Gauge-Higgs Unification In Spontaneously Created Fuzzy Extra Dimensions

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Reference

“Gauge-Higgs Unification In
Spontaneously Created Fuzzy Extra Dimensions"

Kazuyuki Furuuchi (NCTS)
Takeo Inami (Chuo U.)
Kazumi Okuyama (Shinshu U.)

arXiv:1108.4462[hep-ph]

Outline

Introduction & Reviews

- The Era of LHC
- Hierarchy Problem – *Is the Nature Natural?*
- Gauge-Higgs Unification
- Fuzzy Extra Dimensions and D-branes

A Model with Fuzzy Torus

Summary & Future Directions

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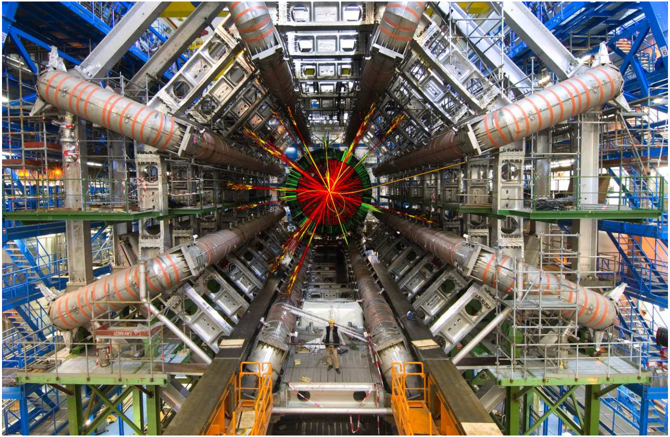
A Model with Fuzzy Torus

Summary & Future Directions

All the HEP people are expecting
the **Large Hadron Collider (LHC)**



ATLAS Experiment (C) 2011 CERN

... Do you know what is expected?

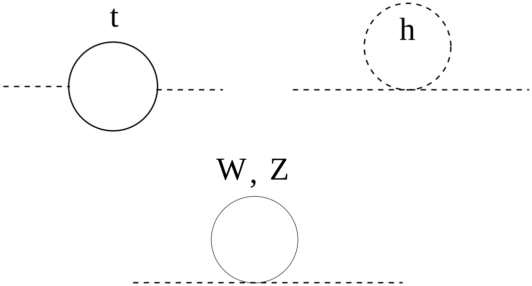
Expecting LHC

LHC is expected to find the **Higgs particle**,
the *last* missing piece of the
Standard Model of Particle Physics (SM).

But **NOT** only the Higgs. Let me explain why.

Hierarchy Problem – *Is the Nature Natural?*

One-loop contributions to the Higgs self-energy in SM:



They roughly tell us

$$\delta_{SM}\, m_H^2 \sim \frac{c_{SM}}{16\pi^2}\,\Lambda^2, \quad c_{SM} \sim 1$$

- Λ should be regarded as modeling new physics effects at high energy.
- Higgs mass is sensitive to the highest energy scale in the theory (**quadratic** dependence on Λ).

For an order estimate, take $m_H \sim 10^2\,GeV$. Then

$$c_\Lambda \sim 16\pi^2 \frac{m_H^2}{\Lambda^2} \sim 10^2 \left(\frac{10^2\,GeV}{\Lambda}\right)^2 \sim \left(\frac{1\,TeV}{\Lambda}\right)^2$$

As an illustration, take $\Lambda \sim$ (Planck Scale) $\sim 10^{19}$ GeV.

$$c_\Lambda \sim \left(\frac{10^3 GeV}{10^{19} GeV}\right)^2 \sim 10^{-32}$$

This is an **Extreme Fine Tuning !**

Taking Λ to be Planck scale is rather extreme.

The formula

$$c_\Lambda \sim \left(\frac{1\,TeV}{\Lambda} \right)^2$$

tells the Standard Model is already **unnatural** if

$$\Lambda \gtrsim 10\,TeV$$

We expect that **the Nature *IS* Natural**,
and thus we expect
New Physics Beyond the SM at the **LHC !**

What are the expected properties of the New Physics in order to maintain the Naturalness ?

