Switching effect upon the quantum Brownian motion

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Outline

- Quantum fluctuation of vacuum field and Brownian motion.
- Model using a test particle method and sudden switching approach
- Our concern and study method
- Model, approach method and result
- Other application like cosmological perturbation
- Future study

Classical Brownian Motion

- The irregular motion of a body arises when it is immersed in a homogeneous fluid made up of much lighter particles.
- System and environment

Newtonian equation:

$$m\frac{d}{dt}v = F$$

Langevin equation:

$$m\frac{d}{dt}v + \xi v = \eta(t)$$

$$\langle \eta(t) \rangle = 0$$
$$\langle \eta(t)\eta(t') \rangle = 2\xi K_B T \delta(t - t')$$

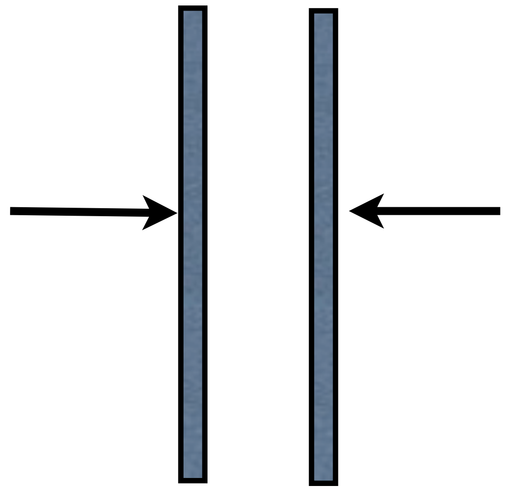
✳When the noise comes from quantum fluctuations, we call it quantum Brownian motion.

✳One major interest is the effect due to color noise.

Quantum vacuum and Casimir effect

- Classical vacuum: Nothing
- Quantum vacuum: Zero-point field, vacuum fluctuation.
- Casimir effect: energy shift in vacuum. No particle creation.
- The sign of Casimir force is still puzzling.

Casimir effect



Dynamical Casimir effect: moving mirror

A perturbed perfectly reflecting mirror in vacuum will change the configuration of zero point field and produce particles.

$$E_{2D,4D} \propto v''$$

For example, an oscillating surface will radiate fewer than 10^-4 photons per oscillation per area. Thus it is a tiny effect under normal condition.

