mathcal{A}(s)$ for 2 → 2 scattering of the longitudinal mode of A_M is

$$\mathcal{A}(s) \propto -\frac{v_4}{v_2^2}(s^2 + \mathcal{O}(s)) \quad \Longrightarrow \quad v_4 < 0 \quad \text{for UV completion}$$

(The sign is kept)

[A. Hashimoto, 08]

ii) <u>The IR obstruction</u>

If $v_4 > 0$, the IR pathology appears as the superluminal fluctuation of massive gauge field around certain (Lorentz symmetry breaking) backgrounds. [Velo et al, 79]

In order to have the causal propagation of massive gauge field, it is again required $v_4 < 0$.

For a model which has a sound IR behavior as well as an origin in sane UV theory, we assume that $v_2, v_4 < 0$ is satisfied.

- In what follows, we set $v_{2n=0}$ for $n > 2$,

$$V_A(\mathcal{A}_M) = v_2 \mathcal{A}_M \mathcal{A}^M + v_4 (\mathcal{A}_M \mathcal{A}^M)^2 \quad \text{with} \quad v_2, v_4 < 0$$

just for simplicity of the analysis.

* Inclusion of higher order terms → we have more parameters to tune

4. Dante's Inferno model with 5D gauge theory origin-2

- Our model: 5D $U(1)$ gauge theory on $M^4 \times S^1$

$$S = \int d^5x \Big[-\frac{1}{4} F^{(A)}_{MN} F^{(A)MN} - \underline{V_A(\mathcal{A}_M)} - \frac{1}{4} F^{(B)}_{MN} F^{(B)MN} - i\bar{\psi}\gamma^M \left(\partial_M + i g_{A5} A_M - i g_{B5} B_M\right)\psi \Big] \quad (M, N = 0, 1, 2, 3, 5)$$

- The effective potential for A, B at the one-loop level is

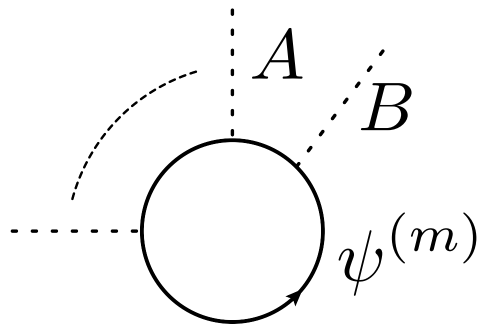
$$V_{1-loop}(A, B) = V_{cl}(A) + V_g(A) + V_f(A, B)$$

$$A,\ B \equiv \sqrt{2\pi L_5} A_5^{(0)},\ \sqrt{2\pi L_5} B_5^{(0)}$$

where $V_{cl}(A)$ is the contribution from the 5D classical potential

$$V_{cl}(A) = \frac{1}{2} m^2 A^2 - \frac{\lambda}{4!} A^4, \qquad -v_2 = \frac{m^2}{2} > 0, \quad -\frac{v_4}{2\pi L_5} = \frac{\lambda}{4!} > 0$$

x
- The one-loop diagram:

$\sum_{A,B} \sum_{m}$ 

+ gauge loop due to the self coupling from $V_A(\mathcal{A}_M)$

- At one-loop level, the fermion contribution $V_f(A,B)$ is

$$V_f(A,B) = \Lambda^4 \sum_{n=1}^{\infty} \frac{1}{n^5} \cos\left\{ n \left(\frac{A}{f_A} - \frac{B}{f_B} \right) \right\}, \qquad \Lambda^4 \propto \frac{1}{L_5^4}.$$

The potential has a discrete shift symmetry $B \to B + 2\pi f_B$

Taking $n = 1$, an appropriate constant shift of B and adding a constant yields Dante's Inferno.

- The parameters in $V_{DI}(A,B)$ relate to 5D gauge theory parameters as

$$f_A = \frac{1}{g_A(2\pi L_5)}, \quad f_B = \frac{1}{g_B(2\pi L_5)}, \quad g_A = \frac{g_{A5}}{\sqrt{2\pi L_5}}, \quad g_B = \frac{g_{B5}}{\sqrt{2\pi L_5}}$$

X V_g is the contribution from the gauge field A_M, and is sub-leading compared with $V_{cl}(A)$ when

$$2\pi L_5 \gtrsim 1 \times 10^2 \qquad (M_P = 1)$$

We do not expect the compactification radius L_5 to be very close to the Planck scale. Therefore this is a natural assumption to make. With this assumption, we will neglect the contribution from V_g.

- Now the potential for Dante's Inferno is obtained from a 5D gauge theory as

$$V_{DI}(A,B) = \frac{m^2}{2}A^2 - \frac{\lambda}{4!}A^4 + \Lambda^4\left\{1-\cos\left(\frac{A}{f_A}-\frac{B}{f_B}\right)\right\}$$

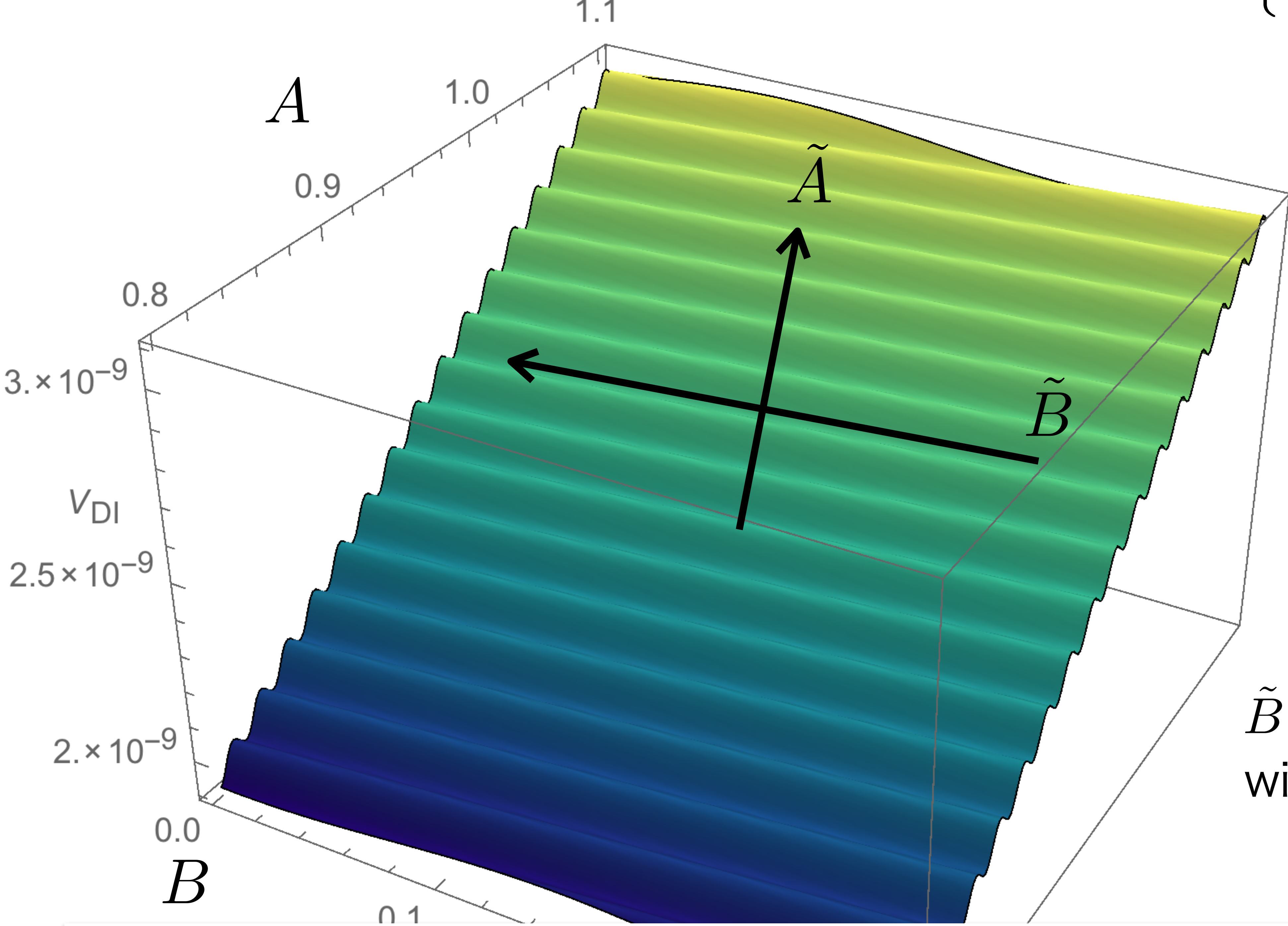
x - To see the inflation, it is convenient to rotate the fields as

