# Searching for the Origin of Accelerating Universe

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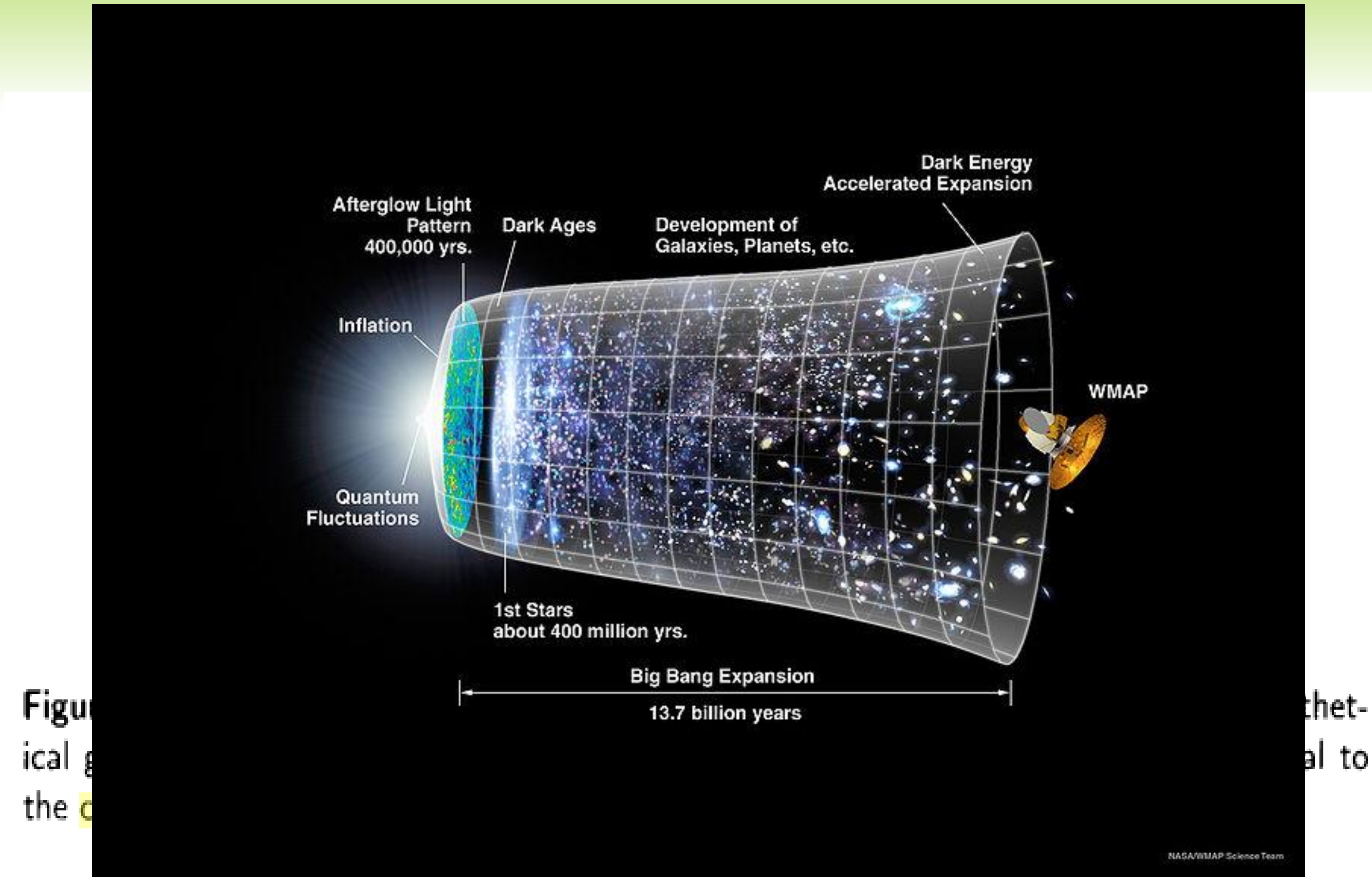
# Outline

- ABC of Cosmology
- Evidences of an Accelerating Universe
- Standard Cosmology (Background Evolution)
- Dark Energy vs. Modified Gravity
- Duality and Discrepancies between DE and MG
- Perturbations (Observables)
- Summary

# ABC of Cosmology

1.Hot Big Bang Model (NASA)

2.Modern Cosmology (by S.Dodelson)



# ABC of Cosmology

- Friedman equations

Modification?

