f(R) Modified Gravity

for *Cosmic Acceleration*

Je-An Gu 顧哲安

臺灣大學梁次震宇宙學與粒子天文物理學研究中心
Leung Center for Cosmology and Particle Astrophysics (LeCosPA), NTU

Collaborators : *Wei-Ting Lin* 林韋廷 @ Phys, NTU

Dark Energy Working Group @ LeCosPA & NCTS-FGCPA

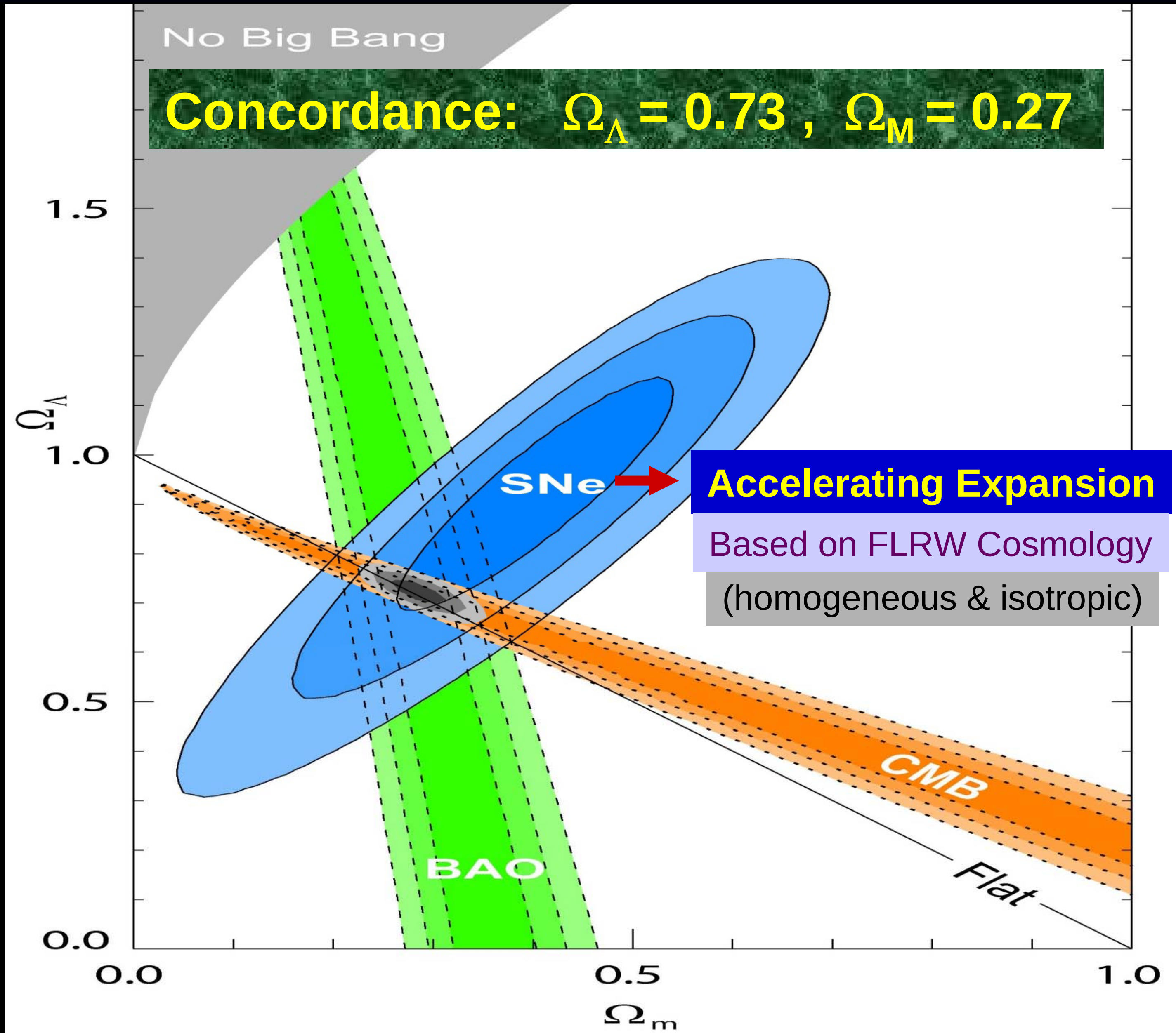
2010/04/13 @ CYCU

Outline

- *Introduction*
- *Modified Gravity*
- *f(R) Modified Gravity*
- "*Designer f(R)*" and our work
- *Results* : *Observational Constraints on "Designer f(R)"*
- *Summary*

Introduction

Observations (which are driving Modern Cosmology)



Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry **Matter/Energy**

Issues:
- (why small) Λ problem
- (why now) coincidence problem

To avoid the issues,
necessary condition:
Energy density changes with time.

- Λ (from vacuum energy)

Many **success factors**:
- fit data
- connections:
  - to famous person: **Einstein**
  - to well known theory: **QFT**
  - to nature of creator: **simple**

Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry — **Matter/Energy**

Issues:
- (why small) Λ problem
- (why now) coincidence problem

To avoid the issues,
necessary condition:
Energy density changes with time.

- Λ (from vacuum energy)
- Quintessence / Phantom

(a simple realization)

Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry | **Matter/Energy**

↑

Dark Energy

- Λ (from vacuum energy)
- Quintessence / Phantom

based on FLRW

1. Einstein GR
2. 3+1 space-time
3. RW metric

Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry — **Matter/Energy**

MG ~ Dark Energy

1. Modified Gravity (MG)
   - special case: $f(R)$
   - gravity action: $R \rightarrow f(R)$
   - Brans-Dicke gravity
   - Scalar-Tensor gravity

- Λ (from vacuum energy)
- Quintessence / Phantom
  - minimal coupling to gravity
  - ↓ (inevitably?)
  - non-minimal coupling to gravity

Einstein GR + Non-Min. Scalar Field ↔ non-minimal coupling to gravity

Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry | **Matter/Energy**

MG ~ Dark Energy

1. Modified Gravity (MG)

- Λ (from vacuum energy)
- Quintessence / Phantom

FLRW

1. Einstein GR
2. 3+1 space-time
3. RW metric

Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry — **Matter/Energy**

↑

Dark Energy

1. Modified Gravity (MG)
2. Extra Dimensions

- Λ (from vacuum energy)
- Quintessence / Phantom

FLRW

1. Einstein GR
2. 3+1 space-time
3. RW metric

Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry | **Matter/Energy**

↑ Dark Energy

1. Modified Gravity (MG)
2. Extra Dimensions

- Λ (from vacuum energy)
- Quintessence / Phantom

isotropic
homogeneous

? *Is FLRW a good approximation ??*

FLRW

1. Einstein GR
2. 3+1 space-time
3. RW metric

Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry — **Matter/Energy**

↑ Dark Energy

1. Modified Gravity (MG)
2. Extra Dimensions
3. Averaging Einstein Equations for an inhomogeneous universe

- Λ (from vacuum energy)
- Quintessence / Phantom

? *Is FLRW a good approximation ??*

FLRW

1. Einstein GR
2. 3+1 space-time
3. RW metric

Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry ↑ Dark Geometry (Gravity)

Matter/Energy ↑ Dark Energy

1. Modified Gravity (MG)
2. Extra Dimensions
3. Averaging Einstein Equations for an inhomogeneous universe

Non-FLRW

- Λ (from vacuum energy)
- Quintessence

FLRW

1. Einstein GR
2. 3+1 space-time
3. RW metric

Candidates: *Dark Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Geometry ↑ Dark Geometry (Gravity)

Matter/Energy ↑ Dark Energy

LeCosPA
DE/MG WG

- Modified Gravity (MG)
- Extra Dimensions
  - Gu and Hwang, Phy.Rev.D (2002);
  - Gu, Hwang and Tsai, Nucl.Phys.B (2004)
- Back reaction of inhomogeneities for an inhomogeneous universe
  - Chuang, Gu and Hwang, Class.Quant.Grav (2008)

- Λ (from vacuum energy)
  - Gu and Hwang, Phys.Rev.D (2006)
  - Chen and Gu, arXiv:0712.2441
  - Gu, arXiv:0801.4737
- Quintessence
  - Gu and Hwang, Phys.Lett.B (2001)
  - Gu, arXiv:0711.36

Dark Energy Working Group

Modified Gravity

Current active members:

[NTU]

2008.09 - present

Je-An Gu (LeCosPA) (DE WG server)

Wei-Ting Lin (MG key worker)

Huitzu Tu (LeCosPA)(DM)

Pao-Yu Wang (DE Pheno)

Tse-Chun Wang (DE Pheno)

[NTNU]

Wolung Lee

Chia-Chun Chang (DE & Structures)

Wetty Chao (DE Pheno)

Vincent Chu (Inhomogeneous Cosmo.)

Huan-Ting Peng (Cosmic Neutrinos)

[NCTU]

Tzuu-Kang Chyi

Friends & **"historically" active members**:

Feng-Yin Chang (LeCosPA, NTU)

Chien-Wen Chen (NTU)

Pisin Chen (LeCosPA, NTU)

Fei-Hung Ho (NCU)

Pei-Min Ho (NTU)

Qing-Guo Huang (KIAS, Korea)

Kwang-Chang Lai (NCTU)

Seokcheon (Sky) Lee (IoPAS)

Guo-Chin Liu (TKU)

Debaprasad Maity (LeCosPA, NTU)

Kin-Wang Ng (IoPAS) (DE WG Leader)

2008.
03-08

Hau-Yu Liu (NTU & ASIAA)

Yen-Wei Liu (NTU)

Tao-Tao Qui (CYCU)

Keiichi Umetsu (ASIAA)

I-Chin Wang (NTNU)

DE / MG Working Group meeting

- Meeting time in 2009 Fall Semester: Wednesday, 7:30 pm — (indefinite)
- Webpage: http://lecospa.ntu.edu.tw/wg_list.php?wgid=2

No.	Time Posted	Poster Name	Article Title	Views	Replies
33	091209	Wei-Ting Lin (NTU)	arXiv:0809.3791 by Zhao, Pogosian, Silvestri, and Zylberberg	5	0
32	091202	Fei-Hung Ho (NCU)	Torsion Cosmology and the Accelerating Universe	9	0
31	091125	Je-An Gu (LeCosPA)	arXiv:0801.2431 by Bertschinger and Zukin	9	0
30	091111	(1) Nien-en Tung (NTU); (2) Fu-Cheng Wang (NTU)	(1) Cook's GR; (2) MCMC	18	0
29	091104	Je-An Gu (LeCosPA)	Modified Gravity: observational constraints	20	0
28	091028	Je-An Gu (LeCosPA)	Modified Gravity: matter density perturbation and gravitational potentials	19	0
27	091021	Chia-Chun Chang (NTNU) and Wei-Ting Lin (NTU)	Modified Gravity: numerical solution of background evolution	13	0
26	091014	(DE WG members)	Modified Gravity: numerical analysis for background evolution	13	0
25	091007	Je-An Gu (LeCosPA)	f(R) Modified Gravity (a beginner's report)	33	0
24	090930	Vincent Chu (NTNU)	Inhomogeneity-Induced Cosmic Acceleration in a Dust Universe	18	0
23	090923	Chia-Chun Chang (NTNU) and Wei-Ting Lin (NTU)	Report on CMBFast	16	0
22	090507	Wei-Ting Lin (NTU) and Je-An Gu (LeCosPA)	Ma-Bertschinger paper [ApJ 455, 7 (1995)], Sec. 7 to Sec. 9 (end)	22	0
21	090430	Chia-Chun Chang (NTNU) and Je-An Gu (LeCosPA)	Ma-Bertschinger paper [ApJ 455, 7 (1995)], Sec. 5.5 to Sec. 6	10	0
20	090423	Vincent Chu (NTNU) and Yen-Wei Liu (NTU)	Ma-Bertschinger paper [ApJ 455, 7 (1995)], Sec. 4 to Sec. 5.4	18	0
19	090416	Je-An Gu (LeCosPA)	Ma-Bertschinger paper [ApJ 455, 7 (1995)], Sec. 1 to Sec. 3	24	0
18	090409	Je-An Gu (LeCosPA)	DE WG meeting on 4/9: status updated and dicussions	24	0
17	090328	Je-An Gu (LeCosPA)	DE WG report for Joint LeCosPA-FGCPA Bi-Monthly Meeting on March 28	21	0
16	090326	Wolung Lee (NTNU) and Wei-Ting Ling (NTU)	Report on Dodelson tesxtbook Chapters 4 & 5	52	0
15	090312	Wolung Lee (NTNU)	Ma-Bertschinger paper: ApJ 455, 7 (1995)	26	0
14	090225	Je-An Gu (LeCosPA)	CMBFAST - preliminary experience	53	0
13	090114	Guo-Chin Liu (TKU)	CMBFAST	76	0
12	090107	Je-An Gu (LeCosPA)	The Future of the Dark Energy Working Group - Part II	88	0
11	081224	Je-An Gu (LeCosPA)	The Future of the Dark Energy Working Group - Part I	97	0
10	081217	Keiichi Umetsu (ASIAA)	Observational probes of dark energy–Part II (emphasis on weak lensing)	126	0
9	081210	Keiichi Umetsu (ASIAA)	Observational probes of dark energy–Part I	306	7
8	081126	Qing-Guo Huang (KIAS)	Holographic dark energy	136	0
7	081119	Chein-Wen Chen (NTU)	Constraining dark energy from SNIa, CMB, BAO observations	170	0
6	081112	Kwang-Chang Lai (NCTU)	Photometric Redshifts and Cosmological Models	134	0
5	081105	Pisin Chen (LeCosPA)	2008 Leopoldina Dark Energy Conference	132	0
4	081022	Wolung Lee (NTNU)	Overview of Dark Energy [current status] – Part II: observational approaches	182	0
3	081015	Wolung Lee (NTNU)	Overview of Dark Energy [current status] – Part I: theoretical approaches	145	0
2	081008	Je-An Gu (LeCosPA)	Type Ia Supernova Data – Part II	126	0
1	081001	Je-An Gu (LeCosPA)	Type Ia Supernova Data – Part I	196	0