$$\begin{pmatrix}\tilde{A}\\ \tilde{B}\end{pmatrix} = \begin{pmatrix}\cos\gamma & -\sin\gamma\\ \sin\gamma & \cos\gamma\end{pmatrix}\begin{pmatrix}A\\ B\end{pmatrix}, \qquad \sin\gamma = \frac{f_A}{\sqrt{f_A^2+f_B^2}}, \quad \cos\gamma = \frac{f_B}{\sqrt{f_A^2+f_B^2}}$$

Then the potential takes the form,

$$V_{DI}(\tilde{A},\tilde{B}) = \frac{m^2}{2}(\tilde{A}\cos\gamma+\tilde{B}\sin\gamma)^2 - \frac{\lambda}{4!}(\tilde{A}\cos\gamma+\tilde{B}\sin\gamma)^4 + \Lambda^4\left(1-\cos\frac{\tilde{A}}{f}\right)$$

x The potential of DI model: $V_{DI}(A,B) = V_A(A) + \Lambda^4 \left\{ 1 - \cos\left(\frac{A}{f_A} - \frac{B}{f_B} \right) \right\}$



$\tilde{B}$ will be identified with the inflaton.

Now, the two condition should be satisfied in DI model:

condition 1 $\quad f_A \ll f_B \lesssim 1. \longrightarrow g_A \gg g_B, \quad \cos\gamma \simeq 1, \quad \sin\gamma \simeq \frac{f_A}{f_B}, \quad f \simeq f_A$

condition 2 $\quad |\partial_{\tilde{A}} V_A(A)|_{A=A_{in}}| \ll \frac{\Lambda^4}{f}$

The inflaton potential

- After $\tilde{A}$ settles down at local minimum, the motion of $\tilde{B}$ leads to the slow-roll inflation. By redefining $\tilde{B} = \phi$, we obtain

$$V_{eff}(\phi) = \frac{m^2}{2}\left(\frac{f_A}{f_B}\tilde{B}\right)^2 - \frac{\lambda}{4!}\left(\frac{f_A}{f_B}\tilde{B}\right)^4 = \frac{m_{eff}^2}{2}\phi^2 - \frac{\lambda_{eff}}{4!}\phi^4$$
$$= \frac{m_{eff}^2}{2}\phi^2\left(1 - c\phi^2\right)$$

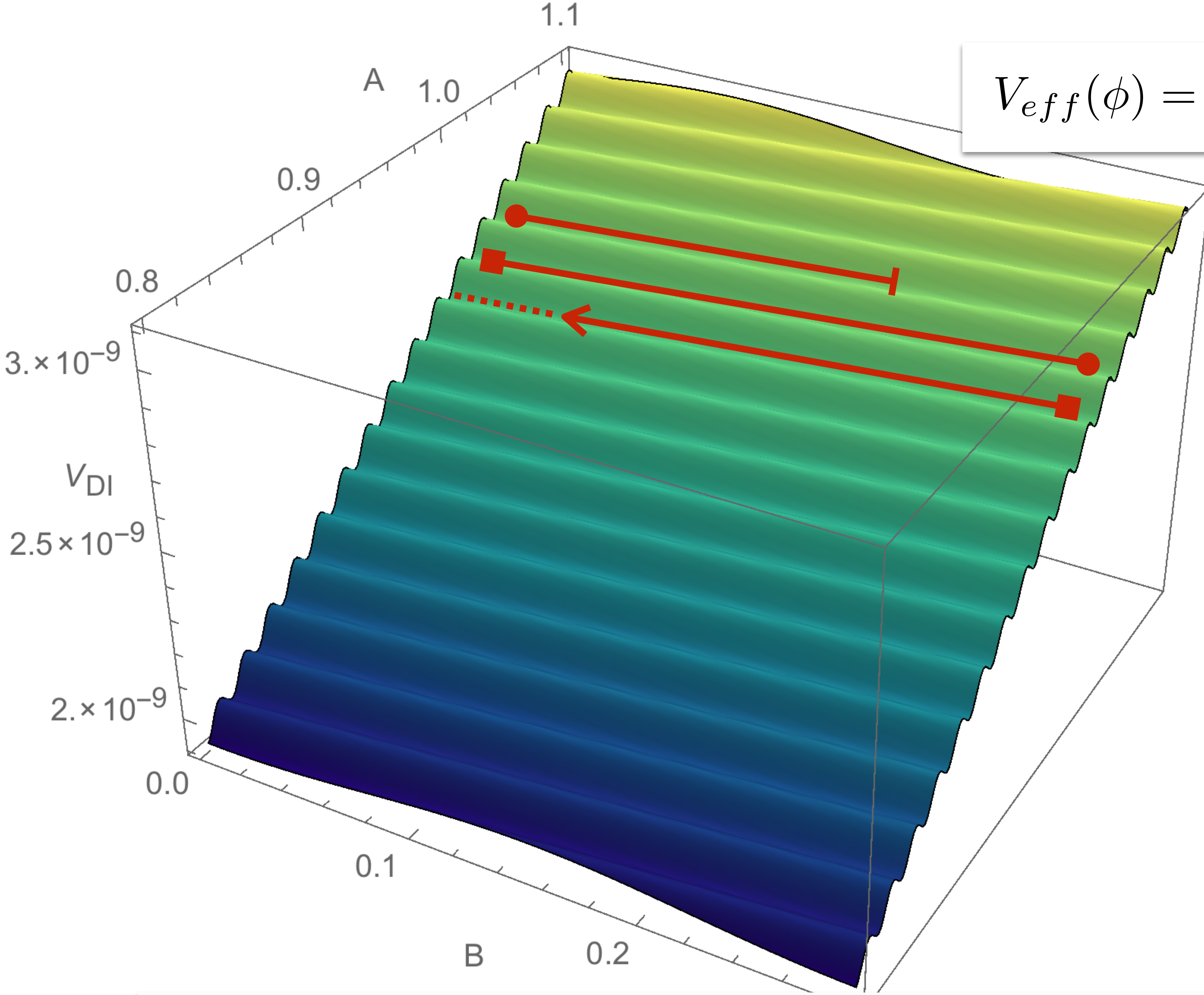
where

$$m_{eff}^2 \simeq \left(\frac{f_A}{f_B}\right)^2 m^2, \qquad \lambda_{eff} \simeq \left(\frac{f_A}{f_B}\right)^4 \lambda\ , \qquad c := \frac{\lambda_{eff}}{12 m_{eff}^2}$$