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k}{a^2} + \frac{\Lambda}{3}$$

acceleration

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i(\rho_i + 3p_i) + \frac{\Lambda}{3}$$

- Critical density and density contrasts

$$\rho_c = \frac{3H^2}{8\pi G}, \quad \rho_\Lambda = \frac{\Lambda}{8\pi G} \qquad \Omega_i = \frac{\rho_i}{\rho_c}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$$

- Curvature

$$\frac{k}{a^2} = H^2(\sum_i \Omega_i + \Omega_\Lambda - 1) \equiv H^2(\Omega_M + \Omega_\Lambda - 1)$$

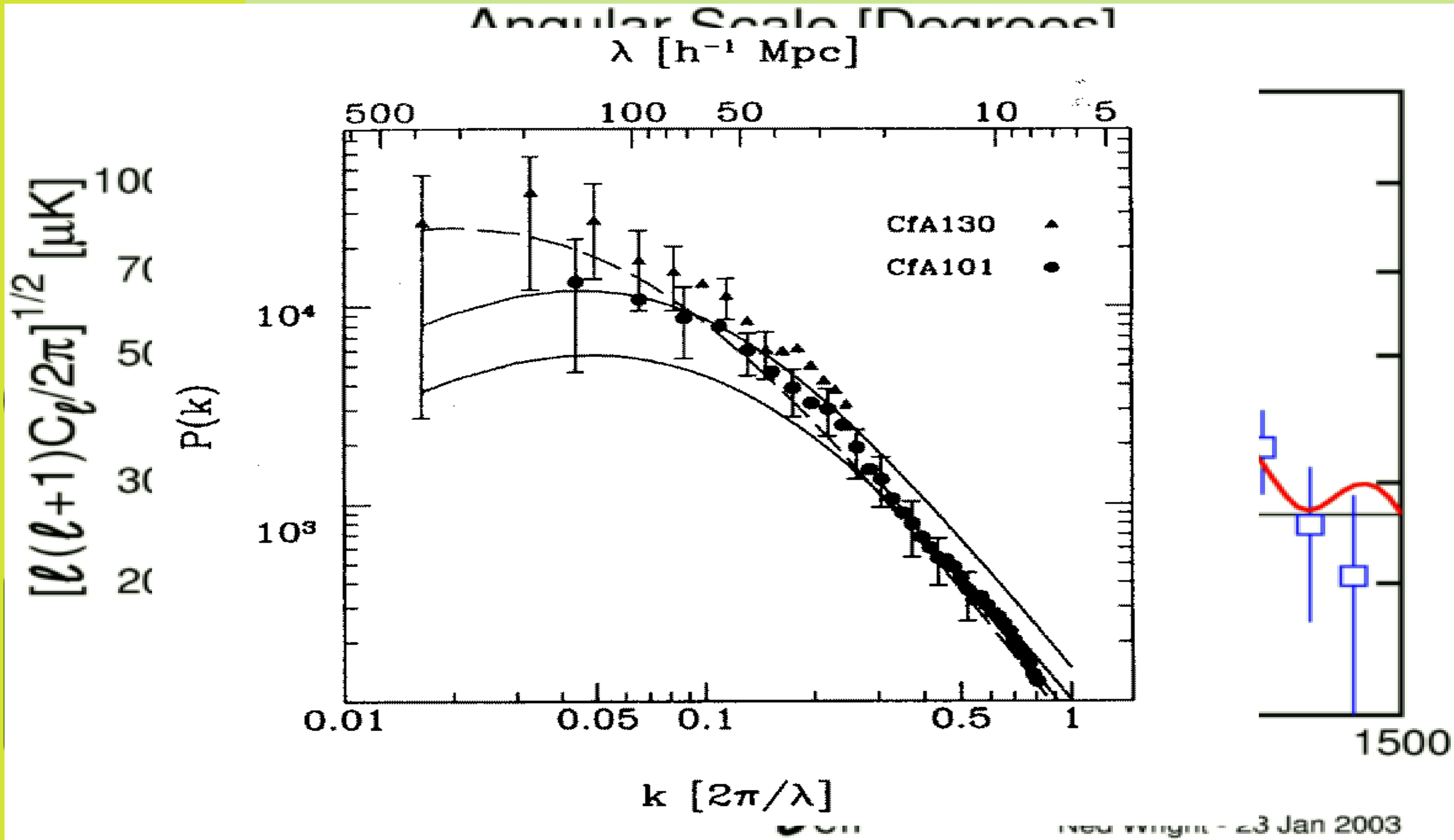
Very small (negligible)

