Hadronic Atoms In Effective Field Theory

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Based on:

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Plan

- ❑ Introduction: Concept of Hadronic Atoms and importance
- ❑ Formalism of Effective Field Theory
- ❑ Framework: Non-Relativistic Effective Field Theory approach
- ❑ Analysis of Kaonic Hydrogen & Kaonic Deuterium
- ❑ Main Results
- ❑ Summary and outlook

Concept Of Hadronic Atoms

- **Hadronic Atom** $\longrightarrow$ *Quasi-stable* bound state of hadrons created predominantly by *Electromagnetic* interactions. E.g. $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\pi^- d$ $K^- p$, $K^- d$
- Bohr radius $R_B \gg R_{strong}$: *Strong interactions effectively much weaker*
- Very small relative momenta $\sim \frac{1}{R_B}$ $\Rightarrow$Non-relativistic approach
- Atomic observables: Shifts of energy levels ΔE_{nl} from purely Coulombic values and the decay widths Γ_{nl}
- *Deser-Trueman formula*: Example $K^- p$ (in the absence of isospin breaking)

$$\Delta E^s_n - \frac{i}{2}\Gamma_n = -\frac{\alpha^3 \mu^3_{KN}}{2\pi M_\pi n^3}\mathcal{T}^{thr}_{KN\to KN}\,, \quad \mathcal{T}^{thr}_{KN\to KN} = 2\pi[1+\frac{M_K}{m_N}](a_0+a_1)$$

- S-wave scattering lengths $a_0\,,\ a_1$ $\Rightarrow$ yields valuable insights into low energy QCD
- Bound states are *systematically* and *consistently* analyzed in the framework of low-energy Effective Field Theory

EFFECTIVE FIELD THEORY: Formalism

(*Weinberg 1979*)

❏ Separation of scales: High and low energy dynamics

- Distinct high and low energy Degrees Of Freedom
- Low energy dynamics in terms of relevant DOFs (eg. pions in QCD)
- High energy dynamics not resolved (heavy DOFs integrated out)
- *Contact interactions* $\Rightarrow$ LECs

❏ Power Counting Scheme $\longrightarrow$ “Effectively” renormalizable

- Re-ordering of standard *Trees* and *Loops* diagrams
- Expansion in powers of small momenta Q over a hard scale Λ $(Q \ll \Lambda)$

$$\boxed{\mathcal{M} = \sum_n \left(\frac{Q}{\Lambda}\right)^n f_n\left(g_i, Q/\mu\right)}$$

$\mu \longrightarrow$ regularization scale $\qquad g_i \longrightarrow$ LECs

$f_n \sim \mathcal{O}(1)$ “naturalness” $\qquad n \longrightarrow$ bounded from below

- *Controlled* and *Systematic* expansion $\Rightarrow$ finite number of counter terms at every order

EFT of Hadronic Atoms: Framework of Approach

- Involves a *hierarchy* of EFTs associated with the various momentum scales present. Broadly, the procedure utilizes three basic steps:

  - **STEP 1**: Construct non-relativistic effective Lagrangian with (complex) couplings and use the usual *Rayleigh-Schrödinger* perturbation theory to obtain the corrections to the exact *Coulomb problem*
  - **STEP 2**: *Matching Procedure* $\Rightarrow$ relate couplings of the effective Lagrangian to QCD threshold parameters e.g. scattering lengths and express the complex energy shift in terms of these parameters
  - **STEP 3**: Extract scattering length(s) from experimentally measured complex energy shift

- A very reliable and accurate method of determining hadron-hadron scattering lengths

Features of Kaonic Hydrogen atom

- **Decay modes**: $(K^- p)_{1s} \to \begin{cases} \pi^0\Lambda, \quad \pi^\pm\Sigma^\mp \quad [\text{strong}] \\ \gamma Y, \quad Y : \Lambda,\, \Sigma^0 \quad [\text{electromagnetic}] \leq 1\% \end{cases}$
- **Inelastic $\bar{K}N$ channels**: $K^- p,\ \bar{K}^0 n,\ \pi^0\Sigma,\ \pi^0\Lambda,\ \eta\Sigma^0,\ \eta\Lambda,\ \pi^\pm\Sigma^\mp,\ K^+\Xi^-,\ \bar{K}^0\Xi^0$
- **Relative momenta**: $\langle p^2 \rangle^{1/2} = \alpha\mu_c \approx$ 2 MeV $\ll \mu_c\ , \quad \mu_c = \frac{m_p M_{K^+}}{m_p + M_{K^+}}$
- **Bohr radius**: $R_B = (\alpha\mu_c)^{-1} \approx$ 100 fm $\gg R_{Strong}$
- **Binding energy**: $E_{1s} = \frac{1}{2}\mu_c\alpha^2 + \ldots \approx$ 8 keV $\ll \mu_c$
- **Width**: $\Gamma_{1s} \approx$ 250 eV $\ll E_{1s}$
- **Unitary cusp**: Large isospin breaking corrections to the LO Deser formula
- **Power counting**: Small parameter, $\boxed{\delta \sim \alpha \sim (m_d - m_u)}$

$$\Delta E^s_n + \frac{i}{2}\Gamma_n = \underbrace{\delta^3}_{\text{LO}} + \underbrace{\delta^{7/2}}_{\text{NLO}} + \underbrace{\delta^4}_{\text{NNLO}} + \ldots \quad \text{(modulo } \ln\delta \text{ terms)}$$

Non-Relativistic Effective Lagrangian for the $\bar{K}N$ system

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \psi^\dagger \left\{ i\mathcal{D}_t - m_p + \frac{\mathcal{D}^2}{2m_p} + \frac{\mathcal{D}^4}{8m_p^3} + \cdots \right. \\
&- \left. c_p^F \frac{e\sigma\mathbf{B}}{2m_p} - c_p^D \frac{e(\mathcal{D}\mathbf{E} - \mathbf{E}\mathcal{D})}{8m_p^2} - c_p^S \frac{ie\sigma(\mathcal{D}\times\mathbf{E} - \mathbf{E}\times\mathcal{D})}{8m_p^2} + \cdots \right\} \psi \\
&+ \chi^\dagger \left\{ i\partial_t - m_n + \frac{\nabla^2}{2m_n} + \frac{\nabla^4}{8m_n^3} + \cdots \right\} \chi \\
&+ \sum_{\pm} (K^\pm)^\dagger \left\{ iD_t - M_{K^+} + \frac{\mathbf{D}^2}{2M_{K^+}} + \frac{\mathbf{D}^4}{8M_{K^+}^3} + \cdots \mp c_K^R \frac{e(\mathbf{DE} - \mathbf{ED})}{6M_{K^+}^2} + \cdots \right\} K^\pm \\
&+ (\bar{K}^0)^\dagger \left\{ i\partial_t - M_{\bar{K}^0} + \frac{\nabla^2}{2M_{\bar{K}^0}} + \frac{\nabla^4}{8M_{\bar{K}^0}^3} + \cdots \right\} \bar{K}^0 \\
&+ \tilde{d}_1 \psi^\dagger\psi (K^-)^\dagger K^- + \tilde{d}_2(\psi^\dagger\chi (K^-)^\dagger \bar{K}^0 + h.c.) + \tilde{d}_3 \chi^\dagger\chi (\bar{K}^0)^\dagger \bar{K}^0 + \cdots
\end{aligned}
$$

proton

neutron

kaon

(**Note :** $\tilde{d}_i$ s are *complex* & determined by *matching* to $\bar{K}N$ threshold amplitude)