The potential $V_{eff}(\phi)$ is not bounded below, but we will only consider the region of ϕ before the potential starts to go down:

$$|\phi| < |\phi|_{max} = \frac{1}{\sqrt{2c}}$$

Trajectory of the slow-roll inflation in DI model



$$V_{eff}(\phi) = \frac{m_{eff}^2}{2}\phi^2\left(1 - c\phi^2\right)$$

— : $\phi = \tilde{B}$

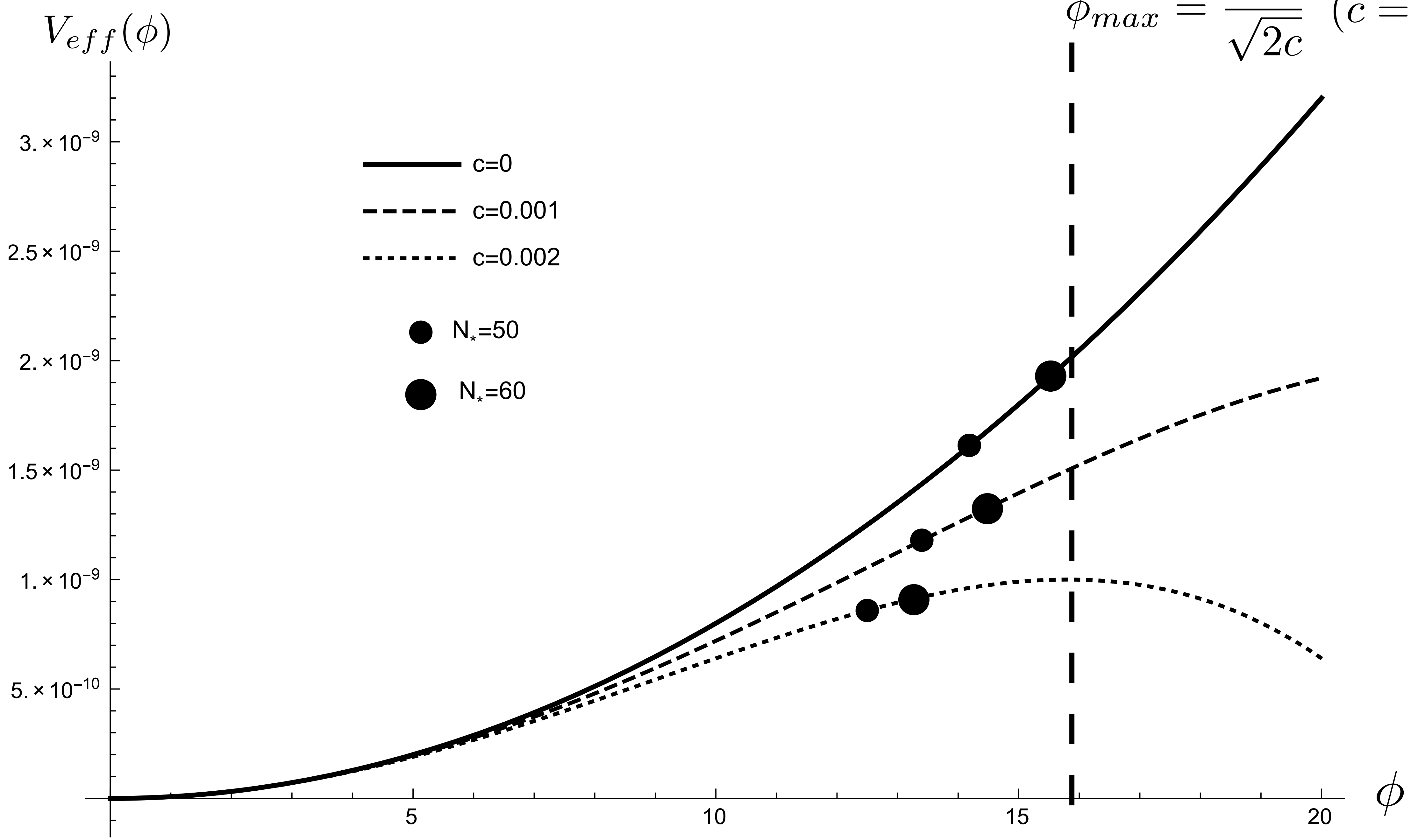
Super-Planckian field excursion $\phi > 1$ is effectively realized by the sub-Planckian fields $A, B < 1$.

5. Prediction of the model

$$V_{eff}(\phi) = \frac{m_{eff}^2}{2}\phi^2\left(1 - c\phi^2\right)$$

The slow-roll inflation is described inside of the region $|\phi| < \phi_{max}$.

$$\phi_{max} = \frac{1}{\sqrt{2c}} \quad (c = 0.001)$$



$M_P = (8\pi G_N)^{-1/2} = 1$

Remark

- We will not worry about the potential beyond $|\phi| > \phi_{max}$

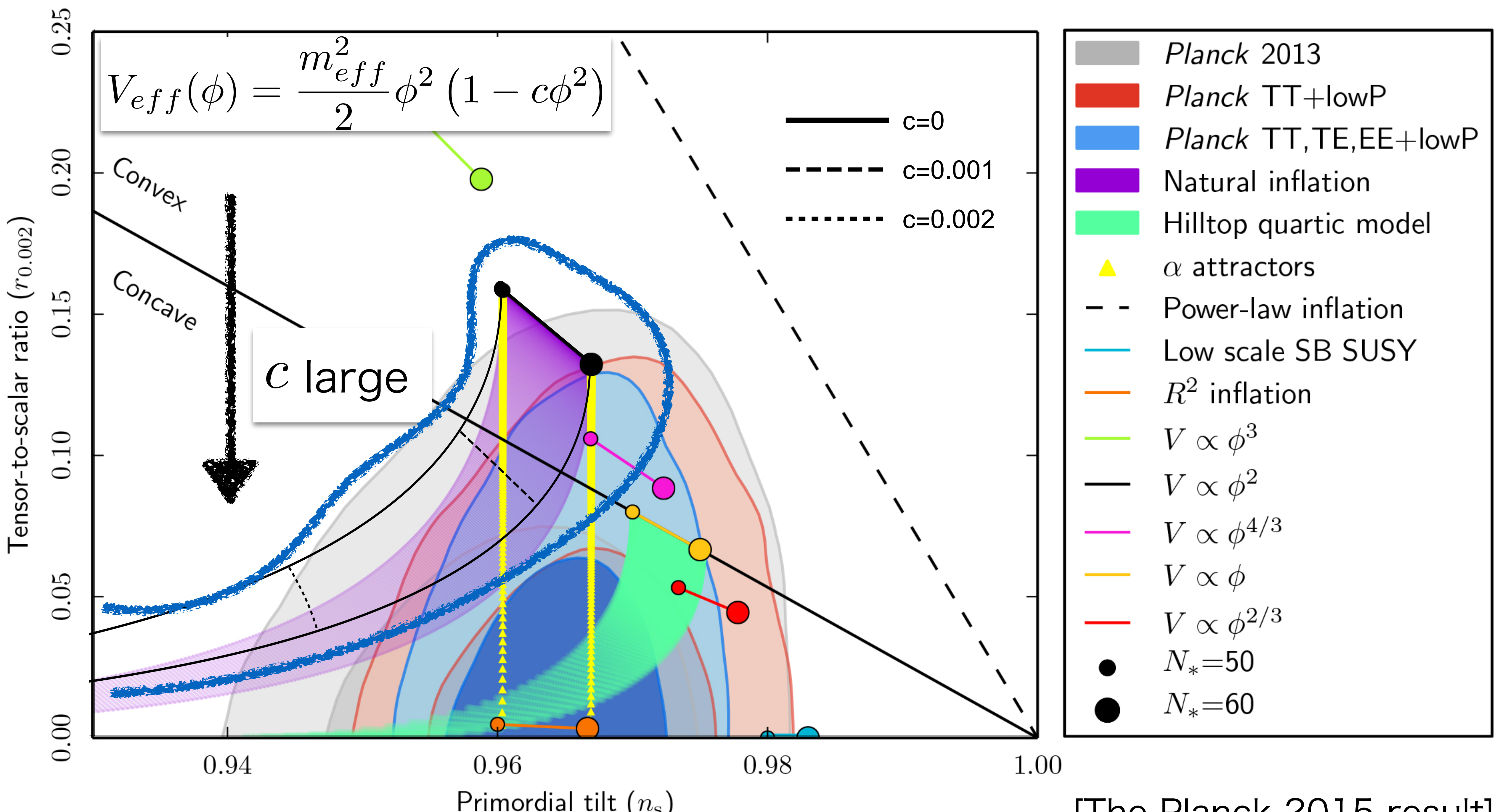
- Actually, it has been shown by A. Hashimoto (08) that massive vector field theories which can be embedded to a UV theory whose S-matrix satisfies canonical analyticity constraints do not have a Lorentz- symmetry-breaking vacuum. $V''_A(\langle \mathcal{A}_M \rangle) > 0$

- In such theories, before the potential starts to go down, the contribution from higher order terms in the potential should come in to prevent Lorentz-symmetry-breaking local minimum, assuming that the potential is bounded from below.

