GENERIC ANALYSIS OF *KINETICALLY* DRIVEN INFLATION

Rio Saitou (LeCosPA, NTU)
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Based on arXiv:1710.04941

Before Inflation…

3 big problems of big bang universe

- Flatness problem

  The early universe is unnaturally flat.

- Horizon problem

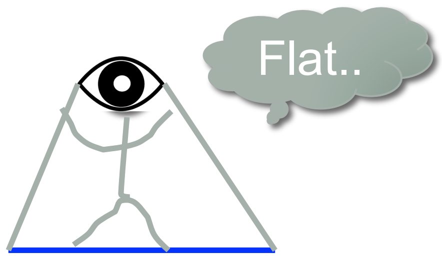
  All of observed causally disconnected CMB photons have the same temperature T~ 2.725K.

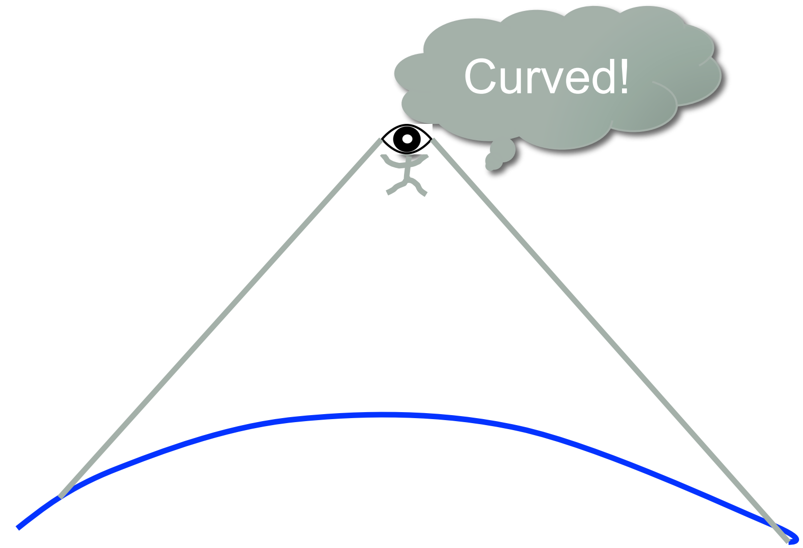
- Monopole problem

  We have not found monopole ever.

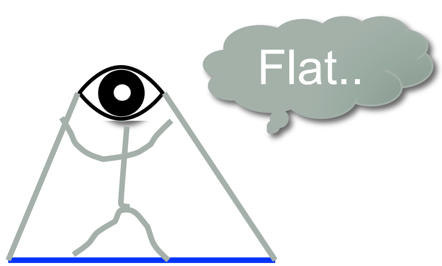
Flatness problem

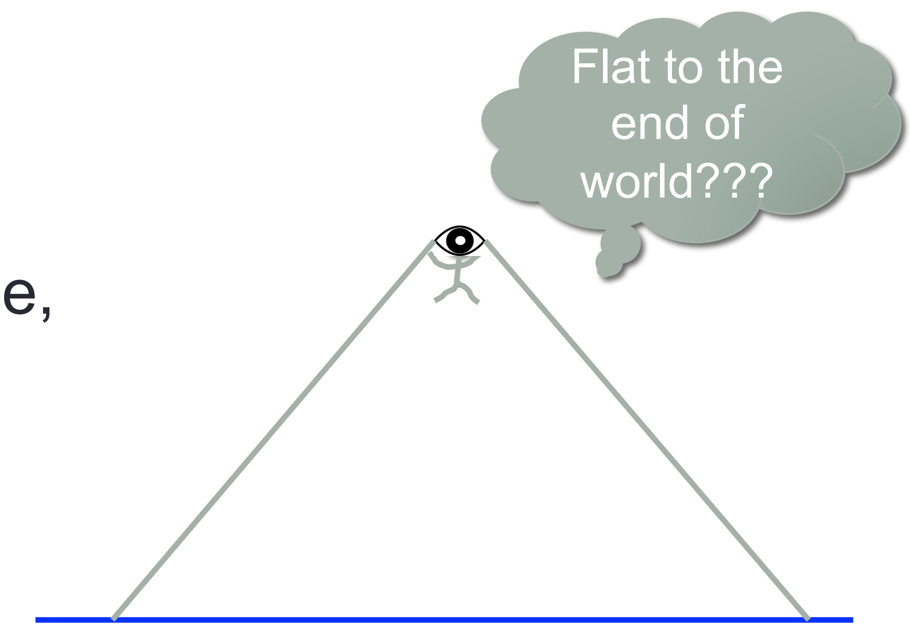
- Curvature of space





- But, in the big bang universe,





Horizon problem

Ref. Jinn-Ouk Gong, IJMPD26 (2016) no.01, 1740003

- Causally disconnected CMB photon

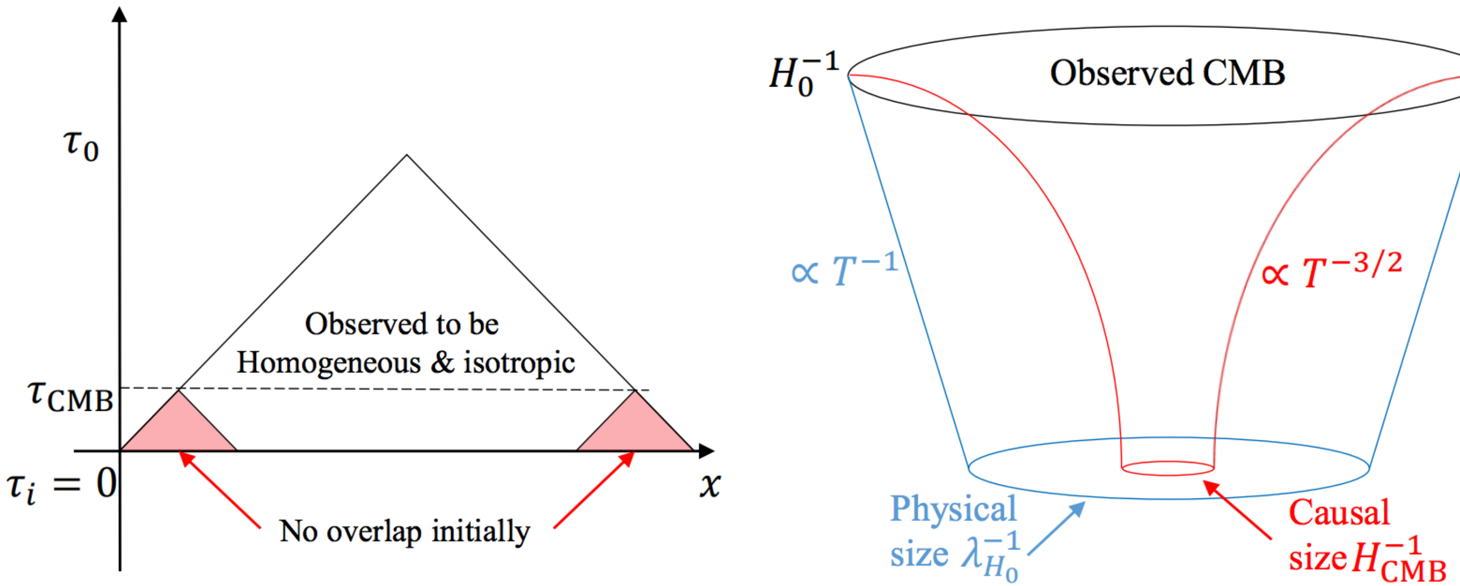
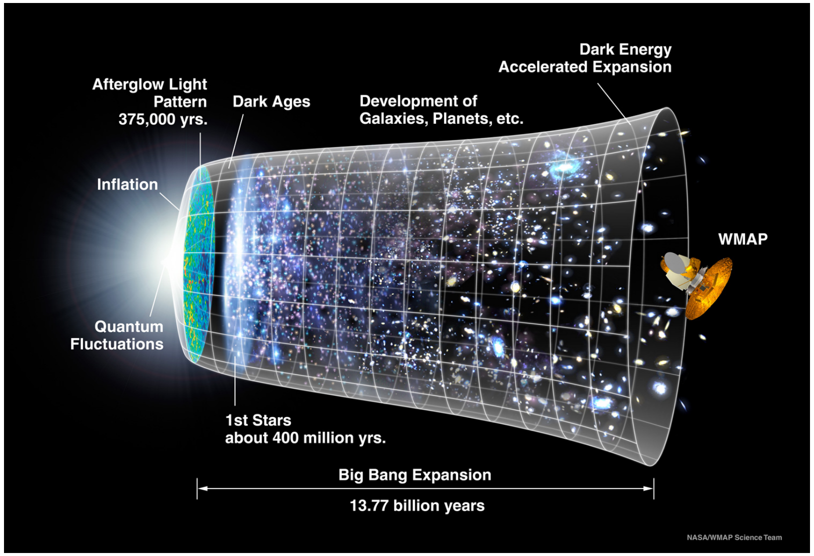