The Electromagnetic form factors:

$$c_p^F = 1 + \mu_p \; ; \; c_p^D = 1 + 2\mu_p + \tfrac{4}{3} m_p^2 \left\langle r_p^2 \right\rangle \; ; \; c_p^S = 1 + 2\mu_p \; ; \; c_K^R = M_K^2 \left\langle r_K^2 \right\rangle$$

Use non-relativistic *Rayleigh-Schrödinger* perturbation theory, to obtain the bound state energy shifts

Perturbation Theory (*Feshbach Formalism*)

- **Full Hamilton**: $H = \underbrace{H_0}_{\text{Free}} + \underbrace{H_C}_{\text{Pure Coulomb}} + \underbrace{V}_{\text{Rest (Perturbations)}}$

- **Schrödinger equation**: $(H_0 + H_C)\,|\Psi_{nljm}(\mathbf{P})\rangle = \bar{E}_n(\mathbf{P})\,|\Psi_{nljm}(\mathbf{P})\rangle$

  where, $\bar{E}_n(\mathbf{P}) = m_p + M_{K^+} + \frac{\mathbf{P}^2}{2(m_p+M_{K^+})} - \frac{\mu_c\alpha^2}{2n^2} = \hat{E}_n + \frac{\mathbf{P}^2}{2(m_p+M_{K^+})}$

- **State Vectors**:

$$|\Psi_{nljm}(\mathbf{P})\rangle = \sum_s \int \frac{d^3\mathbf{q}}{(2\pi)^3} \langle jm|l(m-s)\frac{1}{2}\,s\rangle \times Y_{l(m-s)}(\mathbf{q})\Psi_{nl}(|\mathbf{q}|)\,|\mathbf{P},\mathbf{q},s\rangle$$

where, $|\mathbf{P},\mathbf{q},s\rangle = b^\dagger(\mu_1\mathbf{P}+\mathbf{q},s)a^\dagger(\mu_2\mathbf{P}-\mathbf{q})|0\rangle$

$$\mu_1 = \frac{m_p}{m_p+M_{K^+}}, \quad \mu_2 = \frac{M_{K^+}}{m_p+M_{K^+}}$$

- $\Psi_{nl}(|\mathbf{q}|) \longrightarrow$ Radial Coulomb wavefunction
- $\mathbf{P} \longrightarrow$ CM momenta of the K^-p system
- $a^\dagger \longrightarrow$ Non-relativistic creation operator for K^-
- $b^\dagger \longrightarrow$ Non-relativistic creation operator for p

Green's Functions and Master Equation

- **Free Green's function**: $G_0 = \frac{1}{z-H_0}$
- **Coulomb Green's function**: $G_C = \frac{1}{z-H_0-H_C}$
- **"Pole removed" Coulomb Green's function**:

$$\hat{G}_{nlj}(z) = G_C(z) - \sum_m \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{|\Psi_{nljm}(\mathbf{P})\rangle \langle \Psi_{nljm}(\mathbf{P})|}{z - \bar{E}_n(\mathbf{P})}$$

- **Elastic Transition Operator**:

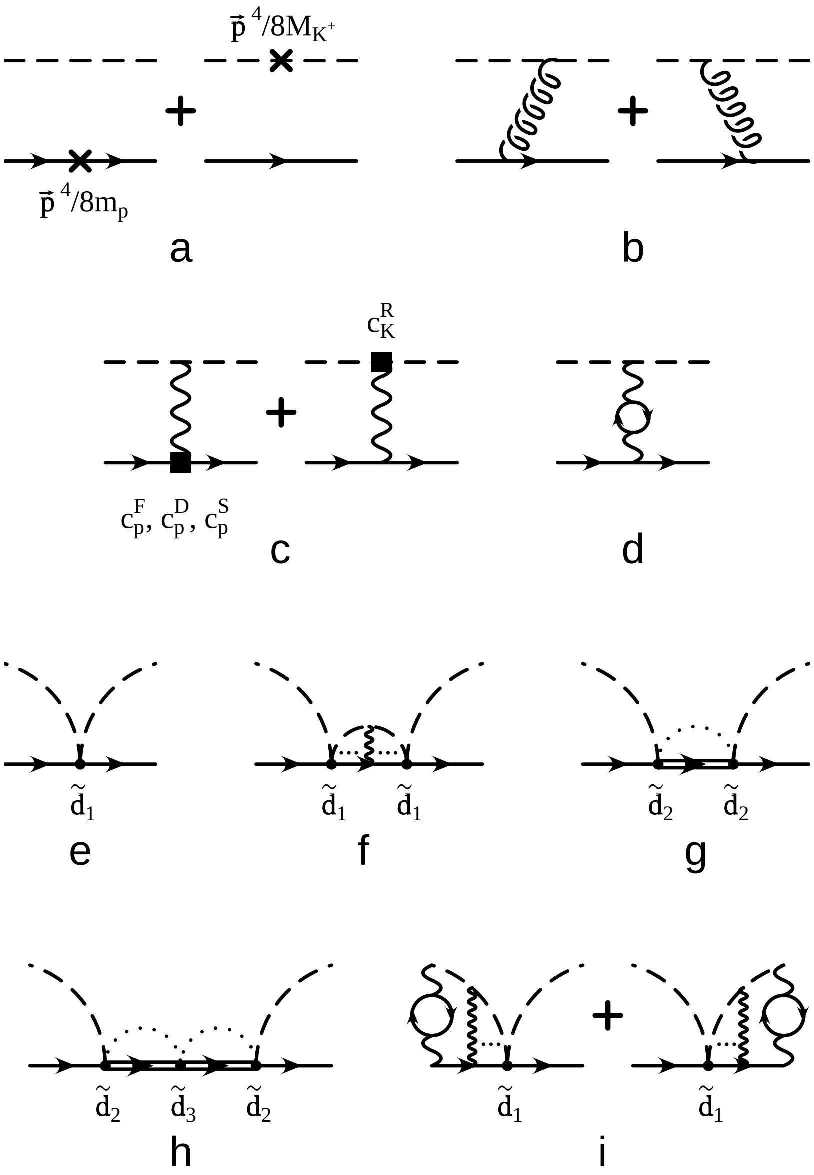
$$\boxed{M_{nlj}(z) = V + V\hat{G}_{nlj}(z)M_{nlj}(z)}$$

$\Rightarrow$ Use iterative technique to solve the master equation

- Look for positions of the shifted poles Δz on the *second Riemann sheet* of the complex z-plane:

$$\boxed{\mathcal{R}e(\Delta z) = \Delta E \qquad \mathcal{I}m(\Delta z) = -\tfrac{1}{2}\Gamma}$$

Feynman Diagrams for Complex Energy Shift



- Use *Coulomb gauge*

- **Electromagnetic Shift**

  (a) Recoil corrections

  (b) Transverse photon exchange

  (c) Finite size corrections

  (d) Vacuum polarization

- **Strong Shift**

  (e) Leading K^-p interaction

  (f) K^-p ints. with Coulomb ladders

  (g) Leading $\bar{K}^0 n$ intermediate state

  (h) Iterated $\bar{K}^0 n$ intermediate state

  (i) Coulomb ladders in the K^-p ints.

