Some applications of gauge/gravity duality in QCD

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& Gravity dual & beta-function

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I Motivation

AdS/CFT conjecture

$$AdS_5 \times S^5 \longleftrightarrow N = 4 \text{ SYM theory}$$

If it's true for any gauge theory
(???)

String theory, quantum gravity	⟷	Non-Abelian gauge theory

Then what's the dual string theory of QCD?
(It is nature to ask the question here)

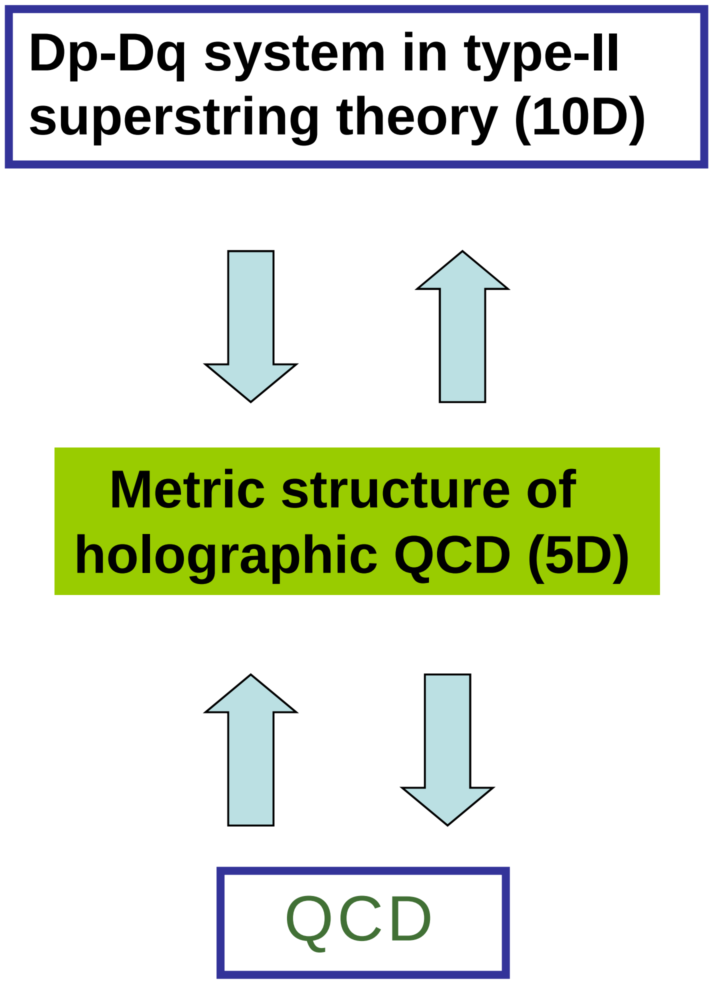
? ⟷ QCD

Question: Is it possible to find a string theory dual to QCD?

Or to find possible holographic description of QCD.

First step, is it possible to find a 5D holographic model of QCD?

Leave the task of deriving the holographic QCD model from string theory. (Top-Down)



What we can do: extract a workable holographic QCD model from the real QCD. (AdS/QCD)

II. Review of Holographic models

Bottom-up models (deform geometry):

hard-wall AdS5 model

soft-wall AdS5 model: quadratic dilaton model

Andreev model: negative quadratic dilaton model

back-reaction model

model resembling QCD running coupling

Top-down models:

SS model(D4-D8-D8), D3-D7, D3-D5, other general Dp-Dq model…

Hard-wall AdS_5 model:

Joseph Polchinski Matthew J. Strassler JHEP 0305:012,2003

L. Da Rold and A. Pomarol, Nucl. Phys. B **721**, 79 (2005)

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

$$ds^2 = e^{2A(z)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \qquad A(z) = -\ln z$$

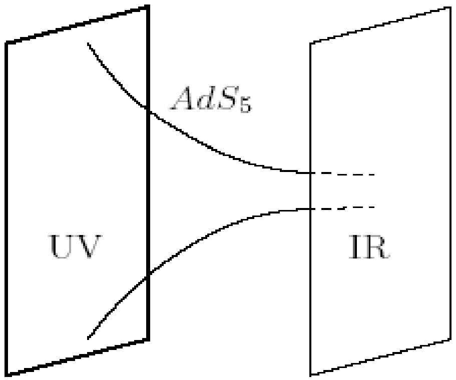


FIG. 1: A slice of AdS_5.

The holographic coordinate range:

$$0 < z \le z_m.$$

Introducing a hard cutoff into the pure AdS background to break the conformal symmetry.

Lowest excitations of hadron (vector and meson): 80-90% agreement with Exp.(advantage)

higher excitations: no Regge behavior. (weak point) m_n^2 grow as n^2.

Soft-wall AdS_5 model or KKSS model

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006)

$$g_{MN}\, dx^M dx^N = e^{2A(z)}(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

Introducing an exponential function as a dilaton field into the action

$$I = \int d^5x \, \sqrt{g}\, e^{-\Phi}\, \mathcal{L},$$

$$A(z) = -\ln z,\ \ \Phi(z) = z^2$$

Introduce a dilaton field to restore Regge behavior

$$\boxed{M^2_{n,S} = 4n + 4S}$$

Andreev-Zakharov model: quadratic correction

O. Andreev, hep-ph/0604204

O. Andreev and V. I. Zakharov, Phys. Rev. D **74**, 025023

$$ds^2 = G_{nm} dX^n dX^m = R^2 \frac{h}{z^2} \left(dx^i dx^i + dz^2 \right)$$

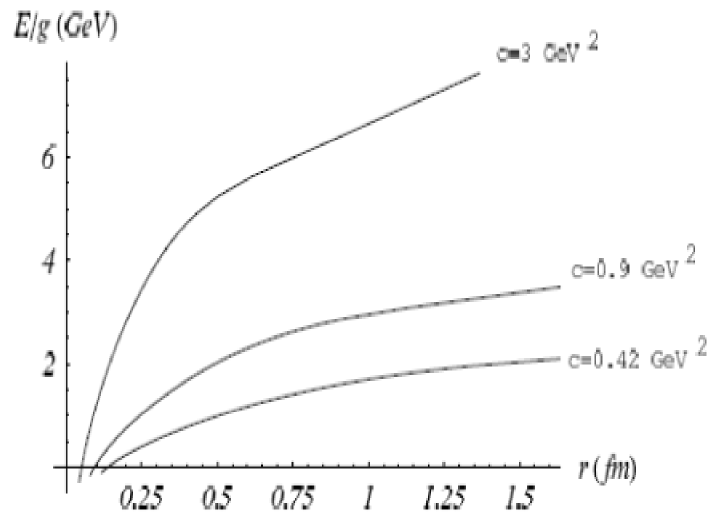
$$h = \mathrm{e}^{\frac{1}{2} c z^2}$$

S. J. Brodsky, G. F. de Teramond and A. Deur, arXiv:1002.3948 [hep-ph]

F. Zuo, arXiv:0909.4240 [hep-ph]

The main difference from KKSS is to introduce the warp factor in the background.

Heavy quark potential



"KKSS" model (effectively):

$$h = e^{-\frac{1}{2} c z^2}$$

The dilaton field in the action of KKSS model is moved to the warp factor effectively.

Dp-Dq model

	0	1	2	3	$\cdots$	p	z	S^{8-p}	
Dp	●	●	●	●	●	●	-	-	
Dq	●	●	●	●	-	-	●	$S^{q-4} \subset S^{8-p}$	-

TABLE II: Spacetime embedding of $Dp - Dq$ system.

Embedding the flavor brane into the color brane (background) without back reaction and turning off fluxes, we investigate the general models to get the hadron (vector and scalar) spectrum

$$ds^2 = e^{2A(z)}\left[\eta_{\mu\nu}dx^\mu dx^\nu + dz^2 + \frac{z^2}{z_0^2}d\Omega_{q-4}^2\right]$$

$$A(z) = -a_0 \ln z, \quad with \quad a_0 = \frac{p-7}{2(p-5)},$$

dilaton field part

$$\Phi(z) \sim d_0 \ln z, \; with \; d_0 = -\frac{(p-3)(p-7)}{2(p-5)}.$$

S. He, M. Huang, Q. S. Yan, Y. Yang Eur.Phys.J. C66 (2010) 187-196

Deformation of Dp-Dq model

To realize the Regge behavior of hardron spectrum , we should m -odify the dilation field by introd -ucing z^2

$$A(z) = -a_0 \ \ln z, \ \ \Phi(z) = d_0 \ln z + c_2 z^2 \, .$$

A. Karch, E. Katz, D. T. Son and M. A. Stephanov,

Phys. Rev. D **74**, 015005 (2006).

Action

$$I = \frac{1}{2} \int d^5 x \sqrt{g} \ e^{-\Phi(z)} \Big\{ \Delta_N \phi_{M_1 \cdots M_S} \Delta^N \phi^{M_1 \cdots M_S} + m_5^2 \phi_{M_1 \cdots M_S} \phi^{M_1 \cdots M_S} \Big\},$$

EOM

$$\partial_z^2 \psi_n - \partial_z B \cdot \partial_z \psi_n + \left(M_{n,S}^2 - m_5^2 e^{2A} \right) \psi_n = 0 \, ,$$

$$B = \Phi - k(2\, S - 1) A = \Phi + c_0 (2\, S - 1) \ln z$$

Deformation of Dp-Dq model

p	3		4			6	
q	5	7	4	6	8	4	6
$k=-\frac{(p-3)(q-5)+4}{p-7}$	1		1	$5/3$	$7/3$	1	7
$a_0=\frac{p-7}{2(p-5)}$	1		$3/2$			$-1/2$	
$c_0=ka_0$	1		$3/2$	$5/2$	$7/2$	$-1/2$	$-7/2$
$d_0=-\frac{(p-3)(p-7)}{2(p-5)}$	0		$-3/2$			$3/2$	
$\mathcal{R}\sim\frac{1}{g_{eff}}$	$1/\sqrt{3}$		$z^{-2}/\sqrt{36\pi}$			$6\sqrt{2}z^6$	

TABLE III: Theoretical results for the $Dp-Dq$ system.

$$k=-\frac{(p-3)(q-5)+4}{p-7}.$$

$$c_0=ka_0=-\frac{(p-3)(q-5)+4}{2(p-5)}.$$

Higgsless model in Deformation of Dp-Dq

Gravity set up

$$A(z) = -c_0 \ln z\,,$$
$$\Phi(z) = c_2 z^2\,,$$

Effective potential

$$V(z) = 2c_2(m-1) + \frac{m^2 - \frac{1}{4}}{z^2} + c_2^2 z^2 + \frac{m_5^2}{z^{2c_0}}\,,$$