Prediction of the slow-roll inflation model

For the slow-roll inflation we have two parameters in the inflaton potential $V_{eff}(\phi)$: m_{eff} and c.

We use $P_s \simeq 2.2 \times 10^{-9}$ to determine m_{eff} as a function of c.



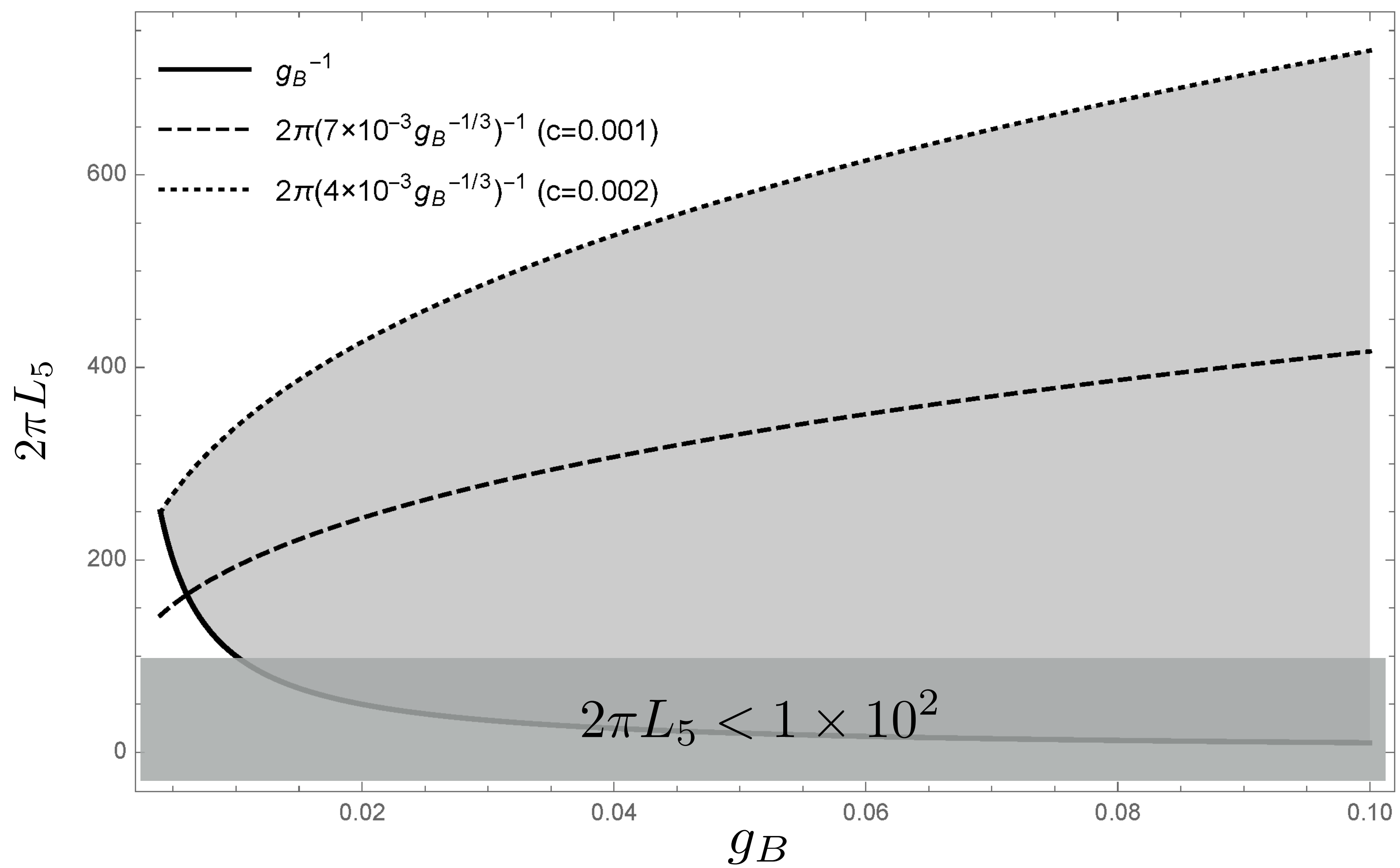
[The Planck 2015 result]

Parameters of the 5D gauge theory

Finally, the following constraints on the parameters of the 5D gauge theory should be satisfied for of DI model:

$$g_A \gtrsim 15 g_B, \quad (f_B \gtrsim 15 f_A),$$

$$7 \times 10^{-3} g_B^{-1/3} < \frac{1}{L_5} \lesssim 2\pi g_B, \quad (N_* = 60,\ c = 0.001).$$



6. Connection to DBI action

- DBI action of D5-brane is

$$S_{D5} = -T_{D5} \int d^6\sigma \sqrt{-\det\left(G_{ab} + \mathcal{F}_{ab}\right)}, \quad (a, b = 0, 1, 2, 3, 5, 6)$$

$$\mathcal{F}_{ab} = B_{ab} - \partial_a C_b + \partial_b C_a$$

- We consider the following background:

$$G_{MN} = \mathrm{diag}(+-----), \quad B_{MN} = 0$$

$\sigma^a = x^a$ static gauge

$\mathcal{F}_{a6} \propto (a_M - \partial_M \theta)$ will be the 5D gauge and the Stueckelberg fields

- After double dimensional reduction twice, in four dimension, we obtain the potential of $A \sim a_5$,