Total Complex Energy Shift

The analytical expression for the total complex energy shift of kaonic hydrogen for a general level with quantum numbers $n = 1, 2, ..., j = \frac{1}{2}, \frac{3}{2}, .., m = -j, ...j$ and $l = j \pm \frac{1}{2}$, could be split up in the following way:

$$\Delta E_{nlj} = \Delta E^{em}_{nlj} + \delta_{l0}(\Delta E^{s}_{n} - \tfrac{i}{2}\,\Gamma_n) + o(\delta^4)$$

- **The Electromagnetic Shift:**

$$\begin{aligned}\Delta E^{em}_{nlj} &= -\frac{m_p^3 + M_{K^+}^3}{8m_p^3 M_{K^+}^3}\left(\frac{\alpha\mu_c}{n}\right)^4 \left\{\frac{4n}{l+\frac{1}{2}} - 3\right\} - \frac{\alpha^4\mu_c^3}{4m_p M_{K^+} n^4}\left\{-4n\delta_{l0} - 4 + \frac{6n}{l+\frac{1}{2}}\right\} \\ &+ \frac{2\alpha^4\mu_c^3}{n^4}\left(\frac{c_p^F}{m_p M_{K^+}} + \frac{c_p^S}{2m_p^2}\right)\left\{\frac{n}{2l+1} - \frac{n}{2j+1} - \frac{n}{2}\,\delta_{l0}\right\} \\ &+ \frac{4\alpha^4\mu_c^3}{n^3}\,\delta_{l0}\left(\frac{c_p^D}{8m_p^2} + \frac{c_K^R}{6M_{K^+}^2}\right) + \Delta E^{(d)}_{nl}\end{aligned}$$

$\Rightarrow$ The general expression for the vacuum polarization contribution (diagram(d)), $\Delta E^{(d)}_{nl}$ is given in the paper by *D.Eiras and J.Soto, Phys. Lett. B* ***491****(2000) 101*

The Strong Energy Shift

$$\Delta E_n^s - \frac{i}{2}\Gamma_n = -\frac{\alpha^3\mu_c^3}{\pi n^3}\left\{\tilde{d}_1 - \frac{\alpha\mu_c^2}{2\pi}\tilde{d}_1^{\,2}(\chi + s_n(\alpha)) - \tilde{d}_2^{\,2}\frac{\mu_0 q_0}{2\pi} + \tilde{d}_2^{\,2}\tilde{d}_3\left(\frac{\mu_0 q_0}{2\pi}\right)^2\right\} + \Delta E_n^{(i)}$$

with, $\mu_0 = \frac{m_n M_{\bar{K}^0}}{(m_n + M_{\bar{K}^0})}$, $q_0 = (2\mu_0(m_n + M_{\bar{K}^0} - m_p - M_{K^+}))^{1/2}$

$$s_n(\alpha) = 2\left(\psi(n) - \psi(1) - \frac{1}{n} + \ln\alpha - \ln n\right), \qquad \psi(x) \doteq \frac{\Gamma'(x)}{\Gamma(x)},$$

$$\chi = \mu^{2(d-4)}\left(\frac{1}{d-4} - \Gamma'(1) - \ln 4\pi\right) + \ln\frac{(2\mu_c)^2}{\mu^2} - 1$$

- **Vacuum polarization** (diagram(i)): $\Delta E_n^{(i)} = -(\alpha^3\mu_c^3/\pi n^3)\,\tilde{d}_1\,\delta_n^{\rm vac} + o(\delta^4)$

- The calculation of $\delta_{n=1}^{\rm vac} \approx 0.87\% \longrightarrow$ *D.Eiras and J.Soto, Phys. Lett. B* ***491****(2000) 101*

- $\tilde{d}_1,\ \tilde{d}_2,\ \tilde{d}_3 \Rightarrow$ matching to the elastic $K^-p \to K^-p$ scatt. amplitude at threshold

$$\boxed{\frac{1}{2M_{K^+}}\mathcal{T}_{KN} = \tilde{d}_1 - \tilde{d}_2^{\,2}\frac{\mu_0 q_0}{2\pi} + \tilde{d}_2^{\,2}\tilde{d}_3\left(\frac{\mu_0 q_0}{2\pi}\right)^2 - \tilde{d}_1^{\,2}\frac{\alpha\mu_c^2}{2\pi}(\chi - 2\pi i)}$$

The Strong Energy Shift

- Matching allows to express the complex strong energy shift in terms of the elastic $\bar{K}N$ threshold amplitude, $\mathcal{T}_{KN}$:

$$\Delta E_n^s - \frac{i}{2}\Gamma_n = -\frac{\alpha^3\mu_c^3}{2\pi M_{K^+} n^3}\mathcal{T}_{KN}\left\{1 - \frac{\alpha\mu_c^2}{4\pi M_{K^+}}\mathcal{T}_{KN}(s_n(\alpha) + 2\pi i) + \delta_n^{\text{vac}}\right\}$$

- $\mathcal{T}_{KN}$ from standard relativistic theory at LO $\mathcal{O}(\delta^0)$:

$$\boxed{\mathcal{T}_{KN} = 2\pi[1 + \tfrac{M_K}{m_N}](a_0 + a_1) + \underbrace{\mathcal{O}(\sqrt{\delta})}_{}}$$

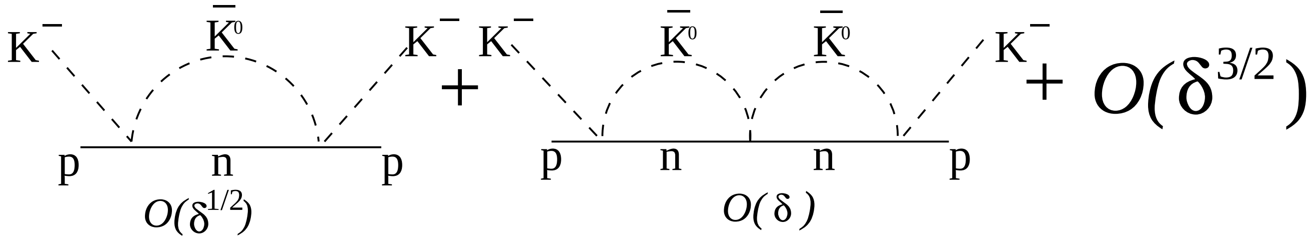
$$\underbrace{\text{Cusp Effect : BIG!!}}_{}$$

$\mathcal{T}_{KN} \longrightarrow$ correct, but not sufficiently accurate for the analysis of experimental data

Explicit inclusion of the Cusp Effect

- **Observation:** Corrections at $\mathcal{O}(\sqrt{\delta})$ arising from **Cusp Effect** could be expressed entirely in terms of a_0 & a_1 $\Rightarrow$ *completely parameter independent*