Figure 3: (Left) conformal diagram of the universe. From the cosmic singularity ($\tau_i = 0$) until the moment of the CMB generation ($\tau_{\rm CMB}$) there was no time for the CMB to achieve causal communication to have the same temperature T_0. (Right) as a sample calculation, we can see that at that time the universe was filled with $10^4 - 10^5$ causally disconnected patches.

Inflation

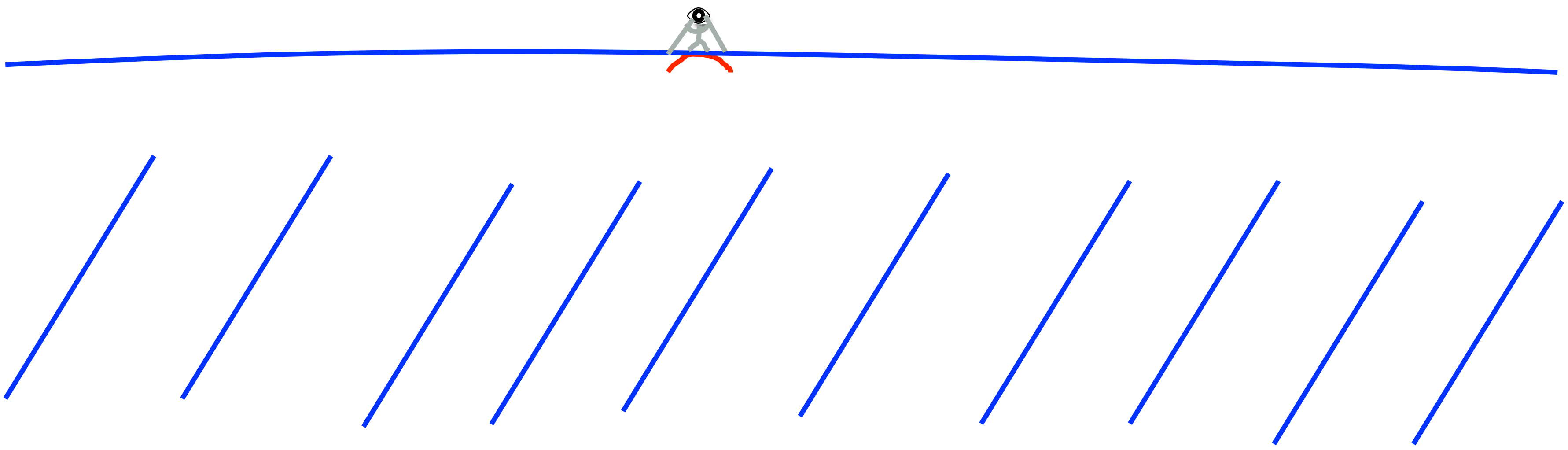
- ✓A solution for the 3 problems of big Bang by accelerating the universe
- ✓The quantum fluctuation created in the inflationary universe provides the seed of the large scale structure
- ✓Graceful exit + reheating



Accelerating expansion

- Making a huge flat and homogeneous region while keeping the same horizon size.

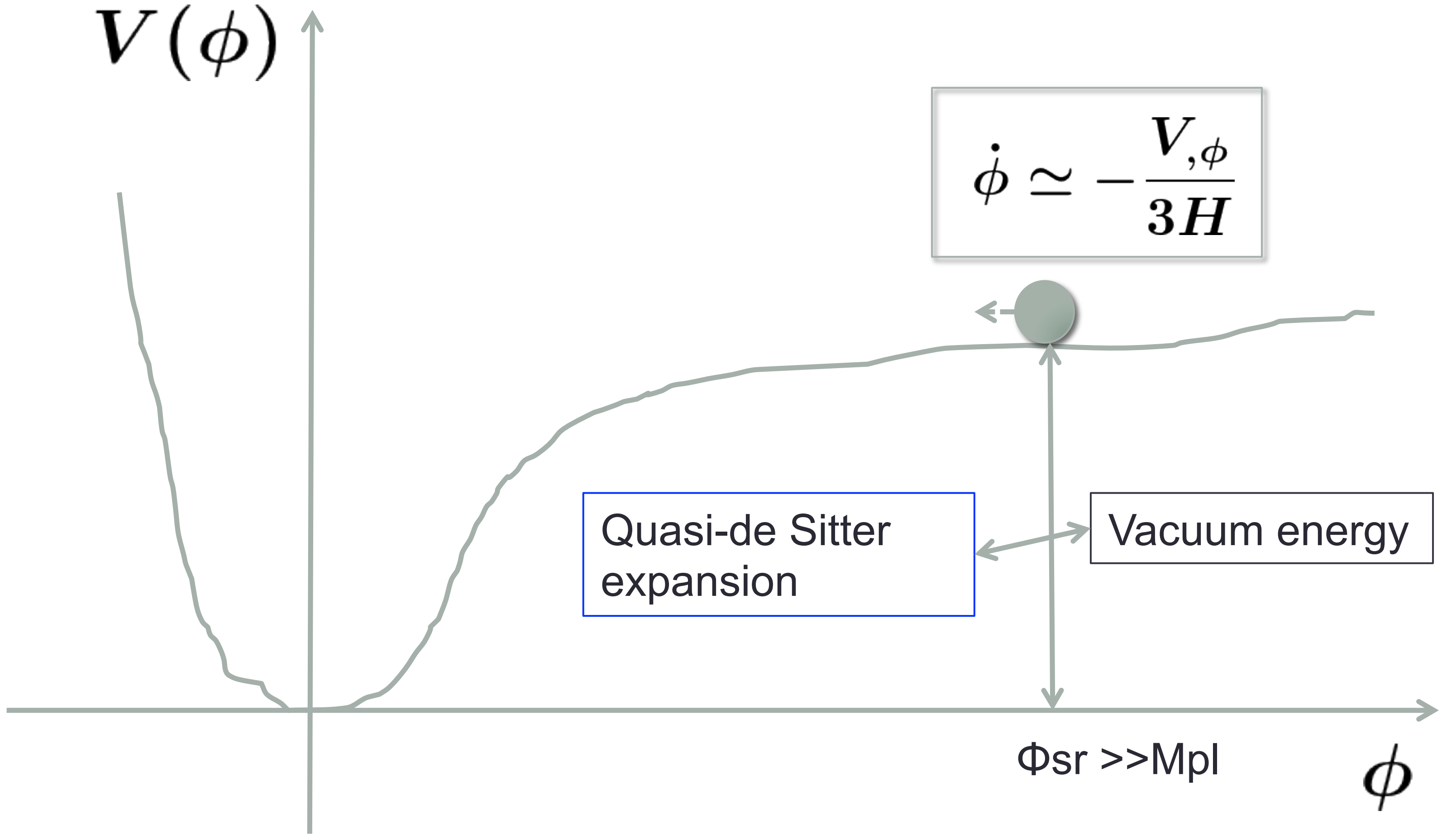
- The vacuum energy can accelerate the universe.