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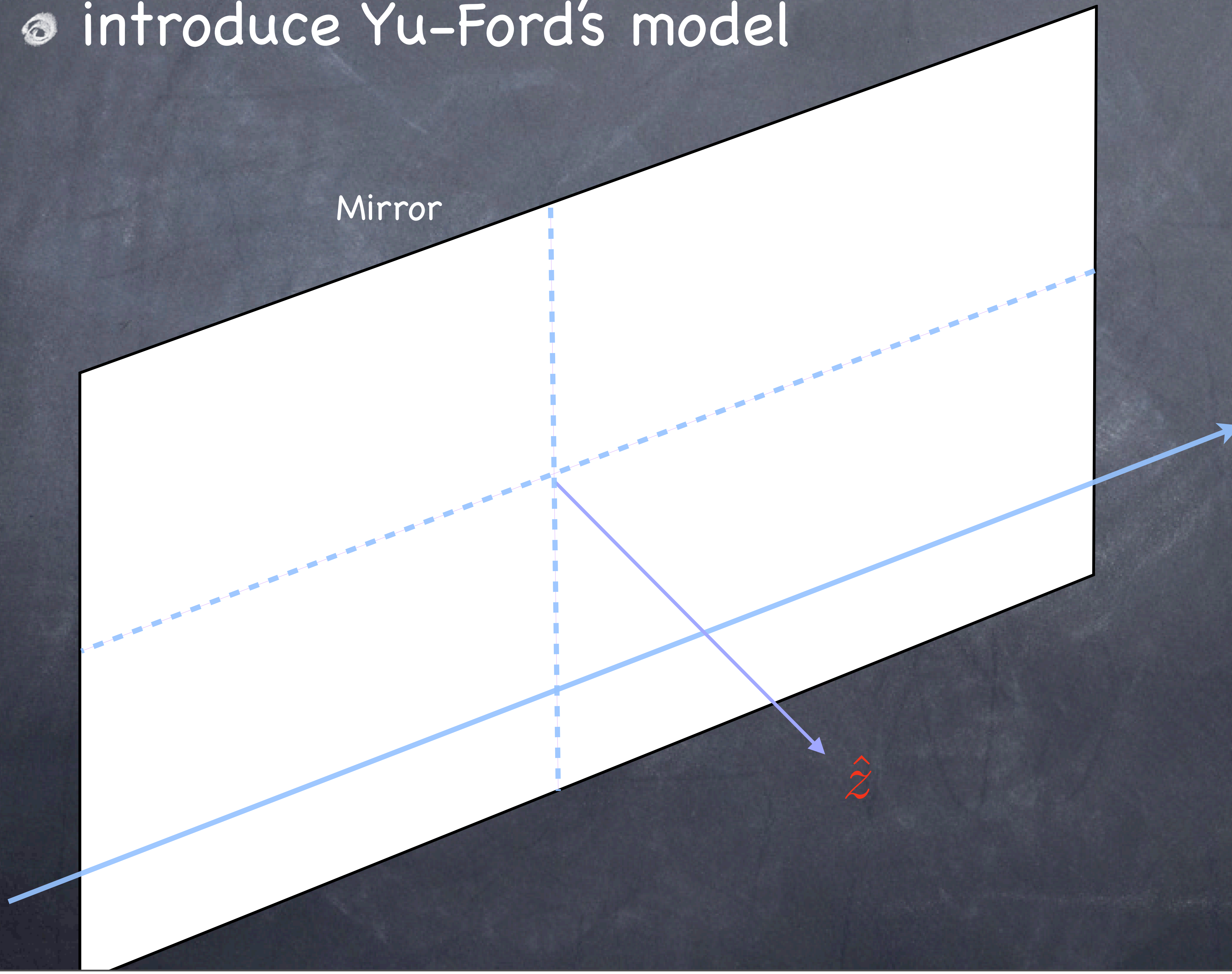
* A charged test particle coupling to the quantum electromagnetic vacuum with boundary

- introduce Yu-Ford's model



A flat, infinitely spreading mirror of perfect reflectivity is installed at $z=0$ and the quantum vacuum of the electromagnetic field is considered inside the half space $z>0$. Then we consider the measurement of the quantum fluctuations of the vacuum by using a classical charged particle with mass m and charge e as a probe. When the velocity of the particle is much smaller than the light velocity c, one can assume that the particle couples solely with the electric field $\vec{E}(\vec{x},t)$. Then the equation of motion for the particle is given by

$$m\frac{d\vec{v}}{dt} = e\vec{E}(\vec{x},t). \tag{1}$$

Furthermore, when the position of the particle does not change so much within the time scale in question, Eq. (1) along with the initial condition $\vec{v}(0)=\vec{v}_0$ is approximately solved to

$$\vec{v}(\tau) \simeq \vec{v}_0 + \frac{e}{m}\int_0^\tau \vec{E}(\vec{x},t)dt. \tag{2}$$

The E field two point function is $\langle E_z(\vec{x},t')E_z(\vec{x},t'')\rangle_R = \frac{1}{\pi^2}\frac{1}{[T^2-(2z)^2]^2},$

For sudden switching, the velocity dispersions are

$$\langle \Delta v_i^2\rangle = \frac{e^2}{m^2}\int_0^\tau dt' \int_0^\tau dt'' \langle E_i(\vec{x},t')E_i(\vec{x},t'')\rangle_R$$

These velocity dispersion in late time limit

$$\langle \Delta v_z^2\rangle \approx \frac{e^2}{4\pi^2 m^2 z^2} + O((z/\tau)^2),$$

$$\langle \Delta v_x^2\rangle = \langle \Delta v_y^2\rangle \approx -\frac{e^2}{3\pi^2 m^2 \tau^2} + O((z/\tau)^2).$$

*The z-component is a constant and was interpreted as a transient effect due to sudden switching!

- Why constant? There is no frictional force! Usually fluctuations grow when there is no dissipation!

→ Color noise and Anti-correlation

- Why negative? These are fluctuation!

→ Active (intrinsic) fluctuation

and

Passive (induced) fluctuation

Our concerns

- Is it the intrinsic property of the new vacuum or just a transient effect?
- A sudden switching model is not realistic when we discuss the effect of quantum fluctuation.
- We thought that a smooth switching would give us some more insight to clear this up.