Two fundamental concepts in the Naturalness criterion
— **Symmetry** and **Effective Field Theory**

- **Symmetry** can forbid terms which do not respect it.
  Softly broken symmetry can protect those from quantum corrections.

  SM does not have any (approx.) symmetry to protect Higgs mass term.

- From the point of view of **effective field theory**, all the terms not forbidden by **symmetry** should appear. Natural magnitude of dim. D operator is Λ^{4-D} (Λ is the energy scale where the effective field theory breaks down).

Further Reading

For interested audiences we refer to

“NATURALLY SPEAKING:
The Naturalness Criterion and Physics at the LHC"
Gian Francesco GIUDICE
arXiv:0801.2562 [hep-ph]

Popular Symmetries for the New Physics Models

- Supersymmetry
- Technicolor – Chiral Symmetry
- **Gauge symmetry**
- etc.

Gauge-Higgs Unification

The inhomogeneous part of the Gauge transformation:

$$A_\mu \to U(x) A_\mu U^\dagger(x) + U(x) i \partial_\mu U^\dagger(x)$$

forbids the mass term

$$-m^2 \mathrm{tr}\, A_\mu A^\mu$$

for the gauge fields.

- Apparently Higgs is a scalar, not a gauge field in 4D.
- But what if the Higgs is a component of a gauge field in **Extra Dimension**?

Example: Gauge-Higgs with S^1 extra D.

The mass of the **zero modes** on S^1 is not completely protected by the gauge symmetry.

Gauge Transformation:

$A_5(x,y) \to U(x,y)A_5(x,y)U^\dagger(x,y) + U(x,y)i\partial_y U^\dagger(x,y)$

($y \sim y + 2\pi R$: Coordinate on S^1.)

The inhomogeneous part of the gauge transformation for the zero-modes are generated by:

$$U_{ij} = \exp[i\lambda_i \delta_{ij} y] \quad (\lambda_i : const.)$$

This is periodic in y only when $\lambda_i = \frac{1}{R}$.

The zero mode transforms as

$$A_{5(0)ii} \to A_{5(0)ii} + \frac{1}{R}$$

The discrete identification allows the periodic potential of the form

$$V(\exp[i\,2\pi R\,A_{5(0)ii}])$$

The Taylor expansion gives **the mass of the zero modes**.

The operator appearing above are nothing but the **Wilson loop** wrapped on the S^1:

$$P\,exp\left[i\oint_0^{2\pi R} dy\,A_5\right]$$

Since the Wilson loop wrapped on S^1 has size $\sim 2\pi R$, it is expected to be insensitive to the physics shorter than this scale.

Thus the mass of the zero-mode is expected to be around

$$m^2 \sim \frac{g_{4D}^2}{16\pi^2}\frac{1}{R^2}$$

Why Gauge-Higgs in Fuzzy Extra Dimensions?

First, motivations:

- Allows purely **4D** description (cf. (De)construction).

Fuzzy Spaces are created through the
Spontaneous Gauge Symmetry Breaking

$\Leftrightarrow$ In ordinary Gauge-Higgs, the Extra D. is given *a priori*.

○ Possibly renormalizable (and asymptotically free) 4D gauge theory at high energy.

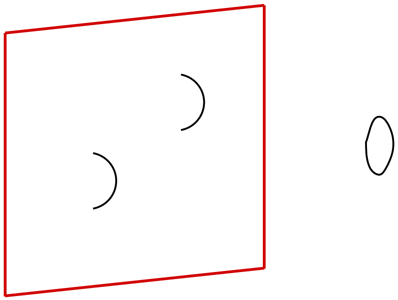
... Particularly nice as a solution to the hierarchy problem.

- Fuzzy spaces are ubiquitous in string theory.

Why String Theory is Good?