Slow-roll inflation

- ✓Scalar field can make a cosmological background.
- ✓The potential energy of scalar field V(Φ) plays a role of the vacuum energy and accelerate the universe.
- ✓Slow-roll inflation ends naturally at $\Phi \sim M_{Pl}$, and reheat the universe through particle decay.
- ✓V(Φ) gets constraints from the observation

Slow-roll attractor



Quantum fluctuation of inflaton

- Perturbation theory around time-*dependent* vacuum expectation value Φ(t)

$$\phi = \phi(t) + \delta\phi(x)$$

QFT on quasi-de Sitter

$$h = v + \eta(x)$$

QFT on M4

- Scalar field is stretched by the expansion, and finally, it approaches to an almost constant value:

$$\delta\phi(x) \sim \frac{H}{2\pi}$$

Observables of inflation

- 3-metric: $^3g_{ij} = a^2(t)\mathrm{e}^{2\zeta}(\delta_{ij} + \gamma_{ij} + \cdots)$
- General coordinate transformation: $\zeta \sim -\frac{H}{\dot{\phi}}\delta\phi$
- 2-point functions of ζ and γij in momentum space

$$P_\zeta(k) = \frac{k^3}{2\pi^2} < \zeta_k \zeta_k > \qquad \left(\quad P_\gamma(k) = \frac{k^3}{\pi^2} < \gamma_{kij}\gamma_k^{ij} > \quad \right)$$

$$n_s - 1 := \frac{d\ln P_\zeta(k)}{d\ln k}$$ ~ Scaling dimension of 2-pt. function

$$r := \frac{P_\gamma(k)}{P_\zeta(k)}$$

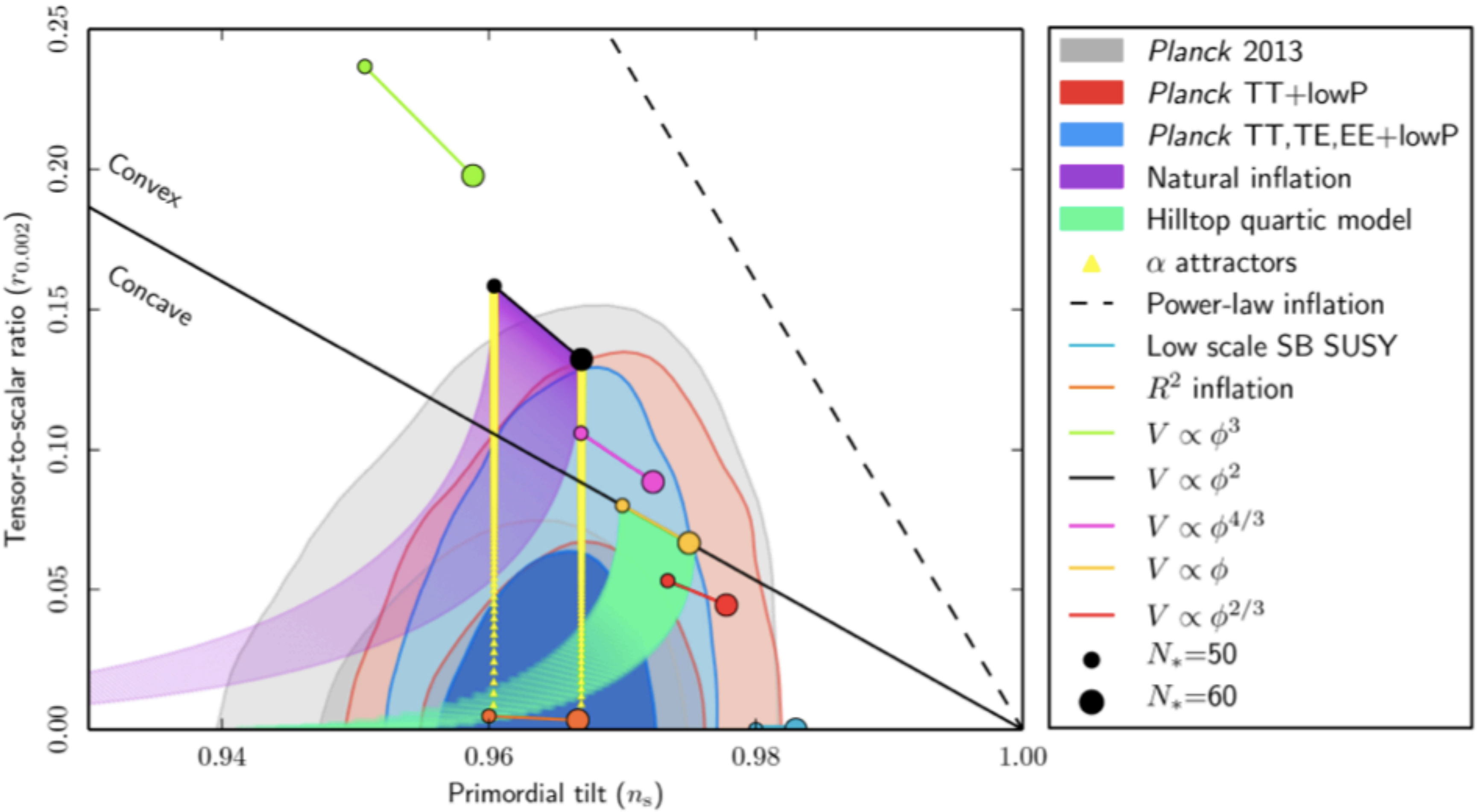


Fig. 12. Marginalized joint 68 % and 95 % CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets, compared to the theoretical predictions of selected inflationary models.

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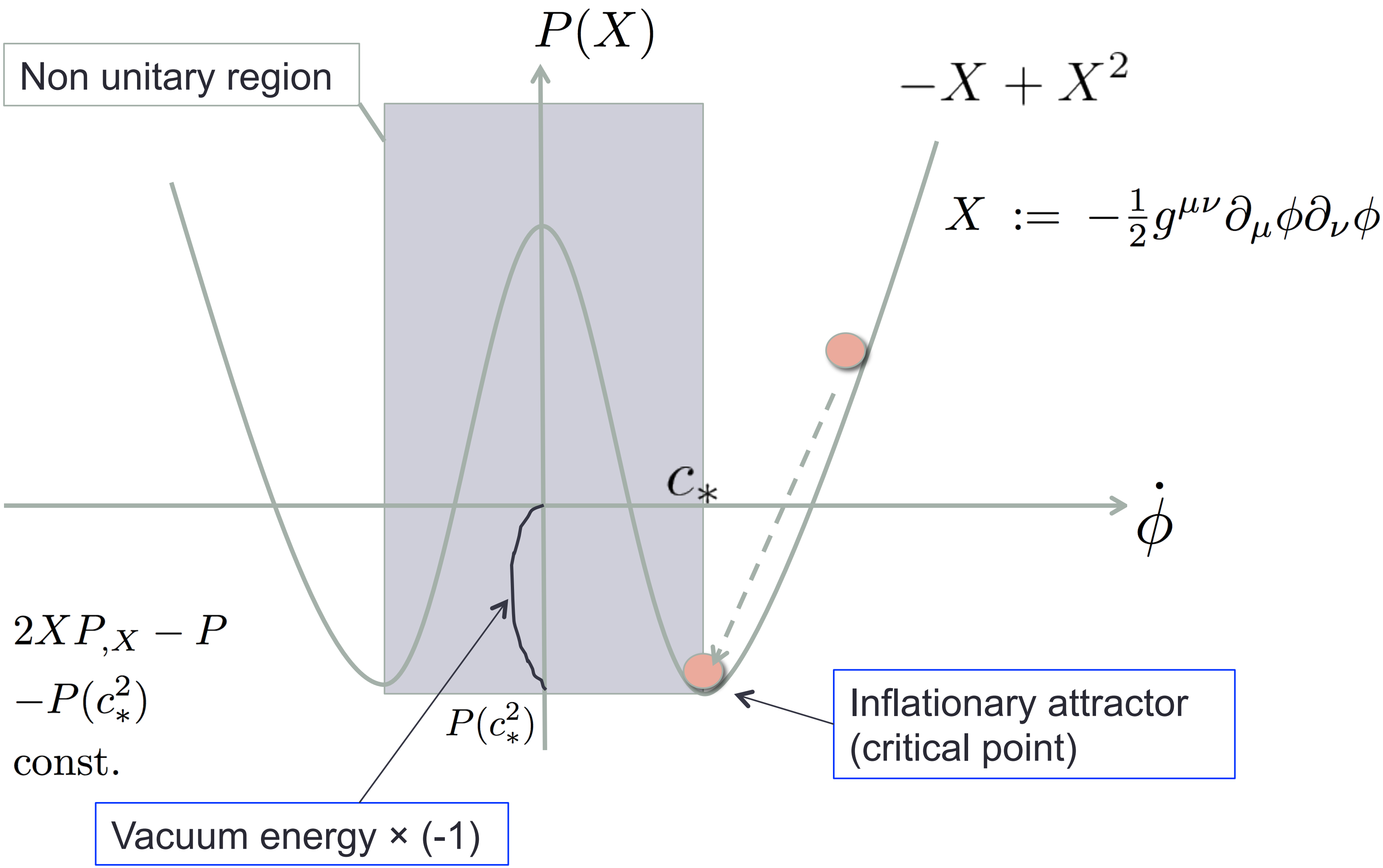
Why slow roll?