Model with switching function

*Switching function $f_\tau(t)$ satisfy the equation

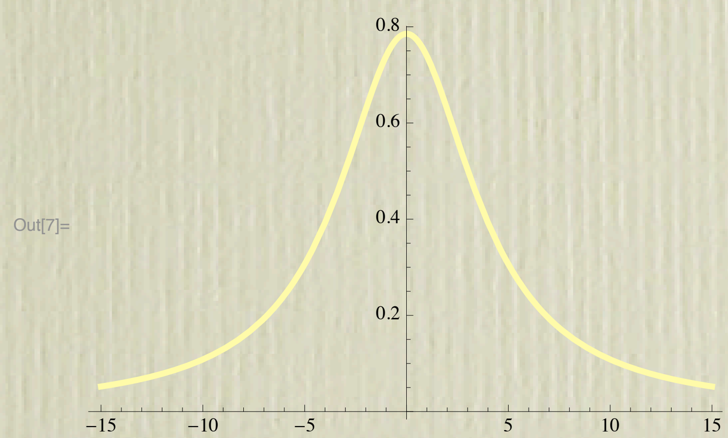
$$\int_{-\infty}^{\infty} f_\tau(t)dt = \tau$$

*The velocity dispersion becomes

$$\langle \Delta v_i^2 \rangle = \frac{e^2}{m^2}\int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt''\, f_\tau(t')f_\tau(t'')\langle E_i(\vec{x},t')E_i(\vec{x},t'')\rangle_R .$$

*e.g. Lorentzian function

$$f_\tau(t) = \frac{1}{\pi}\frac{\tau^2}{t^2+\tau^2},$$



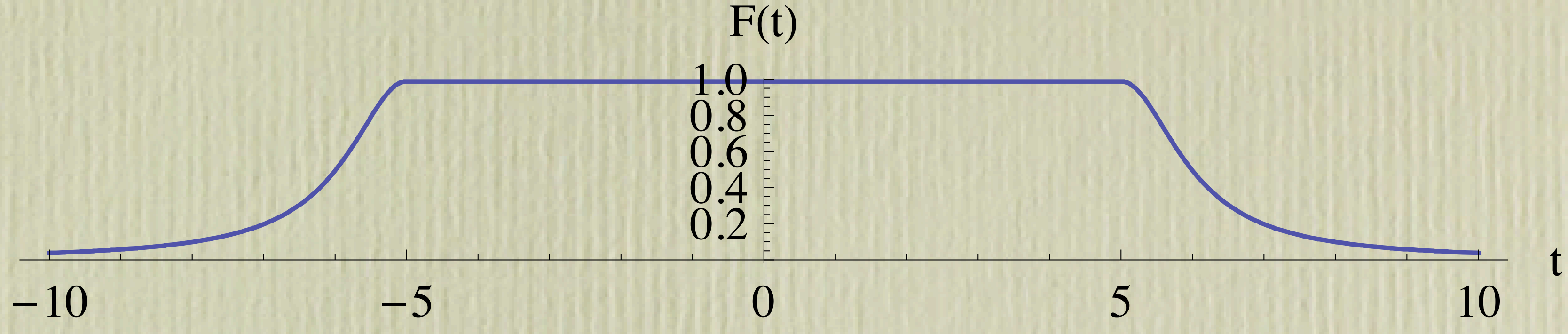
Smooth switching

- When a smooth switching function is considered, the velocity dispersion tends to decay away in late time. It doesn't matter whether a compactified switching function is used or not. For example, the case of Lorentzian function leads to the dispersion

$$\langle \Delta v_z^2 \rangle \approx \frac{e^2}{16\pi^2 m^2 \tau^2}$$

1. It is sensitive to switching function.
2. However it is not a realistic measuring function. There is no stable measuring period.