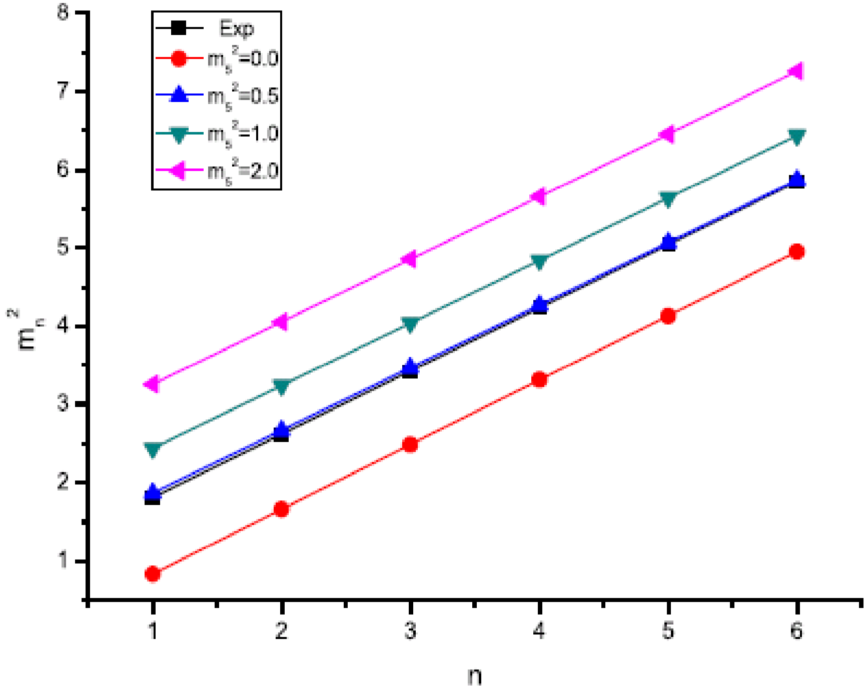
EOM

$$-\psi'' + V(z)\psi = M_n^2 \psi$$

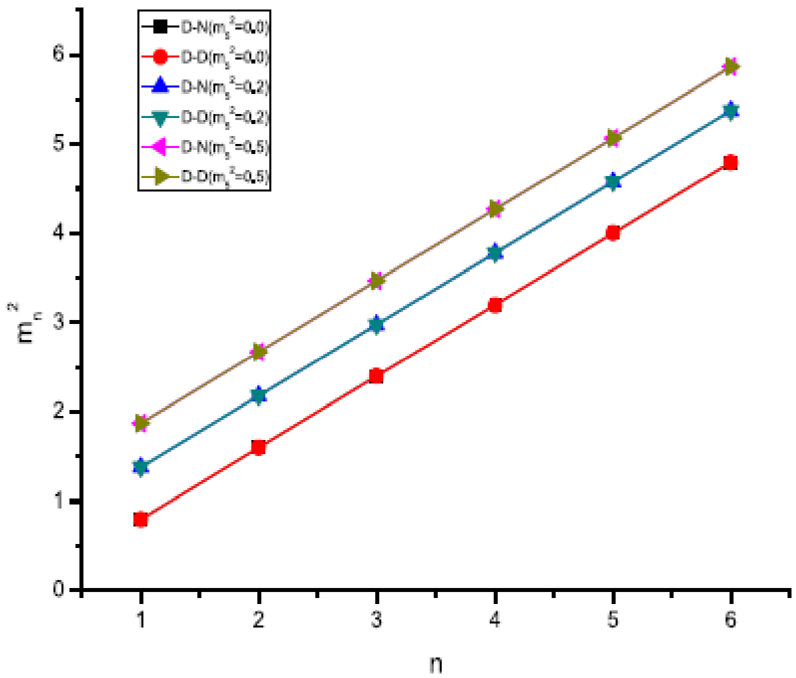
Spectrum

$$M_n^2 = c_2(4n+1) + c_2(2S-1) + c_2\sqrt{[(2S-1)+1]^2 + 4m_5^2}\,.$$

DN boundary condition



DD boundary condition



The spectrum of axion-vector in deformed
Dp-Dq model with constant bulk mass m_5^2
We can realize the Regge behavior of V & A

III Construct holographic QCD model from heavy-quark potential

Deformed warp factor in Pirner-Galow model

H.J.Pirner, B.Galow, arXiv:0903.2701

$$h(z) = \frac{c_2}{\log\left[\frac{1}{z^2+l_s^2}\frac{1}{\Lambda^2}\right]}$$

Identify p~1/z

Starting from the model, we find interesting points :

UV: σz^2

IR: $-\log(z_{IR} - z)$

Our holographic model:

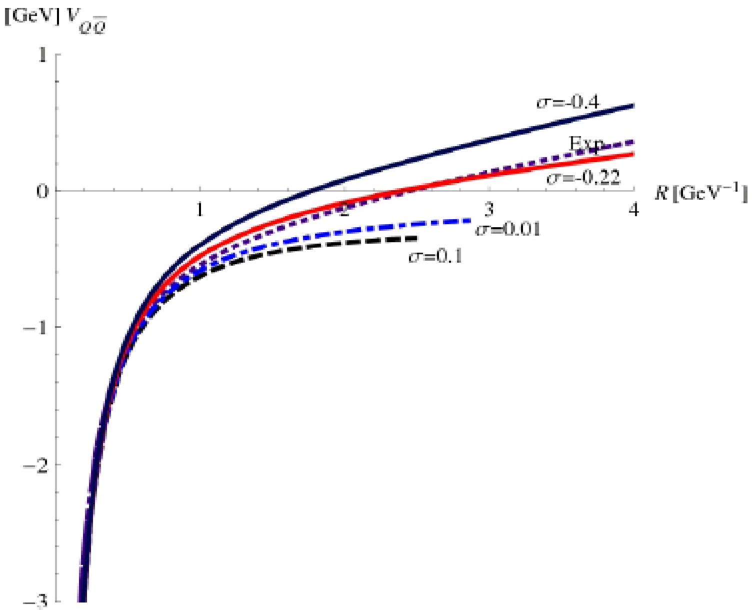
S. He, M. Huang, Q. S. Yan Phys.Rev. D83 (2011) 045034

$$ds^2 = G_{\mu\nu} dX^\mu dX^\nu = \frac{h(z)L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right).$$

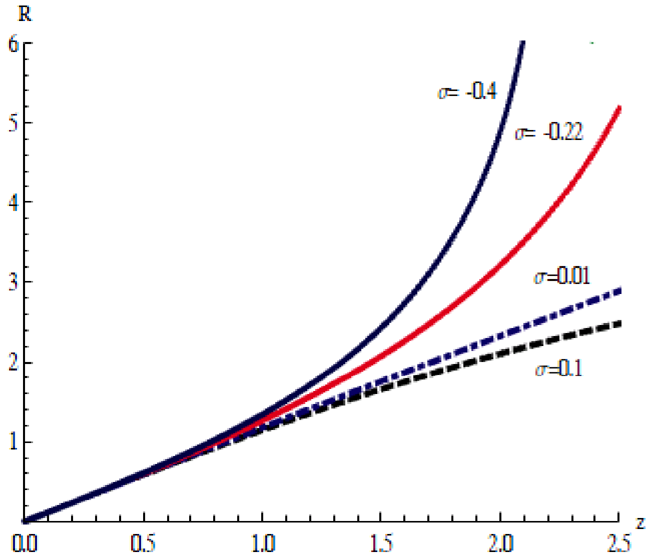
$$h(z) = \exp\left(-\frac{\sigma z^2}{2} - c_0 \log(\frac{Z_{IR} - z}{Z_{IR}})\right)$$

- σ can be either positive or negative
- c_0 plays an important role to control the IR behavior of QCD.
- Z_{IR} is similar to effective cut-off in the bulk.

$c_0 = 0.$ with only quadratic modification



(a)



(b)

$L = 1\mathrm{GeV}^{-1}$ $\sigma_s = 0.38\mathrm{GeV}^{-2}$

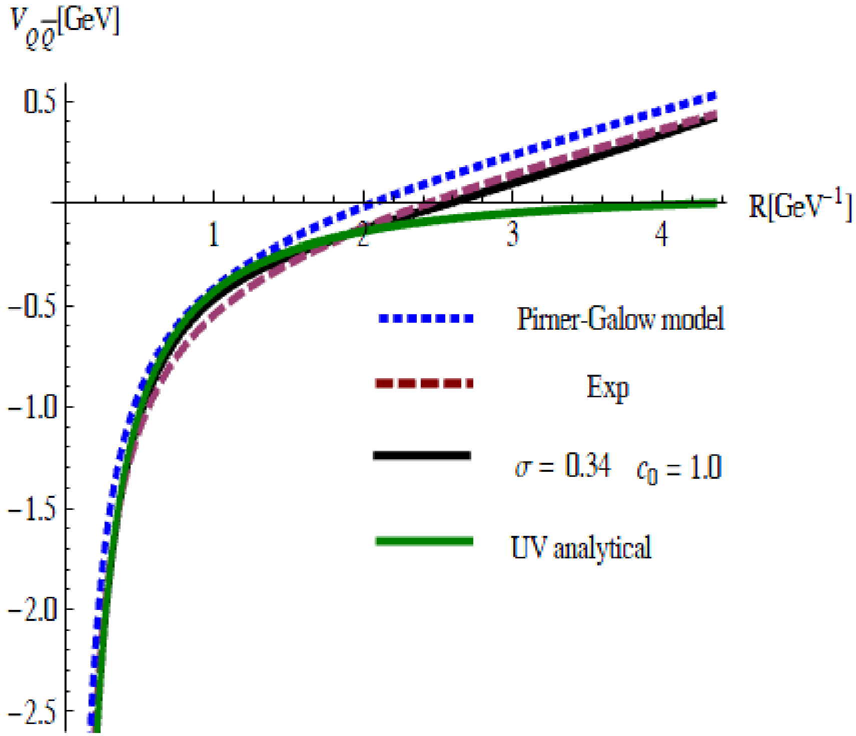
$\sigma = 0.1, 0.01, -0.22, -0.4\mathrm{GeV}^2$

Andreev and Zakharov's model favored, although it does not generate the IR Behavior of real Cornell potential

"KKSS" model cannot reach QCD IR region.