- So far, do you have any reasons why you consider slow-roll inflation only?
- I don’t have.
- If other mechanisms work well for inflating the universe, why not consider them?

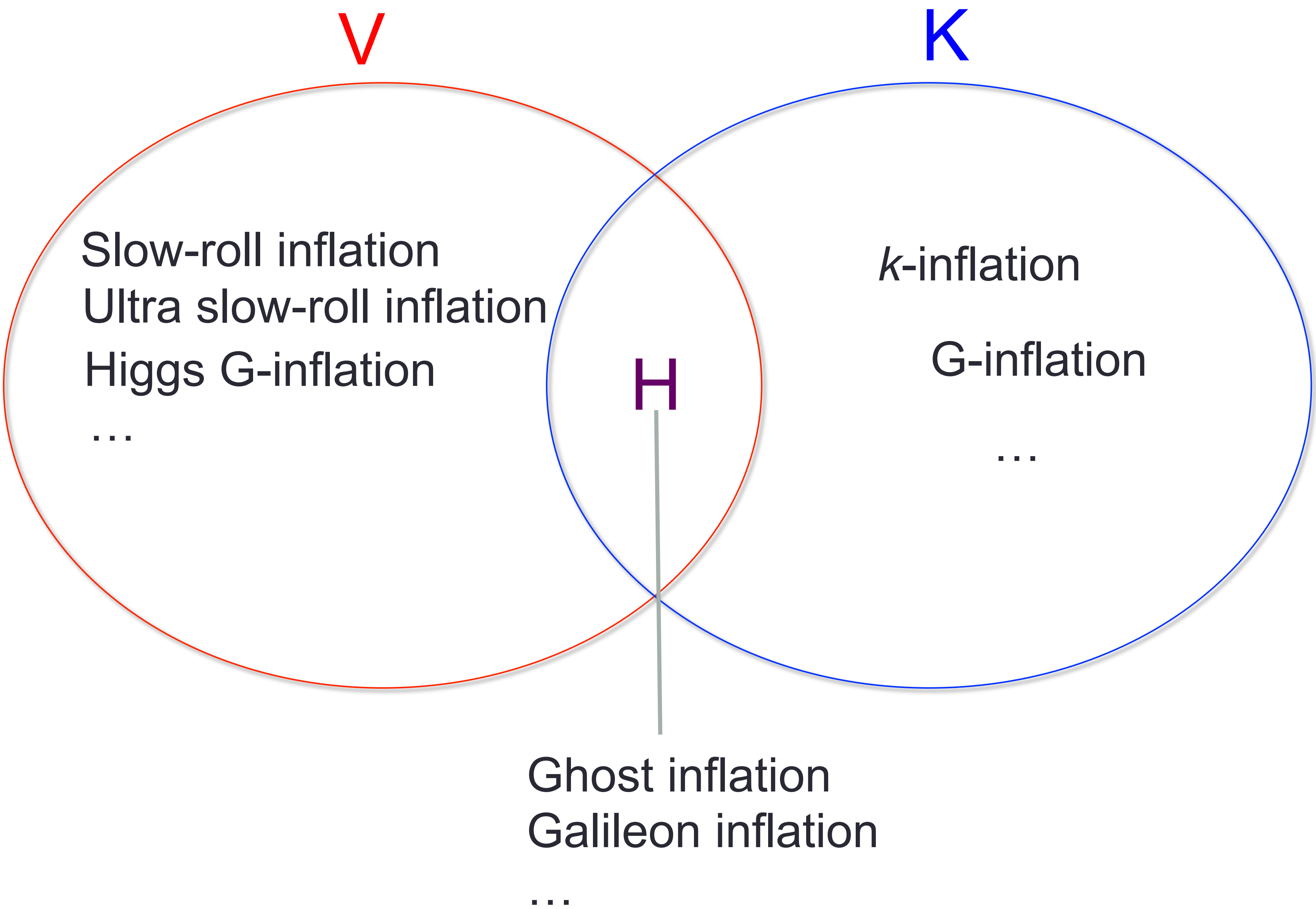
Kinetically driven inflation (KDI)

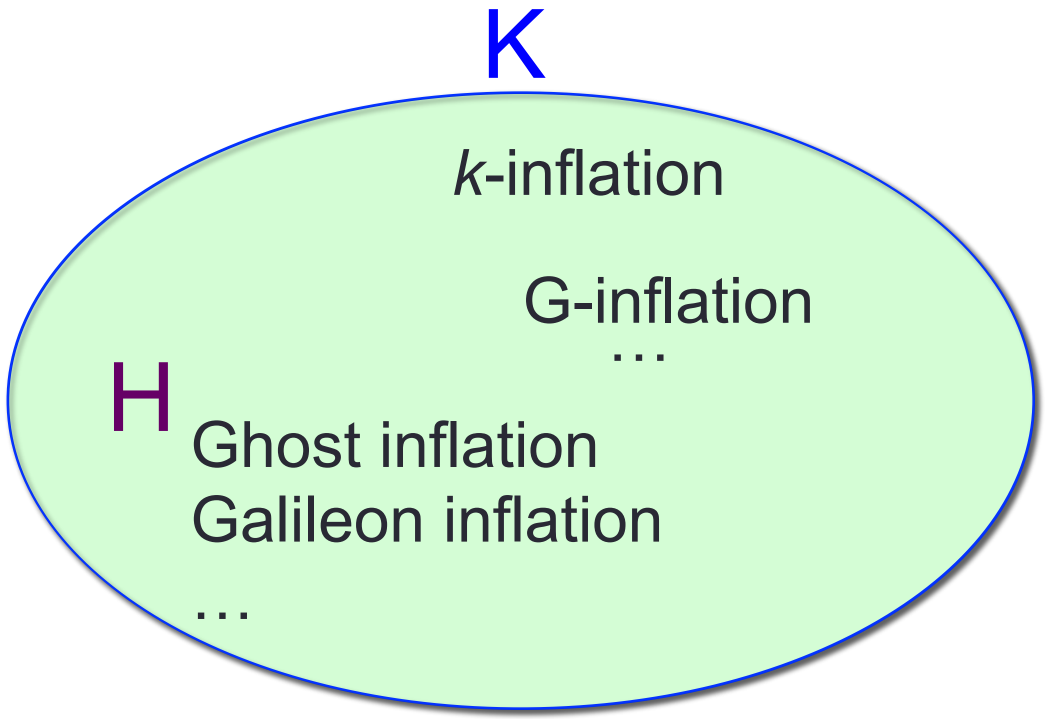
- ✓Another huge class of inflation
- ✓Non-canonical kinetic terms become relevant to inflate the universe.
- ✓Not necessarily need the potential to inflate the universe.
- ✓By tuning functions in the models, inflation can end and transit to the big bang universe.