The Cusp Effect



- **Strategy:** Resummation of the bubbles with $\bar{K}^0 n$ intermediate states gives,

$$\mathcal{T}^{\rm Cusp}_{KN} = 4\pi\left(1+\frac{M_{K^+}}{m_p}\right)\frac{\frac{1}{2}(a_0+a_1)+q_0 a_0 a_1}{1+\frac{q_0}{2}(a_0+a_1)}$$

- We re-define, $\mathcal{T}_{KN} = \mathcal{T}^{\rm Cusp}_{KN} + \frac{i\alpha\mu_c^2}{2M_{K^+}}(\mathcal{T}^{\rm Cusp}_{KN})^2 + \delta\mathcal{T}^{\rm ChPT}_{KN} + o(\delta)$

  where, $\delta\mathcal{T}^{\rm ChPT}_{KN} \sim O(\delta)$ $\longrightarrow$ additional corrections from underlying *chiral* effects

Modified Deser formula to analyze DEAR & KEK data

- Our modified formula, upto-and-including $\mathcal{O}(\alpha^4, \alpha^3(m_d - m_u))$ i.e. NNLO in Isospin breaking:

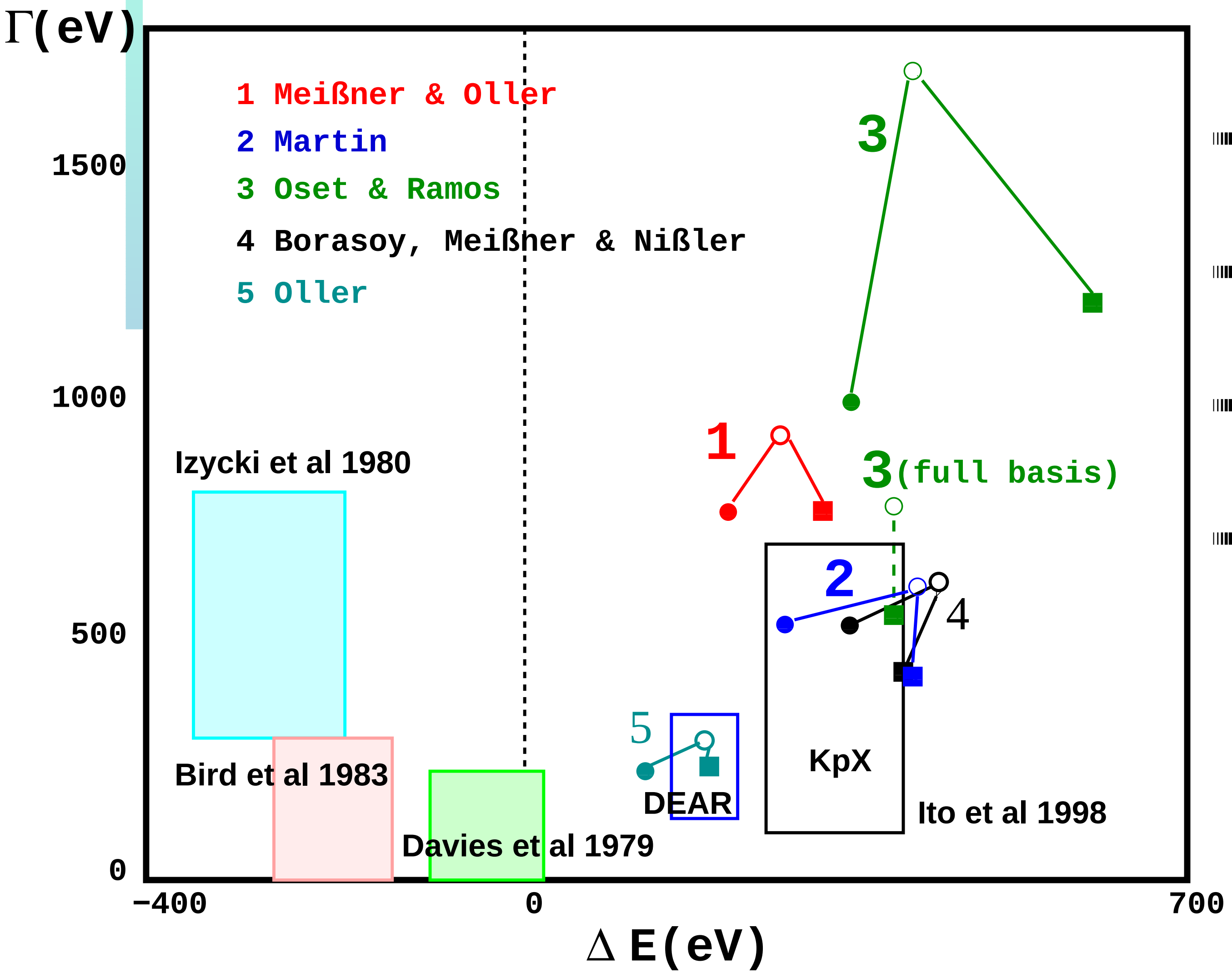
$$\Delta E_n^s - \frac{i}{2}\,\Gamma_n = -\frac{\alpha^3\mu_c^3}{2\pi M_{K^+} n^3}\left(\mathcal{T}_{KN}^{\rm Cusp} + \delta\mathcal{T}_{KN}^{\rm ChPT}\right)\Big\{1 - \underbrace{\frac{\alpha\mu_c^2 s_n(\alpha)}{4\pi M_{K^+}}\,\mathcal{T}_{KN}^{\rm Cusp}}_{\textbf{Coulomb}} + \delta_n^{\rm vac}\Big\}$$

$$\mathcal{T}_{KN}^{\rm Cusp} = 4\pi\left(1+\frac{M_{K^+}}{m_p}\right)\frac{\frac{1}{2}\,(a_0+a_1) + q_0 a_0 a_1}{1+\frac{q_0}{2}\,(a_0+a_1)}\;;\; q_0 = \sqrt{\frac{2m_n M_{\bar{K}^0}}{m_n + M_{\bar{K}^0}}(m_n + M_{\bar{K}^0} - m_p - M_{K^+})}$$

Corrections to the Deser Formula (Rough estimate):

- LARGE (non-analytic in δ, parameter independent) :
  - **Unitary Cusp Effect** $\sim 50\%$ at $\mathcal{O}(\sqrt{\delta}) \longrightarrow$ numerically most dominant (see also: *Dalitz & Tuan*)
  - **Coulomb Effects** $\sim (10$ to $15)\%$ at $\mathcal{O}(\delta \ln \delta)$
- SMALL (analytic in δ) :
  - **Vacuum Polarization** $\sim 1\%$
  - **CHPT** $\sim (-0.5 \pm 0.4)\%$ at $\mathcal{O}(p^2)$(or $\mathcal{O}(\delta)$)

Energy Shift and Width in Kaonic Hydrogen



- ● → LO Deser formula
- ❍ → Including unitary cusp
- ■ → Full modified Deser formula
- Input → $a_0 \& a_1$ from Refs:

MO, Phys. Lett. B 500 (2001) 263

M, Nucl. Phys. B 179 (1981) 33

OR, Nucl. Phys. A 635 (1998) 99

BMN, Phys. Rev C 74 (2006) 055201

O, Eur. Phys. J. A 28 (2006) 63

Recent DEAR data generally not consistent with theoretical analysis based on existing scattering data