The heavy quark potential of the model with both quadratic and logarithmic modification

$c_0 = 1$



$L = 1\text{GeV}^{-1}$ $\sigma_s = 0.38\text{GeV}^{-2}$ $z_{IR} = 2.54\text{GeV}^{-1}$

In our model , we introduce the logarithmic warp factor and we fit the lattice data very well as you see in the figure given above. The value of parameters are listed in the figure.

More compact model with less parameters:
two scales: L (AAdS5 radius)
z_IR (IR cut-off)

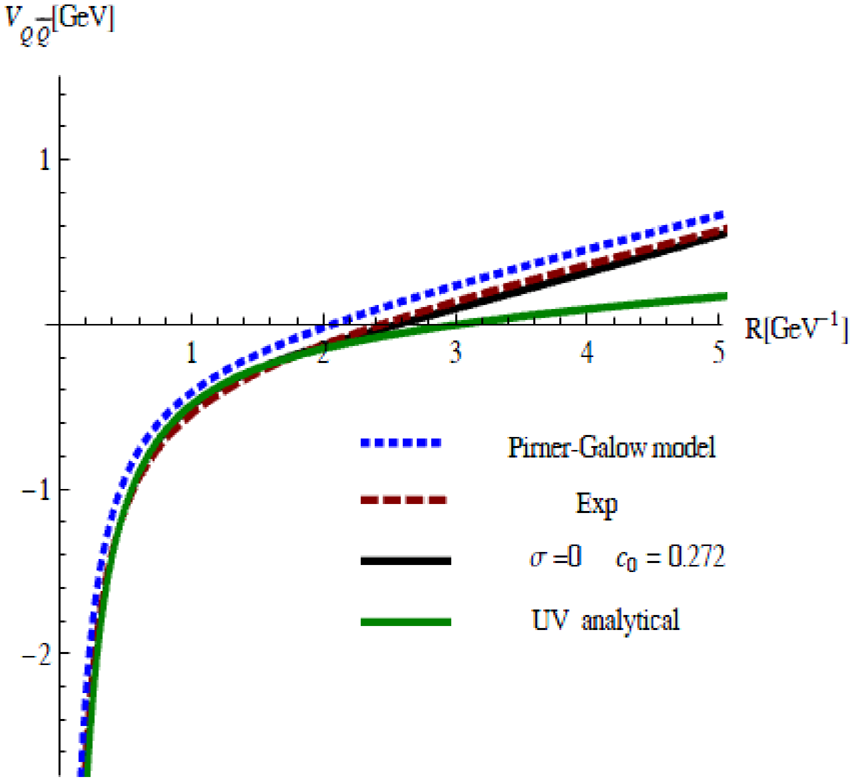
$$ds^2 = G_{\mu\nu}dX^\mu dX^\nu = e^{2\mathcal{A}(z)}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$
$$= \frac{h(z)z_{IR}^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

$$\mathcal{A}(z) = -\log\left(\frac{z}{z_{IR}}\right) - \frac{c_0}{2}\log\left(\frac{z_{IR}-z}{z_{IR}}\right)$$

$$h(z) = \exp\left(-c_0\log\left(\frac{z_{IR}-z}{z_{IR}}\right)\right)$$

In compact model , we only take the logarithmic term into consideration. We can find that it plays an important role in controlling the IR behavior.

$\sigma = 0$ with only logarithmic contribution,
can produce HQ potential,
and the dilaton potential is also stable



$L = 1\text{GeV}^{-1},\ \alpha = 0.38\text{GeV}^{-2}.$

The dual gravity solution of the geometry & Beta function

The ansatz of the metric

$$ds_E^2 = e^{2A(z)}(-dt^2 + d\vec{x}^2 + dz^2),$$

$$e^{2A(z)} = e^{-\frac{4}{3}\phi}\frac{h(z)L^2}{z^2} = e^{-\frac{4}{3}\phi}e^{2\mathcal{A}_s(z)}.$$

$$S_{5D-Gravity} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-G^E}\left(R - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V_B(\phi)\right)$$

Einstein-dilaton equations:

$$3((A'(z))^2 + A''(z)) = -\frac{2}{3}(\phi')^2 - \frac{1}{2}e^{2A(z)}V_B(\phi),$$

$$6(A'(z))^2 = \frac{2}{3}(\phi')^2 - \frac{1}{2}e^{2A(z)}V_B(\phi).$$

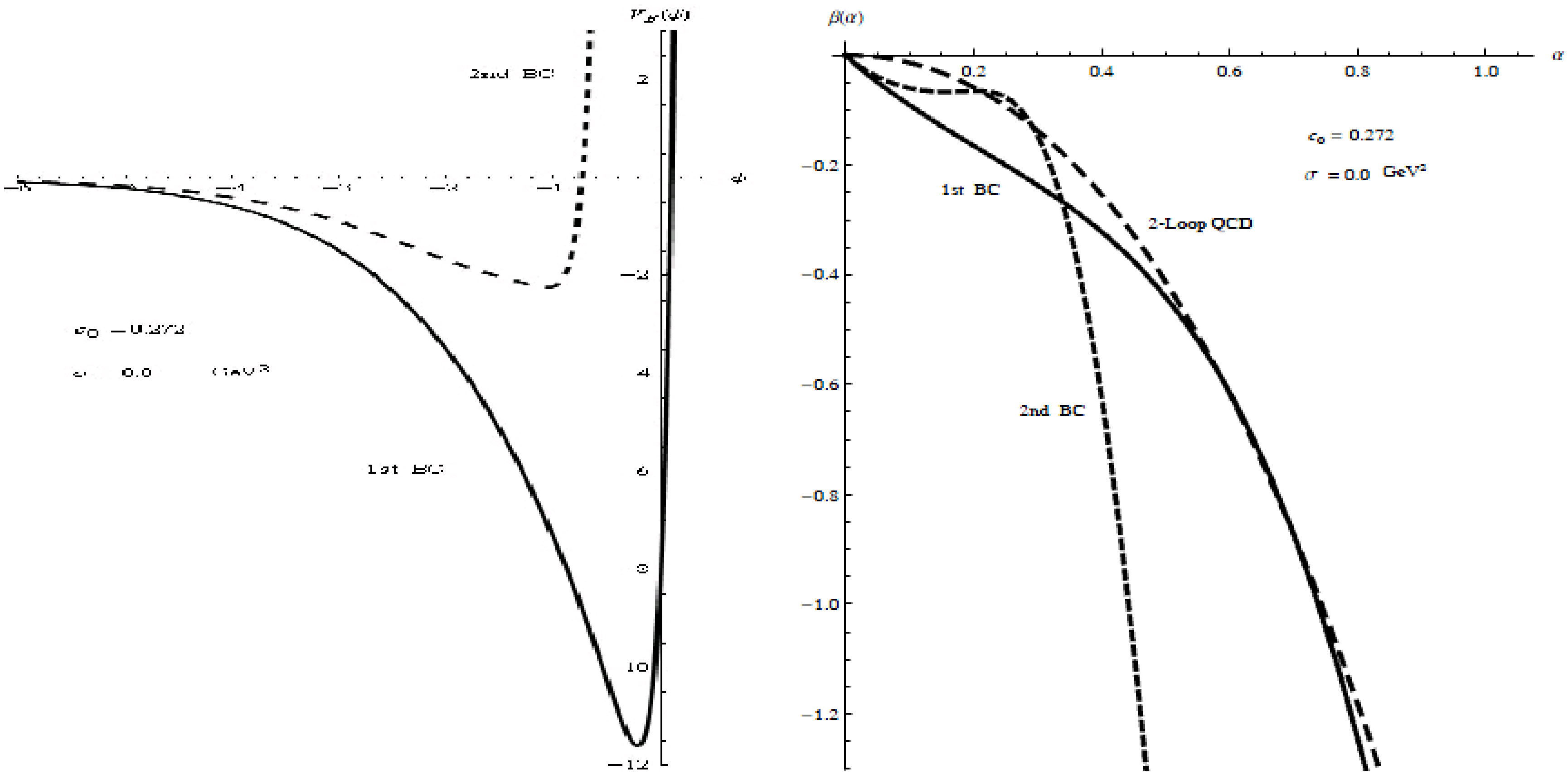
....

$$V_B(\phi(z)) = -4e^{\frac{4}{3}\phi - 2\mathcal{A}_s}[(\phi')^2 + 3(\mathcal{A}_s')^2 - 4\phi'\mathcal{A}_s'],$$

$$\phi'' = \frac{3}{2}\mathcal{A}_s'' + 2\mathcal{A}_s'\phi' - \frac{3}{2}(\mathcal{A}_s')^2.$$

$\sigma = 0$

with only logarithmic contribution,
can produce HQ potential,
and the dilaton potential is also stable



(a) (b)

$c_0 = 0.272\text{GeV}^2$, $z_{IR} = 2.1\text{GeV}^{-1}$

Short summary

I. We deform the AdS geometry with quadratic correction in warped factor and obtain the heavy quark potential.

II. We will improve this hQCD model in noncritical string frame work.

III. We will comment on the effects of logarithmic correction in warped factor later.

$AAdS_5$ gravity solution, thermal dynamics and dual QCD phenomenons

Dan Ning Li, Song He, Mei Huang, Q. S. Yan : JHEP1109:041,2011

Starting from non-critical string action in Einstein frame & String frame

$$\int \sqrt{-g^S} e^{-2\phi} \left(R^S + 4\partial_\mu\phi\partial^\mu\phi - V_S(\phi)\right)$$
$$= \int \sqrt{-g^E} \left[R^E - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V_E(\phi)\right].$$

String frame ansatz

$$ds^2 = \frac{L^2 e^{2A_s}}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i\right)$$

Einstein frame ansatz

$$ds^2 = \frac{L^2 e^{2A_s - \frac{4\phi}{3}}}{r^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i\right)$$

EOMs

$$E_{\mu\nu} + \frac{1}{2} g^E_{\mu\nu}\left(\frac{4}{3}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right) - \frac{4}{3}\partial_\mu\phi\partial_\nu\phi = 0$$

Generating function & General solution

$$\phi(z) = \phi_0 + \phi_1 \int_0^z \frac{e^{2A_s(x)}}{x^2}\, dx + \frac{3A_s(z)}{2} + \frac{3}{2}\int_0^z \frac{e^{2A_s(x)} \int_0^x y^2 e^{-2A_s(y)} A_s'(y)^2 dy}{x^2}\, dx,$$

$$f(z) = f_0 + f_1 \left(\int_0^z x^3 e^{2\phi(x) - 3A_s(x)}\, dx \right),$$

$$V_E(\phi) = \frac{e^{\frac{4\phi(z)}{3} - 2A_s(z)}}{L^2} \left(z^2 f''(z) - 4f(z) \left(3z^2 A_s''(z) - 2z^2 \phi''(z) + z^2 \phi'(z)^2 + 3 \right) \right).$$