DE / MG Working Group meeting

- Meeting time in 2009 Fall Semester: Wednesday, 7:30 pm — (indefinite)
- Webpage: http://lecospa.ntu.edu.tw/wg_list.php?wgid=2

Modified Gravity: numerical solution of background evolution Edit Reply

Poster：Chia-Chun Chang (NTNU) and Wei-Ting Lin (NTU)
Author：
Time：2009-10-21

==
Dark Energy Working Group meeting on 2009.10.21

Reporter : Chia-Chun Chang (NTNU) and Wei-Ting Lin (NTU)
Topic : Modified Gravity: numerical solution of background evolution
Date : October 21st, 2009 (Wednesday)
Time : 7:30 - 9:00 pm
Place : Room 815, New Physics Building, NTU
==
Participants: (5 persons)
Fei-Hung, Vincent, Chia-Chun, Wei-Ting, Je-An

Food [supported by ANNAN Culture and Education (安南文教)]:
- 'Ningyo-yaki (人形燒)' from "Kimura-ya honpo (元祖木村家人形燒本舖)"
in Tokyo Asakusa Nakamise-dori Street (東京淺草仲見世通)
- Coffee bean: 'Signature Blend (冠軍配方)' from "Paul Bassett", Shinjuku, Tokyo (東京新宿)
- Coffee bean: 'Brazil Dattera Yellow Sweet (巴西黃色甜心)' from "Paul Bassett", Shinjuku, Tokyo (東京新宿)
- Tea: Japanese Green Tea (日本煎茶)

Food [supported by ANNAN Culture and Education (安南文教)]:
- 'Ningyo-yaki (人形燒)' from "Kimura-ya honpo (元祖木村家人形燒本舖)"
in Tokyo Asakusa Nakamise-dori Street (東京淺草仲見世通)
- Coffee bean: 'Signature Blend (冠軍配方)' from "Paul Bassett", Shinjuku, Tokyo (東京新宿)
- Coffee bean: 'Brazil Dattera Yellow Sweet (巴西黃色甜心)' from "Paul Bassett", Shinjuku, Tokyo (東京新宿)
- Tea: Japanese Green Tea (日本煎茶)

DE / MG Working Group meeting

- Meeting time in 2009 Fall Semester: Wednesday, 7:30 pm — (indefinite)
- Webpage: http://lecospa.ntu.edu.tw/wg_list.php?wgid=2

Modified Gravit
Poster：Chia-Chun (
Author：
Time：2009-10-21
================
Dark Energy Workin

Reporter : Chia-Chu
Topic : Modified Gra
Date : October 21st,
Time : 7:30 - 9:00 pn
Place : Room 815, N
================
Participants: (5 pers
Fei-Hung, Vincent, C

Food [supported by
- 'Ningyo-yaki (人形燒

- Tea: Japanese Gre

arXiv:0801.2431 by Bertschinger and Zukin

Edit Reply

Poster：Je-An Gu (LeCosPA)
Author：Bertschinger and Zukin
Time：2009-11-25
==
Dark Energy Working Group me

Reporter : Je-An Gu (LeCosPA)
Topic : arXiv:0801.2431 by Bert
Date : November 25th, 2009 (W
Time : 7:40 - 8:40 pm
Place : Room 815, New Physics
========================
Participants: (8 persons)
Wolung, Huitzu, Debu, Fei-Hung

Food [supported by ANNAN Culture and Education (安南文教)]:
- Coffee Beans from Melbourne, Australia
-- 'Eclipse' (blend) from "VENEZIANO Caffe"
-- 'Costa Rica Herbazu' from "ST. ALi"
- 'Amazing Thai Snacks' from Bangkok airport, Thailand
- Bread from "Flavor Field" (@ SOGO)

Food [supported by ANNAN Culture and Education (安南文教)]:
- Coffee Beans from Melbourne, Australia:

Something you may never enjoy unless you attend this meeting !!

- Bread from "Flavor Field" (@ SOGO)

****

Food [supported by ANNAN Culture and Education (安南文教)]:
- 'Ningyo-yaki (人形燒)' from "Kimura-ya honpo (元祖木村家人形燒本舗)" in Tokyo Asakusa Nakamise-dori Street (東京淺草仲見世通)
- Coffee bean: 'Signature Blend (冠軍配方)' from "Paul Bassett", Shinjuku, Tokyo (東京新宿)
- Coffee bean: 'Brazil Dattera Yellow Sweet (巴西黃色甜心)' from "Paul Bassett", Shinjuku, Tokyo (東京新宿)
- Tea: Japanese Green Tea (日本煎茶)

e-Print: arXiv:0909.2045 [astro-ph.CO]

(*Dark Energy & Modified Gravity*) *Phenomenology*

Cosmological Models

Dark Energy
Modified Gravity
Dark Matter
Inflation
⋮

(not limited to DE models)

Inter-medium
(physical quantities)

$d_L(z)$

matter power spectrum

CMB A&P spectrum

Observations

SNe Ia
LSS
Lensing
CMB
⋮

Theoretical prediction

Observational info

Technique / Know-how

(*Dark Energy & Modified Gravity*) *Phenomenology*

Cosmological Models	Inter-medium (physical quantities)	Observations
Dark Energy	$d_L(z)$	SNe Ia
Modified Gravity	matter power spectrum	LSS
Dark Matter	CMB A&P spectrum	Lensing
Inflation		CMB
:		:

Develop *Cosmo.-Pheno.-Tech.*

Theoretical prediction

Observational info

Technique / Know-how

(Dark Energy & Modified Gravity) Phenomenology

- Je-An Gu, Chien-Wen Chen, and Pisin Chen,
  "A new approach to testing dark energy models by observations,"
  New Journal of Physics **11** (2009) 073029 [arXiv:0803.4504].

- Chien-Wen Chen, Je-An Gu, and Pisin Chen,
  "Consistency test of dark energy models,"
  Modern Physics Letters **A 24** (2009) 1649 [arXiv:0903.2423].

- Chien-Wen Chen, Pisin Chen, and Je-An Gu,
  "Constraints on the phase plane of the dark energy equation of state,"
  Physics Letters **B 682** (2009) 267 [arXiv:0905.2738].

Modified Gravity

Candidates: *Modified Gravity* vs. *Dark Energy*

Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Modified Gravity ↘   ↙ Dark Energy

one test

Different effects (predictions) on:

- Cosmic expansion (background evolution): $a(t)$ ← SN BAO CMB
- Evolution of cosmic perturbations: δ_m, Φ, Ψ (k,a) ← CL WL ISW

matter density perturb. (δ_m) | metric perturb. (Φ, Ψ)

∞ tests !!

- Another motivation/goal of MG investigations:
  Cosmological test *of* *alternative gravity theories*

Modified Gravity : $\mathcal{L}_G = R + f(R)$ with flat RW

(modified gravity action)

- Cosmic expansion (background evolution): $a(t)$

one test

$$H^2 = \frac{8\pi G}{3}\rho_m + \left[\frac{1}{6}\left(Rf' - f\right) - H^2 f' - H\dot{f}'\right]$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \left[-\frac{1}{6}f + H^2 f' - \frac{1}{2}H\dot{f}' - \frac{1}{2}\ddot{f}'\right]$$

GR :

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\left(\rho_m + \rho_{DE}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\left(\rho_m + \rho_{DE} + 3p_{DE}\right)$$

- Evolution of cosmic perturbations: δ_{m}, Φ, Ψ (k,a)

∞ tests !!

Sub-horizon, Fourier space

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{eff}\,\rho_m\delta_m \cong 0$$

$$\Psi \cong -4\pi G_{eff}\frac{a^2}{k^2}\rho_m\delta_m$$

$$\frac{\Phi}{\Psi} \cong \left(1 + 2\frac{k^2}{a^2}\frac{f''}{1+f'}\right) \Big/ \left(1 + 4\frac{k^2}{a^2}\frac{f''}{1+f'}\right)$$

$$G_{eff}(k,a) \cong \frac{1}{8\pi(1+f')}\,\frac{1 + 4\dfrac{k^2}{a^2}\dfrac{f''}{1+f'}}{1 + 3\dfrac{k^2}{a^2}\dfrac{f''}{1+f'}}$$

GR : $G_{eff} = \text{const.}$, $\Phi = \Psi$

Modified Gravity : $\mathcal{L}_G = R + f(R)$ with flat RW

(modified gravity action)

- Cosmic expansion (background evolution): $a(t)$

one test

Indistinguishable!!

$$H^2 = \frac{8\pi G}{3}\rho_m + \left[\frac{1}{6}\left(Rf' - f\right) - H^2 f' - \cdots\right]$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \left[-\frac{1}{6}f + H^2 f' - \frac{1}{2}H\dot{f}' - \frac{1}{2}\ddot{f}'\right]$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\left(\rho_m + \rho_{DE}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\left(\rho_m + \rho_{DE} + 3p_{DE}\right)$$

- Evolution of cosmic perturbations: δ_m, Φ, Ψ (k,a)

∞ tests !!

Sub-horizon, Fourier space

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{eff}\rho_m\delta_m \cong 0$$

$$\Psi \cong -4\pi G_{eff}\frac{a^2}{k^2}\rho_m\delta_m$$

$$\frac{\Phi}{\Psi} \cong \left(1 + 2\frac{k^2}{a^2}\frac{f''}{1+f'}\right) \Big/ \left(1 + 4\frac{k^2}{a^2}\frac{f''}{1+f'}\right)$$

$$G_{eff}(k,a) \cong \frac{1}{8\pi(1+f')}\frac{1 + 4\dfrac{k^2}{a^2}\dfrac{f''}{1+f'}}{1 + 3\dfrac{k^2}{a^2}\dfrac{f''}{1+f'}}$$

G_{eff} : function of (k,a)

$\Phi \neq \Psi$

GR : $G_{eff} = \text{const.}$, $\Phi = \Psi$

MG-DE Models vs. Observations

Cosmological Models

Inter-medium
(physical quantities)

Observations

Dark Energy
Modified Gravity

Background Expansion
$H(t)$, $\rho(z)$, $w(z)$, …

Cosmic Structures

(linear cosmological perturbations)
- matter density perturb.: $\delta\rho_m(z)$
- metric perturb.: Φ,Ψ (scalar),…

SNe Ia
CMB
LSS
Lensing

Theoretical prediction

Observational info

(Our work)

f(R) MG vs. *Observations*

Cosmological Models

Inter-medium
(physical quantities)

Observations

f (R)
Modified Gravity

Background Expansion

$H(t)$, $\rho(z)$, $w(z)$, …

SNe Ia
CMB
LSS
Lensing

Cosmic Structures

(linear cosmological perturbations)

- matter density perturb.: $\delta\rho_m(z)$
- metric perturb.: Φ, Ψ (scalar),…

Theoretical prediction

Observational info

(Our work)

f(R) MG vs. *Observations*

Cosmological Models

Inter-medium
(physical quantities)

Observations

f (R)
Modified Gravity

Background Expansion

$H(t)$, $\rho(z)$, $w(z)$, …

SNe Ia
CMB
LSS
Lensing

Cosmic Structures

(linear cosmological perturbations)

- matter density perturb.: $\delta\rho_m(z)$
- metric perturb.: Φ, Ψ (scalar),…

Theoretical prediction

Observational info

(Our work)

f(R) MG vs. *Observations*

Cosmological Models

Inter-medium
(physical quantities)

Observations

"*Designer f (R)*"
Modified Gravity

Background Expansion

$H(t)$, $\rho(z)$, $w(z)$, …

SNe Ia
CMB
LSS
Lensing

Cosmic Structures

(linear cosmological perturbations)

- matter density perturb.: $\delta\rho_m(z)$
- metric perturb.: Φ,Ψ (scalar),…

our plan

(Our work)

f(R) MG vs. *Observations*

Cosmological Models

Inter-medium
(physical quantities)

Observations

Background Expansion

$H(t)$, $\rho(z)$, $w(z)$, …

"*Designer f (R)*"
Modified Gravity

SNe Ia
CMB
LSS
Lensing

Cosmic Structures

(linear cosmological perturbations)

- matter density perturb.: $\delta\rho_m(z)$
- metric perturb.: Φ, Ψ (scalar),…

our plan

f(R) Modified Gravity

f(R) Modified Gravity : $\mathcal{L}_G = R + f(R)$

(modified gravity action)

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + f(R)\right] + \int d^4x\sqrt{-g}\,\mathcal{L}_m$$

metric : $(-,+,+,+)$

$$f_R \equiv \frac{\partial f}{\partial R},\quad f_{RR} \equiv \frac{\partial^2 f}{\partial R^2}$$