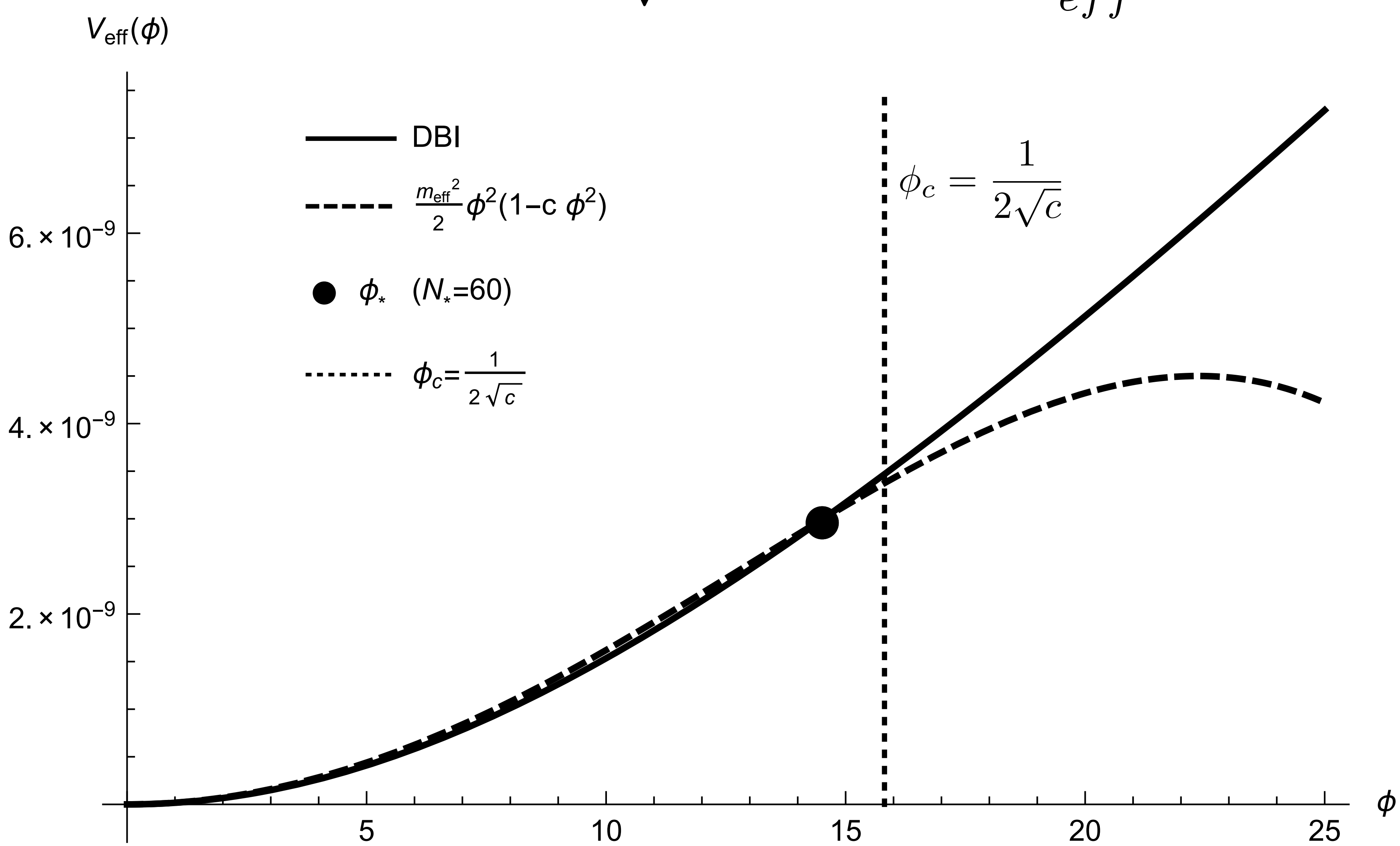
$$V_A(A) \sim \int d^4x \sqrt{1 + const. \times A^2} \qquad \sqrt{1 + A^2} = 1 + \frac{1}{2}A^2 - \frac{1}{8}(A^2)^2 + \frac{1}{16}(A^2)^3 - \cdots .$$

- Through the DI model, the inflaton potential is

$$V_{eff}(\phi) \sim m_{eff}^2 \int d^4x \sqrt{1 + \left(\frac{\phi}{\phi_c}\right)^2} \qquad m_{eff}^2 \propto \frac{1}{\phi_c^2} \qquad \lambda_{eff} \propto \frac{3}{\phi_c^4}$$

ϕ_c is the radius of convergence of the Taylor expansion of the DBI action.

$$\phi_c = \frac{1}{2\sqrt{c}} \qquad c = \frac{\lambda_{eff}}{12 m_{eff}^2}$$



Remark

- We observe that this phenomenological parametrization is a rather good approximation of the potential in the field range where the inflation occurs.

- The inflation does not occur in the field range where the linear approximation at the large field value is valid (linear approximation $\rightarrow$ axion monodromy model)

7. Summary

- We considered $V = m^2\phi^2 - \lambda\phi^4$ in order to accommodate Dante's Inferno model with 5D massive gauge theory origin to the updated upper bound on $r < 0.12$.

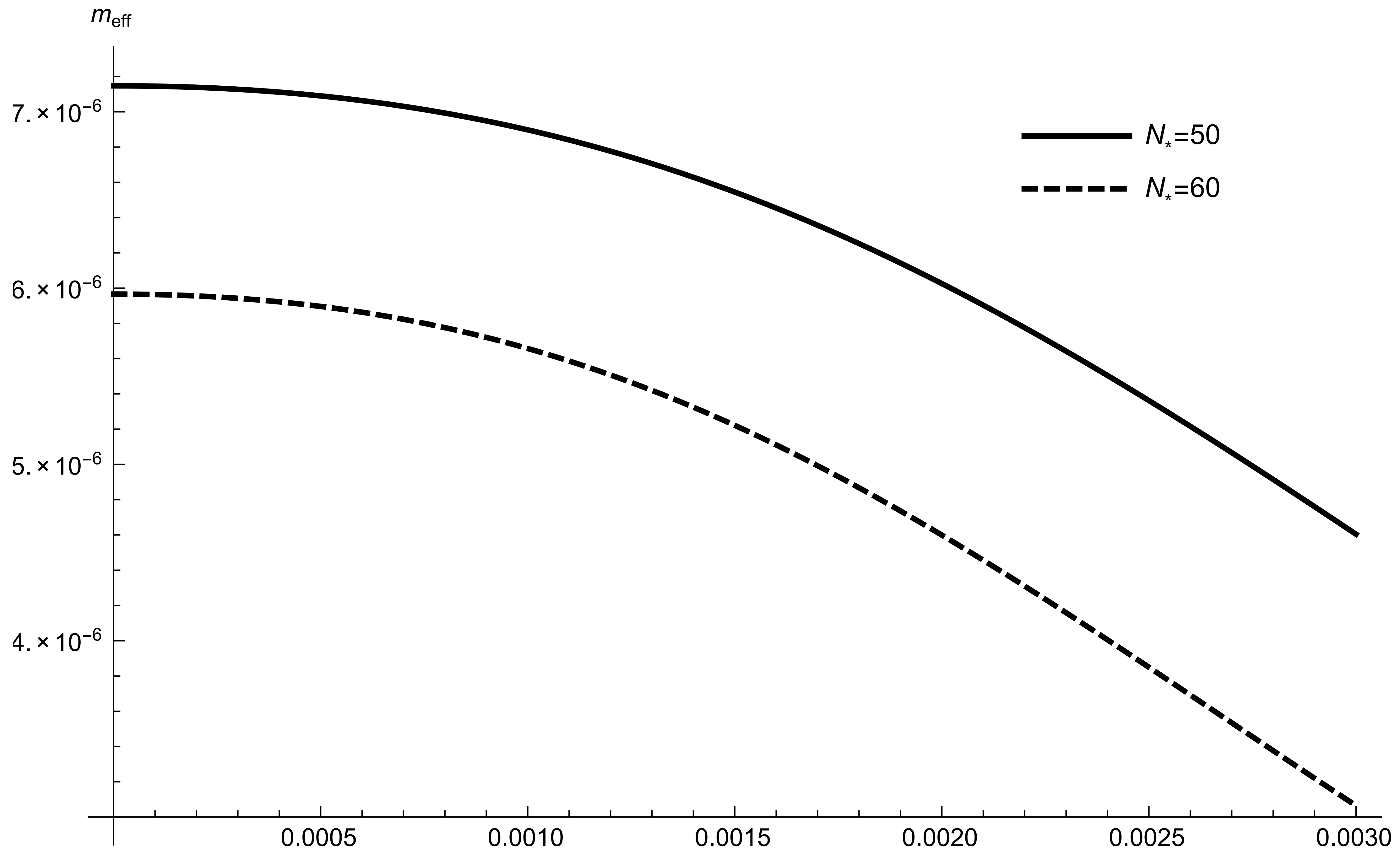
- We examined a criterion for effective field theories to be embedded in a consistent UV theory

  $V_A(\mathcal{A}_M) = v_2 \mathcal{A}_M \mathcal{A}^M + v_4 (\mathcal{A}_M \mathcal{A}^M)^2$ with $v_2, v_4 < 0$