New gravity solution III (semi-analytical form)

$$A_s(z) = ck^2z^2 \qquad \phi(z) = \frac{3}{4}ck^2z^2(1 + H_c(z)),$$

$$f(z) = 1 - f_c^h \int_0^{kz} x^3 \exp\left(\frac{3}{2}cx^2\left(H_c(x/k) - 1\right)\right) dx,$$

with

$$f_c^h = \frac{1}{\int_0^{kz_h} x^3 \exp\left(\frac{3}{2}cx^2\left(H_c(x/k) - 1\right)\right) dx},$$

and

$$H_c(z) = {}_2F_2\left(1, 1; 2, \frac{5}{2}; 2ck^2z^2\right).$$

$$V_E^c(z) = \frac{3e^{ck^2z^2(-1+H_c(z))}}{128L^2k^2z^2}\left(1 - f_c^h \int_0^{kz} e^{\frac{3}{2}cx^2\left(-1+H_c(\frac{x}{k})\right)} x^3 dx\right)\Big[40k^2z^2 + 64ck^4z^4 - 384k^6z^6 + 12\sqrt{2\pi}e^{2ck^2z^2}kz\left(-7 + 20ck^2z^2\right)Erf_c\left(\sqrt{2}kz\right) - 27\pi e^{4ck^2z^2}Erf_c\left(\sqrt{2}kz\right)^2\Big] - \frac{3f_c^h e^{\frac{5}{2}ck^2z^2(-1+H_c(z))}k^3z^3}{16L^2}\left[4kz - 16ck^3z^3 + 3\sqrt{2\pi}e^{2ck^2z^2}Erf_c\left(\sqrt{2}kz\right)\right]$$

Temperature and horizon

$$T = \frac{|f'(z_h)|}{4\pi}.$$

$$T = \frac{k^4 z_h^3 \exp\left(\frac{3}{2}\left(ck^2 z_h^2\, H_c(z_h) - ck^2 z_h^2\right)\right)}{4\pi \int_0^{kz_h} e^{\frac{3}{2}(-ck^2x^2 + ck^2x^2 H_c(x) + \mathrm{Log}[k^2x^2])}\, dx}$$

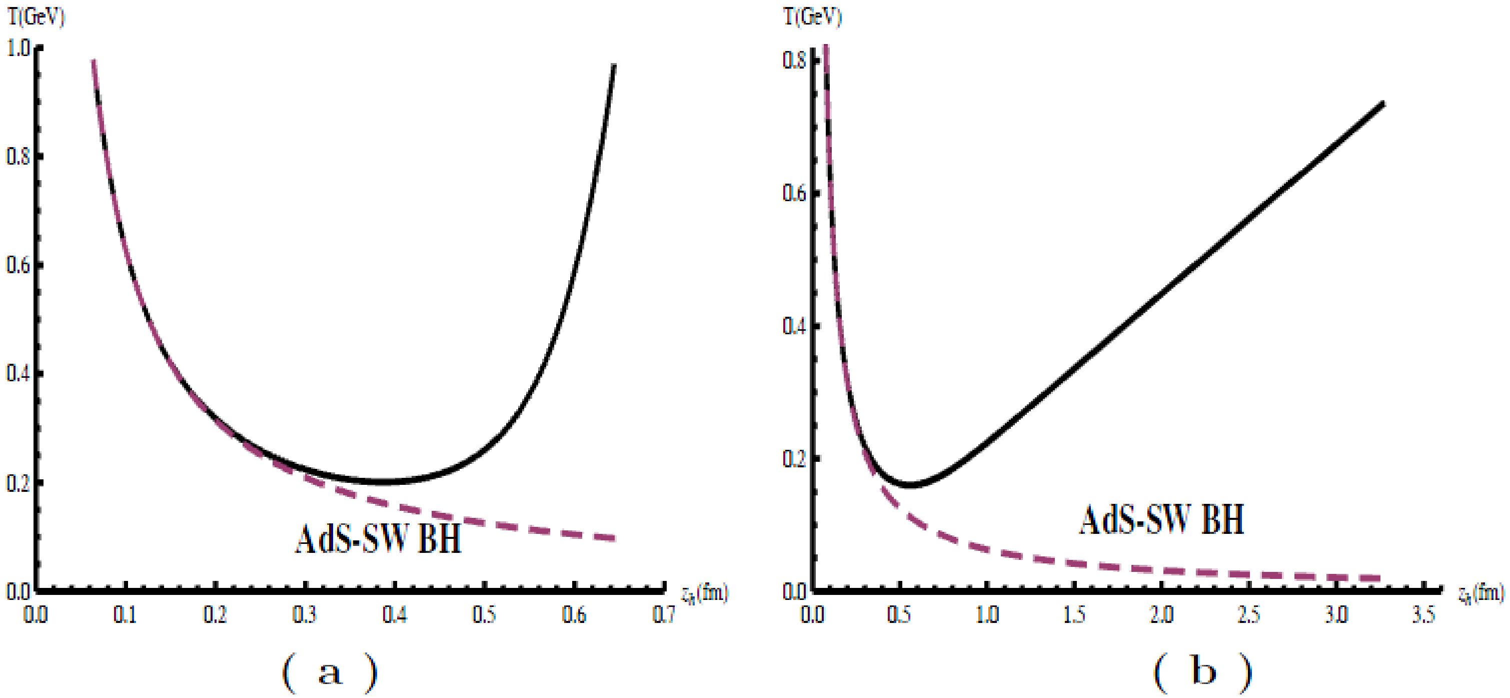


Figure 1: The temperature T as a function of the black-hole horizon z_h with $k = 0.43GeV$ for $c = +$ in (a) and $c = -$ in (b), respectively. The dashed lines are for pure AdS_5 Schwarz black-hole.

Entropy density and Temperature

$$s=\frac{A_{area}}{4G_5V_3}=\frac{L^3}{4G_5}\left(\frac{e^{A_s-\frac{2}{3}\phi}}{z}\right)^3.$$

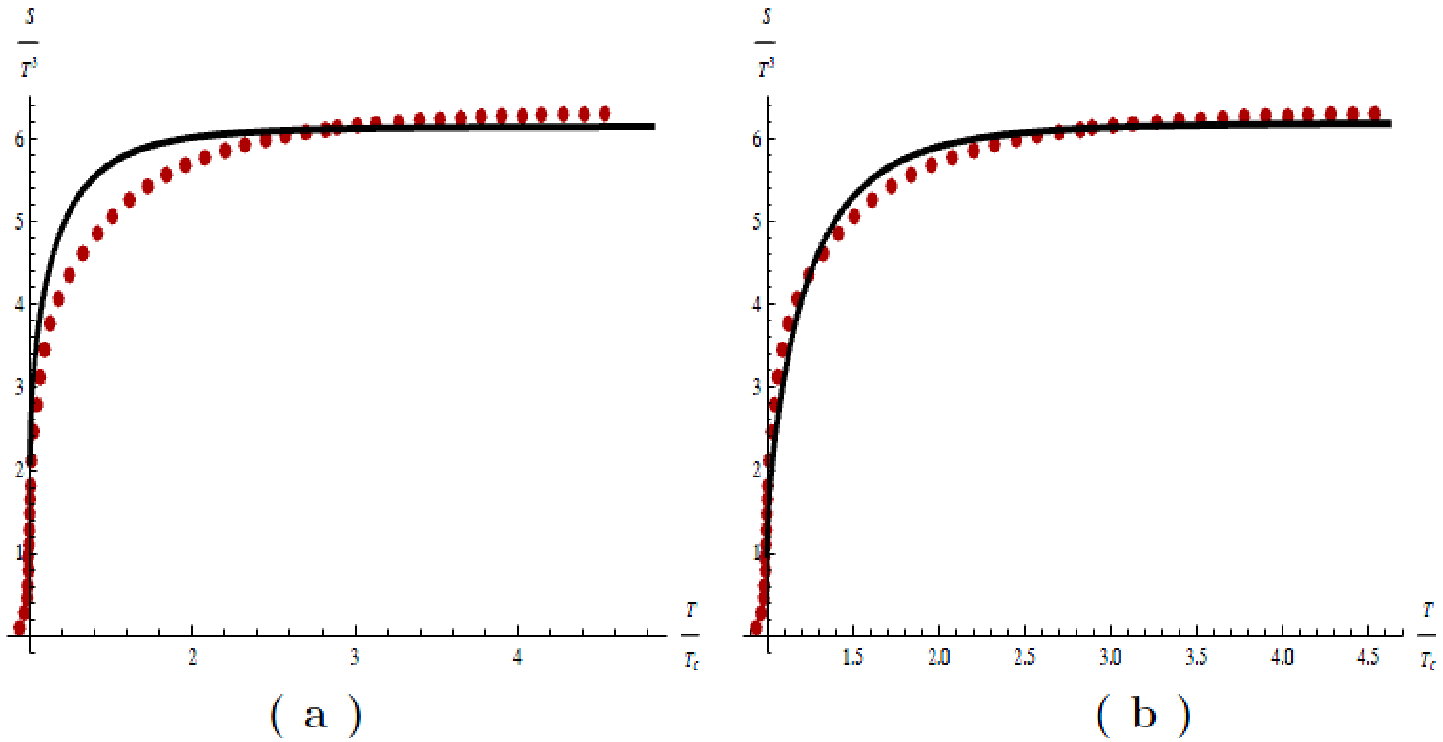


Figure 2: The scaled entropy density s/T^3 as a function of scaled temperature T/T_c with $k=0.43\text{GeV}$ and $G_5/L^3=1.26$ for $c=+$ in (a) and $c=-$ in (b), respectively. The dotted lines in (a) and (b) are lattice results from Nucl. Phys. B **469**, 419 (1996)