- EOS (equation of state) (sorry for typo! P = w*rho)

$$\rho \equiv \omega P, \quad \omega \ : \ \text{equation of state (eos)}, \quad \omega_m = 0 \ : \text{matter}, \quad \omega_r = \frac{1}{3} \ : \text{radiation}$$

$$(\rho + 3P) = (1 + 3\omega)\rho > 0 \ \text{for matter and radiation}$$

# Accelerating Universe



# Background Evolution

- Luminosity Distance (For students, arXiv:astro-ph/9905116)

$$
\begin{aligned}
d_L &= (1+z)|\Omega_k|^{-1/2} \begin{cases} \sinh(|\Omega_k|^{1/2} I) & \text{if} \quad \Omega_k > 0 \\ |\Omega_k|^{1/2} I & \text{if} \quad \Omega_k = 0 \\ \sin(|\Omega_k|^{1/2} I) & \text{if} \quad \Omega_k < 0 \end{cases} \\
I(z) &= \int_0^z \frac{dz'}{H(z')} \\
H(z) &= H_0 \sqrt{(1+z)^3 \Omega_m^{(0)} + (1+z)^2 \Omega_k^{(0)} + f(z) \Omega_x^{(0)}} \\
f(z) &= \exp\left( 3 \int_0^z dz' \frac{(1+\omega_x(z'))}{(1+z')} \right)
\end{aligned}
$$

# Dark Energy vs. Modified Gravity

Bad Management
(original background)
or Someone was here
(source of disturbance)?

# Dark Energy vs. Modified Gravity

$$\delta G_{\mu\nu} + R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}^{\text{fluid}} + 8\pi G T_{\mu\nu}^{\text{DE}}$$

Requirements from the observations

$$\text{DE} : \omega_{\text{DE}} \simeq -1 \text{ and } \Omega_{\text{DE}} \simeq 0.7$$

$$\text{MG} : \text{well-defined infrared limit obeying causality}$$

$$: \zeta_{\text{LW}}^{MG} = \zeta_{\text{LW}}^{GR}$$

- Still require Dark Matter in both theories except TeVeS

# Dark Energy Models I

- Cosmological constant
- Quintessence

| Quintessence Potential | Reference | $\omega$ |
|---|---|---|
| $V_0 \exp(-\lambda\phi)$ | Ratra & Peebles (1988), Wetterich (1988), Ferreira & Joyce (1998) | $\omega = \lambda^2/3 - 1$ <br> $\lambda > 5.5 - 4.5, \Omega < 0.1 - 0.15$ |
| $V_0/\phi^\alpha, \alpha > 0$ | Ratra & Peebles (1988) | $\omega > -0.7$ |
| $m^2\phi^2, \lambda\phi^4$ | Frieman et al (1995) | PNGB $M^4[\cos(\phi/f) + 1]$ |
| $V_0(\exp M_p/\phi - 1)$ | Zlatev, Wang & Steinhardt (1999) | $\Omega_m \geq 0.2, \omega < -0.8$ |
| $V_0 \exp(\lambda\phi^2)/\phi^\alpha$ | Brax & Martin (1999,2000) | $\alpha \geq 11, \omega \simeq -0.82$ |
| $V_0(\cosh\lambda\phi - 1)^p$ | Sahni & Wang (2000) | $p < 1/2, \omega < -1/3$ |
| $V_0 \sinh^{-\alpha}(\lambda\phi)$ | Sahni & Starobinsky (2000), Ureña-López & Matos (2000) | early time : inverse power <br> late time : exponential |
| $V_0(e^{\alpha\kappa\phi} + e^{\beta\kappa\phi})$ | Barreiro, Copeland & Nunes (2000) | $\alpha > 5.5, \beta < 0.8, \omega < -0.8$ |
| $V_0[(\phi - B)^\alpha + A]e^{-\lambda\phi}$ | Albrecht & Skordis (2000) | $\omega \sim -1$ |
| $V_0 \exp[\lambda(\phi/M_p)^2]$ <br> $V_0 \cosh[\lambda\phi/M_p]$ | Lee, Olive, & Pospelov (2004) | $\omega \sim -1$ <br> $\omega \sim -1$ |