Field eqn: $$(1+f_R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R+f) + \left(g_{\mu\nu}\nabla^\alpha\nabla_\alpha - \nabla_\mu\nabla_\nu\right)f_R = 8\pi G T_{\mu\nu}$$

Energy-momentum tensor: $T_{\mu\nu} = (\rho + P)U_\mu U_\nu + P g_{\mu\nu}(R+f)$

- Cosmic expansion (background evolution): $a(t)$

Flat RW metric: $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$

$$H^2 = \frac{8\pi G}{3}\rho_m + \left[\frac{1}{6}(Rf_R - f) - H^2 f_R - H\dot{f}_R\right] \rightarrow \rho_{eff}$$

$$8\pi G\rho_{eff} = \frac{1}{2}(Rf_R - f) - 3H^2 f_R - 3H\dot{f}_R$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \left[-\frac{1}{6}f + H^2 f_R - \frac{1}{2}H\dot{f}_R - \frac{1}{2}\ddot{f}_R\right] \rightarrow \rho_{eff} + 3p_{eff}$$

$$R = 6\left(\frac{\ddot{a}}{a} + H^2\right) = 6\left(\dot{H} + 2H^2\right)$$

$$8\pi G P_{eff} = \frac{1}{2}f - \frac{1}{6}Rf_R - H^2 f_R + 2H\dot{f}_R + \ddot{f}_R$$

f(R) Modified Gravity : $\mathcal{L}_G = R + f(R)$

(modified gravity action)

metric : $(-,+,+,+)$

- Cosmic expansion (background evolution): $a(t)$

$$R = 6\left(\frac{\ddot{a}}{a} + H^2\right) = 6\left(\dot{H} + 2H^2\right)$$

Flat RW metric: $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$

$$f_R \equiv \frac{\partial f}{\partial R} \qquad f_{RR} \equiv \frac{\partial^2 f}{\partial R^2}$$

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{eff}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_m + \rho_{eff} + 3P_{eff}\right)\rho_m$$

$$\rho_{eff} = \frac{1}{8\pi G}\left[\frac{1}{2}\left(Rf_R - f\right) - 3H^2 f_R - 3H\dot{f}_R\right]$$

$$f_R \equiv \frac{\partial f}{\partial R} = \frac{\dot{f}}{\dot{R}}$$

$$P_{eff} = \frac{1}{8\pi G}\left[\frac{1}{2}f - \frac{1}{6}Rf_R - H^2 f_R + 2H\dot{f}_R + \ddot{f}_R\right]$$

$$w_{eff} = -1 + \frac{Rf_R/3 - 4H^2 f_R - H\dot{f}_R + \ddot{f}_R}{\left(Rf_R - f\right)/2 - 3H^2 f_R - 3H\dot{f}_R}$$

f(R) Modified Gravity : $\mathcal{L}_G = R + f(R)$

(modified gravity action)

metric : $(-,+,+,+)$

❖ Cosmic expansion (background evolution): $a(t)$

$$R = 6\left(\frac{\ddot{a}}{a} + H^2\right) = 6\left(\dot{H} + 2H^2\right)$$

Flat RW metric: $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$

$$f_R \equiv \frac{\partial f}{\partial R}$$

$$f_{RR} \equiv \frac{\partial^2 f}{\partial R^2}$$

$$\begin{cases} H^2 = \dfrac{8\pi G}{3}\left(\rho_m + \rho_{eff}\right) \\ \dfrac{\ddot{a}}{a} = -\dfrac{4\pi G}{3}\left(\rho_m + \rho_{eff} + 3P_{eff}\right)\rho_m \end{cases}$$

$$\begin{cases} \rho_{eff} = \dfrac{1}{8\pi G}\left[\dfrac{1}{2}\left(Rf_R - f\right) - 3H^2 f_R - 3H\dot{f}_R\right] \\ P_{eff} = \dfrac{1}{8\pi G}\left[\dfrac{1}{2}f - \dfrac{1}{6}Rf_R - H^2 f_R + 2H\dot{f}_R + \ddot{f}_R\right] \end{cases}$$

$$f_R \equiv \frac{\partial f}{\partial R} = \frac{\dot{f}}{\dot{R}}$$

a category of f(R)

For a given expansion history $H(t)$, we can (always?) find a $f(R)$ that generates the required $H(t)$.

different initial condition → different $f(R)$

The $f(t)$ satisfies a 2nd-order diff. eq. given by Friedmann eq.

f(R) Modified Gravity : $\mathcal{L}_G = R + f(R)$

(modified gravity action)

- Evolution of cosmic perturbations: $\delta_{\rm m}$, Φ, Ψ (k,a)
  - Perturbed metric with scalar perturbations in a longitudinal gauge:

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1-2\Phi)\delta_{ij}dx^i dx^j$$

  - Decompose quantities into background and inhomogeneous parts:

$$T_{\rm m}{}^0{}_0 = -(\rho_{\rm m} + \delta\rho_{\rm m}),\ T_{\rm m}{}^0{}_i = -\rho_{\rm m} v_{m,i}$$

v_m: velocity potential

Sub-horizon, Fourier space

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{eff}\rho_m\delta_m \cong 0$$

$$\Psi \cong -4\pi G_{eff}\frac{a^2}{k^2}\rho_m\delta_m$$

$$\frac{\Phi}{\Psi} \cong \left(1+2\frac{k^2}{a^2}\frac{f''}{1+f'}\right)\Big/\left(1+4\frac{k^2}{a^2}\frac{f''}{1+f'}\right)$$

$$G_{eff}(k,a) \cong \frac{1}{8\pi(1+f')}\frac{1+4\dfrac{k^2}{a^2}\dfrac{f''}{1+f'}}{1+3\dfrac{k^2}{a^2}\dfrac{f''}{1+f'}}$$

G_{eff} : function of (k,a)

$\Phi \neq \Psi$

GR : $G_{eff} = \text{const.}$, $\Phi = \Psi$

Designer f(R)

(Our work)

f(R) MG vs. *Observations*

Cosmological Models

Inter-medium
(physical quantities)

Observations

"*Designer f (R)*"
Modified Gravity

Background Expansion

$H(t)$, $\rho(z)$, $w(z)$, …

SNe Ia
CMB
LSS
Lensing

Cosmic Structures

(linear cosmological perturbations)

- matter density perturb.: $\delta\rho_m(z)$
- metric perturb.: Φ, Ψ (scalar),…

our plan

(Our work)

f(R) MG vs. *Observations*

Cosmological Models

Inter-medium
(physical quantities)

Observations

"*Designer f (R)*"
Modified Gravity

Background Expansion

$w_{eff}(z)$ = different constant
different initial conditions

SNe Ia
CMB
LSS
Lensing

Cosmic Structures

(linear cosmological perturbations)

- matter density perturb.: $\delta\rho_m(z)$
- metric perturb.: Φ,Ψ (scalar),…

our plan

(Our work)

f(R) MG vs. *Observations*

Cosmological Models

Inter-medium
(physical quantities)

Observations

"*Designer f (R)*" Modified Gravity

Background Expansion

$w_{eff}(z)$ = different constant
different initial conditions

Cosmic Structures

SNe Ia
CMB
LSS
Lensing

- Lombriser, Slosar, Seljak, and Hu, arXiv:1003.3009
  "Constraints on f(R) gravity from probing the large-scale structure"
- Giannantonio, Martinelli, Silvestri, and Melchiorri, arXiv:0909.2045
  "New constraints on parametrised modified gravity from correlations of the CMB with large scale structure"

(Our work)

f(R) MG vs. *Observations*

- Lombriser, Slosar, Seljak, and Hu, arXiv:1003.3009

  "Constraints on f(R) gravity from probing the large-scale structure"

$$0 < B_0 < 1.1 \times 10^{-3}$$

95% C.L.

$$B_0 \equiv \frac{f_{RR}}{1+f_R} R' \frac{H}{H'} \qquad \left(' \equiv \frac{d}{d \ln a} \right)$$

- Giannantonio, Martinelli, Silvestri, and Melchiorri, arXiv:0909.2045

  "New constraints on parametrised modified gravity from correlations of the CMB with large scale structure"

$$1 < \frac{\Psi}{\Phi} < 1.996$$

95% C.L.

$$\frac{\Psi}{\Phi} \cong \frac{1 + 4 \frac{k^2}{a^2} \frac{f_{RR}}{1+f_R}}{1 + 2 \frac{k^2}{a^2} \frac{f_{RR}}{1+f_R}}$$

Results

(Our work)

f(R) MG vs. *Observations*

Cosmological Models

Inter-medium
(physical quantities)

Observations

"*Designer f (R)*"
Modified Gravity

Background Expansion

$w_{eff}(z) = -1 \sim -0.87$
different initial conditions

SNe Ia
CMB
LSS
Lensing

Cosmic Structures

(linear cosmological perturbations)

- matter density perturb.: $\delta\rho_m(z)$
- metric perturb.: Φ, Ψ (scalar),…

our plan

(Our work)