Schematic of the typical KDI



$$\rho = 2XP_{,X} - P$$
$$= -P(c_*^2)$$
$$= \text{const.}$$


V
K
Slow-roll inflation
Ultra slow-roll inflation
Higgs G-inflation
…
k-inflation
G-inflation
…
H
Ghost inflation
Galileon inflation
…



- No unified formulation for KDI ever.

- ***Research object***

To develop a unified formulation for evaluating the whole behavior of KDI systematically.

2. Inflationary attractor and perturbative expansion

A model-independent framework

- We intend to derive features of KDI in a model-independent manner.

- 3 basic requirements for the theory

✓The action consists of the scalar field Φ and the metric field $g_{\mu\nu}$ only: $S[\phi,\, g_{\mu\nu}]$

✓The degrees of freedom is 1 scalar mode + 2 tensor modes only.

✓The theory has the flat Friedmann-Robertson-Walker (FRW) solution.

Equations of motion in general form

- Flat FRW background metric

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\vec{x}^2$$

- Equations of motion

$$\frac{\delta S}{\delta N} = 0 \rightarrow \quad \mathcal{E}(\underline{\phi,\ \dot{\phi},\ H,\ \cdots}) = 0$$

A constraint equation, ~Friedmann equation

$$\frac{\delta S}{\delta \phi} = 0 \rightarrow \quad \dot{J} + 3HJ = P_{,\phi}$$

~ Klein-Gordon eq. for background field Φ(t)

$$J = J(\underline{\phi,\ \dot{\phi},\ H,\ \cdots}),$$

$$P = P(\underline{\phi,\ \dot{\phi},\ \ddot{\phi},\ H,\ \dot{H},\ \cdots})$$

$$P_{,\phi} = \partial P / \partial \phi$$

Inflationary attractor of KDI

- If $P_{,\phi} \simeq 0$,

$$\dot{J} + 3HJ = \cancel{P_{,\phi}} \simeq \mathbf{0}$$

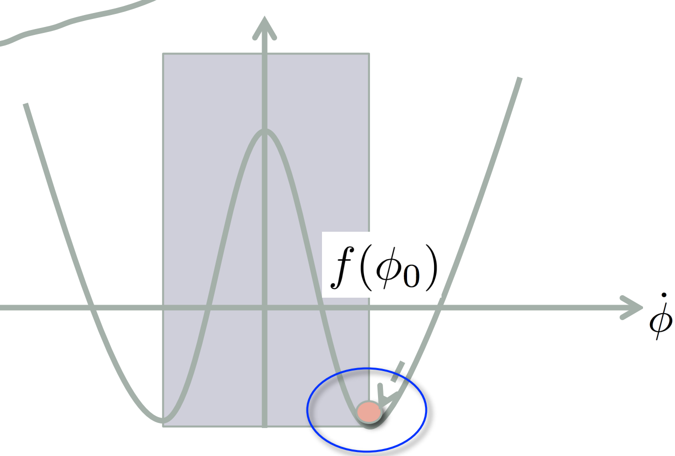
$$J \simeq c_J a^{-3} \to 0 , \qquad \mathcal{E}(\phi,\, \dot{\phi},\, H,\, \cdot$$

If H_0 varies sufficiently slowly with respect to Φ_0, the root solution becomes a quasi-de Sitter attractor.

- A root solution for $J = 0$ and $\mathcal{E} = 0$

$$\boxed{\dot{\phi}_0 = f(\phi_0) \neq 0, \quad H_0 = H_0(\phi_0)}$$

Inflationary attractor of KDI



An example of KDI attractor

C. Armendariz-Picon et. al.
PLB458 (1999) 209

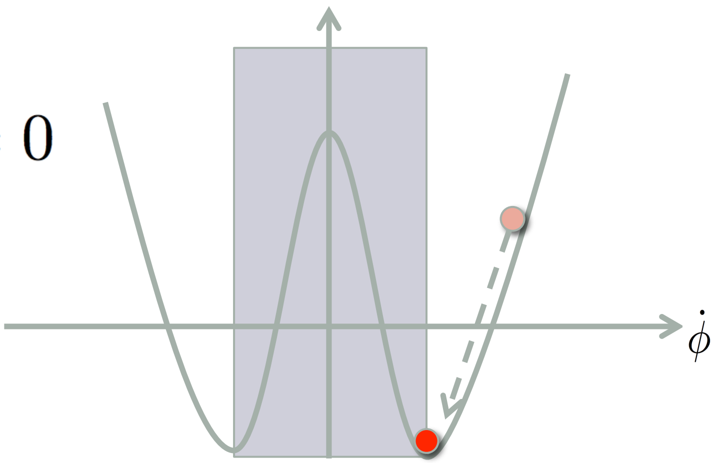
- Action and EoMs

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - K(\phi)X + X^2\right]$$

$$\mathcal{E} = -KX + 3X^2 - 3M_{\rm Pl}^2H^2 = 0$$

$$\dot{J} + 3HJ = \cancel{-K_{,\phi}X,}$$

$$J = \dot{\phi}(-K + 2X) \to \mathbf{0}$$



- KDI attractor

$$\dot{\phi}_0 = \pm\sqrt{K(\phi_0)}, \quad H_0 = \frac{K(\phi_0)}{\sqrt{3}M_{\rm Pl}}$$

Next-to-leading solution

- KDI attractor is just a leading solution of the full solution.

- Actual motion of scalar field deviates slightly from KDI attractor.

- How do we evaluate the deviation?
  → consider isotropic and homogeneous perturbations of background

$$\phi(t) = \phi_0(t) + \phi_1(t), \quad H(t) = H_0(t) + h_1(t)$$

An ansatz for the derivatives w.r.t. Φ

- We *sprit* the motion of system to KDI attractor and the perturbations around it.

$$\dot{J} + 3HJ = P_{,\phi} \quad \Rightarrow \quad \begin{aligned} J_0 &= 0 \\ \dot{J}_1 + 3H_0 J_1 &= P_{,\phi} \end{aligned}$$

- To treat the derivatives w.r.t. Φ as perturbations, we make an ansatz:

$$|\xi| \ll 1 \qquad \{\phi_1,\, h_1,\, A_{,\phi}\} = O(\xi),$$

$$A = \sum_{n=0}^{\infty} \left(\frac{\partial^n}{\partial \phi^n}\right) A_n,$$

NOT ONLY *P*.

where $\{A_n\}$ are arbitrary functions of the backgrounds without differentiation by ϕ.