# DE II : Quartessence (Unification of DM & DE)

| Model | $\rho$ | Ref |
|---|---|---|
| Modified polytropic Cardassian | $[Aa^{3q(\nu-1)} + Ba^{-3q}]^{\frac{1}{q}}$ | [10] |
| New generalized Chaplygin gas | Same | [11] |
| ΛCDM | $q = 1, \nu = 1$ | [12] |
| Cardassian expansion | $q = 1$ | [13] |
| Polytropic Cardassian | $\nu = 1$ | [11] |
| generalized Chaplygin gas | $\nu = 2$ | [14] |
| variable Chaplygin gas | $q = 2$ | [15] |
| Chaplygin gas | $\nu = 2, q = 1$ | [16] |
| Modified Chaplygin gas | $(A + Ba^{-3})^q$ | [17] |
| Exponential Cardassian | $(Aa^{-3} + B)\exp[(\frac{qB}{Aa^{-3}+B})^\nu]$ | [18] |
| Extra dimension inspired | $Aa^{-3}[1 + \exp(-Ba^{-3})]^q$ | [19] |
| Phenomenological approach | $A(1 + Ba^{-1})^{q-\nu}[1 + Ca^{-\nu}]$ | [20] |
| Leaking gravity (DGP) | $Aa^{-3} + B - \sqrt{B^2 + ABa^{-3}}$ | [21] |

# Quartessence II

[10] K. Freese, Nucl. Phys. Proc. Suppl. **124**, 50. 2003 [hep-ph/0208264].
[11] X. Zhang, JCAP **0601**, 003. 2006 [0411221].
[12] T. Koivisto, H. Kurki-Suonio, and F. Ravndal, Phys. Rev. **D 71**, 064027. 2005. [astro-ph/0409163]
[13] K. Freese and M. Lewis, Phys. Lett. **B540**, 1. 2002 [astro-ph/0201229].
[14] M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. **D66**, 043507, 2002 [gr-qc/0202064].
[15] Z.-K. Guo and Y.-Z. Zhang, [astro-ph/0506091].
[16] A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. **B511**, 265 2001 [gr-qc/0103004].
[17] L. P. Chimento, Phys. Rev. **D69**, 123517 2004 [astro-ph/0311613].
[18] D.-J. Liu, C.-B. Sun, and X.-Z. Li, Phys. Lett. **B634**, 442. 2006 [astro-ph/0512355].
[19] Y.-G. Gong, X.-M. Chen, and C.-K. Duan, Mod. Phys. Lett. **A19**, 1933. 2004 [astro-ph/0404202].
[20] V. F. Cardone, A. Troisi, and S. Capozziello, Phys. Rev. **D69**, 083517. 2004 [astro-ph/0402228].
[21] C. Deffayet, G. R. Dvali, and G. Gabadadze, Phys. Rev. **D65**, 044023. 2002 [astro-ph/0105068].

# MODIFIED GRAVITY I

- A broad class of alternative gravity theories

  $\Psi$ : matter fields

- $\phi$ : a scalar field

$$S = \int d^n x \sqrt{-g} \left[ \frac{1}{2} f(R, \phi) + \mathcal{L}_\phi(g_{\mu\nu}, \phi, \partial\phi) + \mathcal{L}_m(g_{\mu\nu}, \Psi) \right]$$

$$\mathcal{L}_\phi = -\frac{M^2}{2}\omega(\phi)(\partial\phi)^2 - V(\phi)$$

$$(\partial\phi)^2 = \nabla_\mu \phi \nabla^\mu \phi. \; F(R, \phi) = \partial f(R, \phi)/\partial R.$$