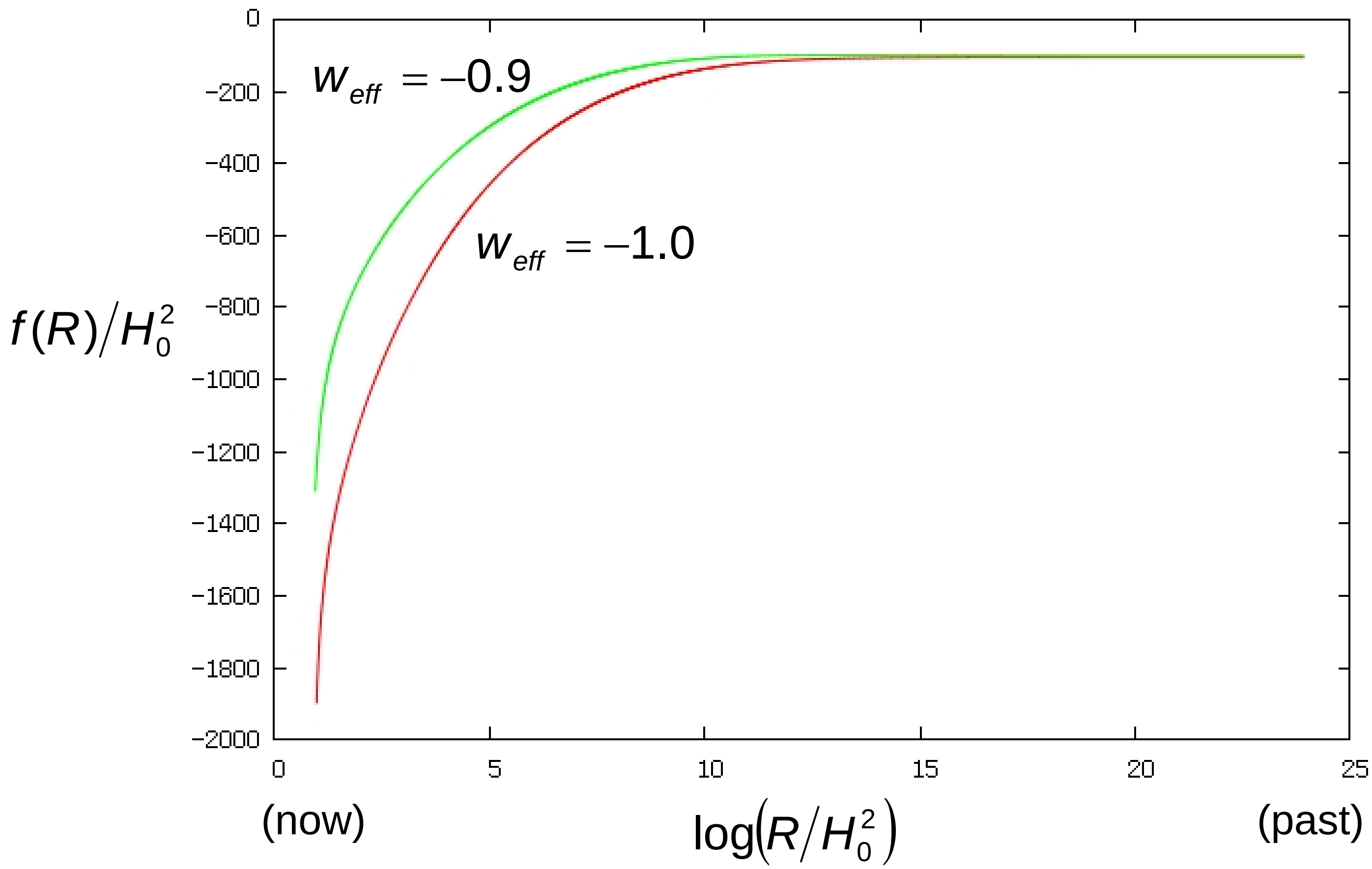
"Designer f(R)" MG

$w = -1.0$ & -0.9

$a_i = 10^{-8}$

$w_{eff} = -1.0: \quad f(a_i)/H_0^2 = -104, \; f_R(a_i) = 5.0 \times 10^{-28}$

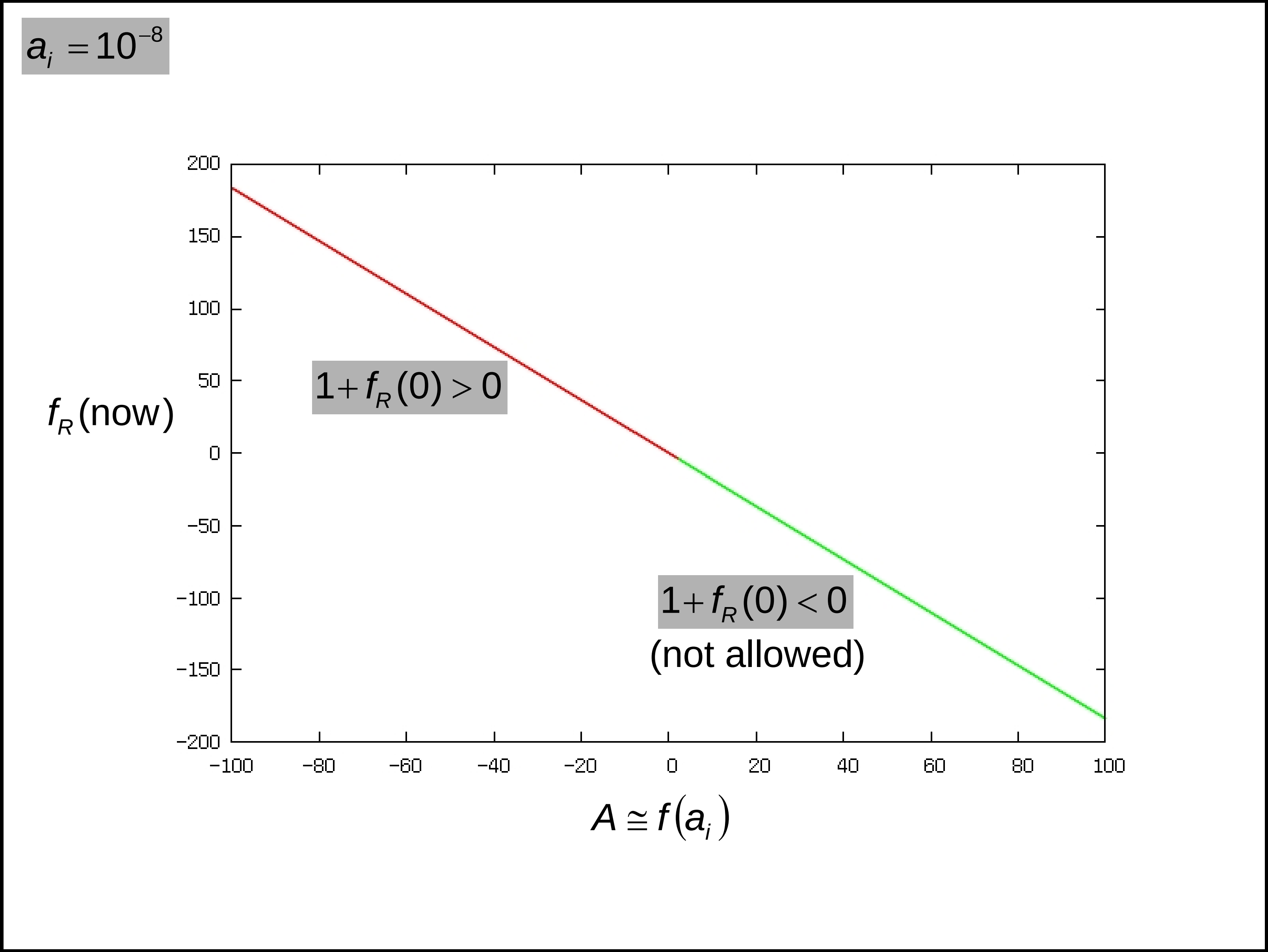
$w_{eff} = -0.9: \quad f(a_i)/H_0^2 = -100, \; f_R(a_i) = -5.4 \times 10^{-27}$



(Our work)

“Designer f(R)” MG

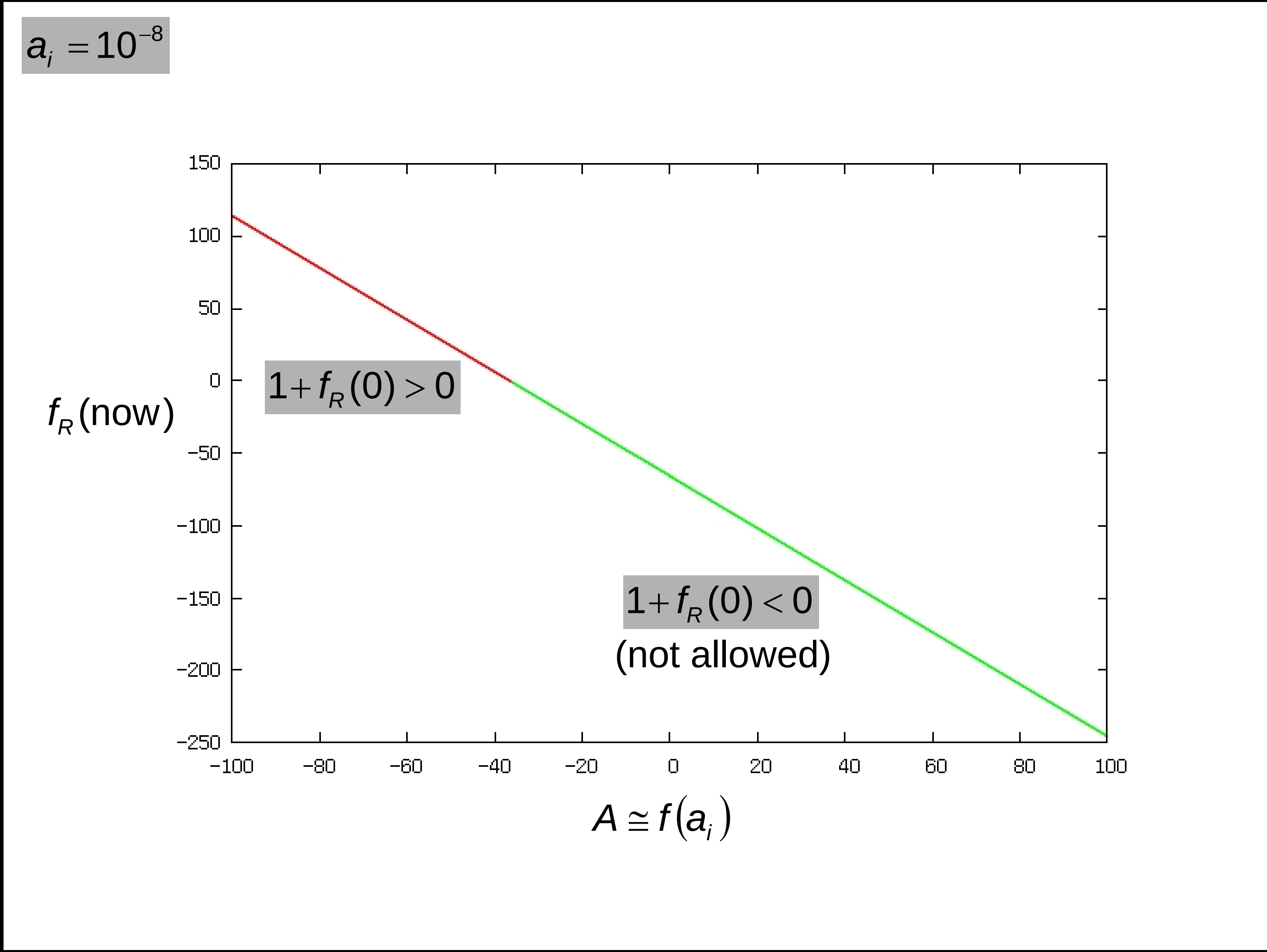
$w = -1.0$



(Our work)

"Designer f(R)" MG

$w = -0.9$

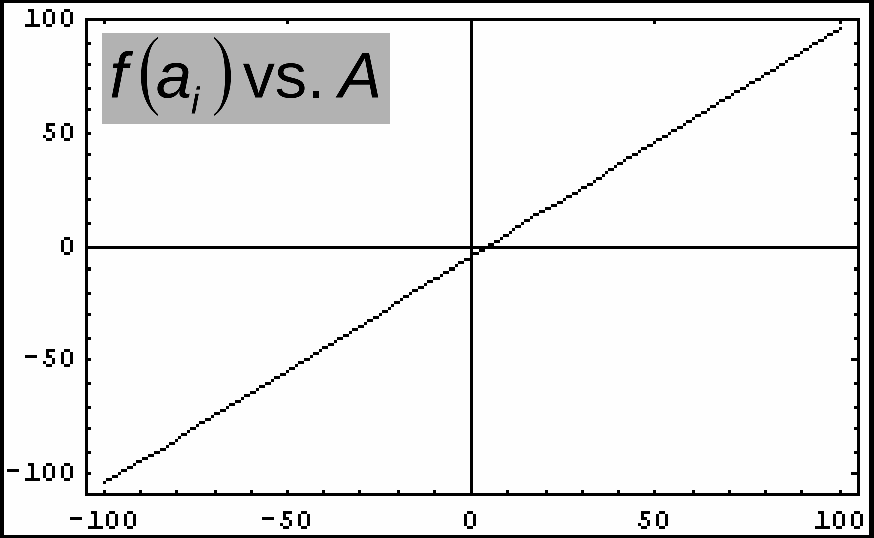


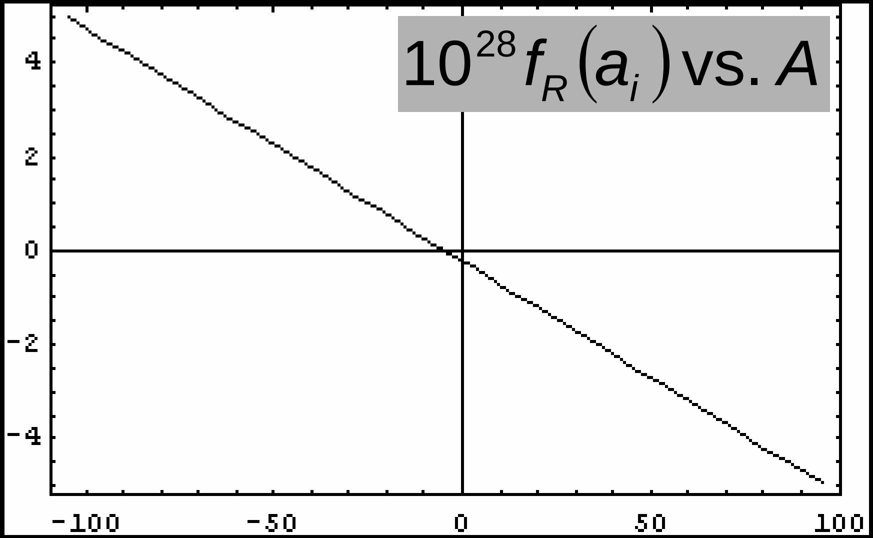
(Our work)