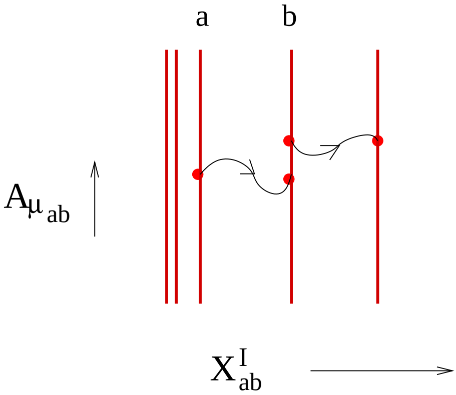
- **Simple** Principles:
  - Just extending Quantum Field Theory based on Particles to Strings
  - Based on QFT and general coordinate tr. invariance
  - No parameter ... (if ideally formulated)
- **Rich** Consequences:
  - Unification
  - Highly constrained ... space-time dimensions, gauge group, matter contents (reps.) ...
  - Almost unique UV completion of quantum gravity, so far known
  - Microscopic explanation of Black Hole Thermodynamics
  - Extra D. and SUSY naturally incorporated
  - Reduces to QFT in low energy, but NOT arbitrary
  - etc. etc.

D-branes, Open Strings (and Closed Strings)



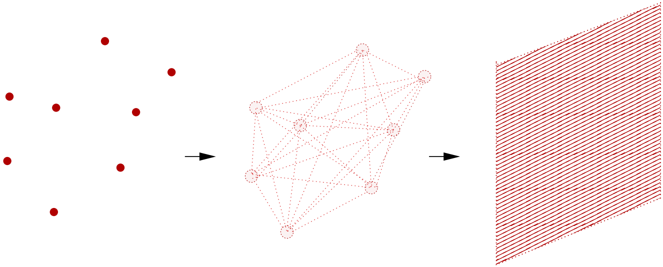
- Closed strings (∋ graviton) propagate in 10D.
- Open strings (∋ gauge field) are constrained on D(irichlet)-branes.
- Closed string interactions are weak when the (true) Extra D. is much larger than the string scale.

Matrix Coordinates of D-branes



X^I_{ab}: $N \times N$ Hermite Matrices (N: # of D-branes)
(adjoint rep. of $U(N)$ gauge group).

D-branes Are Fuzzy !



$[X^I, X^J] = 0$ $\quad$ $[X^I, X^J] \neq 0$ $\quad$ $[X^I, X^J] = i\,\theta^{IJ}$

Generically **non-commutative** (or **Fuzzy**) !

- Commuting matrices can be simultaneously diagonalized by $U(N)$ gauge rotation.
- Then, each diagonal component can be interpreted as a position of a D-brane.

$$X^I = \begin{pmatrix} x_1^I & 0 & \cdots & 0 \\ 0 & x_2^I & 0 & \vdots \\ \vdots & 0 & \ddots & \\ 0 & \cdots & & x_N^I \end{pmatrix}$$

Example: Non-commutative $\mathbb{R}^2$

$$[X_c^1, X_c^2] = i\theta^{12} \quad (const.)$$

- Realized by $\infty \times \infty$ matrices
  (cf. Quantum Mechanics).
- Infinitely many Dp-branes with θ
  = D(p+2)-brane with b.g. 2-form field B_{ij} ($\theta \sim B^{-1}$).

Define

$$\hat{\partial}_I \equiv -iB_{IJ}X^J_c$$

Then

$$[\hat{\partial}_I, X^J_c] = \delta_I{}^J$$

$\Rightarrow \hat{\partial}_I$ is a derivative operator on NC $\mathbb{R}^2$.

Expand the D-brane matrix coordinate field $X^I(x)$:

$$\begin{aligned} X^I(x) &= X^I_c - \theta^{IJ}A_J(x) \\ &= i\theta^{IJ}\left(\hat{\partial}_J + iA_J(x)\right) \end{aligned}$$

The original gauge transformation:

$$X^I(x) \to U(x) X^I(x) U^\dagger(x)$$

Expand around the vacuum and fix the vacuum part in the gauge transformation:

$$\begin{aligned} & X^I(x) = i\theta^{IJ}\left(\hat{\partial}_J + iA_J(x)\right) \\ \to \quad & U(x) i\theta^{IJ}\left(\hat{\partial}_J + iA_J(x)\right) U^\dagger(x) \\ & = i\theta^{IJ}\left(\hat{\partial}_I + U(x)[\hat{\partial}_I, U(x)^\dagger] + iU(x)A_J(x)U^\dagger(x)\right) \end{aligned}$$

$A_I(x)$ transforms as a **gauge field** on NC $\mathbb{R}^2$!