Equations of motion up to O(ξ)

- The constraint equation

$$\mathcal{E} = \cancel{\mathcal{E}_0} + \left(\mathcal{E}_{,\dot{\phi}}\right)_0 \dot{\phi}_1 + \left(\mathcal{E}_{,H}\right)_0 h_1 = 0,$$

$$h_1 = -\left(\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0 \dot{\phi}_1$$

- The field equation for Φ1

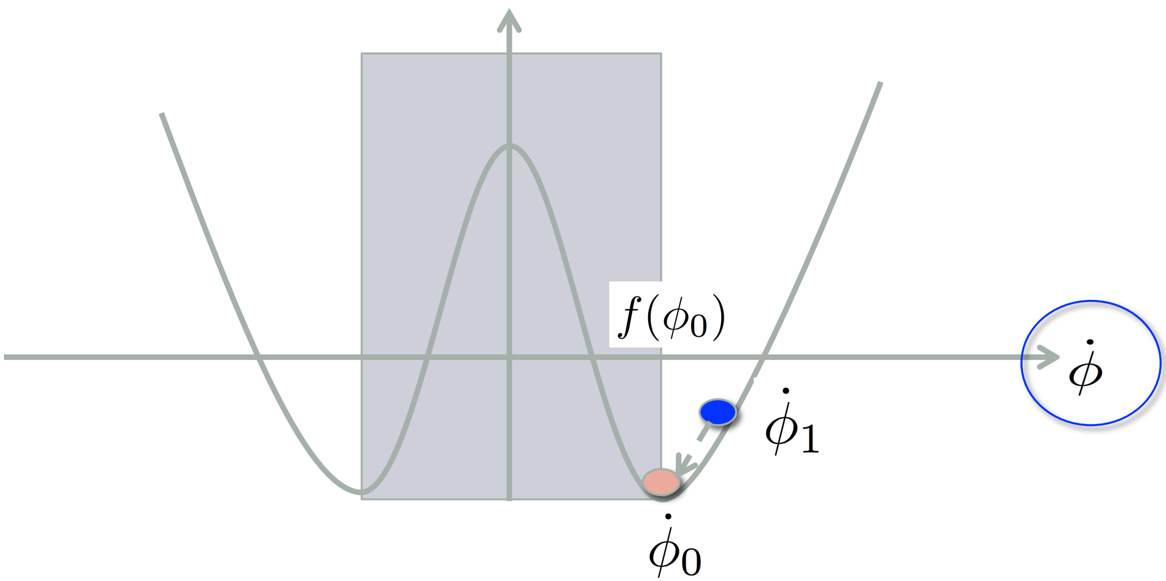
$$J = \cancel{J_0} + \left(J_{,\dot{\phi}}\right)_0 \dot{\phi}_1 + \left(J_{,H}\right)_0 h_1, \quad J = \left\{\left(J_{,\dot{\phi}}\right)_0 - \left(J_{,H}\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0\right\}\dot{\phi}_1$$

$$\dot{J} + 3HJ = P_{,\phi}$$

$$\boxed{\begin{aligned} \ddot{\phi}_1 + 3H_0\dot{\phi}_1 &= \left(\widetilde{P_{,\phi}}\right)_0, \\ \widetilde{P_{,\phi}} &:= \frac{P_{,\phi}}{\left(J_{,\dot{\phi}}\right)_0 - \left(J_{,H}\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0}. \end{aligned}}$$

Validity of the perturbative solution

- The evolution of KDI attractor is characterized by not the field value but the field velocity (or the functional form of *f*): $\dot{\phi}_0 = f(\phi_0) \neq 0$.



Perturbative expansion

$$|\dot{\phi}_0| \gg |\dot{\phi}_1|$$

3. Quantum fluctuation

2nd order action of the scalar mode

- We assume the form of the 2nd order action as

$$S_2^{(S)} = \int dt d^3x a^3 \left[\mathcal{G}_s \dot{\zeta}^2 - \frac{\mathcal{F}_s}{a^2} (\vec{\nabla}\zeta)^2 + \cancel{\cdots} \right] ,$$

$$\mathcal{F}_s = \mathcal{F}_s(\phi,\, \dot{\phi},\, \ddot{\phi},\, H\,,\, \dot{H},\, \cdots),$$

$$\mathcal{G}_s = \mathcal{G}_s(\phi,\, \dot{\phi},\, \ddot{\phi},\, H,\, \dot{H},\, \cdots)\ .$$

Scalar fluctuation with a usual scaling. ex.) Horndeski theory

- ζ may be regarded as the comoving curvature perturbation, for example.

Approximation to conformal time

- Def.) slow-roll parameters and the conformal time

$$\epsilon_1 := -\frac{\dot{H}}{H^2}, \quad \epsilon_2 := \frac{\dot{\epsilon}_1}{H\epsilon_1},$$
$$f_{s1} := \frac{\dot{\mathcal{F}}_s}{H\mathcal{F}_s}, \quad f_{s2} := \frac{\dot{f}_{s1}}{Hf_{s1}},$$
$$g_{s1} := \frac{\dot{\mathcal{G}}_s}{H\mathcal{G}_s}, \quad g_{s2} := \frac{\dot{g}_{s1}}{Hg_{s1}},$$

$$d\tau_s := \frac{c_s}{a}dt$$

- Integration of dτs and the approximation we employ:

$$\begin{aligned}\tau_s &= -\int^{\tau_s} \frac{\frac{d}{d\tau_s'}\left(\frac{c_s}{aH}\right)}{\left\{1-\epsilon_1-\frac{1}{2}(f_{s1}-g_{s1})\right\}} d\tau_s' \\ &= -\frac{c_s}{aH}\frac{1}{\left\{1-\epsilon_1-\frac{1}{2}(f_{s1}-g_{s1})\right\}} \\ &\quad \cancel{+\int^{\tau_s} \frac{\epsilon_1\epsilon_2+\frac{1}{2}(f_{s1}f_{s2}-g_{s1}g_{s2})}{\left\{1-\epsilon_1-\frac{1}{2}(f_{s1}-g_{s1})\right\}^2} d\tau_s'}\end{aligned}$$

Mode function

- Mukhanov-Sasaki equation

$$\partial_{\tau_s}^2 \zeta_k - \frac{2\nu_s - 1}{\tau_s} \partial_{\tau_s} \zeta_k + k^2 \zeta_k = 0,$$

$$\nu_s := \frac{3 - \epsilon_1 + g_{s1}}{2 - 2\epsilon_1 - f_{s1} + g_{s1}}.$$

- Solution(s) using the approximation

$$\zeta_k = \frac{1}{2}\sqrt{\frac{\pi}{2}}(-k\tau_s)^{3/2} \frac{H\left(1 - \epsilon_1 - \frac{1}{2}(f_{s1} - g_{s1})\right) \mathcal{G}_s^{1/4}}{k^{3/2} \mathcal{F}_s^{3/4}} \times H_{\nu_s}^{(1)}(-k\tau_s) \quad (3$$

Spectrum and spectral index for the superhorizon modes

$$P_\zeta := \frac{k^3}{2\pi^2}|\zeta_k|^2 \qquad n_s - 1 := \frac{d\ln P_\zeta}{d\ln k}$$

	P_ζ	$n_s - 1$	ζ
$\nu_s > 0$	$\frac{\gamma_s}{2}(-k\tau_s)^{3-2\nu_s}\frac{\mathcal{G}_s^{1/2}}{\mathcal{F}_s^{3/2}}\frac{H^2}{4\pi^2}$	$3-2\nu_s$	const.
$\nu_s < 0$	$\frac{\|A\|^2\left(1-\epsilon_1-\frac{1}{2}(f_{s1}-g_{s1})\right)^2}{2}(-k\tau_s)^{3+2\nu_s} \times \frac{\mathcal{G}_s^{1/2}}{\mathcal{F}_s^{3/2}}\frac{H^2}{4\pi^2},$	$3+2\nu_s$	growing
$\nu_s = 0$	$\frac{\left(1-\epsilon_1-\frac{1}{2}(f_{s1}-g_{s1})\right)^2}{\pi}(-k\tau_s)^3\left\{\ln(-k\tau_s)\right\}^2 \times \frac{\mathcal{G}_s^{1/2}}{\mathcal{F}_s^{3/2}}\frac{H^2}{4\pi^2},$ (4	$\simeq 3$	growing

4. Case I: Shift symmetric KDI

Background evolution of shift symmetric system

- Shift symmetry of the scalar field

$$\phi \to \phi + c \quad \Longrightarrow \quad S[\partial_\mu \phi,\ g_{\mu\nu}]$$