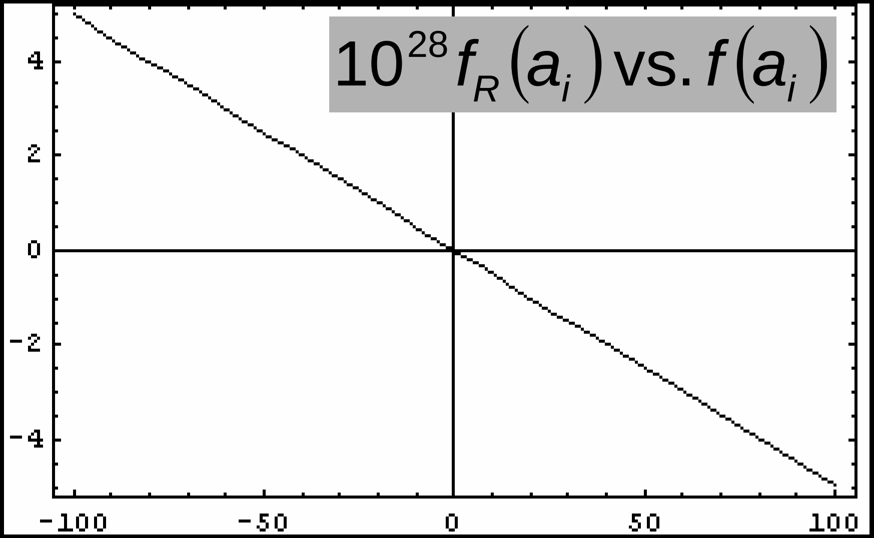
"Designer f(R)" MG

$w = -1.0$

$a_i = 10^{-8}$





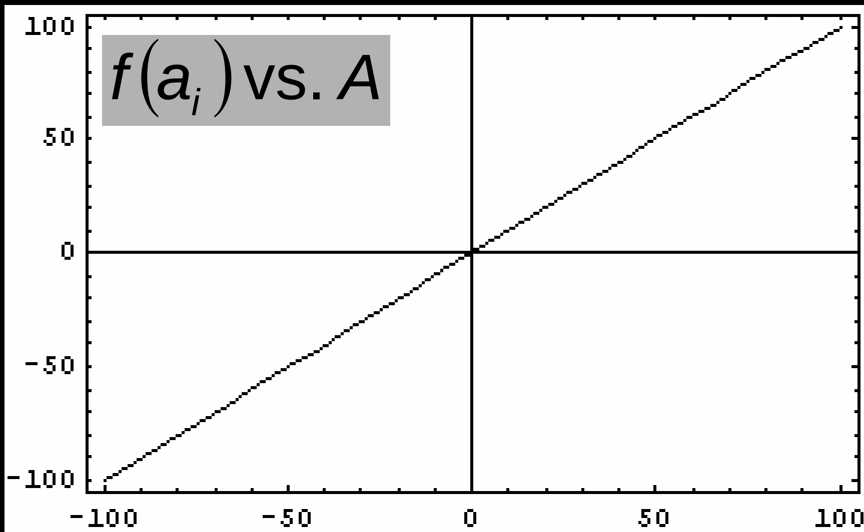


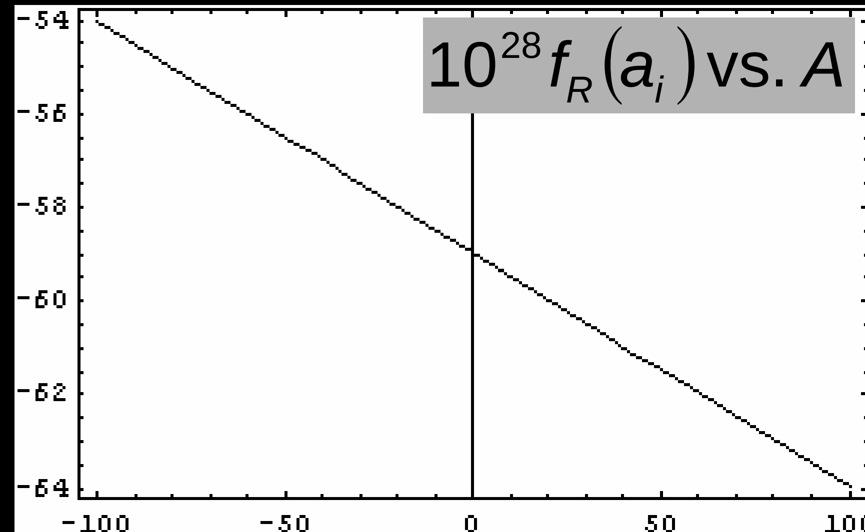
(Our work)

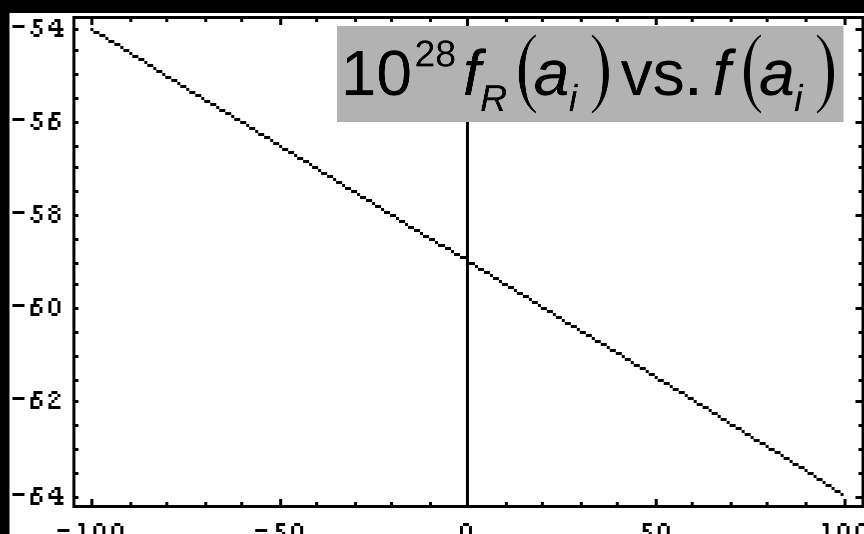
“Designer f(R)” MG

$w = -0.9$

$a_i = 10^{-8}$







(Our work)

f(R) MG vs. *Observations*

Cosmological Models	*Inter-medium* (physical quantities)	*Observations*

"*Designer f(R)*" Modified Gravity

Background Expansion

$w_{eff}(z)$ = different constant
different initial conditions

Cosmic Structures

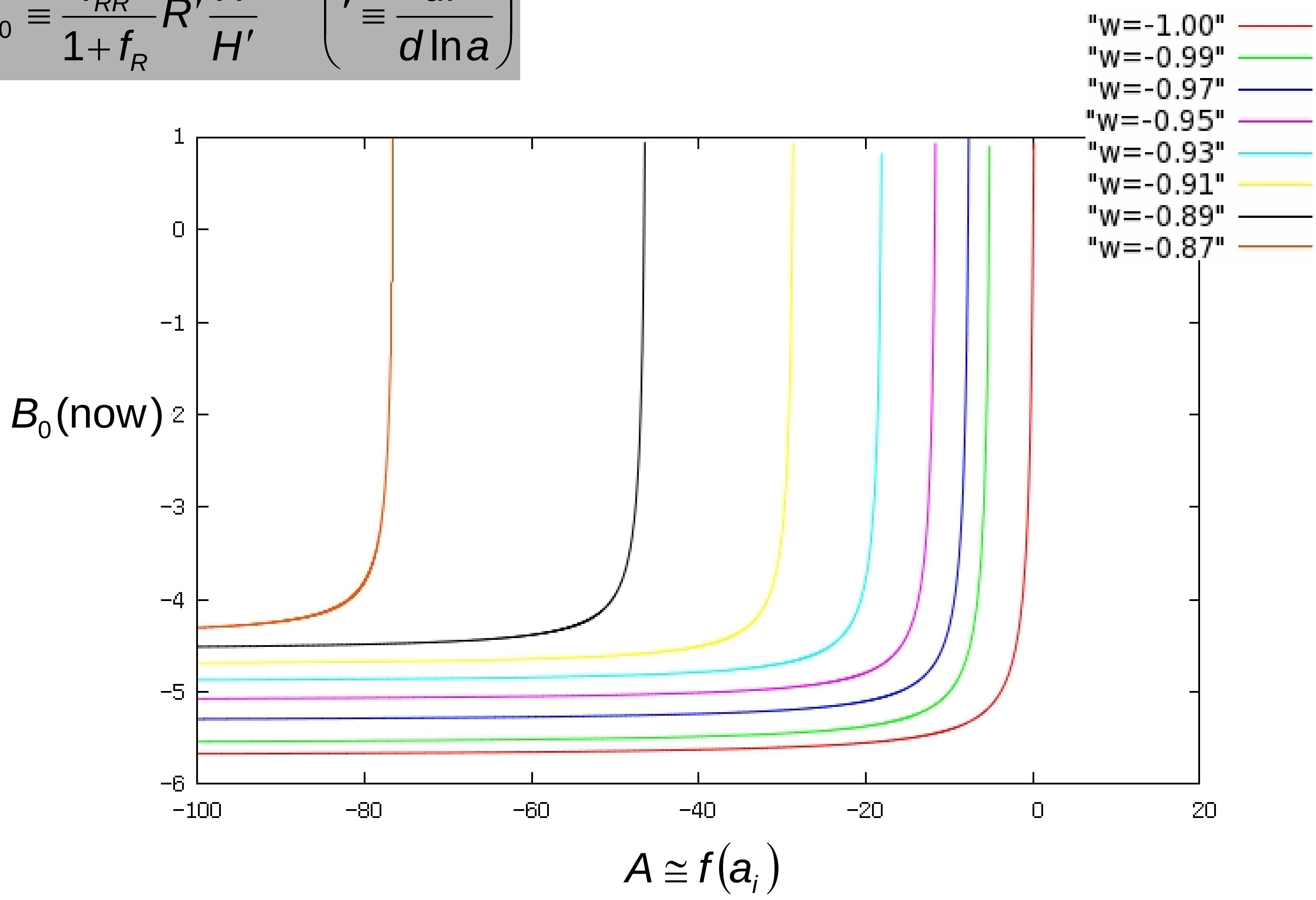
SNe Ia
CMB
LSS
Lensing

- Lombriser, Slosar, Seljak, and Hu, arXiv:1003.3009 $0 < B_0 < 1.1 \times 10^{-3}$
  "Constraints on f(R) gravity from probing the large-scale structure"
- Giannantonio, Martinelli, Silvestri, and Melchiorri, arXiv:0909.2045
  "New constraints on parametrised modified gravity from correlations of the CMB with large scale structure" $1 < \frac{\Psi}{\Phi} < 1.996$

(Our work)

"Designer f(R)" MG

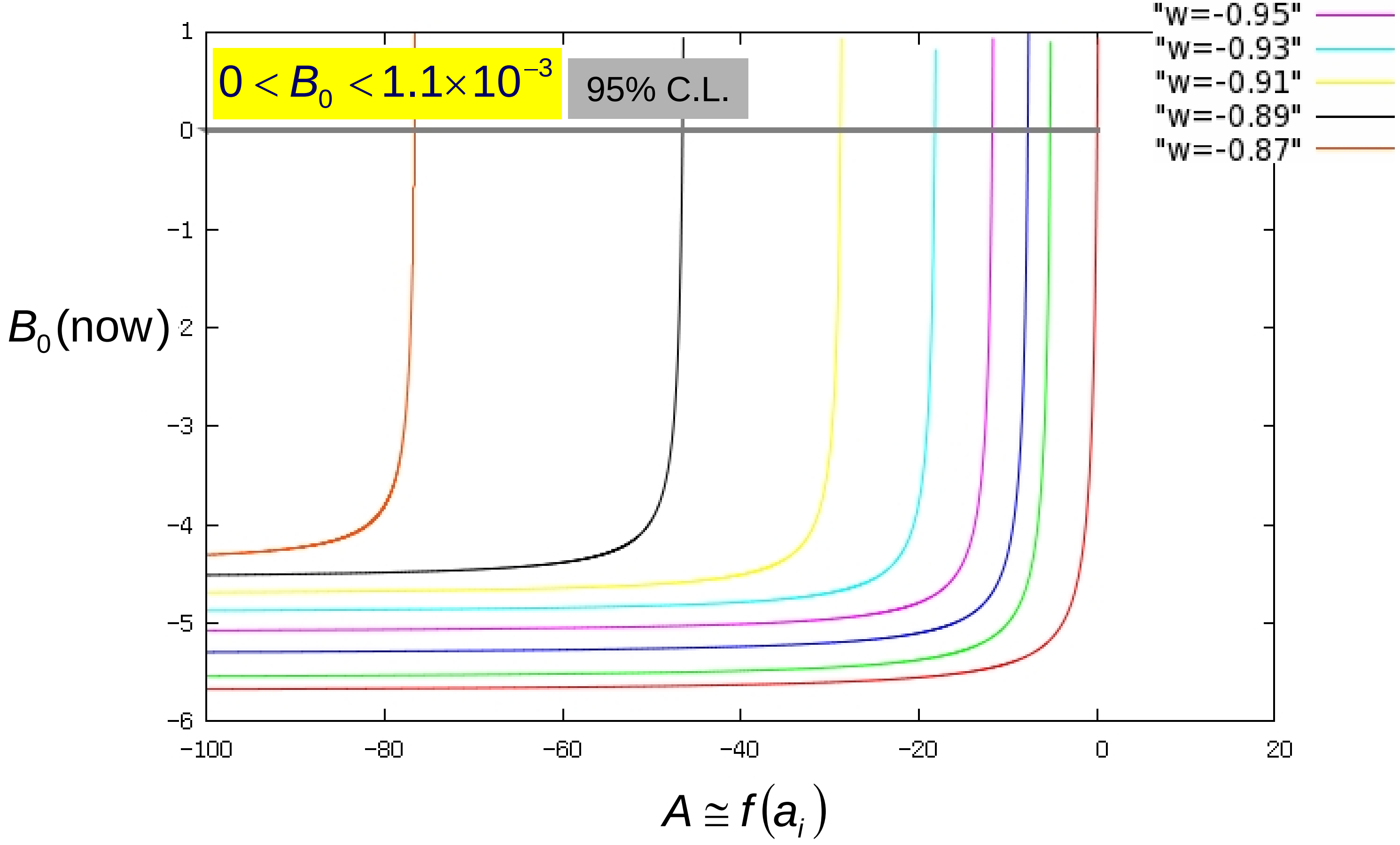
$$B_0 \equiv \frac{f_{RR}}{1+f_R} R' \frac{H}{H'} \qquad \left(' \equiv \frac{df}{d\ln a} \right)$$



(Our work)