# Modified Gravity II

| Generalized gravity | $\frac{1}{2}f(R,\phi)$ | $\mathcal{L}_\phi(\phi,\partial\phi)$ | $p(R,\phi)$ | $\varphi$ | $\hat{V}(\varphi)$ | Ref |
|---|---|---|---|---|---|---|
| Nonlinear gravity | $\frac{1}{2}f(R)$ | $\omega=0,\ V=0$ | $p=F(R)$ | $\sqrt{\frac{3}{2}}\ln F$ | $\frac{FR-f}{2F^2}$ | [1] |
| $R^2$-gravity | $\frac{1}{2}(R+\alpha R^2)$ | $\omega=0,\ V=0$ | $p=1+2\alpha R$ | $\sqrt{\frac{3}{2}}\ln F$ | $\frac{FR-f}{2F^2}$ | [2] |
| $1/R$-gravity | $\frac{1}{2}(R-\mu^4/R)$ | $\omega=0,\ V=0$ | $p=1+\mu^4/R^2$ | $\sqrt{\frac{3}{2}}\ln F$ | $\frac{FR-f}{2F^2}$ | [3] |
| Scalar-tensor theory | $\frac{1}{2}F(\phi)R$ | $\omega(\phi),\ V(\phi)$ | $p=F(\phi)$ | $\int\sqrt{\frac{\omega}{F}+\frac{3}{2}\frac{F'^2}{F^2}}d\phi$ | $\frac{V}{F^2}$ | [4] |
| Brans-Dicke theory | $\phi R$ | $\omega(\phi)=2\frac{\omega}{\phi},\ V=0$ | $p=\phi$ | $\int\sqrt{\frac{\omega}{F}+\frac{3}{2}\frac{F'^2}{F^2}}d\phi$ | 0 | [5] |
| Dilaton | $\frac{1}{2}e^{-\phi}R$ | $\omega(\phi)=e^{-\phi},\ V=0$ | $p=e^{-\phi}$ | $\frac{5}{2}\phi$ | 0 | [6] |
| NMC scalar | $\frac{1}{2}(1+\xi\phi^2)R$ | $\omega=1,\ V(\phi)$ | $p=1+\xi\phi^2$ | $\int\frac{\sqrt{1+\xi(6\xi-1)\phi^2}}{1-\xi\phi^2}d\phi$ | $\frac{V}{1-\xi\phi^2}$ | [7] |
| CC ($\xi=\frac{1}{6}$) | $\frac{1}{2}(1+\frac{1}{6}\phi^2)R$ | $\omega=1,\ V(\phi)$ | $p=1+\frac{1}{6}\phi^2$ | $\sqrt{6}\tanh^{-1}\frac{\phi}{\sqrt{6}}$ | $\frac{V}{1-\frac{1}{6}\phi^2}$ | [8] |
| Induced Gravity | $\frac{1}{2}\epsilon\phi^2 R$ | $\omega=1,\ V(\phi)$ | $p=\epsilon\phi^2$ | $\sqrt{6+\frac{1}{\epsilon}}\ln\phi$ | $\frac{V}{\epsilon\phi^2}$ | [9] |
| GR with a scalar | $\frac{1}{2}R$ | $\omega=1,\ V(\phi)$ | $p=1$ | $\phi$ | $V$ | |

# Duality of DE and MG

- Friedman equations

$$H^2 - \delta H = \frac{8\pi G}{3}\rho_m$$

$$H^2 \equiv \frac{8\pi G}{3}\left(\rho_m + \rho_{\mathrm{DE}}\right)$$

$$\omega_{\mathrm{DE}} = -1 - \frac{1}{3}\frac{d\ln\delta H}{d\ln a}$$

| | $\delta H$ | $\omega_{\mathrm{DE}}$ |
|---|---|---|
| BD | $\frac{\omega_{\mathrm{BD}}}{6}\frac{\dot\phi^2}{\phi^2} - H\frac{\dot\phi}{\phi}$ | $-1 + \frac{\frac{\ddot\phi}{\phi} - H\frac{\dot\phi}{\phi} + \omega_{\mathrm{BD}}\frac{\dot\phi^2}{\phi^2}}{\frac{\omega_{\mathrm{BD}}}{2}\frac{\dot\phi^2}{\phi^2} - 3H\frac{\dot\phi}{\phi}}$ |
| DGP | $\frac{H}{r_0}$ | $-\frac{1}{1+\Omega_{\mathrm{m}}}$ |
| Metric $f(R)$ | $\frac{1}{3F_0}\left(\frac{1}{2}(FR-f) - 3H\dot F + 3H^2(F_0-F)\right)$ | $-1 + \frac{2\ddot F - 2H\dot F - 4\dot H(F_0-F)}{FR - f - 6H\dot F + 6H^2(F_0-F)}$ |
| Palatini $f(R)$ | $\frac{1}{3F_0}\left(\frac{1}{2}(FR-f) + \frac{3}{2}\ddot F + \frac{3}{2}H\dot F - \frac{3}{2}F\frac{\dot F^2}{F^2} + 3H^2(F_0-F)\right)$ | $-1 + \frac{2\ddot F - 2H\dot F - 3F\frac{\dot F^2}{F^2} - 4\dot H(F_0-F)}{FR - f + 3\ddot F + 3H\dot F - 3F\frac{\dot F^2}{F^2} + 6H^2(F_0-F)}$ |

# Discrepancies DE and MG

- Perturbational Evolutions

Table 2. Comparisons of $H(z)$ and evolution of the linear perturbation $\delta_m$ with the effective gravitational constant $G_{\rm eff}$ in each model. In metric and Palatini formalism the perturbation calculations are held for subhorizon scale (i.e. $\frac{k}{a} > H$). Where $Q = -\frac{2F_{,R}}{F}\frac{k^2}{a^2}$ and $F'(T) = \frac{\partial F(T)}{\partial T}$.