Pressure and Temperature

$$\frac{dp(T)}{dT} = s(T).$$

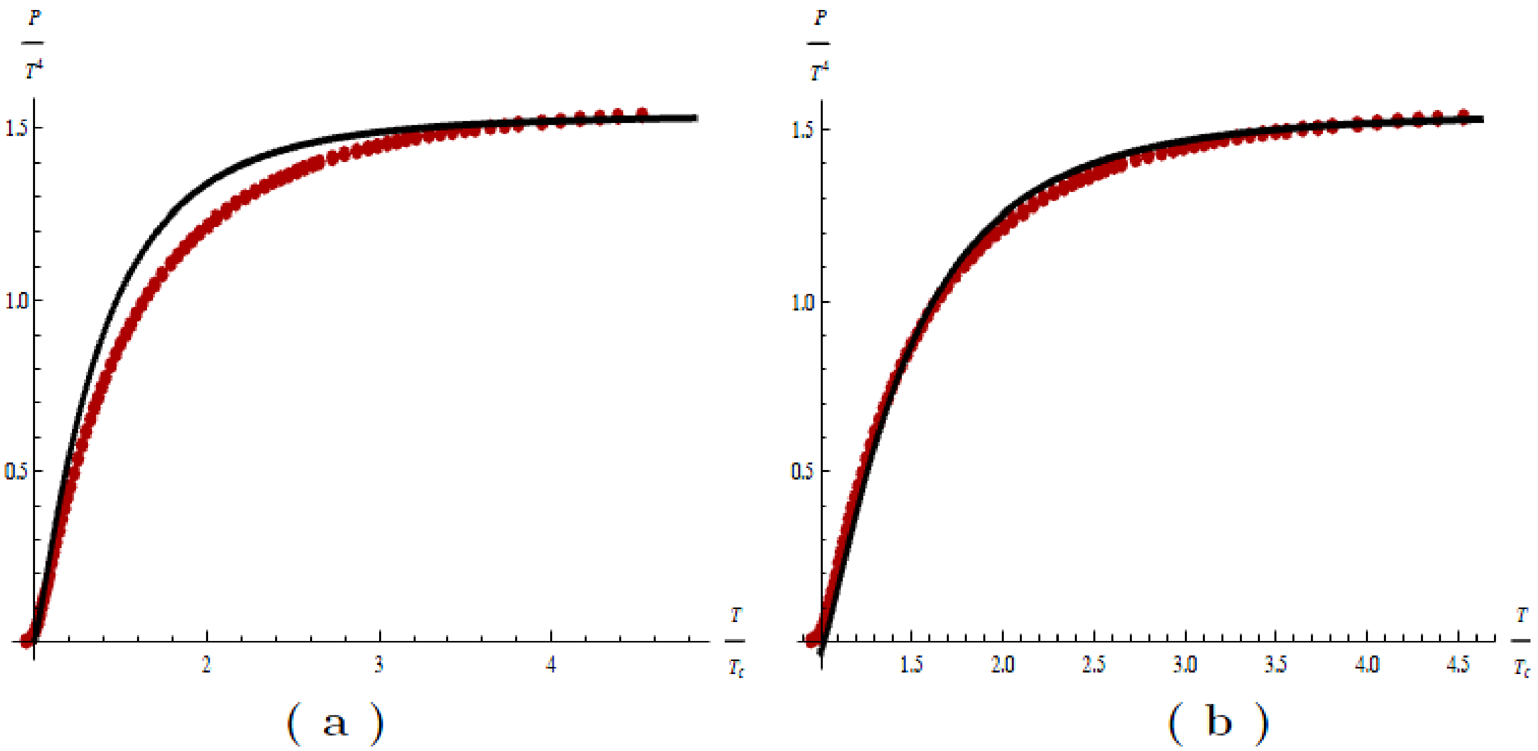


Figure 3: The scaled pressure density p/T^4 as a function of scaled temperature T/T_c with $k = 0.43GeV$ and $G_5/L^3 = 1.26$ for $c = +$ in (a) and $c = -$ in (b), respectively. The dots are the lattice data from Nucl. Phys. B **469**, 419 (1996)

Energy density and Temperature

$$\epsilon = -p + sT.$$

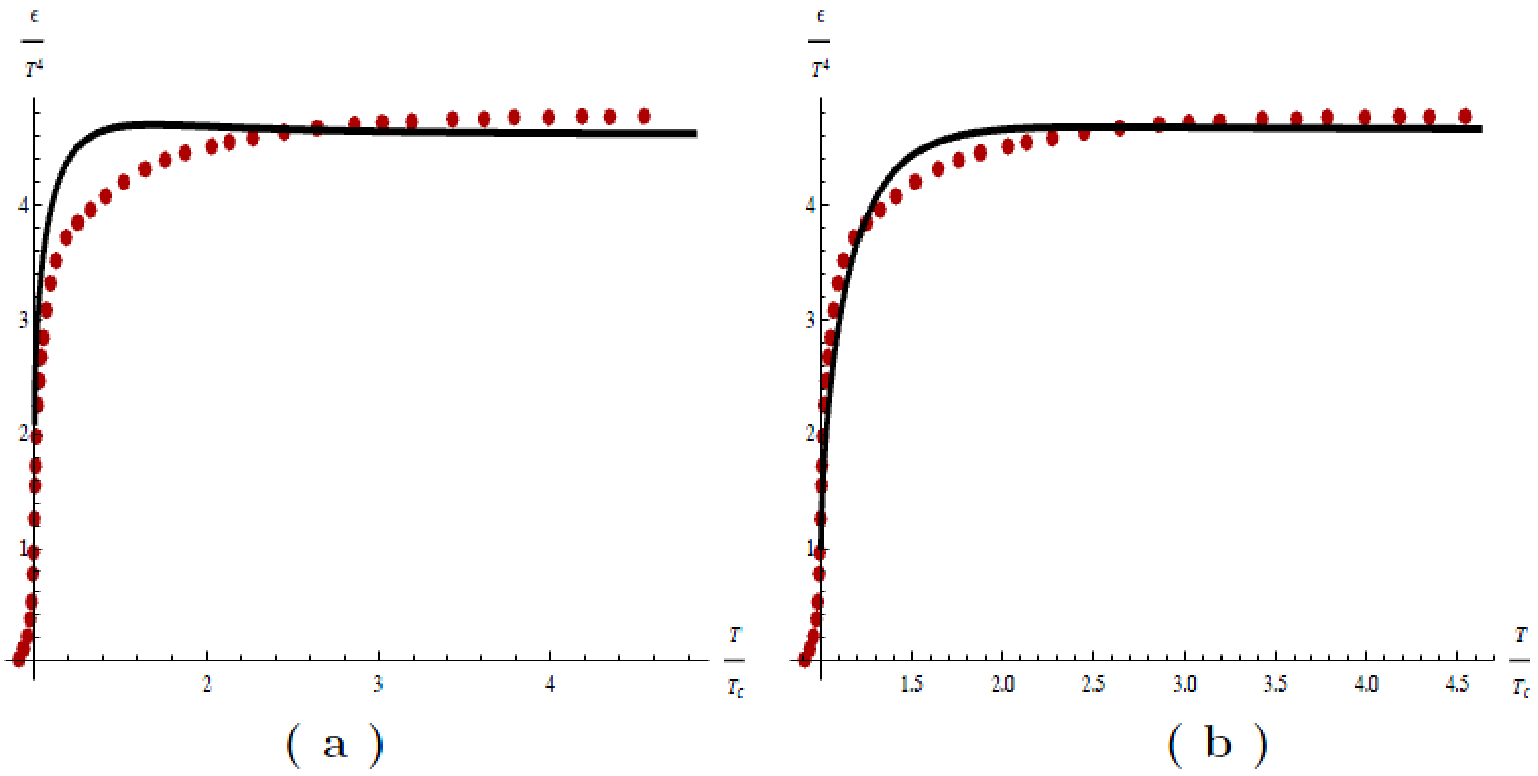
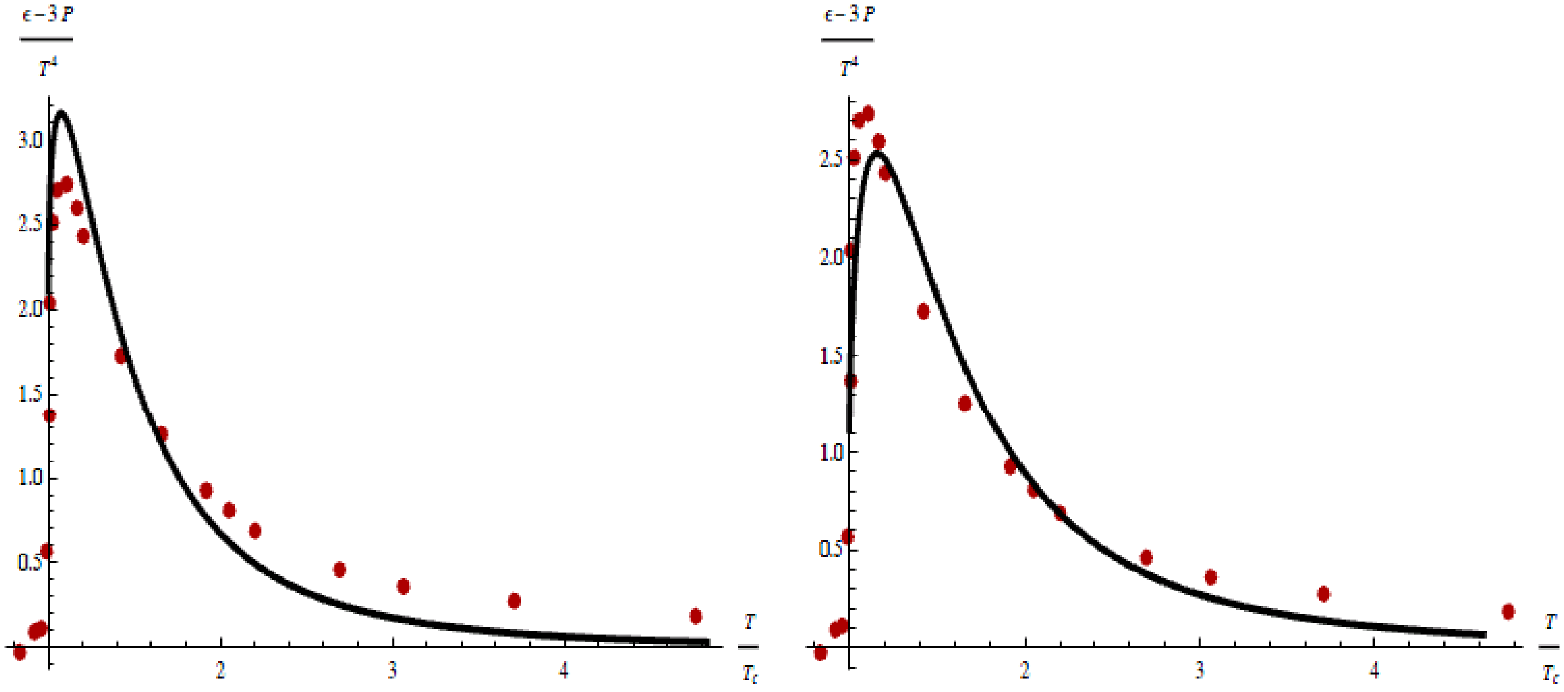