"Designer f(R)" MG

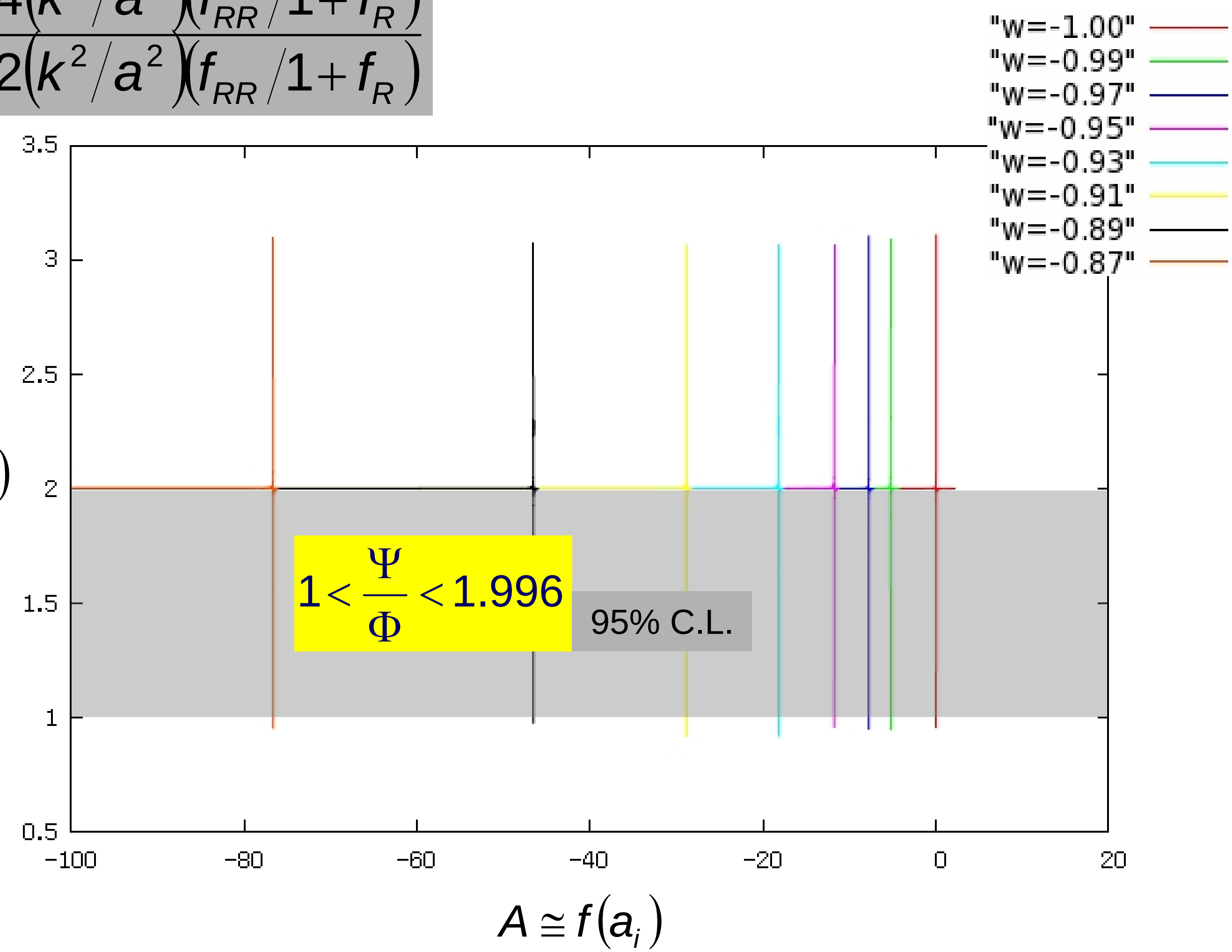
$$B_0 \equiv \frac{f_{RR}}{1+f_R} R' \frac{H}{H'} \qquad \left(' \equiv \frac{df}{d\ln a} \right)$$



(Our work)

"Designer f(R)" MG

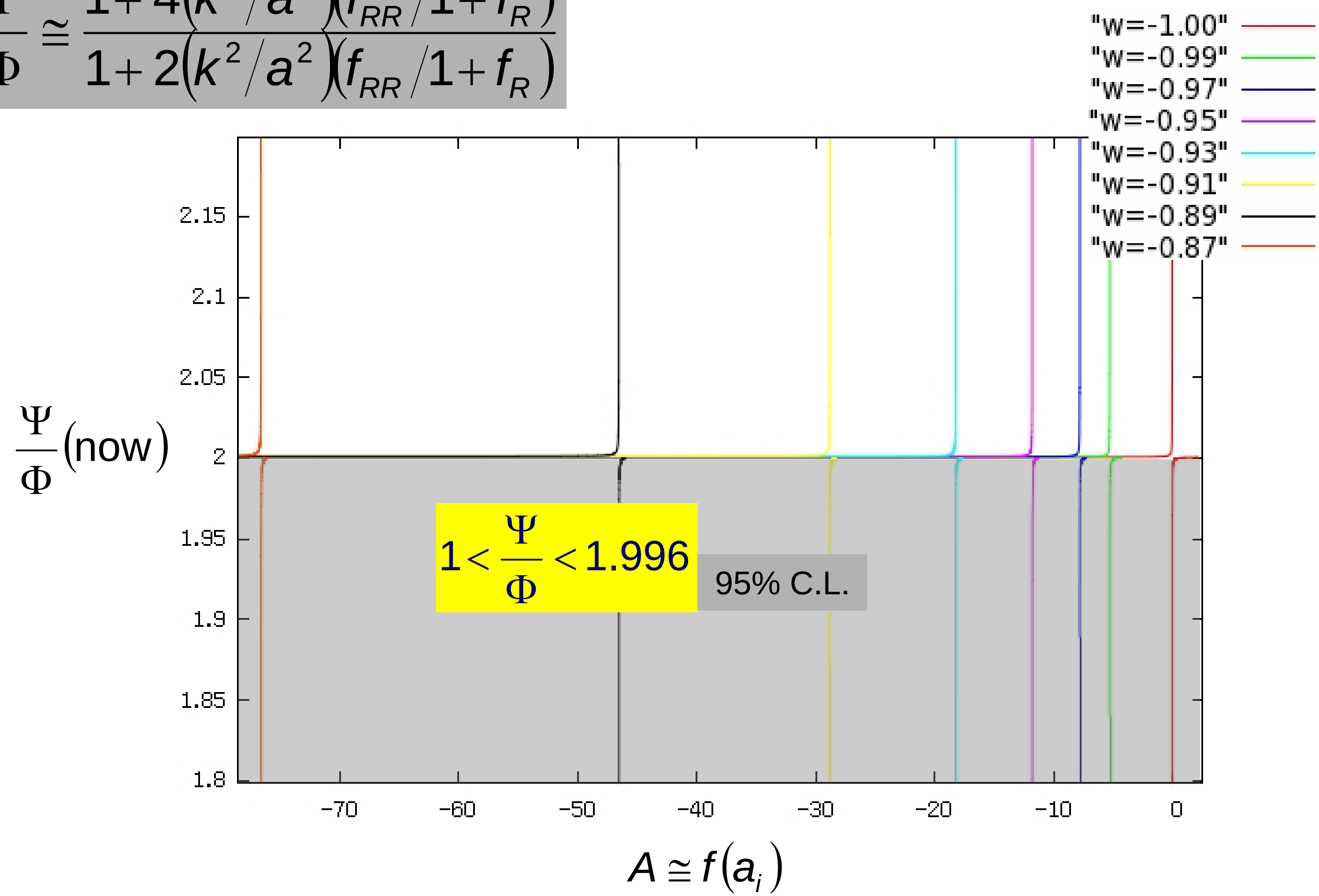
$$\frac{\Psi}{\Phi} \cong \frac{1+4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1+2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



(Our work)

"Designer f(R)" MG

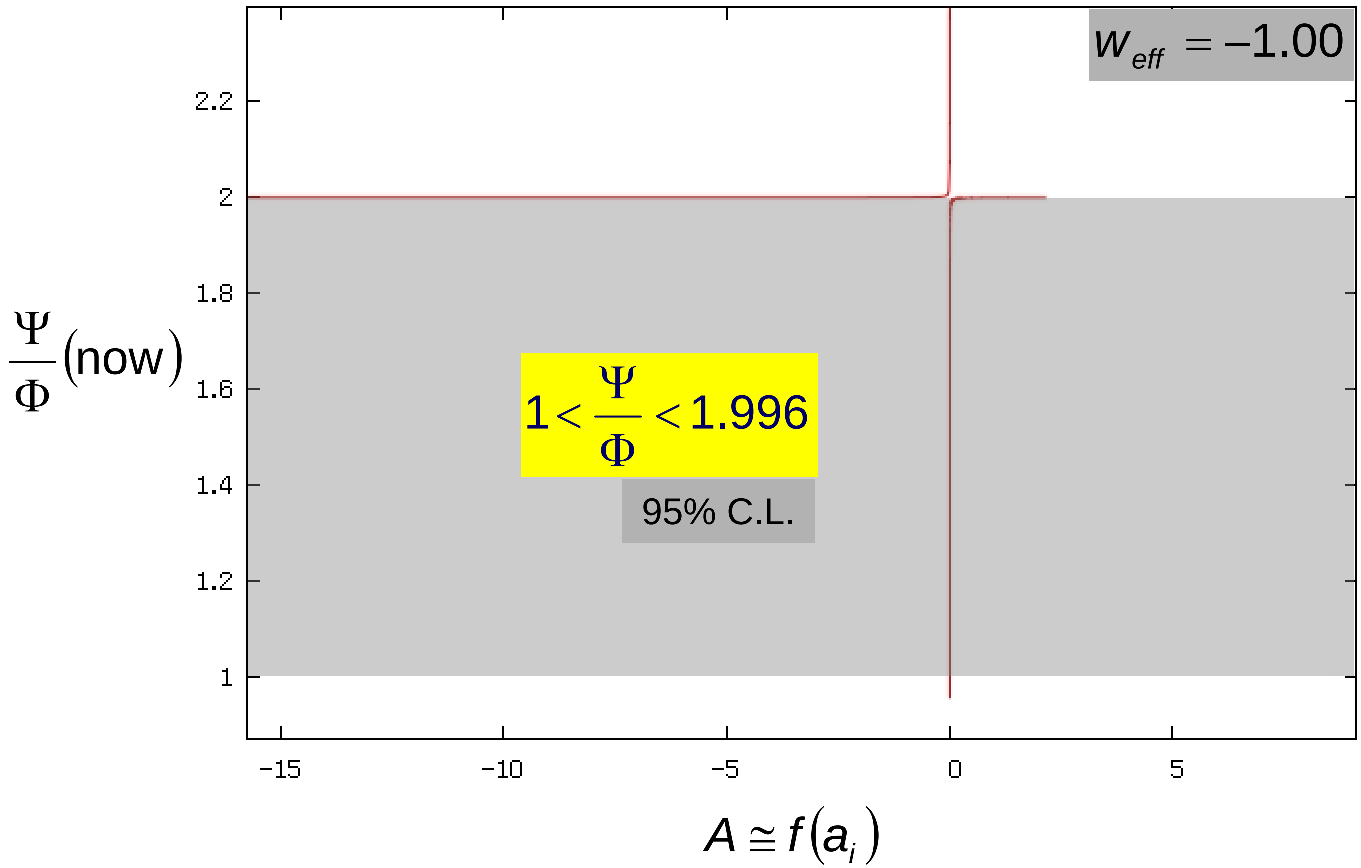
$$\frac{\Psi}{\Phi} \cong \frac{1+4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1+2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



(Our work)

"Designer f(R)" MG

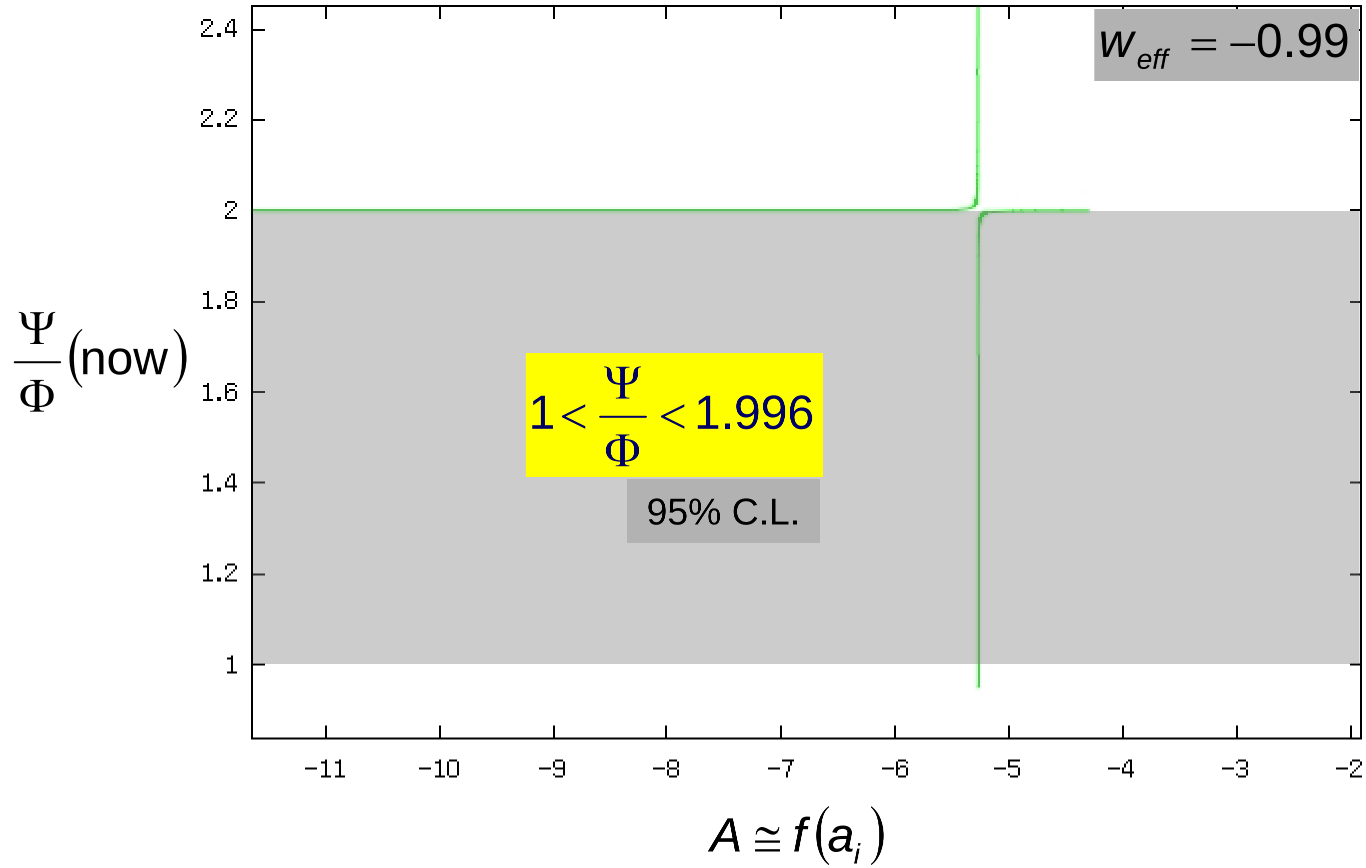
$$\frac{\Psi}{\Phi} \cong \frac{1 + 4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1 + 2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