Lorentz-Plateau switching function



We here construct a switching function which is characterized by a stable measuring part (of a time scale τ_1) and two switching tails describing the turn-on and the turn-off processes (of a total time scale τ_2). This "Lorentz-plateau" function $F_{\tau\mu}(t)$ is a blend of a plateau part and the Lorentz function (see Fig. 1), characterized by two parameters τ and μ and defined as

$$F_{\tau\mu}(t) = 1 \quad (\text{for } |t| \leqslant \tau/2)$$
$$= \frac{\mu^2}{(|t|/\tau - 1/2)^2 + \mu^2} \quad (\text{for } |t| > \tau/2). \qquad (18)$$

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\begin{aligned}
\langle \Delta v_z^2 \rangle &= \langle \Delta v_z^2 \rangle_M + \langle \Delta v_z^2 \rangle_S + \langle \Delta v_z^2 \rangle_{MS} \\
&\approx \left\{ 1 + \frac{\pi^4 z^2 \tau_2^2}{4(\pi^2 z^2 + \tau_2^2)^2} - O(1)\frac{8}{3}\left(\frac{z}{\tau_1}\right)^2 \frac{\tau_2}{\tau_1} \right\} \langle \Delta v_z^2 \rangle_M .
\end{aligned}
$$

(i) When $\tau_2 \ll 2z \ll \tau_1$, $\langle \Delta v_z^2 \rangle \approx \langle \Delta v_z^2 \rangle_M$.

(ii) When $\tau_2 \approx 2z \ll \tau_1$, $\langle \Delta v_z^2 \rangle \approx \frac{3}{2} \langle \Delta v_z^2 \rangle_M$.

When the time scale τ_2 of the switching tails is much shorter than the time scale $2z$, the velocity dispersion $\langle \Delta v_z^2 \rangle$ reduces to the result of the sudden switching case given in Ref. [2] [case (i)]. As the time scale τ_2 increases up to around the time scale $2z$, however, $\langle \Delta v_z^2 \rangle$ becomes around $3/2$ times of $\langle \Delta v_z^2 \rangle_M$ [case (ii)]. It means that the contribution from the switching tails, $\langle \Delta v_z^2 \rangle_S$, is almost of the same order as the contribution from the measuring part, $\langle \Delta v_z^2 \rangle_M$. Hence the condition for the switching to be regarded as the "sudden switching" is $\tau_2 \ll 2z$, i.e., the switching time scale is much smaller than the scale characterizing the system configuration.

(iii) When $2z \ll \tau_1 \ll \tau_2$ and $\tau_2 / \tau_1 = O((\tau_1 / 2z)^2)$,

$$\langle \Delta v_z^2 \rangle \approx \langle \Delta v_z^2 \rangle_S .$$

Next, as the switching time τ_2 increases the velocity dispersion decreases, reducing to the Lorentzian switching case [Eq. (11)] at around $\tau_2 \sim O((\tau_1 / 2z)^2)\tau_1$ [case (iii)]. This occurs mainly due to the cancellation of the M term by the negative contribution from the MS term, which is actually the correlation between the switching part and the main measuring part.

Anti-correlation effect

$$\langle \Delta v_z^2 \rangle = \langle \Delta v_z^2 \rangle_M + \langle \Delta v_z^2 \rangle_S + \langle \Delta v_z^2 \rangle_{MS}$$

The interesting finding here is that $\langle \Delta v_z^2 \rangle_{MS}$ is negative!

After playing around three cases:

1. Main part and one tail
2. Single switching tail
3. Only switching tails

-> We conclude that the negative value is really the anti-correlation effect between the the switching tail and the main part!

#Is it easy to produce a sudden switch procedure?

The smooth switching function discussed in the paper would be interpreted as a mathematical description of this process of shooting a probe from a far distance. In this situation, the switching time scale is around z/v_0 and the intrinsic time scale determined by the system configuration is z. It is obvious that the switching time scale cannot be smaller than the intrinsic time scale z in this case, since $v_0 < 1$. This is just one example, but it at least shows that the sudden-switching approximation is not always valid and that we should be more careful about the switching effect in the process of the quantum vacuum.

Summary

- The velocity dispersion is sensitive to the switching effect. The criterion for sudden switching is when the switching time is much smaller than the intrinsic time scale.
- The non-vanished z-component in the sudden switching approach is the fluctuation due to the Casimir vacuum, but is not transient effect.
- The anti-correlation between the switching tail and the main measuring part provides a way to manipulate the total fluctuation.

Future works

- We are nervous about the point particle model and would like to study the effect using a wave packet.
- The boundary condition here is also needed to be improved.

Application: Cosmology

- Similar situations happen in elsewhere. e.g. The passive cosmological perturbation.