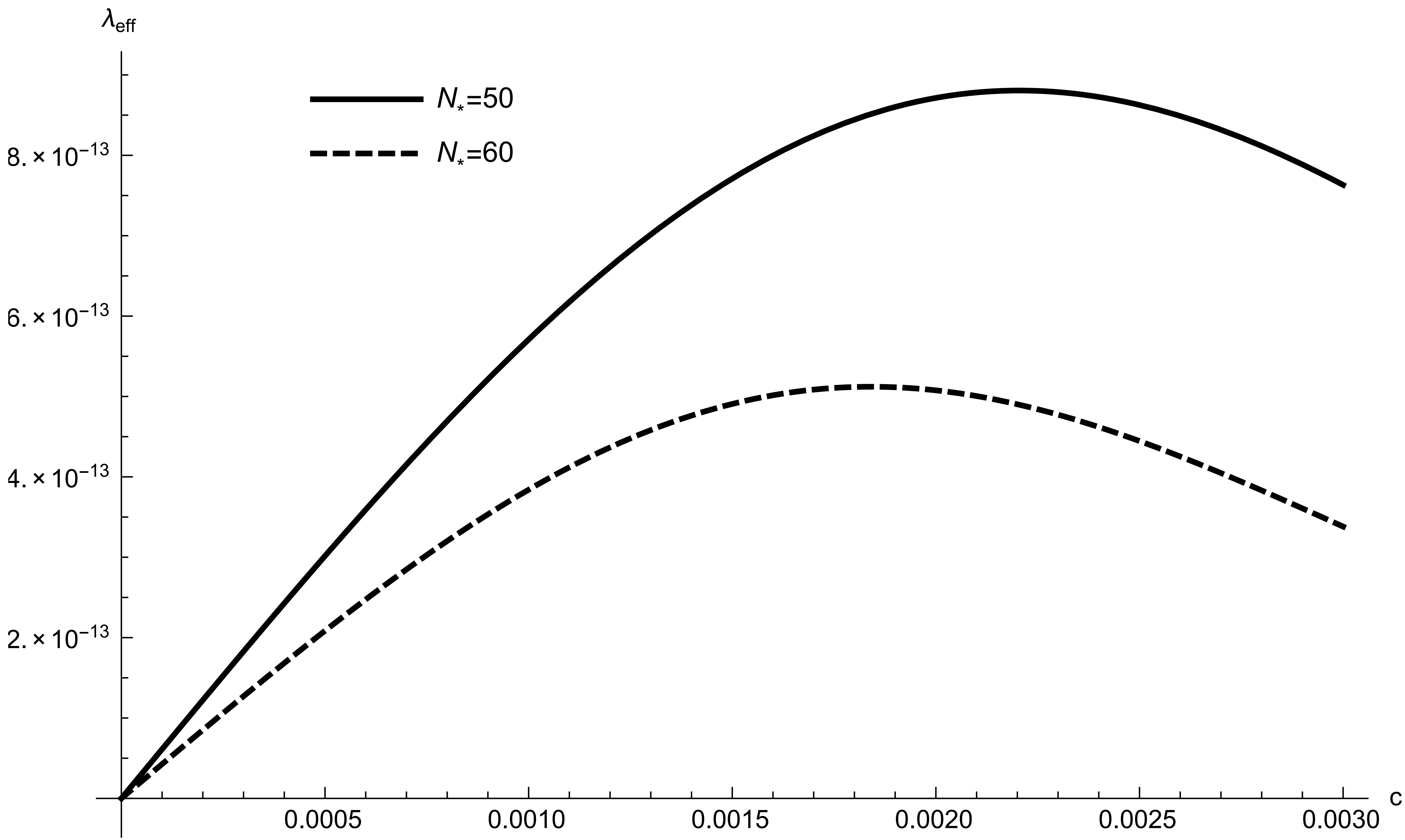
- We gave a possible connection of our DI model to DBI action of D5-brane.

Backup slides

Plot of m_eff as a function of c



Plot of λ_eff as a function of c



Single Axion monodromy

[Silverstein et al, 08,14]

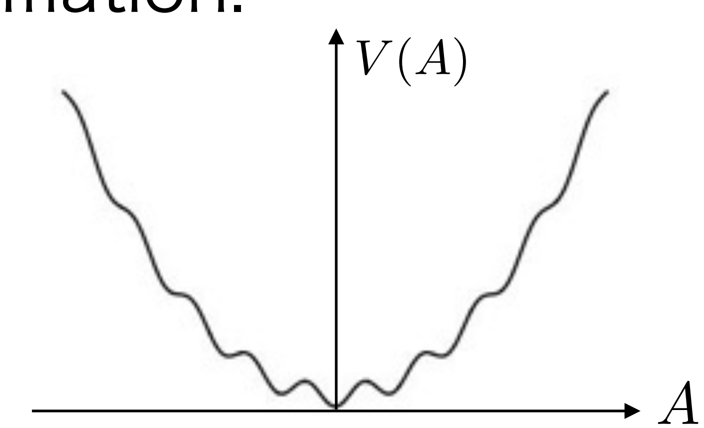
$$V(A) = \frac{1}{2}m^2A^2 + \Lambda^4\left(1 - \cos\left(\frac{A}{f}\right)\right)$$

- Due to the quadratic term, the potential energy does not return the same under the shift $A \to A + 2\pi f$

- Even if the fundamental field has sub-Planckian period $2\pi f < M_P$, the trans-Planckian excursion can be effectively achieved by traverses many cycle. The potential energy increases over each cycle but much of the remaining physics essentially repeats itself

- This model effectively reduces to chaotic model $V \sim \frac{1}{2}m^2A^2$ when the slope of the sinusoidal potential is much smaller than that of the mass term during inflation.

$\Lambda^4/f \ll m^2 A_*$ $*$: at horizon exit



The potential can be derived from a 5D $U(1)$ gauge theory on $M^4 \times S^1$

$$S = \int d^5x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 \left(A_\mu - g_5 \partial_\mu \theta \right)^2 + (\text{matters}) \right] \qquad (\mu = 0, \cdots, 3, 5)$$

- The inflaton field A comes from the zero mode of A_5 after S^1 compactification.

$$A = \sqrt{2\pi L} A_5^{(0)} \qquad L : S^1 \text{ radius}$$

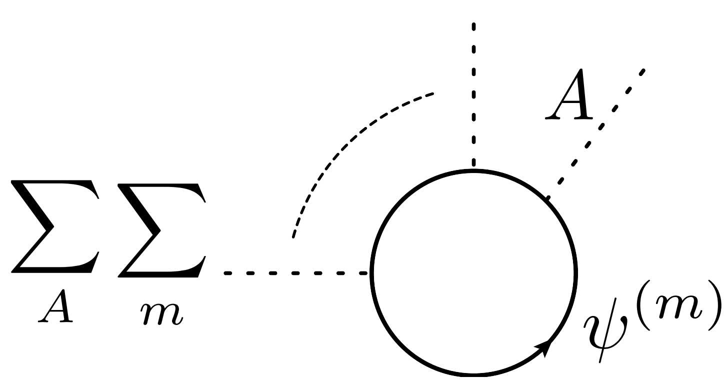
- We introduced the Stueckelberg filed θ and the Stueckelberg mass term which gives rise to the quadratic term in the effective potential.

$$A_\mu \to A_\mu + \partial_\mu \Lambda, \quad \theta \to \theta + 1/g_5 \Lambda \qquad \text{gauge trf.}$$

- The one-loop effective potential of $A_5^{(0)}$ is obtained as

$$V(A_5^{(0)})_{1-\text{loop}} = \frac{m^2}{2} A_5^{(0)2} + \frac{3}{\pi^2 (2\pi L)^4} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos\left(n A_5^{(0)} (2\pi g L) \right) + \text{const.}$$

$$g = \frac{g_5}{\sqrt{2\pi L}} \quad : \text{4d gauge coupling}$$



All KK modes m are taken into account.