DEAR & KEK restrictions to a_0 & a_1

- Modified *Deser* formula for kaonic hydrogen is used to extract the elastic threshold scattering amplitude $a_p(K^- p \to K^- p)$:

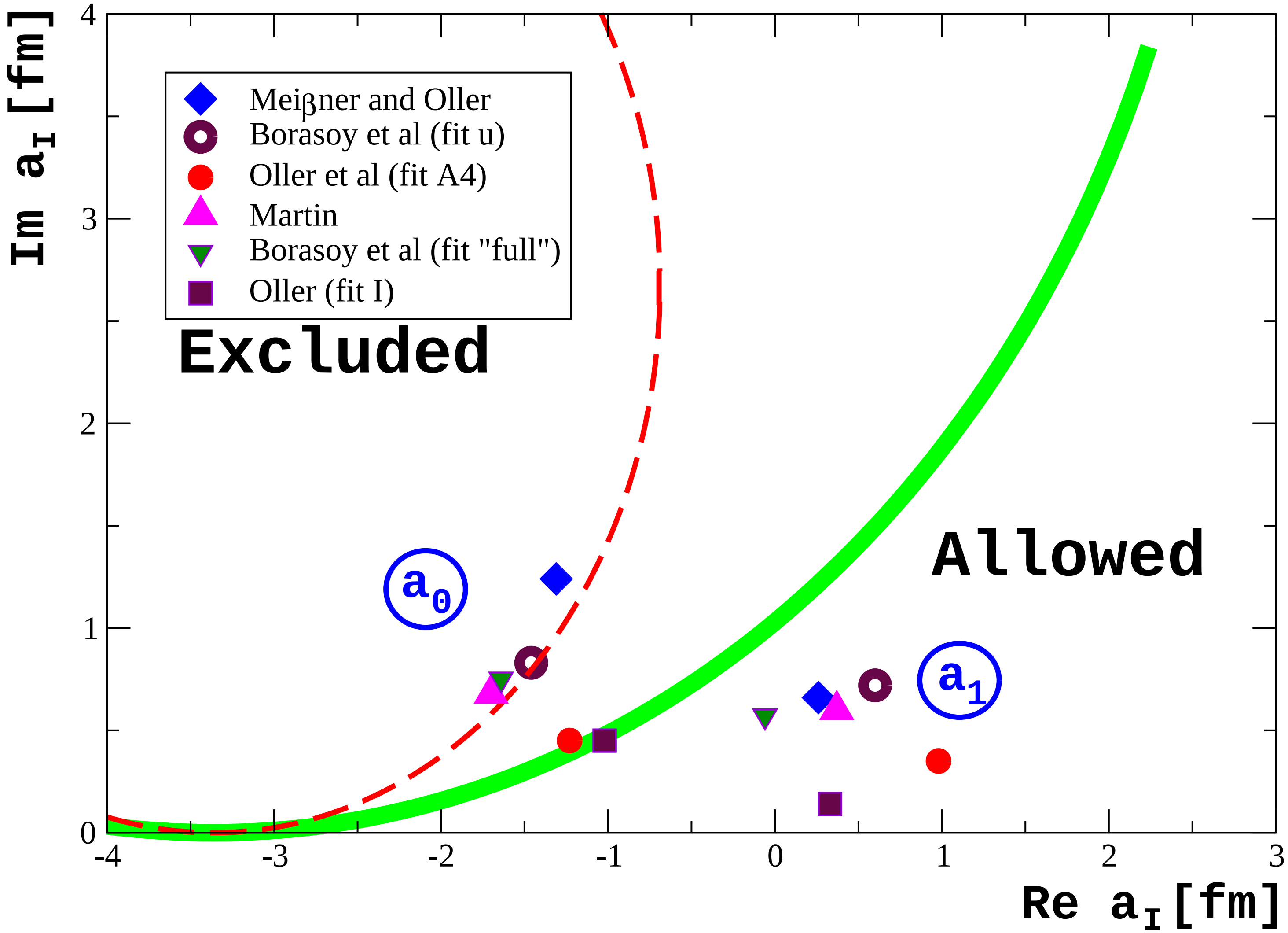
$$\Delta E_{1s} - \frac{i}{2}\Gamma_{1s} = -2\alpha^3\, \mu_{Kp}^2\, a_p\, \{1 - 2\alpha\mu_{Kp}(\ln\alpha - 1)a_p\}$$

- DEAR results (central): $\Delta E_{1s} = 193$ eV ; $\Gamma_{1s} = 249$ eV
- KEK results (central): $\Delta E_{1s} = 323$ eV ; $\Gamma_{1s} = 407$ eV
- **Constraints:**
  - **Isospin breaking (Cusp Effect)** $\longrightarrow$ eqn. of a circle

$$\boxed{a_0 + a_1 + \frac{2q_0}{1-q_0 a_p}\, a_0 a_1 - \frac{2a_p}{1-q_0 a_p} = 0} \quad ; \quad a_p = \frac{\frac{1}{2}(a_0+a_1)+q_0 a_0 a_1}{1+\frac{q_0}{2}(a_0+a_1)}$$

  - **Unitarity condition**

$$\boxed{\mathrm{Im}\, a_I \geq 0 \quad , \quad I = 0, 1}$$


Im a_I [fm]
Re a_I [fm]
Meiβner and Oller
Borasoy et al (fit u)
Oller et al (fit A4)
Martin
Borasoy et al (fit "full")
Oller (fit I)
Excluded
Allowed
a0
a1
4
3
2
1
0
-4
-3
-2
-1
0
1
2
3

The Kaonic Deuterium

- **Hadronic Atom** $\longrightarrow$ Quasi-stable bound state of a kaon (K^-) and a deuteron (d)
  $\longrightarrow$ Interactions predominantly *electromagnetic*
  $\longrightarrow$ Strong ints. are *effectively much weaker*
- $R_{\rm int} \sim 70\,{\rm fm} \gg R_{\rm strong}$
- $E^d_{1s} \simeq \frac{1}{2}\,\mu_{kd}\,\alpha^2 = 10.4$ keV ; $\Gamma^d_{1s} = \frac{1}{\tau} \simeq 1.2$ keV
- *Deser-Trueman* formula at LO: $\Delta E^d_{1s} - \frac{i}{2}\Gamma^d_{1s} = -2\alpha^3\mu^2_{kd}A_{\bar{K}d}$
- Our *modified Deser formula* up to and including NNLO in Isospin breaking:

$$\Delta E^d_{1s} - \frac{i}{2}\Gamma^d_{1s} = -2\alpha^3\mu^2_{kd}\underbrace{A_{\bar{K}d}}\left\{1 - \underbrace{2\alpha\,\mu_{kd}\,A_{\bar{K}d}(\ln\alpha - 1)} + \underbrace{\delta^{\rm vac}}\right\}$$

Unitary Cusp $\sim$ **small**. **Coulomb** $\sim 20\%$ $\sim 1\%$

Why study Kaonic Deuterium?

- **AIM:** $a_0 \,\&\, a_1 \longrightarrow$ Complex quantities

  $\longrightarrow$ 4 independent measurement of physical quantities (at least!!)