(Notice the appearance of the inhomogeneous term.)

The potential term on D-branes is turned into
the kinetic term of the gauge field
in the directions of Fuzzy Extra Dimension:

$$
\begin{aligned}
V(X) &\sim g_{IK} g_{JL} [X^I, X^J][X^K, X^L] \\
&\rightarrow G^{IK} G^{JL} \left(F_{IJ} + B_{IJ}\right)\left(F_{KL} + B_{KL}\right)
\end{aligned}
$$

$$F_{IJ} \equiv \partial_I A_J - \partial_J A_I + i[A_I, A_J]$$

where

$$G^{IJ} \equiv (\theta g \theta)^{IJ}$$

(open string metric)

Summary of Fuzzy Spaces (Created via SSB)

- Ubiquitous in D-brane systems
- Allows lower dimensional QFT description
- Gauge fields appear from the fluctuations around the fuzzy vacua

$\Rightarrow$ Provides a natural setting for the **Gauge**-**Higgs** Unification in **Extra D.** because it gives

Unified description of
Space, Gauge field and Higgs !

Outline

A Model with Fuzzy Torus

Torus was a good setup for Gauge-Higgs Unifications.

⇒ Let's try **Fuzzy Torus** !

(Circle is simpler, but we don't have fuzzy circle since we need at least 2 coordinates to define a fuzzy space)

4D Action (Effective field theory with cut-off Λ):

$$S = \int d^4x \, \mathrm{tr}_{U(kN)} \Big[- \frac{1}{2} F_{\mu\nu}(x) F^{\mu\nu}(x) + f^2 \sum_{I=1,2} D_\mu U_I(x) D^\mu U_I^\dagger(x) + g^2 f^4 \left| [e^{i\theta^{12}} U_1 U_2 - U_2 U_1] \right|^2 + \ldots \Big]$$

First we choose

$$\theta^{12} \equiv \theta = \frac{2\pi}{N}$$

Naturalness of this value is an issue discussed later.
(We also suppressed order one coefficients for the simplicity of presentations.)

$U_I(x)$ $(I = 1, 2)$: Unitary Matrix Fields.

Covariant Derivative:

$$D_\mu U_I(x) = \partial_\mu U_I(x) - ig[A_\mu(x), U_I(x)]$$

Gauge Field Strength:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

Two familiar ways to look at the model:

1. System of two pion-like fields coupled to a gauge field (chiral perturbation theory).
2. (Twisted) lattice gauge theory in Extra D. directions.

The cut-off of this model should be taken around

$$\Lambda \sim 4\pi f$$

Don't be confused with the cut-off for the SM.
The cut-off for the SM should be taken as

$$\Lambda_{SM} \sim \frac{2\pi g f}{N} \equiv \frac{1}{R}$$

since $1/R$ is the new physics scale.

Symmetries of the model

- 4D Poincare Symmetry
- Gauge Symmetry
- $U(1)^2$ Global Symmetry (Center Symmetry):

$$U_I \to e^{i\alpha_I} U_I \quad (I = 1, 2)$$

Approx. symmetries of the model

- Global $(U_L(kN) \times U_R(kN))^2$ "chiral" symmetry (broken by the coupling to the gauge field)

$$U_I \to L_I U_I R_I^\dagger \qquad (I = 1, 2)$$

- Reflections in Extra Dimensions:

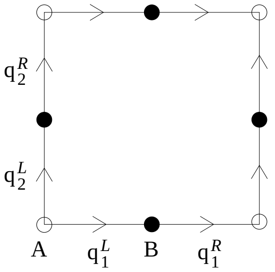
$$P_1 \quad : \quad U_1 \to U_1^{-1} = U_1^\dagger$$
$$P_2 \quad : \quad U_2 \to U_2^{-1} = U_2^\dagger$$

- CP

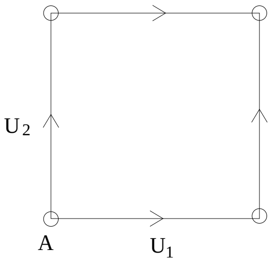
$$A_\mu \to A_\mu^T$$
$$U_I \to U_I^T$$

Why “chiral"?