| | $\frac{H(z)}{H_0}$ | $\delta_m$ | $G_{\rm eff}$ |
|---|---|---|---|
| BD | $\sqrt{\frac{\delta H}{H_0^2} + \frac{\phi_0}{\phi}\Omega_m^{(0)}(1+z)^3}$ | $\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0$ | $\left(\frac{2\omega_{\rm BD}+4}{2\omega_{\rm BD}+3}\right)\frac{1}{\phi_0}$ |
| DGP | $\frac{1}{2}\left(\frac{1}{r_0 H_0} + \sqrt{\left(\frac{1}{r_0 H_0}\right)^2 + 4\Omega_m^{(0)}(1+z)^3}\right)$ | $\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0$ | $G\left(1 + \frac{1}{3[1+2r_0 H\omega_{\rm DE}]}\right)$ |
| Metric $f(R)$ | $\sqrt{\frac{\delta H}{H_0^2} + \Omega_m^{(0)}(1+z)^3}$ | $\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0$ | $\frac{2(1-2Q)}{2-3Q}\frac{G}{F}$ |
| Palatini $f(R)$ | $\sqrt{\frac{\delta H}{H_0^2} + \Omega_m^{(0)}(1+z)^3}$ | $\ddot{\delta} + H\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0$ | $-\frac{1}{4\pi}\frac{F'}{2F+3F'\rho}\frac{k^2}{a^2}$ |

# Perturbations

- To get a well defined infrared limit, time and space dependence of perturbations must be factorized for long wavelengths.

$$ds^2 = a^2(\tau)\left(-(1+2\Phi)d\tau^2 + (1-2\Psi)d\vec{x}^2\right)$$

Due to consistency (to RW metric) condition

$$\frac{1}{a^2}\frac{\partial}{\partial\tau}\left(\frac{a^2\Psi}{\mathcal{H}}\right) + \Phi - \Psi = \left[\frac{1}{a}\frac{\partial}{\partial\tau}\left(\frac{a}{\mathcal{H}}\right) + \mathcal{O}(k^2)\right]\zeta$$

$$\Phi(\vec{k},t) = F(a)\zeta(\vec{k}) + \mathcal{O}(k^2\zeta)$$

$$\Psi(\vec{k},t) = \gamma(\vec{k},a)\Phi(\vec{k},t) + \mathcal{O}(k^2\zeta)$$

Two categories : $\gamma(\vec{k},a)$ or $\gamma(a)$

# Perturbation Equations

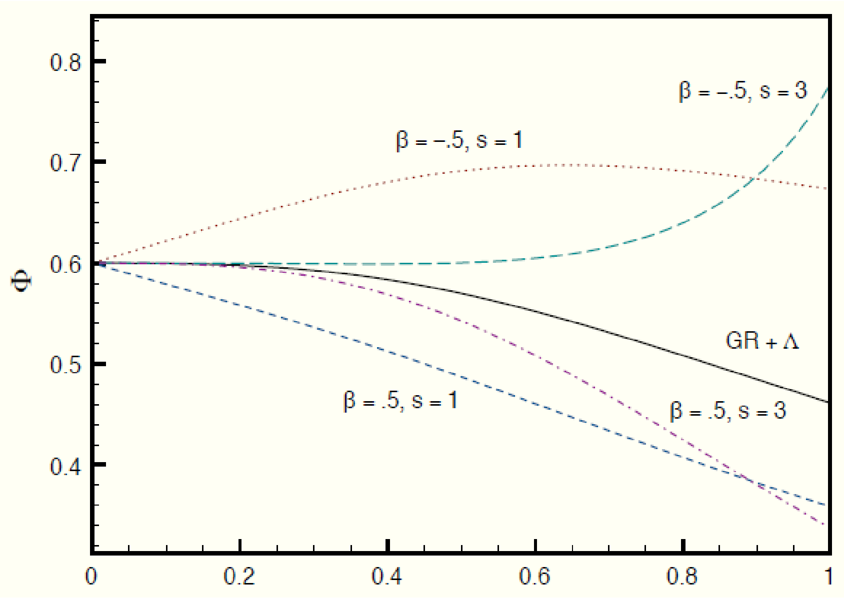
Two categories : $\gamma(a)$ or $\gamma(\vec{k}, a)$