- At one-loop, the Stueckelberg mass m is not renormalized.
- Stueckelberg field does not contribute to the potential at one loop
- The parameters in the axion monodromy model are related to the parameters in the 5D gauge theory as

$$f = \frac{1}{g(2\pi L)}, \quad \Lambda^4 = \frac{c}{\pi^2(2\pi L)^4}, \quad c \sim \mathcal{O}(1), \quad m = m$$ Stueckelberg mass

Then, $\Lambda^4/f \ll m^2 A_*$ (effectively chaotic) and CMB data with $r = 0.16$ require

$$1.0 \times 10^{14}\,\mathrm{GeV} < \frac{1}{L} < 3.2 \times 10^{16}\,\mathrm{GeV}, \quad m^2 \sim 10^{26}\mathrm{GeV} \ll H_*^2 \quad (H_* \simeq 10^{14}\mathrm{GeV})$$

and $g \sim \mathcal{O}(1)$

$\Delta A > M_P$: trans-Planckian field excursion of fundamental field

- The small m^2 is natural in the sense of 't Hooft if the shift symmetry $A \to A + C$ is a good symmetry at the Planck scale. But it is beyond the scope of higher-dim. gauge theory so we can not ensure the naturalness of small $m < H$.

Axion Alignment & Axion Hierarchy:
improvement of natural inflation

[Kim et al, 08, Ben-Dayan et al. 14]

Both models can be described by the potential of the form

$$V(A,B) = \Lambda_1^4\left(1-\cos\left(\frac{m_1}{f_A}A+\frac{n_1}{f_B}B\right)\right)+\Lambda_2^4\left(1-\cos\left(\frac{m_2}{f_A}A+\frac{n_2}{f_B}B\right)\right)$$

The main feature of these two models is to acquire a large effective decay const. $f_{\rm eff} > M_P$ from the (small) scales f_A and f_B by defining the eigenvectors of the mass matrix. $\Delta A, \Delta B < M_P$ is satisfied.

$$\begin{pmatrix} \phi_s \\ \phi_l \end{pmatrix} = \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}$$

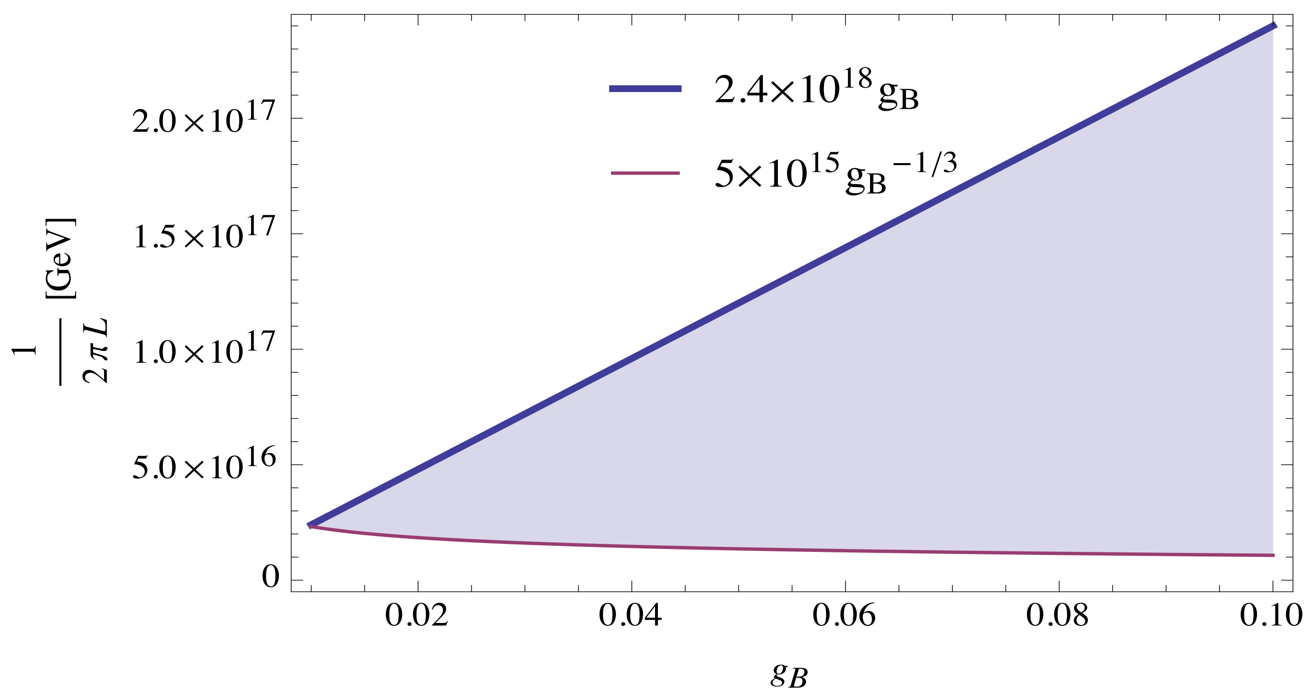
where

$$\cos\zeta = \frac{f_s}{f_A}m_1, \quad \sin\zeta = \frac{f_s}{f_B}n_1, \qquad f_s = \frac{1}{\sqrt{\frac{m_1^2}{f_A^2}+\frac{n_1^2}{f_B^2}}}$$

Dante's Inferno with quadratic potential from 5D gauge theory

- The constraints for Dante's Inferno is written in terms of the parameters of the 5D gauge theory as

$$g_A > 14 g_B, \quad g_B^{-1/3} \times 3.2 \times 10^{16}\,\mathrm{GeV} < \frac{1}{L} \lesssim g_B \times 2.4 \times 10^{18}\,\mathrm{GeV}$$



The allowed values of the gauge couplings and the compactification radius are rather restricted.

The gauge couplings are in the range $0.04 - \mathcal{O}(1)$

- In terms of two physical fields

$$V(\phi_s, \phi_l) = \Lambda_1^4 \left(1 - \cos\left(\frac{\phi_s}{f_s}\right)\right) + \Lambda_2^4 \left(1 - \cos\left(\frac{\phi_s}{f_s'} + \frac{\phi_l}{f_l}\right)\right),$$

effective decay constant: $f_l = \dfrac{\sqrt{m_1^2 f_B^2 + n_1^2 f_A^2}}{m_1 n_2 - m_2 n_1}$

Axion alignment model

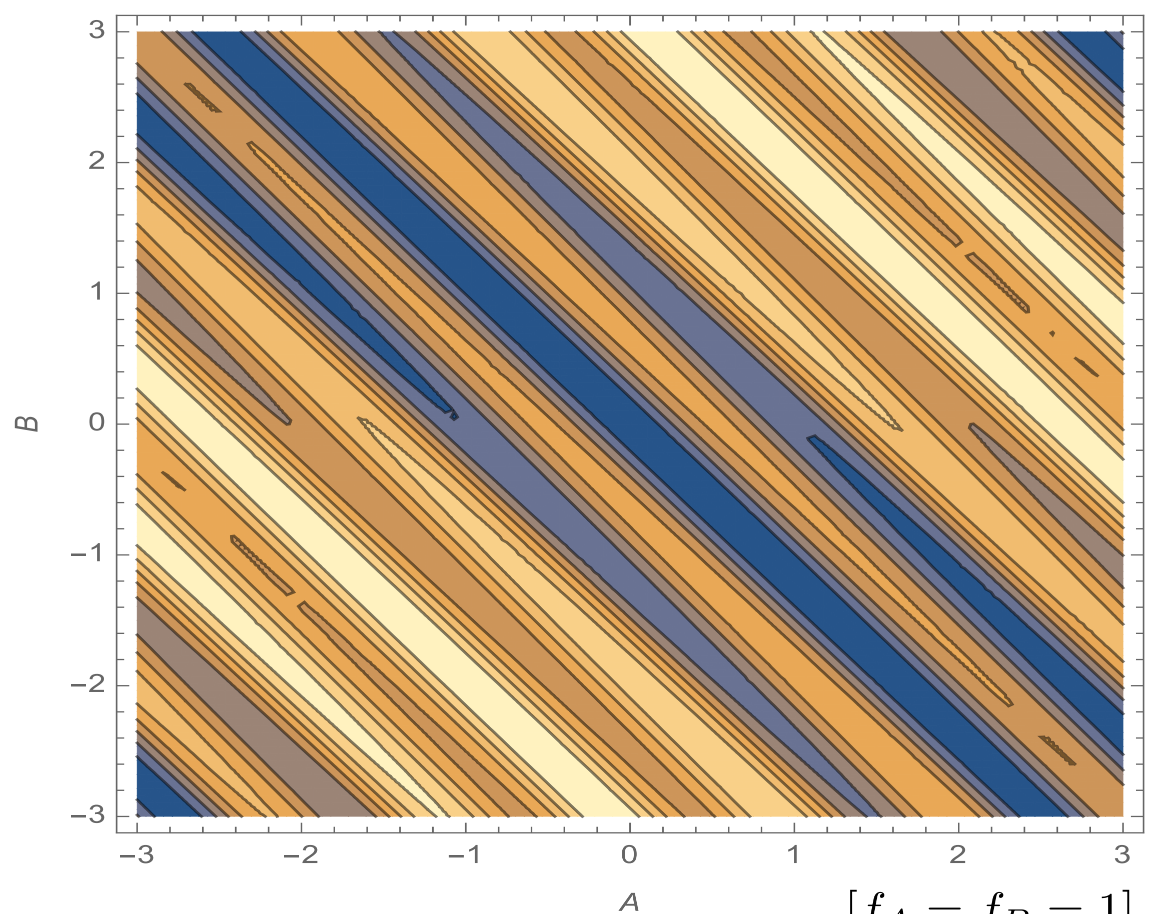
$$|m_1 n_2 - m_2 n_1| \ll |m_1|, |n_1| \;\; \Rightarrow \;\; |f_l| \gg f_A, f_B \;\; (f_s, f_s')$$