Figure 4: The scaled energy density ϵ/T^4 as a function of scaled temperature T/T_c with $k = 0.43GeV$ and $G_5/L^3 = 1.26$ for $c = +$ in (a) and $c = -$in (b), respectively. The dots are lattice data from Nucl. Phys. B **469**, 419 (1996)

Sound velocity and Temperature

$$c_s^2 = \frac{d\log T}{d\log s} = \frac{s}{T ds/dT}$$



(a)   (b)

Figure 5: The trace anomaly $\epsilon - 3p$ as a function of scaled temperature T/T_c with $k = 0.43GeV$ and $G_5/L^3 = 1.26$ for $c = +$ in (a) and $c = -$ in (b), respectively. The dots are the lattice data from Nucl. Phys. B **469**, 419 (1996)

Trace anomaly and Temperature

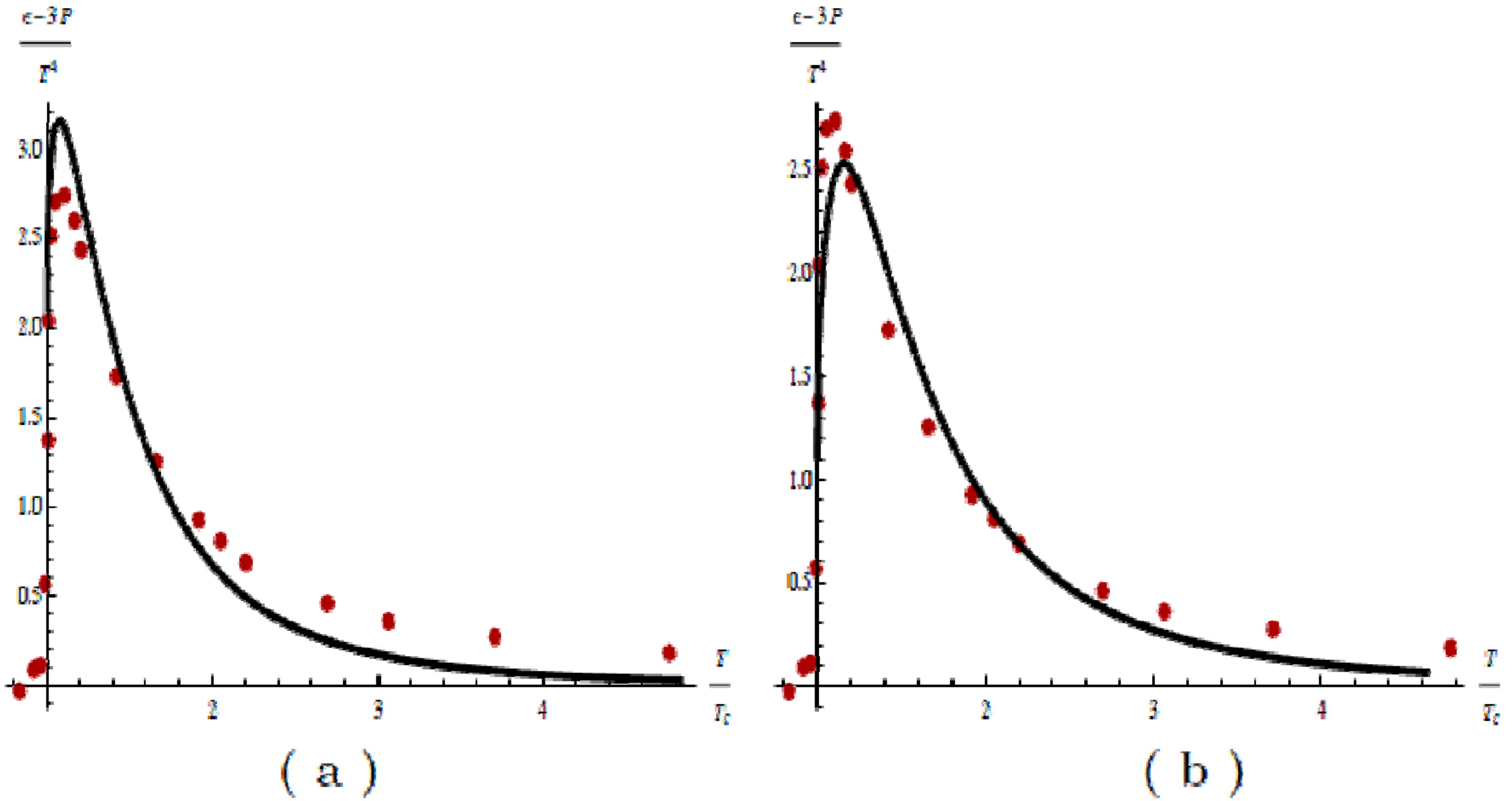
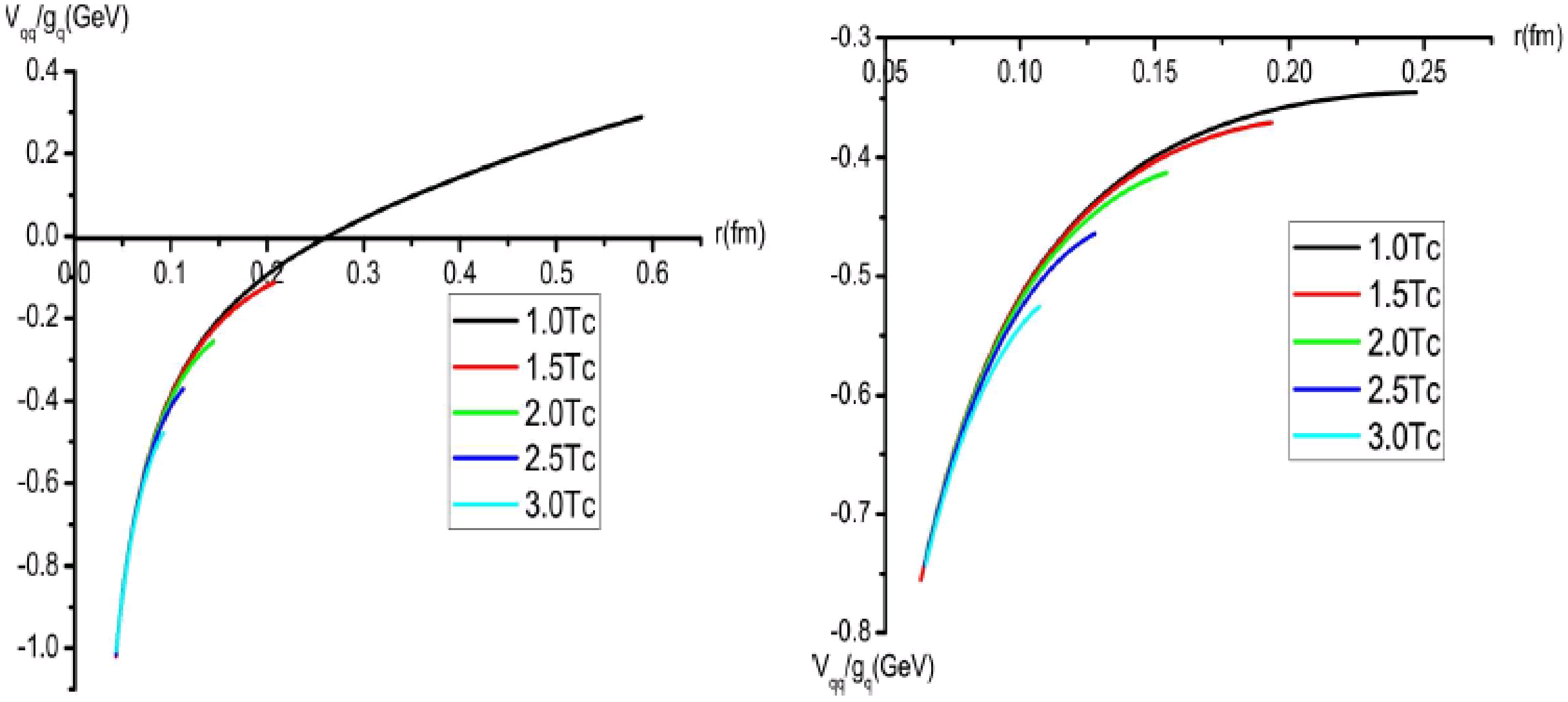


Figure 5: The trace anomaly $\epsilon - 3p$ as a function of scaled temperature T/T_c with $k = 0.43 GeV$ and $G_5/L^3 = 1.26$ for $c = +$ in (a) and $c = -$ in (b), respectively. The dots are the lattice data from Nucl. Phys. B **469**, 419 (1996)

Heavy quark potential and emperature



(a) (b)

Figure 7: The heavy quark potential $V_{Q\bar{Q}}/g_q$ as a function of the separation distance r for different temperatures for c=+ (a) and $c = -$ (b), respectively. Where $k = 0.43\text{GeV}$ is used.