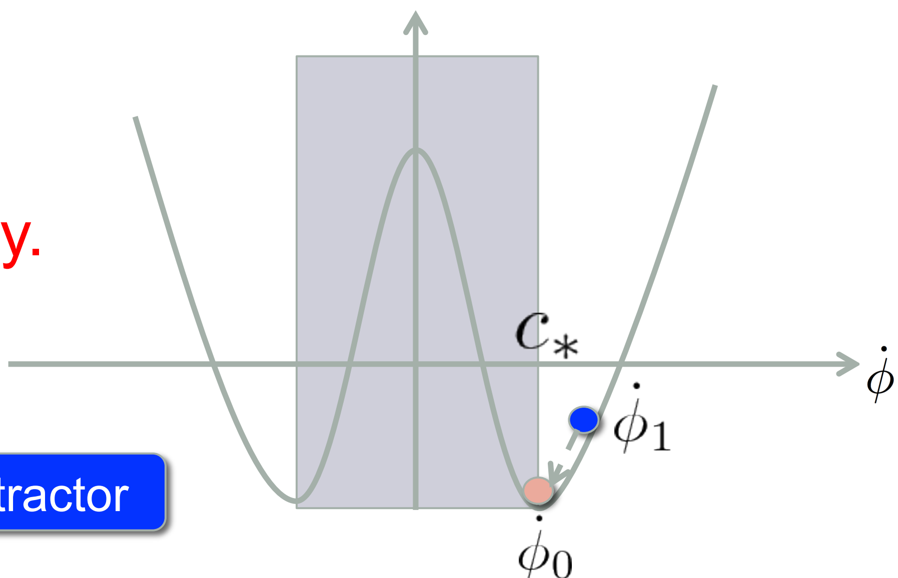
- KDI attractor of shift symmetric system

$$\dot{\phi}_0 = c_* = \mathrm{const} \neq 0, \qquad H_0 = \mathrm{const} > 0.$$

- Perturbation of background

$$\ddot{\phi}_1 + 3H_0\dot{\phi}_1 = \left(\widetilde{P_{,\phi}}\right)_0$$ = 0 exactly.

$$\boxed{\dot{\phi}_1 \simeq \frac{c_1}{a^3}}$$ → 0



~ Ultra slow-roll motion around KDI attractor

Speed of sound

- Expansion of coefficient functions

$$\begin{aligned}\mathcal{F}_s &= \mathcal{F}_{s0} + \left(\mathcal{F}_{s,\dot{\phi}}\right)_0 \dot{\phi}_1 + \left(\mathcal{F}_{s,\ddot{\phi}}\right)_0 \ddot{\phi}_1 \\ &\quad + \left(\mathcal{F}_{s,H}\right)_0 h_1 + \left(\mathcal{F}_{s,\dot{H}}\right)_0 \dot{h}_1 \\ &= \mathcal{F}_{s0} + 3H_0 \left(\mathcal{F}_{s1} - \mathcal{F}_{s2}\right) \dot{\phi}_1 + \mathcal{F}_{s2} \left(\widetilde{P_{,\phi}}\right)_0 , \\ \mathcal{G}_s &= \mathcal{G}_{s0} + \left(\mathcal{G}_{s,\dot{\phi}}\right)_0 \dot{\phi}_1 + \left(\mathcal{G}_{s,\ddot{\phi}}\right)_0 \ddot{\phi}_1 \\ &\quad + \left(\mathcal{G}_{s,H}\right)_0 h_1 + \left(\mathcal{G}_{s,\dot{H}}\right)_0 \dot{h}_1 \\ &= \mathcal{G}_{s0} + 3H_0 \left(\mathcal{G}_{s1} - \mathcal{G}_{s2}\right) \dot{\phi}_1 + \mathcal{G}_{s2} \left(\widetilde{P_{,\phi}}\right)_0 ,\end{aligned}$$

$$\begin{aligned}\mathcal{F}_{s0} &:= \mathcal{F}_s(\phi_0,\, \dot{\phi}_0,\, H_0,\, \cdots), \\ \mathcal{F}_{s1} &:= \frac{1}{3H_0}\left\{\left(\mathcal{F}_{s,\dot{\phi}}\right)_0 - \left(\mathcal{F}_{s,H}\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0\right\}, \\ \mathcal{F}_{s2} &:= \left(\mathcal{F}_{s,\ddot{\phi}}\right)_0 - \left(\mathcal{F}_{s,\dot{H}}\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0 , \\ \mathcal{G}_{s0} &:= \mathcal{G}_s(\phi_0,\, \dot{\phi}_0,\, H_0,\, \cdots), \\ \mathcal{G}_{s1} &:= \frac{1}{3H_0}\left\{\left(\mathcal{G}_{s,\dot{\phi}}\right)_0 - \left(\mathcal{G}_{s,H}\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0\right\}, \\ \mathcal{G}_{s2} &:= \left(\mathcal{G}_{s,\ddot{\phi}}\right)_0 - \left(\mathcal{G}_{s,\dot{H}}\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0 .\end{aligned}$$

- Speed of sound and Stability condition

$$\begin{aligned}c_s^2 &:= \frac{\mathcal{F}_s}{\mathcal{G}_s} \\ &= \frac{\mathcal{F}_{s0} + 3H_0 \left(\mathcal{F}_{s1} - \mathcal{F}_{s2}\right) \dot{\phi}_1 + \mathcal{F}_{s2} \left(\widetilde{P_{,\phi}}\right)_0}{\mathcal{G}_{s0} + 3H_0 \left(\mathcal{G}_{s1} - \mathcal{G}_{s2}\right) \dot{\phi}_1 + \mathcal{G}_{s2} \left(\widetilde{P_{,\phi}}\right)_0}\end{aligned}$$

$$\mathcal{F}_s \geq 0, \quad \mathcal{G}_s > 0, \quad \mathcal{G}_s \geq \mathcal{F}_s$$

Classification by speed of sound

- Speed of sound for the shift symmetric system

$$c_s^2 \simeq \frac{\mathcal{F}_{s0} + 3H_0c_1\left(\mathcal{F}_{s1} - \mathcal{F}_{s2}\right)a^{-3}}{\mathcal{G}_{s0} + 3H_0c_1\left(\mathcal{G}_{s1} - \mathcal{G}_{s2}\right)a^{-3}}$$

- 3 types satisfied with the stability condition

$$\mathcal{F}_s \geq 0, \quad \mathcal{G}_s > 0, \quad \mathcal{G}_s \geq \mathcal{F}_s$$

(i) $\mathcal{G}_{s0} \geq \mathcal{F}_{s0} > 0$

(ii) $\mathcal{F}_{s0} = 0$ and $\mathcal{G}_{s0} > 0$

(iii) $\mathcal{F}_{s0} = \mathcal{G}_{s0} = 0$

Second order products of slow-roll parameters

- For the type (i) theories, $\quad \text{(i)}\ \mathcal{G}_{s0} \geq \mathcal{F}_{s0} > 0$

$$\epsilon_1 \propto f_{s1} \propto g_{s1} \propto a^{-3}, \qquad \underline{\epsilon_2 \simeq f_{s2} \simeq g_{s2} \simeq -3}$$

- The conformal time

$$\tau_s = -\int^{\tau_s} \frac{\frac{d}{d\tau_s'}\left(\frac{c_s}{aH}\right)}{\left\{1-\epsilon_1-\frac{1}{2}(f_{s1}-g_{s1})\right\}} d\tau_s'$$

$$= -\frac{c_s}{aH}\frac{1}{\left\{1 \cancel{-\epsilon_1-\frac{1}{2}(f_{s1}-g_{s1})}\right\}}$$

$$\boxed{+\cancel{\int^{\tau_s} \frac{\epsilon_1\epsilon_2+\frac{1}{2}(f_{s1}f_{s2}-g_{s1}g_{s2})}{\left\{1-\epsilon_1-\frac{1}{2}(f_{s1}-g_{s1})\right\}^2} d\tau_s'} \simeq \frac{3c_s}{4aH}\cancel{\left\{\epsilon_1+\frac{1}{2}(f_{s1}-g_{s1})\right\}}}$$

Sum of the shift symmetric KDI

	ν_s	$n_s - 1$	spectrum	ζ
Type (i)	3/2	0	scale-invariant	const.
Type (ii)	3/5	9/5	Blue	const.
Type (iii)	0	$\simeq 3$	Blue	growing
Others	—	—	—	Unstable

Sum of the shift symmetric KDI

	ν_s	$n_s - 1$	spectrum	ζ
Others	—	—	—	Unstable

The shift symmetric system within our generic framework ***cannot*** create the observed red spectrum.