The model may be obtained from a (chiral) **quiver gauge theory**



Moose before chiral symmetry breaking



Moose after chiral symmetry breaking

The chiral symmetry will be crucial for the suppression of the mass of the to be Higgs field.

Minimum of the Potential

$$U_1 = U \equiv W_1 \otimes \mathbf{1}_k$$
$$U_2 = V \equiv W_2 \otimes \mathbf{1}_k$$

W_1 and W_2 are so-called ’t Hooft-Weyl matrices which generate the **Fuzzy Torus**:

$$W_1 W_2 = e^{-i\theta} W_2 W_1$$

(See below.)

The fuzzy torus vacuum breaks the symmetries as follows:

- Gauge symmetry: $U(kN) \to U(k)$
- Center symmetry: $U(1)^2 \to \mathbb{Z}_N \times \mathbb{Z}_N$

The unbroken part of the center symmetry is crucial for the suppression of the mass of the to be Higgs field.

Fuzzy Torus

$$W_1 W_2 = e^{-i\theta} W_2 W_1$$

$$W_1 = \begin{pmatrix} 1 & & & & \\ & e^{-i\theta} & & & \\ & & e^{-i2\theta} & & \\ & & & \ddots & \\ & & & & e^{-i(N-1)\theta} \end{pmatrix}$$

$$W_2 = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & 0 & 1 \\ 1 & & & & 0 \end{pmatrix}$$

“Fourier expansion" on the fuzzy torus:

$$\varphi \sim \sum_m \sum_n \varphi_{(m,n)} e^{imn\theta} W_1^m W_2^n$$

The sums run integers in $-\frac{N}{2} \leq m, n < \frac{N}{2}$.

Define

$$\begin{aligned}
\delta_1 \varphi &\equiv W_1 \varphi W_1^\dagger - \varphi \\
\delta_2 \varphi &\equiv W_2 \varphi W_2^\dagger - \varphi
\end{aligned}$$

We have

$$\begin{aligned}
\delta_1 (W_1^m W_2^n) &= (e^{-in\theta} - 1)(W_1^m W_2^n) \\
\delta_2 (W_1^m W_2^n) &= (e^{im\theta} - 1)(W_1^m W_2^n)
\end{aligned}$$

Define ("lattice spacing")

$$a \equiv \frac{1}{gf}$$

In $N \to \infty$ limit with $2\pi R \equiv aN$ **fixed**:

$$\frac{1}{a}\delta_1 W_2^n \quad \to \quad -i\frac{n}{R}W_2^n \longleftrightarrow \partial_{\phi_2} e^{-i\frac{n}{R}\phi_2}$$

$$\frac{1}{a}\delta_2 W_1^m \quad \to \quad i\frac{m}{R}W_1^m \longleftrightarrow \partial_{\phi_1} e^{i\frac{m}{R}\phi_1}$$

In this limit $W_{1,2}$ are identified with coordinates on ordinary torus:

$$W_1 \to e^{-i\frac{\phi_1}{R}}, \quad W_2 \to e^{i\frac{\phi_2}{R}}$$

The mass spectrum around the fuzzy torus vacuum is obtained as

$$
\begin{aligned}
& m^2_{(m,n)} \\
\equiv \quad & \left(\frac{2}{a}\right)^2 \sin^2\frac{m\theta}{2} + \left(\frac{2}{a}\right)^2 \sin^2\frac{n\theta}{2} \\
= \quad & \frac{2}{a^2}(1-\cos m\theta) + \frac{2}{a^2}(1-\cos n\theta)
\end{aligned}
$$

In the large N limit with **fixed** $2\pi R \equiv aN$, it reduces to that of the KK modes on the ordinary torus with radius R:

$$
m^2_{(m,n)} \to \left(\frac{m}{R}\right)^2 + \left(\frac{n}{R}\right)^2
$$

To see the mass of the zero-modes,

Calculate 1PI effective potential for the
zero modes on the fuzzy torus.

Below we study the case $\boxed{k=2}$ as an illustration.