Polyakov loop and Temperature

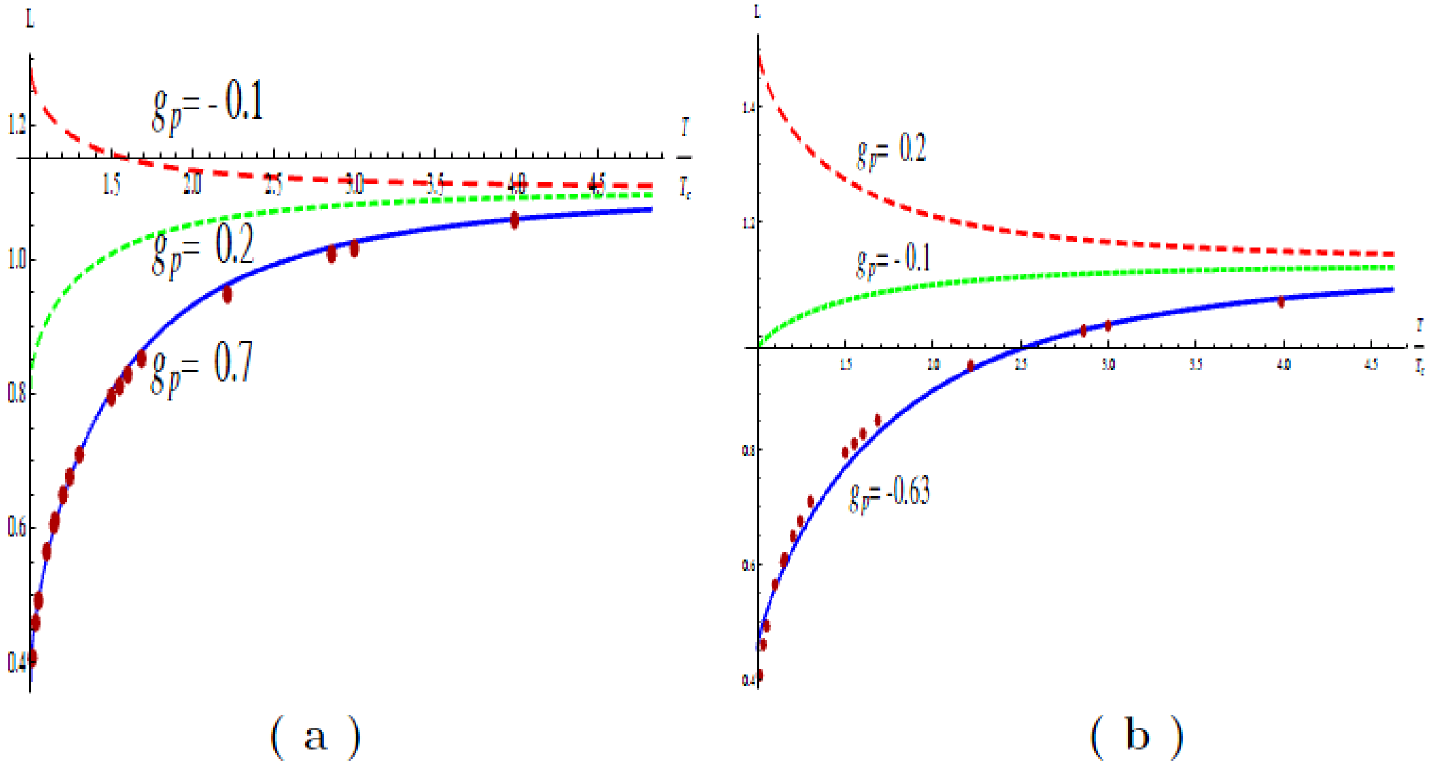


Figure 8: The expectation value of the Polyakov loop as a function of the temperature: (a) in the case of $c = +$ with $k = 0.43GeV$, $C_p = 0.1$ and $g_p = -0.1, 0.2, 0.7$, (b) in the case of $c = -$ with $k = 0.43GeV$, $C_p = 0.12$, and $g_p = 0.2, -0.1, -0.63$. The dots are lattice data taken from Phys. Rev. D **77**, 034503 (2008)

Spatial wilson loop and Temperature

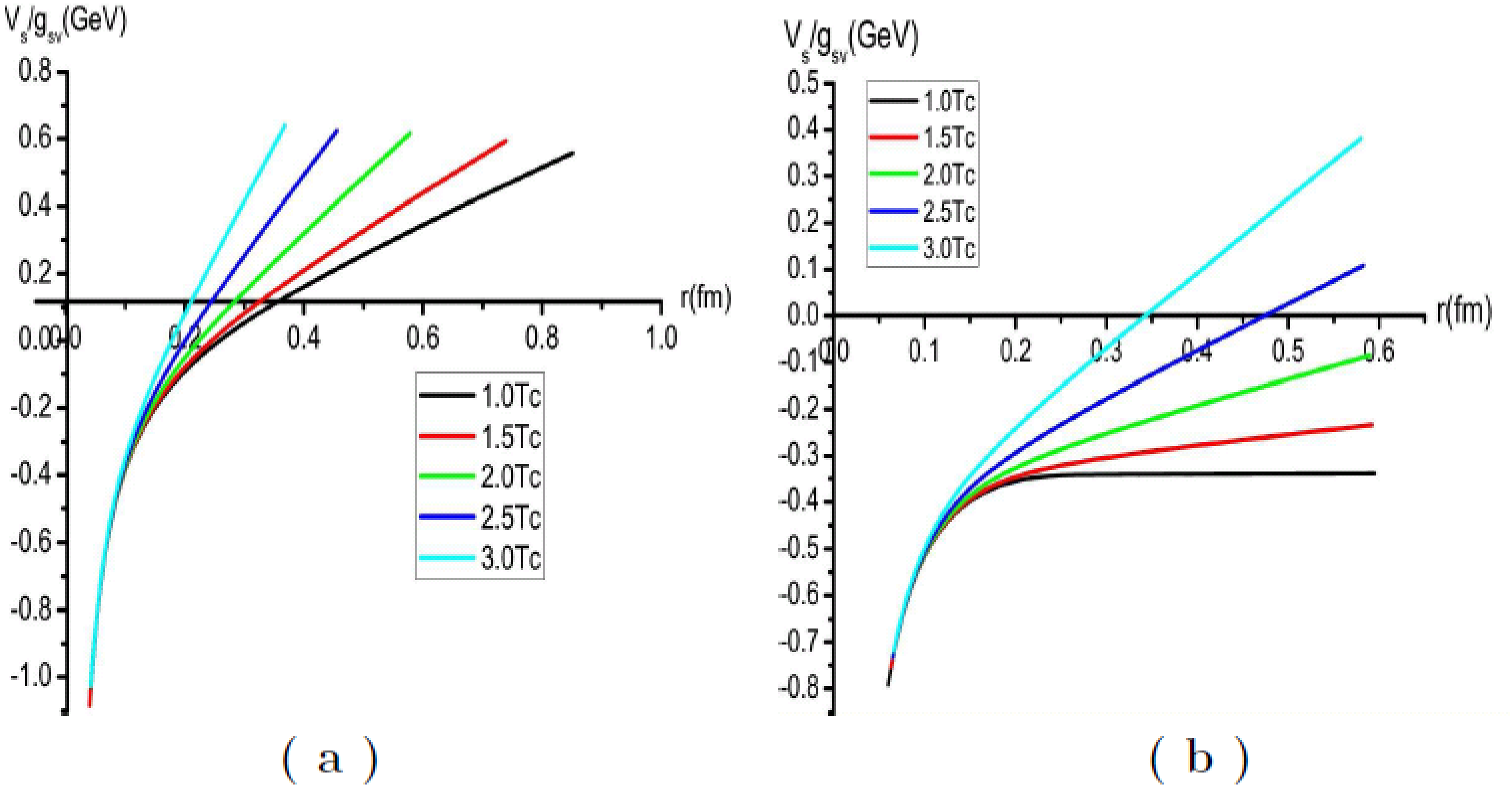
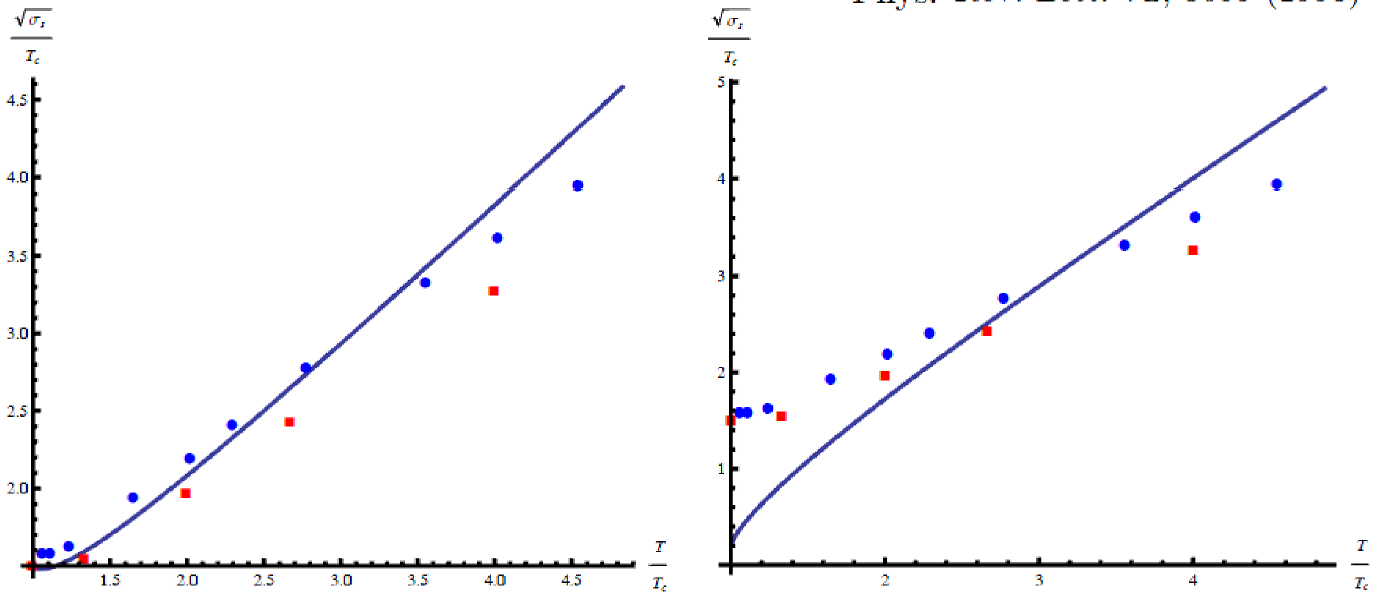


Figure 9: The spatial heavy quark potential V_s/g_{sv} as a function the distance between the quark anti-quark r: (a) for the case of $c = +$ with $k = 0.43\text{GeV}$, (b) for the case of $c = -$ with $k = 0.43\text{GeV}$.

Spatial string tensioin VS Lattice data

[1] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier and B. Petersson, Nucl. Phys. B **469**, 419 (1996)

[2] G. S. Bali, J. Fingberg, U. M. Heller, F. Karsch and K. Schilling, Phys. Rev. Lett. **71**, 3059 (1993)



(a) (b)

Figure 10: The scaled spatial string tension $\sqrt{\sigma_s}/T_c$ as a function of the scaled temperature T/T_c for the case of $c = +$ in (a) with $k = 0.43$GeV and $g_{sv} = 0.55$ are used, and $c = -$ in (b) with $k = 0.43$GeV and $g_{sv} = 0.7$ are used, respectively. The blue dots stands for lattice data for pure $SU(3)$ gauge theory fron [1] . The red dots are lattice data for pure $SU(2)$ gauge theory which is from [2].