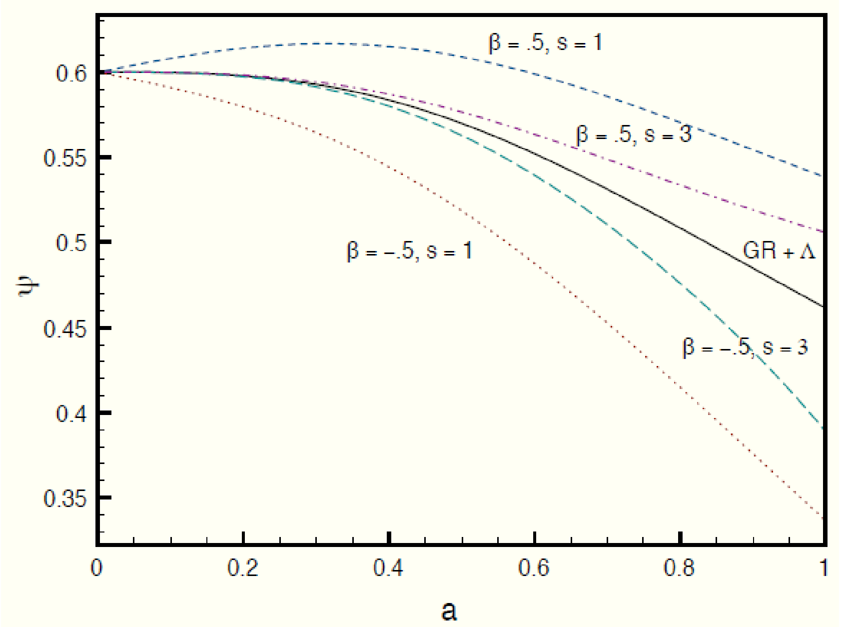
Scale independent MG : $\gamma(a) \equiv 1 + \beta a^s$ , $G_\Psi = \gamma G_\Phi$

Scale dependent MG : $\gamma(\vec{k}, a) \equiv \frac{1+\beta_1 k^2 a^s}{1+\beta_2 k^2 a^s}$ , $\frac{G_\Phi}{G} = \frac{1+\alpha_1 k^2 a^s}{1+\alpha_2 k^2 a^s}$

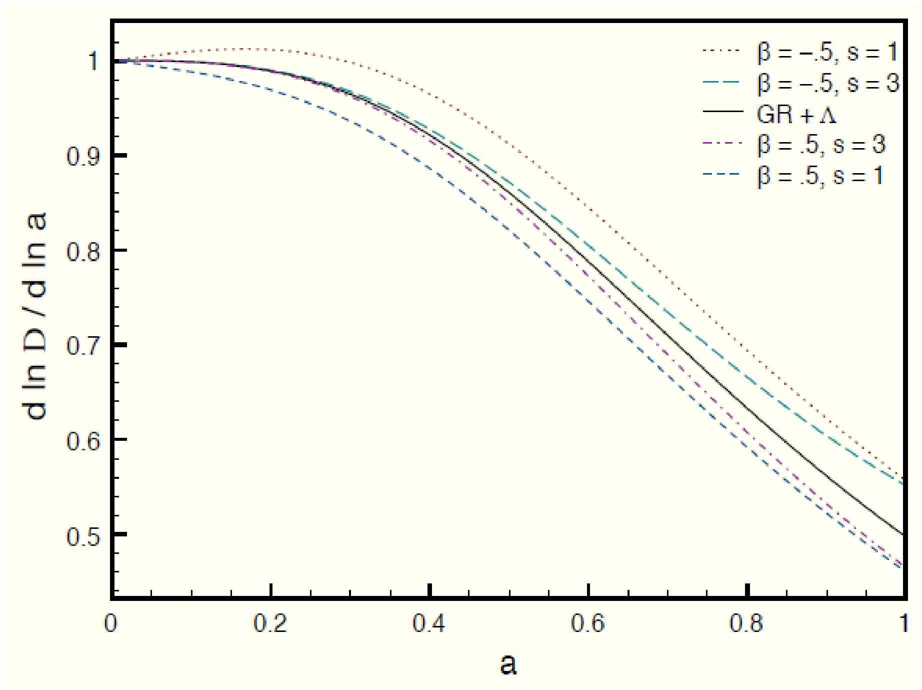
| | | |
|---|---|---|
| density fluctuations | $\dot{\delta} \simeq -(1+\omega)\frac{\theta}{a} - 3H\frac{\delta P}{\delta\rho}\delta\rho + 3H\omega\delta$ | $\dot{\delta}_{MG} \simeq -\frac{\theta_{MG}}{a}$ |
| velocity | $\dot{\theta} = -H(1-3\omega)\theta - \frac{\dot{\omega}}{1+\omega}\theta + \left(\frac{\delta P/\delta\rho}{1+\omega}\delta - \sigma + \Phi\right)\frac{k^2}{a}$ | $\dot{\theta}_{MG} = -H\theta_{MG} + \frac{k^2\Phi}{a}$ |
| Poisson equation | $k^2\Phi \simeq -4\pi G a^2 \rho\delta$ | $k^2\Phi \simeq -4\pi G_{eff} a^2 \rho\delta$ |
| Anisotropy | $k^2(\Phi - \Psi) \simeq 12\pi G a^2 (1+\omega)\rho\sigma$ | $k^2(\Phi + \Psi) \simeq -8\pi \tilde{G}_{eff} a^2 \rho\delta_{MG}$ |