- Measurement of $a_p(K^-p \rightarrow K^-p)$ from DEAR collaboration

  $\longrightarrow$ Energy shift & Decay width of kaonic hydrogen

- Measurement of $A_{Kd}(K^-d \rightarrow K^-d)$ from (proposed) SIDDHARTA collaboration

  $\longrightarrow$ Energy shift & Decay width of kaonic deuterium

- **INVERSE PROBLEM:**
  - Multiple-scattering series: $A_{Kd} = \underbrace{\frac{1}{2}(a_0 + 3a_1)}_{\text{Impulse approx.}} + \underbrace{(\text{double scattering}) + \cdots}_{\text{corrections}}$
  - Physical basis: $a_p = \frac{\frac{1}{2}(a_0+a_1)+q_0 a_0 a_1}{1+\frac{q_0}{2}(a_0+a_1)}$ (includes LO isospin breaking)

- **Strategy I:** Assume *synthetic data* from available theoretical predictions for A_{Kd}, in the absence of experimental data presently

Multiple Scattering Formula

- Perturbative formula for A_{Kd} is divergent $\Rightarrow$ Large size of the $\bar{K}N$ scattering lengths
- **Strategy II:** Partial re-summation in the *Static Limit* (Fixed Center Approximation)

$$\left(1+\frac{M_K}{M_d}\right)A_{Kd} = \int_0^\infty dr\ \left(u^2(r)+w^2(r)\right)\ \hat{a}_{kd}(r) \quad ; \quad \int_0^\infty dr\ \left(u^2(r)+w^2(r)\right) = 1\,,$$

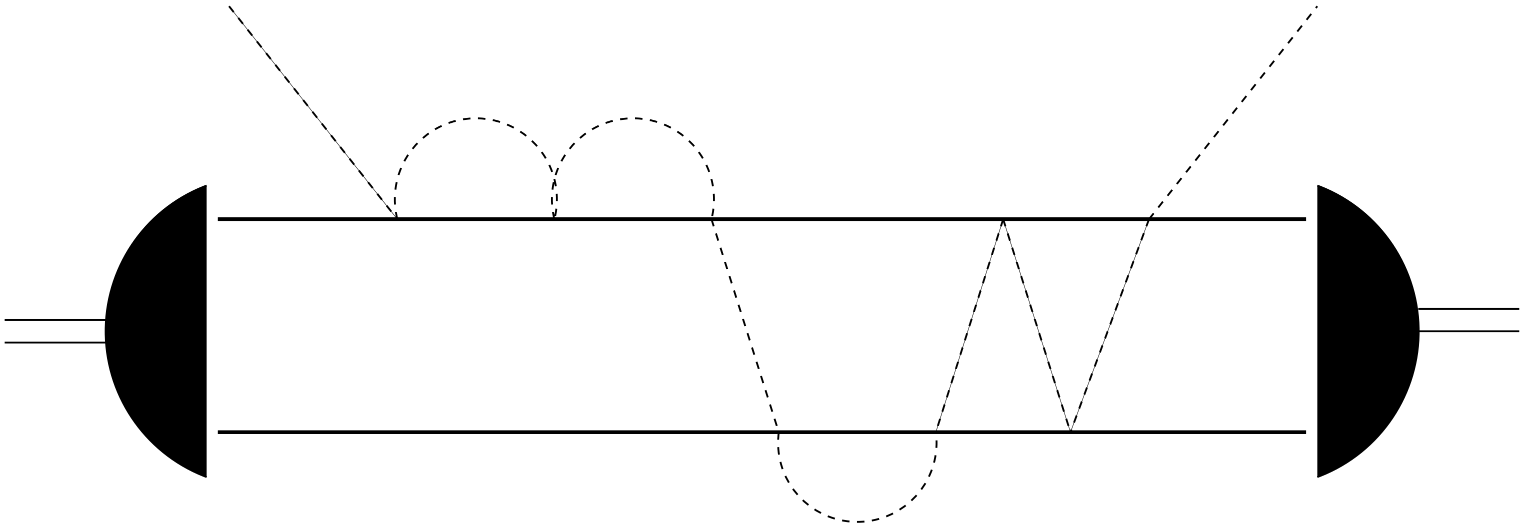
$$\hat{a}_{kd}(r) = \frac{\mathcal{K}(a_p+a_n)+\mathcal{K}^2(2a_pa_n-b_x^2)/r-2\mathcal{K}^3b_x^2a_n/r^2}{1-\mathcal{K}^2a_pa_n/r^2+\mathcal{K}^3b_x^2a_n/r^3} + \delta_{3-\mathrm{body}}$$

$$b_x^2 = a_x^2/(1+\mathcal{K}a_u/r) \quad ; \quad \mathcal{K} = \left(1+\frac{M_K}{m_p}\right)$$

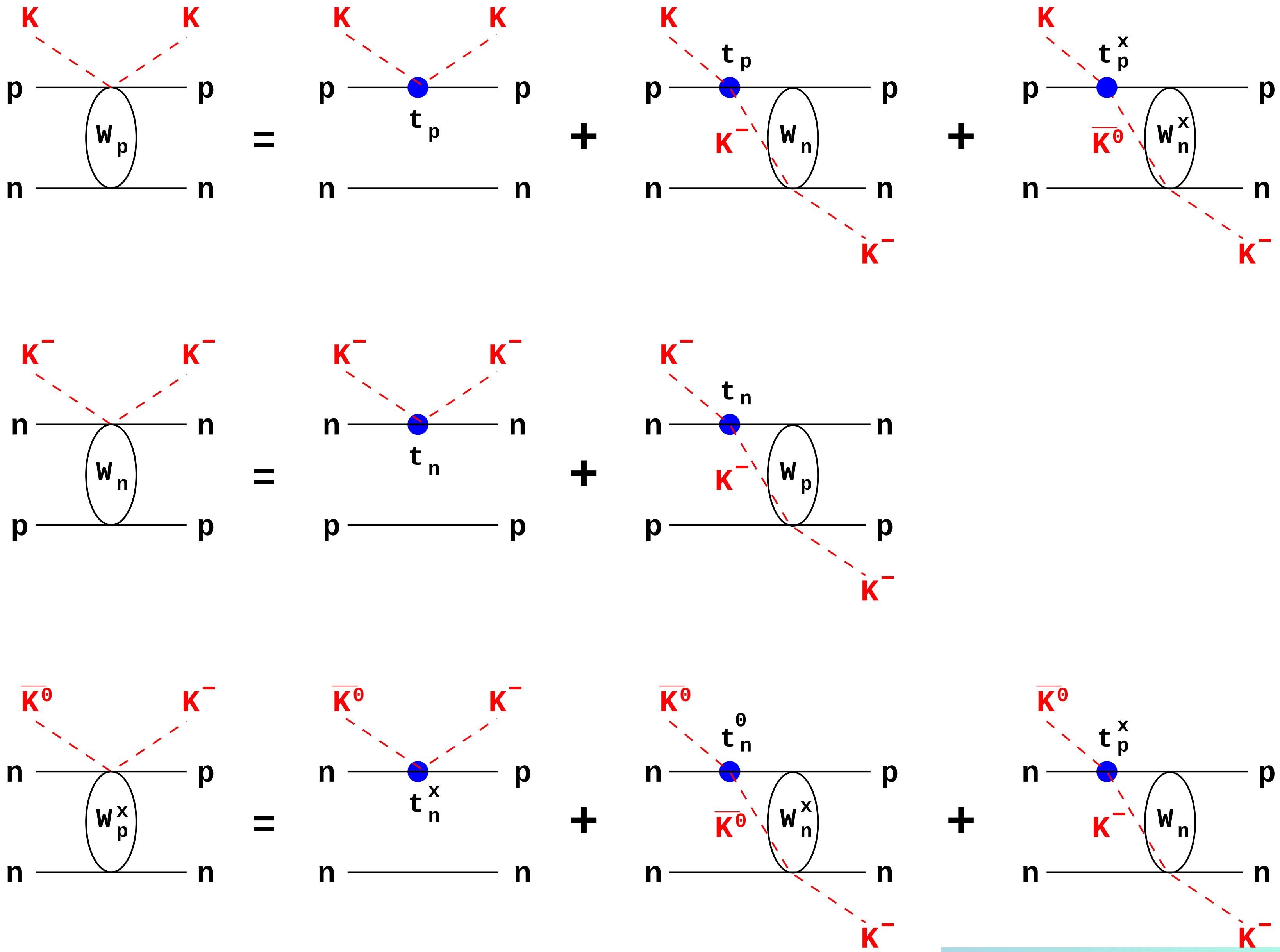
- Nucleon is infinitely heavy: $M_K/m_N \sim 0$ (self-energy corrections~ 0)
- $u(r),\ w(r)$: Deuteron S-, D-wavefunctions
- $\delta_{3-\mathrm{body}} \sim 0 \longrightarrow$ deuteron wave-functions with non-perturbative pions
- NLO Chiral EFT wavefunction (*Epelbaum, et al.*)