→ We need to introduce other sources for the scalar fluctuation, or, ***break the shift symmetry*** to create the spectral tilt consistent to the observational value $n_s - 1 \sim -0.04$.

5. Case II: Φ-dependent KDI

Background evolution of Φ-dependent system

- KDI attractor and EoM

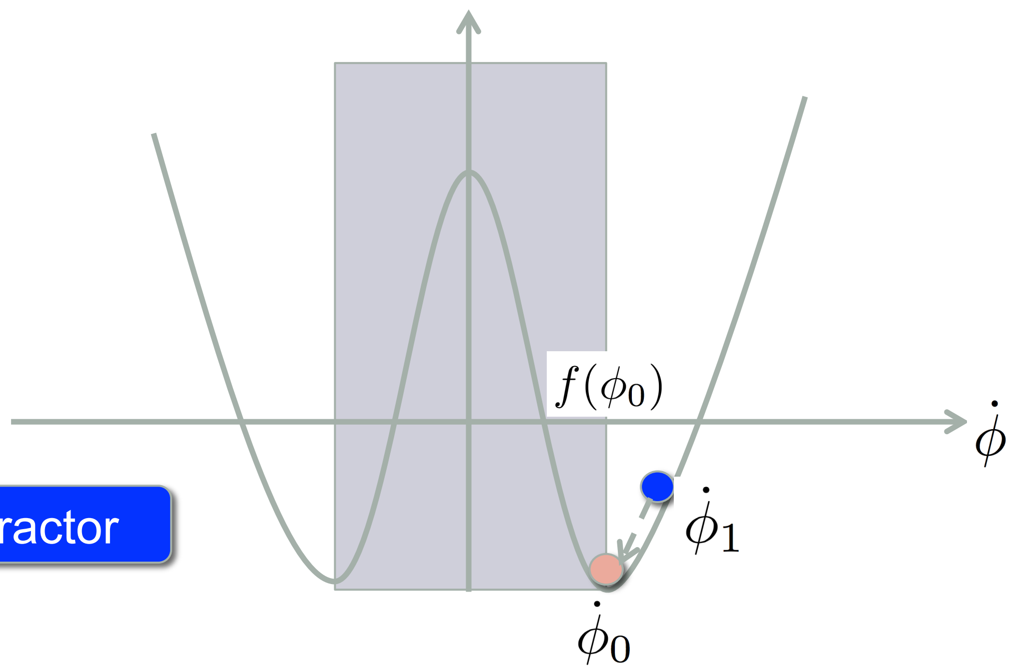
$$\dot{\phi}_0 = f(\phi_0), \quad H_0 = H_0(\phi_0),$$

$$\cancel{\ddot{\phi}_1} + 3H_0\dot{\phi}_1 = (\widetilde{P_{,\phi}})_0 \ .$$

- If $|\ddot{\phi}_1| \ll |(\widetilde{P_{,\phi}})_0|$,

$$\boxed{\dot{\phi}_1 \simeq \frac{(\widetilde{P_{,\phi}})_0}{3H_0}}$$

~Slow-roll motion around KDI attractor



3 conditions for slow-roll like motion

1. "Slow-roll" condition for Φ1: $|\ddot{\phi}_1| \ll |(\widetilde{P_{,\phi}})_0|$

$$\eta_\phi := \frac{f(P_{,\phi\phi})_0}{3H_0(P_{,\phi})_0}, \quad |\eta_\phi| \ll 1$$

$$\{\phi_1,\, h_1,\, A_{,\phi}\} = O(\xi),$$

$$A = \sum_{n=0}^{\infty}\left(\frac{\partial^n}{\partial\phi^n}\right) A_n,$$

2. Leading >> sub-leading: $|\dot{\phi}_0| \gg |\dot{\phi}_1|$

$$\epsilon_\phi := \frac{(P_{,\phi})_0}{3fH_0\left\{(J_{,\dot{\phi}})_0 - \left(J_{,H}\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0\right\}}, \quad |\epsilon_\phi| \ll 1$$

3. Quasi-de Sitter attractor

$$\epsilon_1 \simeq -\frac{f}{H_0^2}\left\{H_{0,\phi_0} - 3H_0\left(\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0 \epsilon_\phi\eta_\phi\right\}$$

$$\simeq -\frac{fH_{0,\phi_0}}{H_0^2}, \quad |\epsilon_1| \ll 1$$

A condition for the graceful exit

- THE END OF INFLATION: $|\epsilon_1| \sim 1$

$$\boxed{|\epsilon_1| \simeq \left|\frac{f H_{0,\phi_0}}{H_0^2}\right| \ll 1} \Rightarrow \boxed{|\epsilon_1| \simeq \left|\frac{f H_{0,\phi_0}}{H_0^2}\right| \sim 1 \quad \& \quad \text{Continuation of the condition}}$$

Inflation

Graceful exit

- To end up with the graceful exit, we have to tune H_0 as

✓inflating the universe enough

✓changing its value rapidly and stopping the acceleration

✓escaping from the KDI attractor

Other constraints

- Stability condition
- Conditions for ignoring the second order products of slow-roll parameters
- Observational constraints to n_s, r, f_{NL} etc.

Highly model-dependent

We could satisfy all of the constraints by tuning the Φ-dependent functions in the action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - K(\phi)X + X^2\right]$$

Sum of Φ-dependent KDI

Requirements	Equations	Conditions up to O(ξ)
Slow-roll condition for Φ_1	$\lvert\ddot{\phi}_1\rvert \ll \lvert(\widetilde{P_{,\phi}})_0\rvert$	$\eta_\phi := \frac{f(P_{,\phi\phi})_0}{3H_0(P_{,\phi})_0}, \quad \lvert\eta_\phi\rvert \ll 1$
KDI attractor >> perturbation	$\lvert\dot{\phi}_0\rvert \gg \lvert\dot{\phi}_1\rvert$	$\epsilon_\phi := \frac{(P_{,\phi})_0}{3fH_0\left\{(J_{,\dot{\phi}})_0 - \left(J_{,H}\frac{\mathcal{E}_{,\dot{\phi}}}{\mathcal{E}_{,H}}\right)_0\right\}}, \quad \lvert\epsilon_\phi\rvert \ll 1$
Quasi-de Sitter expansion	$-\frac{\dot{H}}{H^2} \ll 1$	$\epsilon_1 \simeq -\frac{fH_{0,\phi_0}}{H_0^2}, \quad \lvert\epsilon_1\rvert \ll 1$
Graceful exit	$-\frac{\dot{H}}{H^2} \sim 1$ at late time	$\left\lvert\frac{fH_{0,\phi_0}}{H_0^2}\right\rvert \sim 1$ & its continuation
Others	Highly model-dependent	

6. Summary

- We have performed a model-independent analysis of KDI within a generic framework, which includes many of previous models.

- The shift symmetric KDI is described as the perturbation behaves as ultra slow-roll inflation around the exact de Sitter attractor, but they cannot create the observed quantum fluctuation.

- The Φ-dependent KDI is described as the perturbation "slow-rolls" around the quasi-de Sitter attractor. They take 4 essential conditions to inflate the universe and end in the graceful exit. We could construct viable models by tuning the Φ-dependent functions in the models.