The zero modes:

$$U_1 = U_0 \equiv U e^{i\frac{1}{f\sqrt{4N}}u(x)\Sigma}, \quad U_2 = V_0 \equiv V e^{i\frac{1}{f\sqrt{4N}}v(x)\Sigma}$$

with

$$\Sigma = \mathbf{1}_N \otimes \frac{1}{\sqrt{2}}\sigma_3$$

σ_i $(i=1,2,3)$: Pauli matrices $(\because k=2)$.

After appropriate gauge fixing and inclusion of the ghost contribution,

$$\begin{aligned} V_{1-loop}(u,v) &= i\log\det((D^0)^2)^{-6/2} + i\log\det((D^0)^2)^{+1} \\ &= -2i\,\mathrm{Tr}\log((D^0)^2) \end{aligned}$$

$$(D^0)^2 \equiv \partial_\mu\partial^\mu + \Delta_I^0\Delta_I^0$$

$$\Delta_1^0\varphi \equiv \frac{1}{a}\left(U_0\varphi U_0^\dagger - \varphi\right), \quad \Delta_2^0\varphi \equiv \frac{1}{a}\left(V_0\varphi V_0^\dagger - \varphi\right)$$

After Wick rotation,

$$
\begin{aligned}
& V_{1-loop}(u,v) \\
= \;& 2\sum_{m}\sum_{n}\sum_{i,j=1}^{2}\int \frac{d^4k}{(2\pi)^4}\log\left(k^2+m^2_{(m,n)(i,j)}(u,v)\right)
\end{aligned}
$$

where ...

$$
\begin{aligned}
& m^2_{(m,n)(i,j)}(u,v) \\
\equiv\ & \left(\frac{2}{a}\right)^2 \sin^2 \frac{1}{2}\left(m\theta + (u_i - u_j)\right) \\
& + \left(\frac{2}{a}\right)^2 \sin^2 \frac{1}{2}\left(n\theta + (v_i - v_j)\right) \\
=\ & \frac{2}{a^2}\left(1 - \cos\left(m\theta + (u_i - u_j)\right)\right) \\
& + \frac{2}{a^2}\left(1 - \cos\left(n\theta + (v_i - v_j)\right)\right),
\end{aligned}
$$

$$
u_1 = -u_2 = \frac{1}{\sqrt{2Nf}}u, \quad v_1 = -v_2 = \frac{1}{\sqrt{2Nf}}v
$$

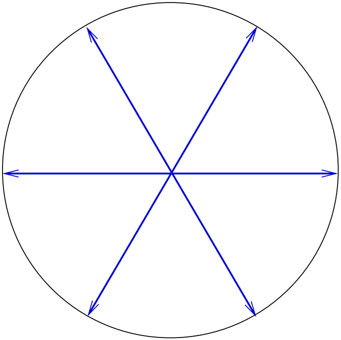
With the momentum cut-off at Λ,

$$\begin{aligned} & V_{1-loop}(u, v) \\ = & \sum_{m,n} \sum_{i,j=1}^{2} \left[\frac{\Lambda^2}{8\pi^2} m^2_{(m,n)(i,j)}(u, v) \right. \\ & \left. + \frac{1}{16\pi^2} (m^2_{(m,n)(i,j)}(u, v))^2 \log \frac{m^2_{(m,n)(i,j)}(u, v)}{\Lambda^2} + \mathcal{O}(\Lambda^{-2}) \right] \end{aligned}$$

Notice that the sum of $m^2_{(m,n)(i,j)}(u, v)$ over m and n depends neither on $u(x)$ nor $v(x)$ for $N \geq 2$ due to the cancellations between phases.

Also, the sum of $(m^2_{(m,n)(i,j)}(u, v))^2$ over m and n depends neither on $u(x)$ nor $v(x)$ for $N \geq 3$.

Cancellation of phases ($N = 6$)



$$\sum_{m=0}^{N-1} \exp\left[\frac{2\pi m}{N}\right] = 0$$

Thus, there's **no quadratic "divergence" for $N \geq 2$**, and there's **no log "divergence" for $N \geq 3$** in the potential (other than the constant part which we neglect).