Non critical string & HQCD model

Einstein-Maxwell-Dilaton system

Motivation

Rong-Gen Cai, Song He, Danning Li **JHEP 1203 (2012) 033**

To cover the degree of freedom in QCD phase Diagram.
Quarks (chemical potential) & gluons(dilaton potential)

Gravity Action

$$S_{5D} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^S}e^{-2\phi}\left(R^S + 4\partial_\mu\phi\partial^\mu\phi - V_S(\phi) - \frac{1}{4g_g^2}e^{\frac{4\phi}{3}}F_{\mu\nu}F^{\mu\nu}\right),$$

$$S_{5D} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^E}\left(R - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V_E(\phi) - \frac{1}{4g_g^2}F_{\mu\nu}F^{\mu\nu}\right)$$

EMD system

Ansatz

$$ds_E^2 = \frac{L^2 e^{2A_s - \frac{4\phi}{3}}}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^i dx^i\right).$$

EOM

$$E_{\mu\nu} + \frac{1}{2}g^E_{\mu\nu}\left(\frac{4}{3}\partial_\mu\phi\partial^\mu\phi + V_E(\phi)\right) - \frac{4}{3}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2g_g^2}\left(F_{\mu\lambda}F_\nu^\lambda - \frac{1}{4}g^E_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right) = 0,$$

$$\frac{8}{3}\partial_z\left(\frac{L^3 e^{3A_s(z)-2\phi}f(z)}{z^3}\partial_z\phi\right) - \frac{L^5 e^{5A_s(z)-\frac{10}{3}\phi}}{z^5}\partial_\phi V_E = 0.$$

$$\frac{1}{\sqrt{-g^E}}\partial_\mu\sqrt{-g^E}F^{\mu\nu} = 0.$$

EMD system

General solution

$$\phi(z) = \int_0^z \frac{e^{2A_s(x)}\left(\frac{3}{2}\int_0^x y^2 e^{-2A_s(y)} A_s'(y)^2\, dy + \phi_1\right)}{x^2}\, dx + \frac{3A_s(z)}{2} + \phi_0,$$

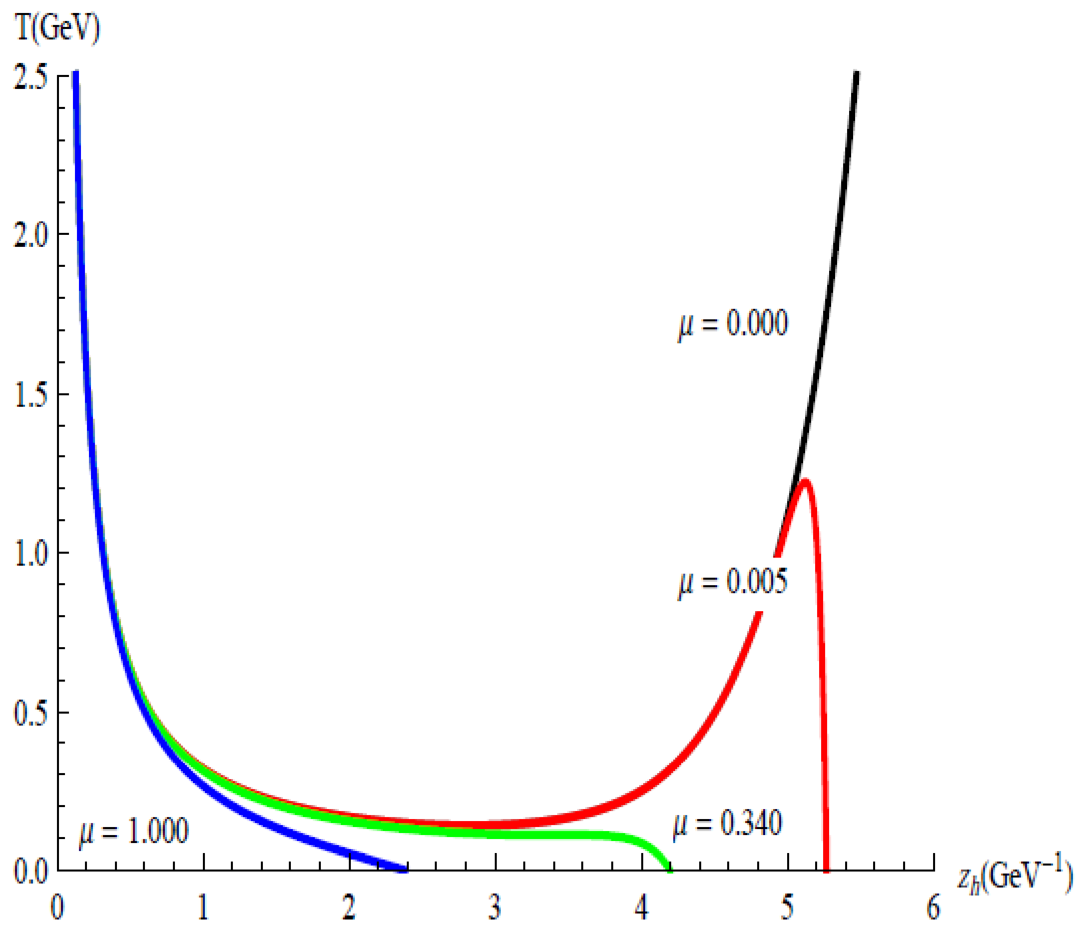
$$A_t(z) = A_{t0} + A_{t1}\left(\int_0^z y e^{\frac{2\phi(y)}{3} - A_s(y)}\, dy\right),$$

$$f(z) = \int_0^z x^3 e^{2\phi(x) - 3A_s(x)}\left(\frac{{A_{t1}}^2\left(\int_0^x y e^{\frac{2\phi(y)}{3} - A_s(y)}\, dy\right)}{g_g^2 L^2} + f_1\right) dx + f_0,$$

$$V_E(z) = \frac{e^{\frac{4\phi(z)}{3} - 2A_s(z)}}{L^2}\Big(r^2 f''(z) - 4f(z)\left(3z^2 A_s''(z) - 2z^2\phi''(z) + z^2\phi'(z)^2 + 3\right) - \frac{3z^4 e^{\frac{4\phi(z)}{3} - 2A_s(z)} A_t'(z)^2}{2L^2 g_g^2}\Big),$$

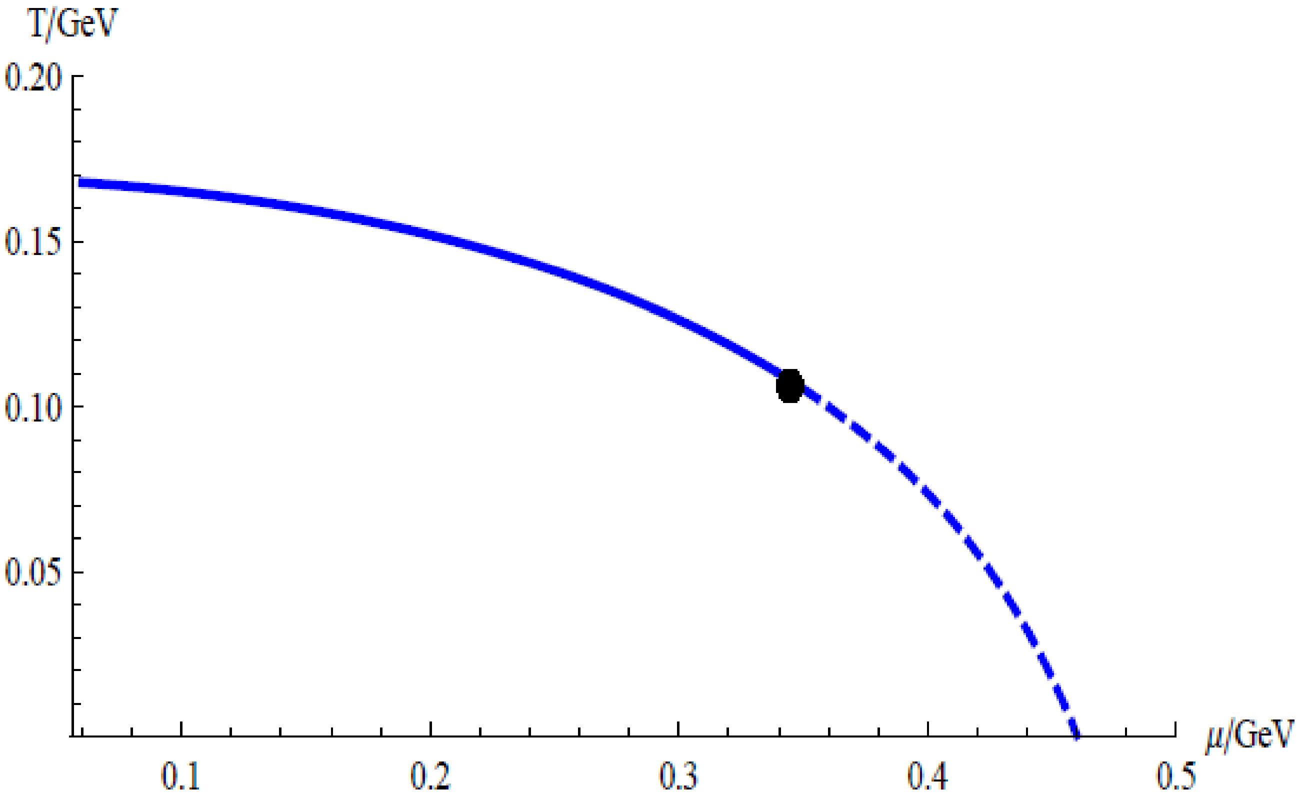
HQCD model and gravity dual

$$A_s(z) = k^2 z^2$$



QCD Phase diagram

$$\langle \mathcal{P} \rangle = e^{-\frac{1}{2T} F^{\infty}(T)}$$



Discussion

I. **We proposal a new noncritical string frame work to get analytical solution of graviton-dilaton system.**

II. **In this frame work, we obtain the analytical solution of HQCD model with positive/negative quadratic warp factor.**

III. **Considering the nontrival dilaton configuration, We study the equation of state of the system dual to the graviton-dilaton system. The equation of state are not sensitive to the sign of the warp factor.**

IV. **In this frame work, we study heavy quark potential, Polyakov loop and spatial string tension. We find these loop operator in HQCD model with positive warp factor can reproduce the lattice data.**

V. **We realize the QCD phase diagram in graviton-dilaton-Maxswell system.**

VI. **How to embed our frame work in critical string theory?**

Thanks for your attention