Fixed Center Approximation (Static Limit)

- *R. Chand and R. H Dalitz, Ann. Phys. 20 (1962) 1*
  S. Kamalov, E. Oset and A. Ramos, Nucl. Phys. A690 (2001) 494
- Validity of this approximation $\longrightarrow$ *Fäldt, Bahaoui, et al., Baru, et al.*
  $\longrightarrow$ Pretty good for $\pi^- d$ $\longrightarrow$ few percent
  $\longrightarrow$ $(20-30)\%$ for $K^- d$



Faddeev partition diagrams



LO Isospin Breaking: Cusp effect in $\bar{\mathrm{K}}\mathrm{N}$ system

Channel Amplitudes	Isospin Sym.	Isospin Breaking
$a_p(K^-p \to K^-p)$	$\frac{1}{2}(a_0 + a_1)$	$\frac{\frac{1}{2}(a_0+a_1)+q_0 a_0 a_1}{1+\frac{q_0}{2}(a_0+a_1)}$
$a_n(K^-n \to K^-n)$	a_1	a_1
$a_x(K^-p \to \bar{K}^0 n)$	$\frac{1}{2}(a_0 - a_1)$	$\frac{\frac{1}{2}(a_0-a_1)}{1-\frac{iq_c}{2}(a_0+a_1)}$
$a_u(\bar{K}^0 n \to \bar{K}^0 n)$	$\frac{1}{2}(a_0 + a_1)$	$\frac{\frac{1}{2}(a_0+a_1)-iq_c a_0 a_1}{1-\frac{iq_c}{2}(a_0+a_1)}$

$$q_c = \sqrt{2\mu_c \Delta}\,, \quad q_0 = \sqrt{2\mu_0 \Delta}\,,$$

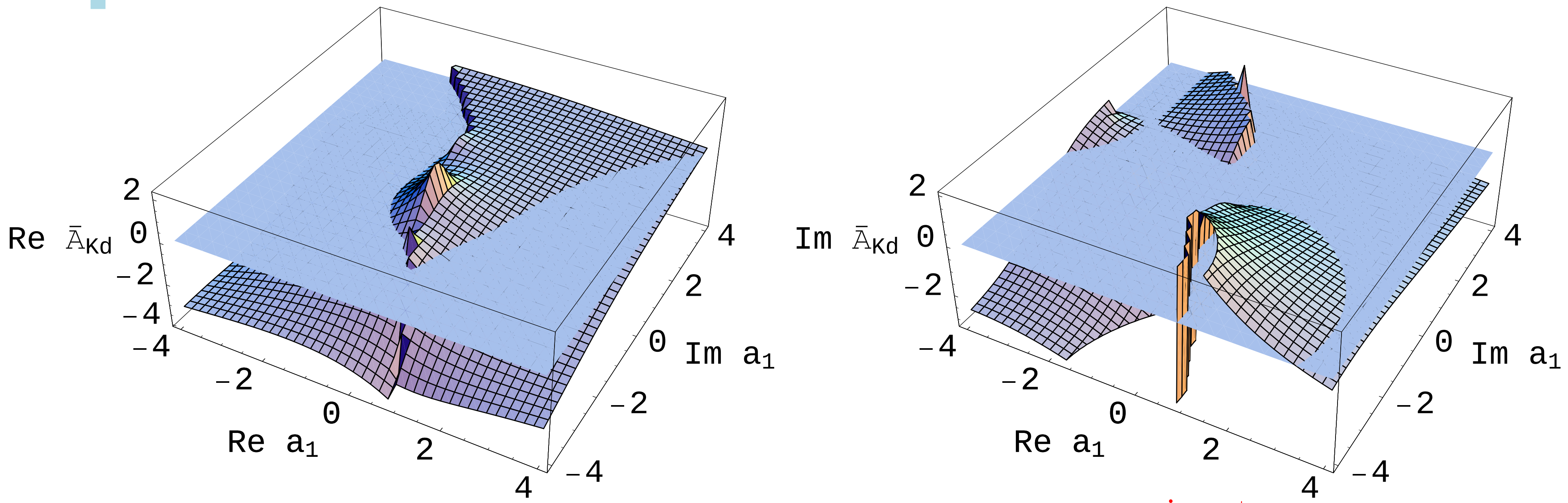
$$\Delta = m_n + M_{\bar{K}^0} - m_p - M_K\,,$$

$$\mu_c = \mu_{Kp} = \frac{m_p M_{K^+}}{m_p + M_{K^+}}\,, \quad \mu_0 = \mu_{Kn} = \frac{m_n M_{\bar{K}^0}}{m_n + M_{\bar{K}^0}}$$