The masses of the zero-modes turn out to be

$$m^2 \sim \frac{g_{4D}^2}{16\pi^2}\frac{1}{R^2}$$

as expected.

The potential for the zero-modes can be regarded as a potential for discretized version of the Wilson loops.

Comments

This particular model has some similarity with the **(De)construction** by Arkani-Hamed-Cohen-Georgi'01 (latticizing extra dims. by quiver gauge theory)

But ...

- Large N reduction (Eguchi-Kawai model) '82
- Twisted Eguchi-Kawai model '83
- Myers effect (fuzzy sphere from D-branes) '99

... The fuzzy versions of the (de)construction are already known *before* the (de)construction !

Another large class of 4D description of Extra D.

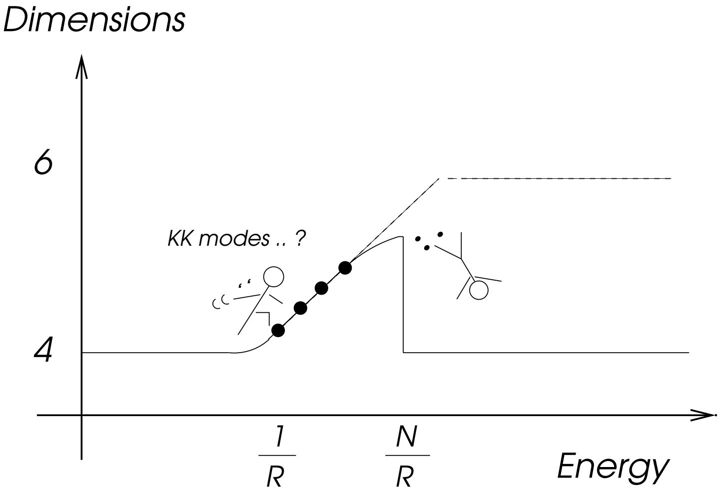
Of course, we respect that they first applied the (de)construction to the hierarchy problem.

Outline

Summary, so far

- Constructed a **4D** model in which Fuzzy Extra D. are created through the spontaneous gauge symmetry breaking.
- Mass spectrum through the Higgs mechanism mimics the low-lying KK modes.
- Identified the Higgs with the extra dimensional component of the gauge field **emerged** on the Fuzzy Extra D. vacuum.
- **Mechanism to solve the hierarchy problem**
  - No Quadratic “divergence" at 1-loop.
  - No Log divergence for $N \geq 3$, which is rather small.
- Goes back to 4D in high-energy.

(Cartoon)



Further Results in Our Paper

1. All loop analysis based on symmetries:
   Unbroken part of the Center symmetry and
   Chiral symmetry are crucial.
2. N dependence:
   N/R is a UV cut-off in the extra D. directions.
3. Rigidness of the results under small variation of θ,
   and the naturalness of its value.
4. Realization on D-branes.

... Please read the original paper for the detail.

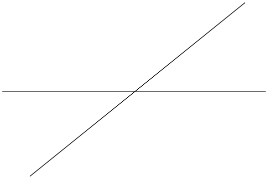
Future Directions

1. Gauge representation
   ($SU(2)$ doublet from $U(3)$ adjoint off-diagonal)
2. Wine-bottle potential
   (May be solved together with 1.)
3. Coupling to SM fermions
   - 3.1 In Gauge-Higgs, Yukawa originates from gauge coupling. Notice that gauge coupling is more restricted than usual Yukawa.
   - 3.2 Fuzziness also constrains possible gauge reps.
   - 3.3 Fuzziness may also introduce interesting structures (cf. FK-Okuyama '10 – Yukawa texture from fuzzy intersections) $\Leftrightarrow$ new ingredients for 3.1
4. Generalization to other fuzzy spaces
   (Non-trivial. But similar if it has non-trivial 1-cycle on which Wilson loop can wrap?)

We have to construct a model with the condition that it has a fuzzy vacuum.

Little bit more about 3.3

In FK-Okuyama (as well as many D-brane models), the Yukawa couplings appear from intersection of D-branes.



ordinary overlap

fuzzy overlap

The **fuzzy** overlap may introduce particular structures to the Yukawa couplings.

Thank You !