(Our work)

"Designer f(R)" MG

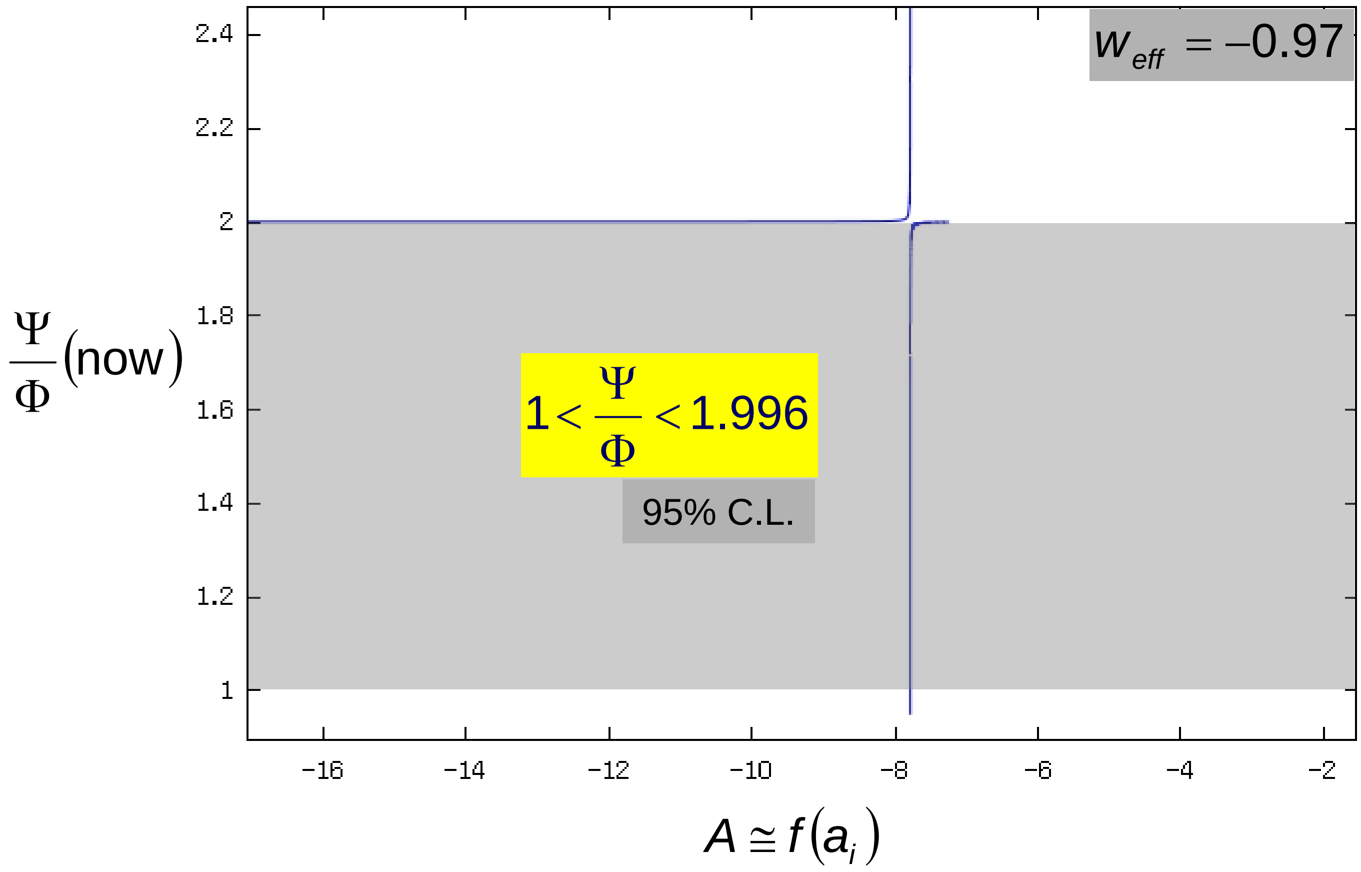
$$\frac{\Psi}{\Phi} \cong \frac{1+4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1+2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



(Our work)

"Designer f(R)" MG

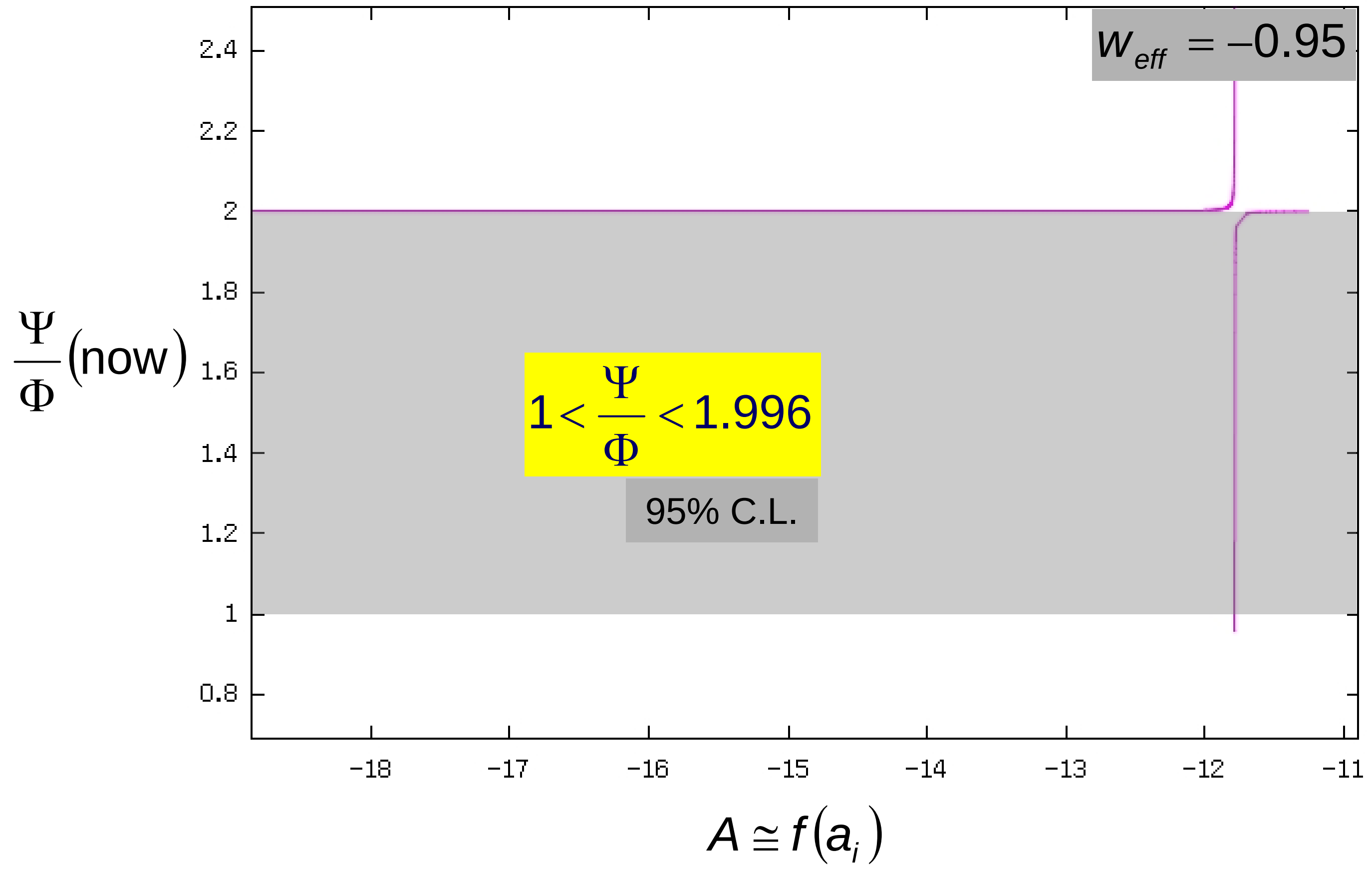
$$\frac{\Psi}{\Phi} \cong \frac{1+4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1+2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



(Our work)

"Designer f(R)" MG

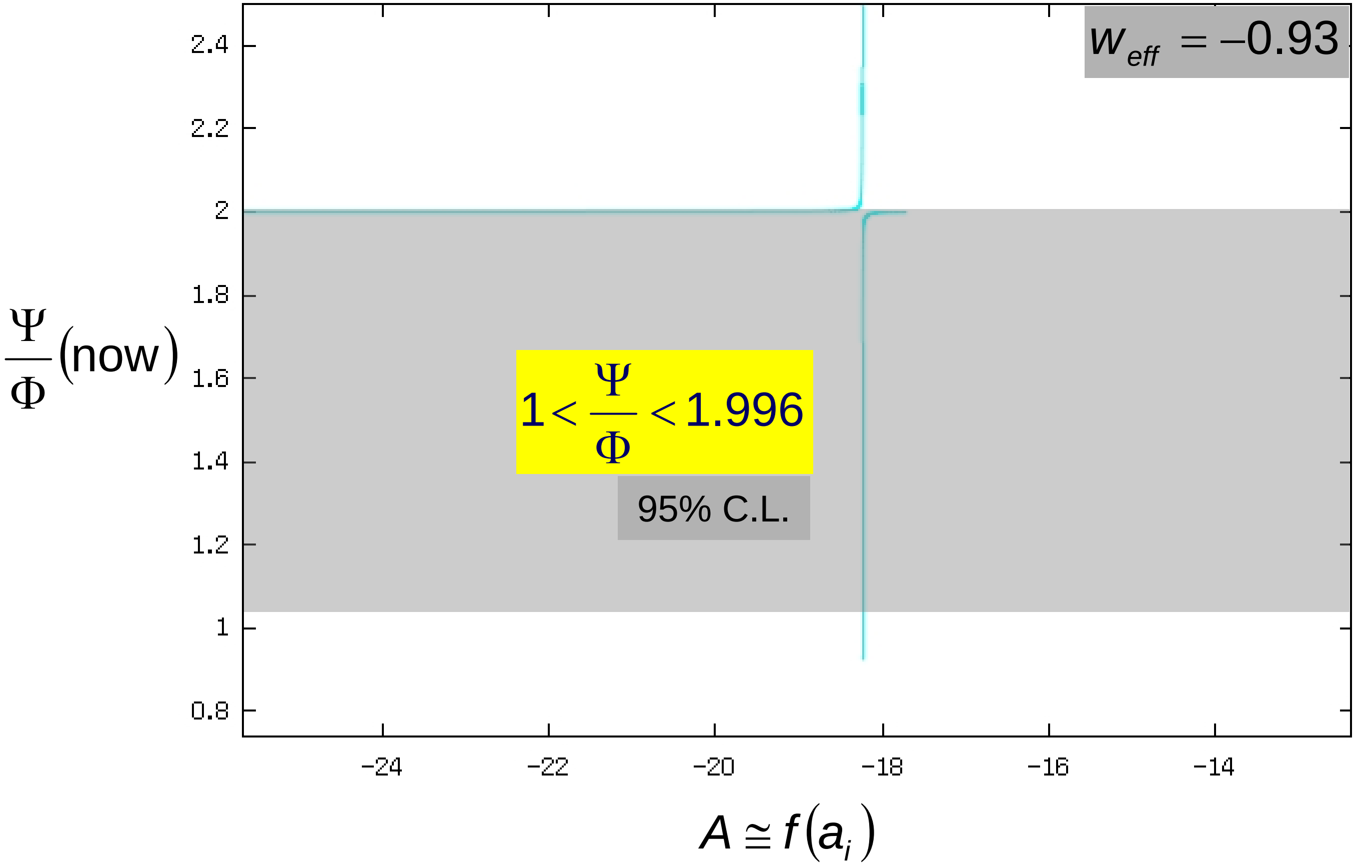
$$\frac{\Psi}{\Phi} \cong \frac{1 + 4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1 + 2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



(Our work)