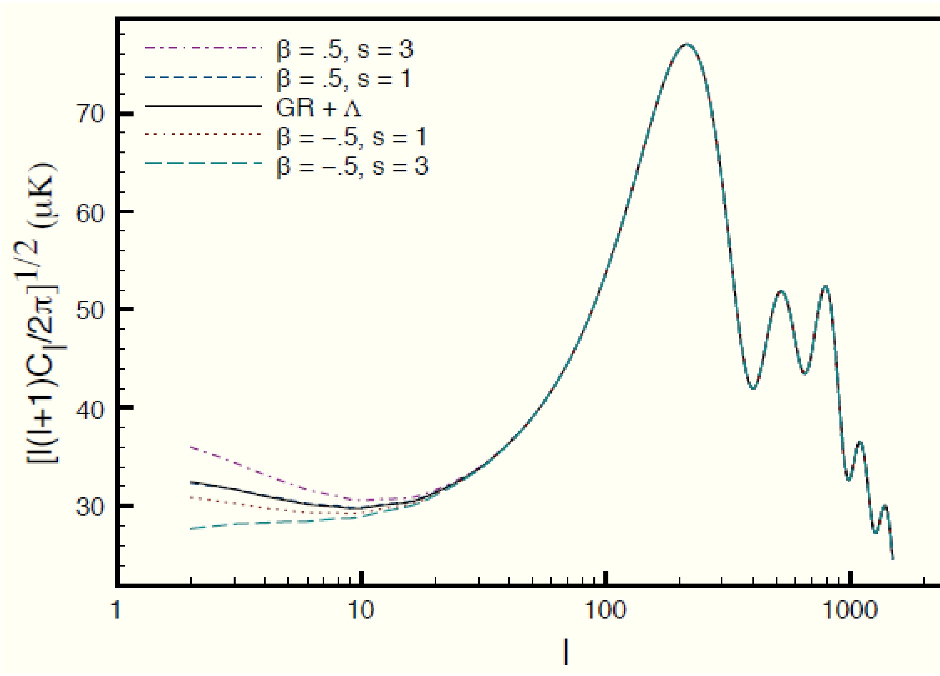
# Perturbation Quantities



# PERTURBATION QUANTITIES (CONTINUED)



# Observables

- Observables

| | | |
|---|---|---|
| Matter Power Spectra | $P_\delta = \frac{k^4}{(4\pi G)^2 a^4 \rho^2} P_\Psi$ | $P_{\delta,MG} = \frac{k^4}{(8\pi G_{eff})^2 a^4 \rho^2_{MG}} P_{\Psi+\Phi}$ |
| Velocity Spectra | $P_v = \frac{f^2 a^2 H^2}{k^2} P_\delta$ | $P_{v,MG} = \frac{f^2 a^2 H^2}{k^2} P_{\delta,MG}$ |
| ISW | $C_l^{ISW} = \int P_{\dot\Phi+\dot\Psi}(k,\chi) a^2 \frac{d\chi}{\chi^2}$ | |
| Weak Lensing | $P_l^\kappa = \int P_{\Phi+\Psi}(k,\chi) W^2(\chi) \frac{l^4}{\chi^4} d\chi$ | |

- Evidences (?)

: (1) High peculiar velocity (~ 400km/s) [0809.4041]

(2) Bright High z SNe [0811.2802]

# Summary

- The current Universe is under accelerating expansion.
- Either(/Both) Dark Energy or(/and) Modified Gravity may explain the accelerating universe.
- Dark Enegy models and Modified Gravity theories can be degenerated in the background evolutions.
- Accurate measurements for the background evolutions to constraint the parameters of theories.
- Compare with perturbational observables.
- Probably we can distinguish the origin of acceleration from both background and perturbation evolutions.