3-d plots of $\bar{A}_{Kd}$ vs Re a_1 & Im a_1 e.g., *Torres et al.*

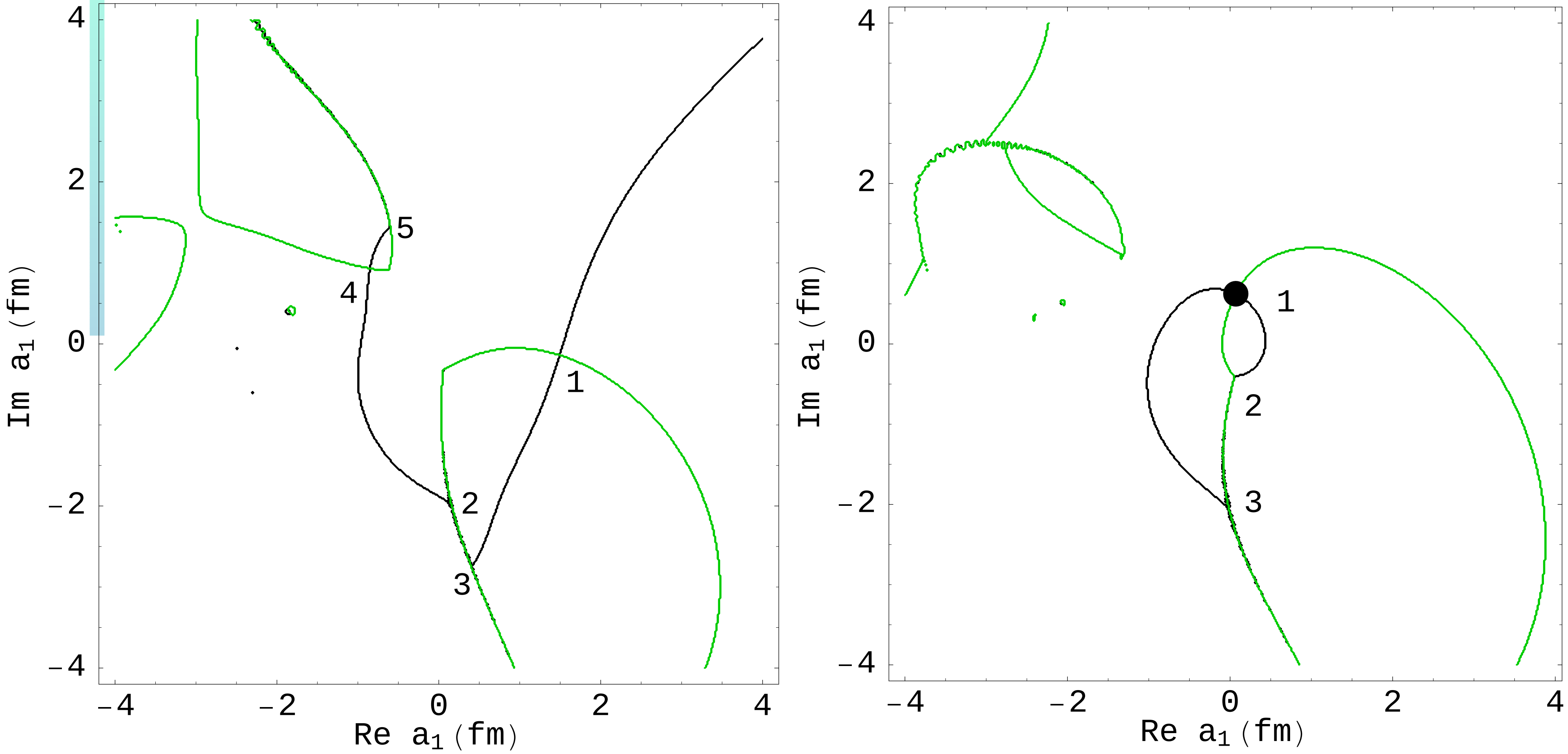
Take $\longrightarrow$ $A_{Kd}^{\rm input} = -1.34 + i1.04$ fm

$a_p^{\rm input}$ (DEAR/KEK) & $a_0 = a_0(a_1, a_p^{\rm input})$



Define $\longrightarrow$ $\bar{A}_{Kd}(a_1) = A_{Kd}(a_1) - A_{Kd}^{\rm input}$

Solutions for a_0 & a_1 with $A_{Kd}^{\rm input}$ from Torres *et al*



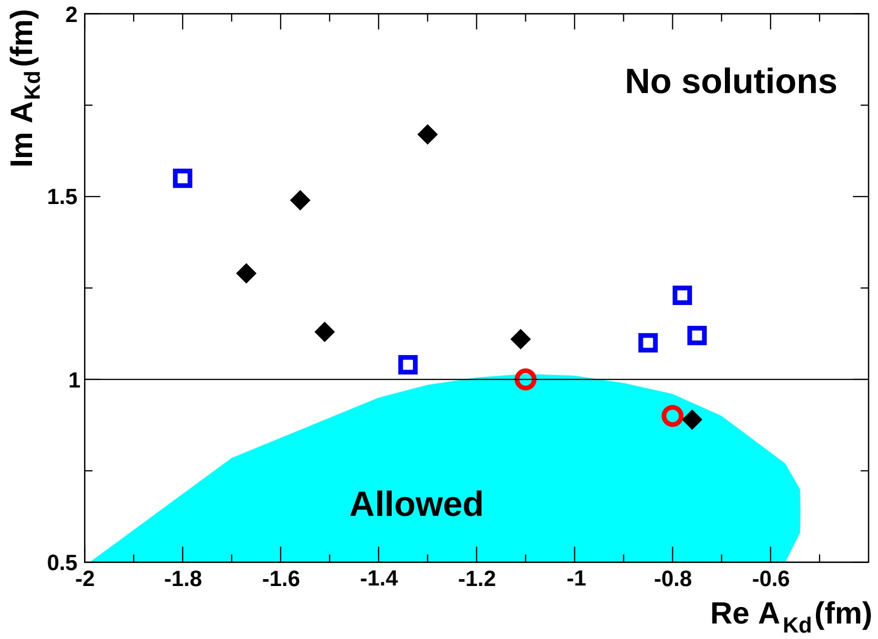
DEAR $\longrightarrow$ No physical solution

KEK $\longrightarrow$ $a_1 = 0.07 + i\,0.62$

$\longrightarrow$ $a_0 = -1.35 + i\,0.60$

Results from the study of Kaonic Deuterium

- Isospin breaking in A_{Kd} turn out to be very mild (see also *Dalitz*)
- Highly non-linear nature of the inverse problem $\Rightarrow$ *very restrictive solutions*
  - strong constraints on the input DEAR data
  - much milder restriction for input KEK data
- Allowed region in the $(\mathrm{Re}\, A_{Kd}, \mathrm{Im}\, A_{Kd})$ plane for $a_0 \,\&\, a_1$ consistent with DEAR data, using the NLO EFT wave-function with cut off $\Lambda = 600\ \mathrm{MeV}$ (say)



Summary & Outlook

- Systematic analysis of hadronic atoms in the framework of a *Non-relativistic EFT*
- Modified Deser formula is proposed for a better analysis of DEAR/KEK data that incorporates the large nonanalytic corrections, in a parameter independent manner (in terms of $a_0 \,\&\, a_1$ only):
  - *Unitary Cusp*
  - *Coulomb corrections*
- The analytic corrections are much smaller
- Existing $K^- p$ scattering data not consistent with recent DEAR data
- Isospin breaking effects in the Kd system was found to be small
- Analysis of kaonic deuterium poses stringent constraints on the underlying $\bar{K}N$ dynamics which might eventually help to accurately extract $a_0 \,\&\, a_1 \Rightarrow$ SIDDHARTA
- **Outlook**: Calculations of isospin breaking corrections, *beyond* LO in ChPT and inclusion of $\Lambda(1405)$ resonance in the $\bar{K}N$ channel to further reduce systematic errors.