"Designer f(R)" MG

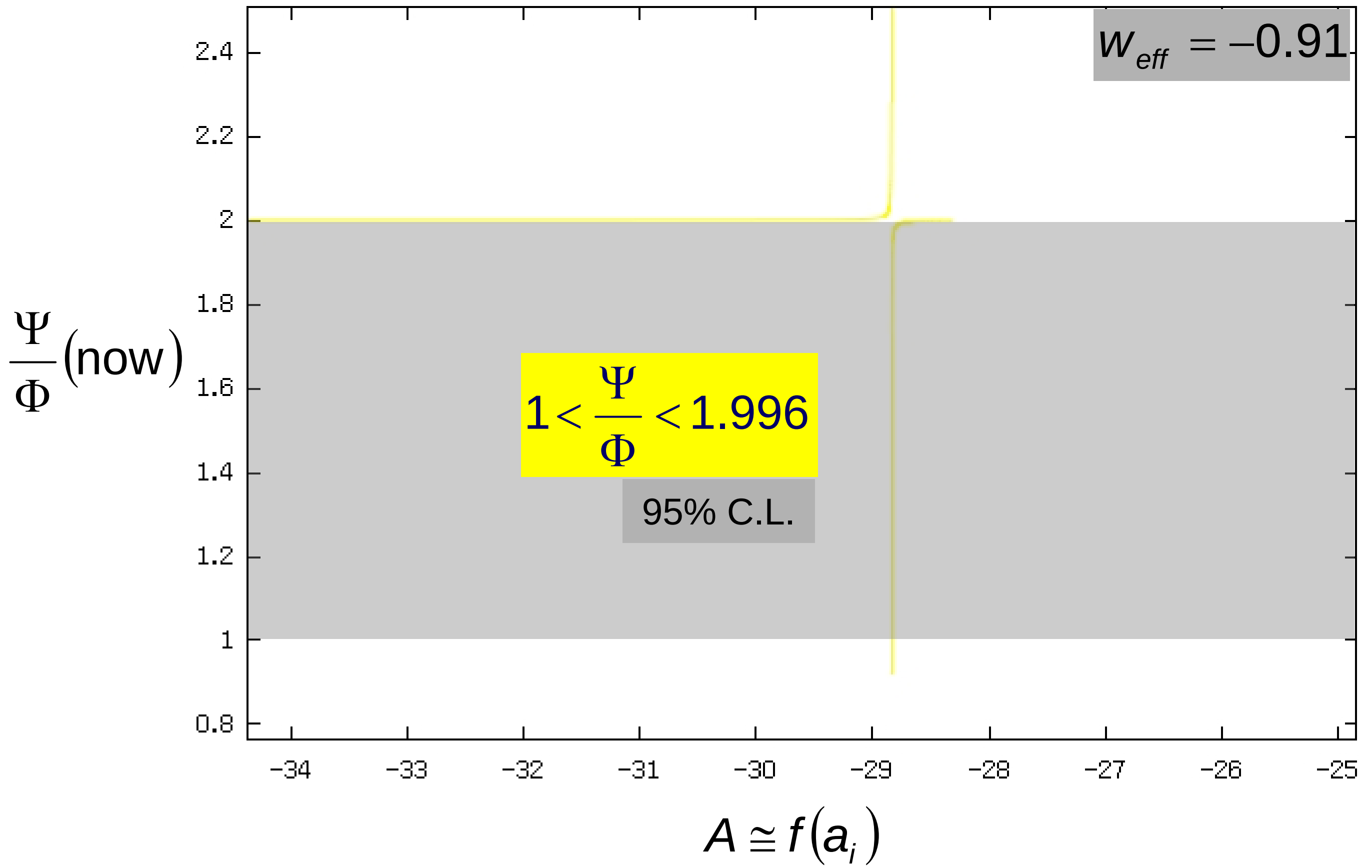
$$\frac{\Psi}{\Phi} \cong \frac{1+4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1+2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



(Our work)

"Designer f(R)" MG

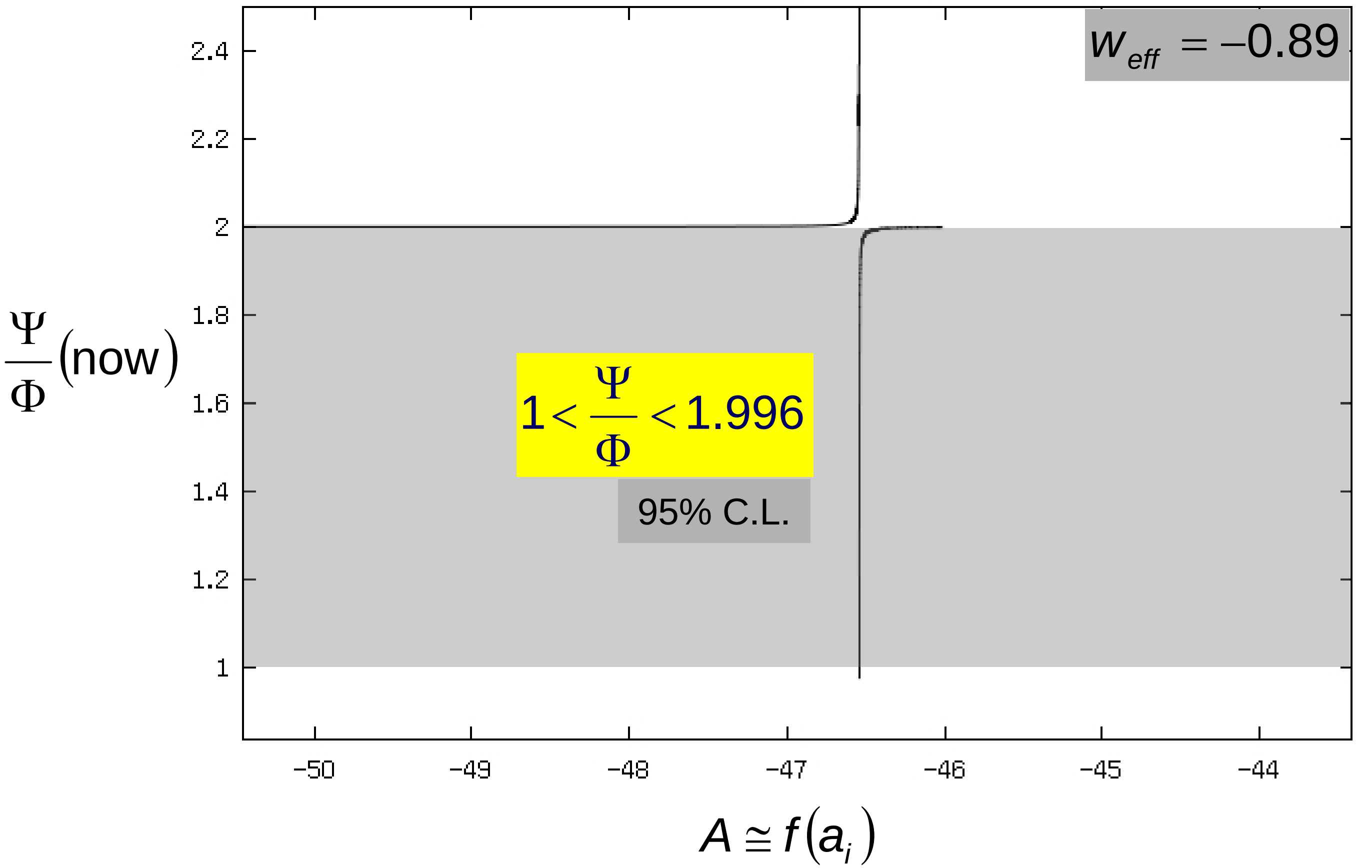
$$\frac{\Psi}{\Phi} \cong \frac{1+4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1+2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



(Our work)

"Designer f(R)" MG

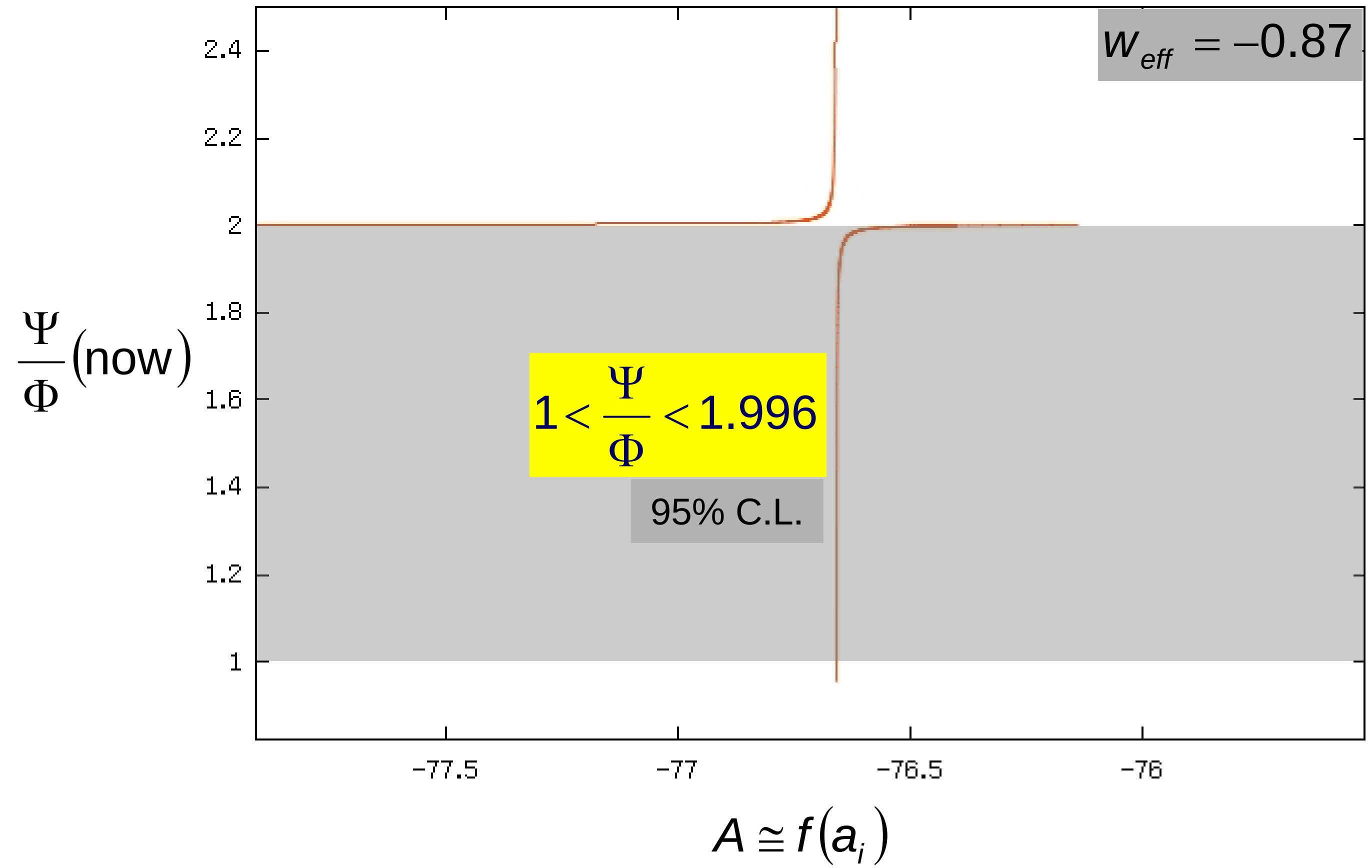
$$\frac{\Psi}{\Phi} \cong \frac{1+4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1+2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



(Our work)

"Designer f(R)" MG

$$\frac{\Psi}{\Phi} \cong \frac{1+4\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}{1+2\left(k^2/a^2\right)\left(f_{RR}/1+f_R\right)}$$



"Designer f(R)" MG : Observational Constraints

	initial condition		
w_{eff}	A	$f(a_i)$	$10^{28}\, f_R(a_i)$
– 1.00	$-10^{-7} \sim 10^{-4}$	– 4.379±ε	$-10^{-6} \sim 10^{-8}$
– 0.99	– 5.271±ε	– 5.273±ε	– 0.119±ε
– 0.97	– 7.789±ε	– 7.791±ε	– 0.780±ε
– 0.97	– 7.789±ε	– 7.791±ε	– 0.780±ε
– 0.95	– 11.787±ε	– 11.790±ε	– 0.780±ε
– 0.93	– 18.241±ε	– 18.250±ε	– 10.068±ε
– 0.91	– 28.838±ε	– 28.862±ε	– 32.241±ε
– 0.89	– 46.542±ε	– 46.608±ε	– 101.04±ε
– 0.87	– 76.660±ε	– 76.841±ε	– 313.49±ε

Summary

Summary

- We have studied "*designer f*(*R*)" for various constant w_{eff} and various initial conditions. We compare those models with the observational constraints about cosmic structures.

- There exist "designer *f*(*R*)" models consistent with data. However, we need to *fine tune* the initial conditions in order to have those observation-consistent models

- Most of the models under our consideration predict $\Psi/\Phi \simeq 2$, which is consistent with the current constraint (95% C.L.): $1<\Psi/\Phi<2$.

  If the future observations can give an upper bound significantly smaller than 2, many *f*(*R*) models will be ruled out; on the contrary, for a lower bound significantly larger than 1, **GR** will be ruled out.

Thank you.