ϕ_s : heavy, $m_{\phi_s} > H$,

irrelevant to inflation

ϕ_l : light, $m_{\phi_l} < H$,

identified with the inflaton



$[f_A = f_B = 1]$

Axion hierarchy model $(n_2 = 0)$

$$\left|\frac{f_A}{m_1}\right| \ll \frac{f_A}{|m_2|}, \frac{f_B}{|n_1|} \quad \Rightarrow \quad |f_l| = \frac{\sqrt{m_1^2 f_B^2 + n_1^2 f_A^2}}{|m_2 n_1|} \simeq \left|\frac{m_1}{n_1 m_2}\right| f_B$$

This model requires simple hierarchy $m_1 \gg m_2$ to obtain $f_l \gg f_B \gg f_A$

- The two models reduce to natural inflation with the effective decay constant f_l.

- The potential is derived from the 5D action with two kinds of matters.

$$S=\int d^5x\Big[-\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu}-\frac{1}{4}F^B_{\mu\nu}F^{B\mu\nu}$$
$$-i\bar{\psi}\gamma^\mu(\partial_\mu+ig_{A5}m_1A_\mu+ig_{B5}n_1B_\mu)\psi-i\bar{\chi}\gamma^\mu(\partial_\mu+ig_{A5}m_2A_\mu-ig_{B5}n_2B_\mu)\chi\Big]$$

$$\Rightarrow\quad V(A,B)=\Lambda_1^4\left(1-\cos\left(\frac{m_1}{f_A}A+\frac{n_1}{f_B}B\right)\right)+\Lambda_2^4\left(1-\cos\left(\frac{m_2}{f_A}A+\frac{n_2}{f_B}B\right)\right)$$

In the 5D gauge theory, m_1,m_2,n_1,n_2 correspond to the charges of $U_{A,B}(1)$.

Assumption: m_1,m_2,n_1,n_2 are all integers.
We assume that charges are quantized.

The model parameters are given by

$$\Lambda_{1,2}\simeq\frac{3}{\pi^2}\frac{1}{2\pi L^4}\ ,\quad f_{A,B}=\frac{1}{g_{A,B}(2\pi L)}\ ,\quad g_{A,B}=\frac{g_{A5,B5}}{\sqrt{2\pi L}}$$

- WGC and natural inflation + $r = 0.16$ require $2\pi f_{A,B} \lesssim M_P$ and $|f_l| \gtrsim 20 M_P$

Axion alignment

$$|m_1 n_2 - m_2 n_1| \ll |m_1|, |n_1| \qquad f_l = \frac{\sqrt{m_1^2 f_B^2 + n_1^2 f_A^2}}{m_1 n_2 - m_2 n_1}$$

→ $\max(|m_1|, |n_1|) \gtrsim 20 \times 2\pi$, matter with large charge at least $\sim \mathcal{O}(100)$

A matter with such a large charge seems to us quite unnatural, considering that the energy scale under consideration is rather high ($H \simeq 10^{14}\mathrm{GeV}$).

Axion hierarchy

$$|f_l| \simeq \left| \frac{m_1}{n_1 m_2} \right| f_B \quad \rightarrow \quad |m_1| \gtrsim 20 |n_1 m_2| \times 2\pi$$

A large hierarchy between the charges in the same gauge group $U_A(1)$

Such a large hierarchy ($\mathcal{O}(100)$) between the charges in the same gauge group seems quite unnatural.

- Various axion inflation models can be derived from the higher-dimensional (5D) gauge theories.
- The allowed range of the gauge theory parameters are quite constrained. - CMB data and WGC
- Among the models studied, Dante's Inferno model appears as the most natural model in this framework, the gauge couplings are in the range $0.04 - \mathcal{O}(1)$.
- Single field axion monodromy leaves the problem that whether the shift symmetry is a good symmetry or not to its UV completion theory.

- Comment on the anionic coupling $\frac{\alpha}{4f}\phi\tilde{F}_{\mu\nu}F^{\mu\nu}$

In 5D U(1) gauge theory we could get the same type of interaction from the CS action which breaks the Z_2 symmetry in 5th direction.
After S^1 compactification, [Furuuchi and Jackson (13)]

$$\frac{g_4^2 k}{8\pi}\frac{A_5^{(0)}}{2\pi f}\tilde{F}_{\mu\nu}^{(0)}F^{\mu\nu(0)}$$

It results in the corrections to the power spectrum and the non-Gaussian parameter [Barnaby etal (2010), Ferreira etal (2014)]

$$\mathcal{P}_{\zeta\,\mathrm{one-loop}} = 7.5\times 10^{-5}\cdot\mathcal{P}_\zeta\frac{e^{4\pi\xi}}{\xi^6} \qquad f_{NL}^{\mathrm{eq}} = 4.4\times 10^{10}\cdot\mathcal{P}_\zeta^3\frac{e^{6\pi\xi}}{\xi^9}$$

$$\xi_i = \frac{\alpha_i\dot{\phi}_i}{2f_iH} \quad \text{with} \quad \alpha_i = \frac{k_i}{\pi}\frac{g_i^2}{4\pi}, \quad f = \frac{1}{2\pi g_i L}$$

- If we require

$$\mathcal{P}_\zeta \simeq \mathcal{P}_\zeta^{\rm total} \simeq 2.2 \times 10^{-9} \ (\mathcal{P}_\zeta \gg \mathcal{P}_{\zeta\,\rm one-loop})$$

$|f_{NL}^{\rm equil}| < 117$ by PLANCK → $\xi_i < 3$

- Dantes Inferno case, $L \sim 10^{-17}{\rm GeV}^{-1}$

$$\xi_i = \frac{k_i g_i^3}{4\pi}\dot{\phi}_i \cdot 10^{-31}{\rm GeV}^{-2} \qquad (i = A, B)$$

It is evaluated as follows $|\xi_A| \simeq \frac{f_A}{f_B}\frac{k_A g_A^3}{4\pi}, \quad |\xi_B| \simeq \frac{k_B g_B^3}{4\pi}$

We find that $\xi_i < 3$ can easily be satisfied within our parameters space for

$$k_A \sim \mathcal{O}(1) - \mathcal{O}(10), \quad k_B \sim \mathcal{O}(1) - \mathcal{O}(100)$$

$$\because \quad f_A/f_B = g_B/g_A < 0.07, \quad g_B \lesssim 0.2$$