Trapping effect on inflation

Chung Yuan Christian University

May 24 , 2012

I-Chin Wang

Tamkang University

Collaborators: Wolung Lee, Chun-Hsien Wu and Kin-Wang Ng

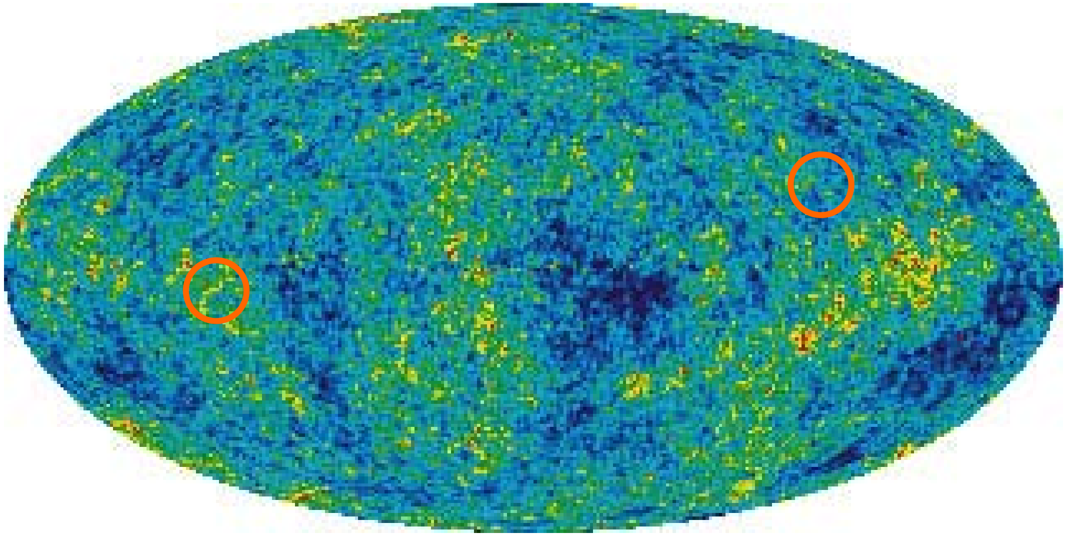
Outline

- Inflation
- Trapped inflation
- Trapping effects on inflation
- Numerical results
- Summary

Standard model for cosmology is established by

- Hubble's law in 1929

Horizon problem



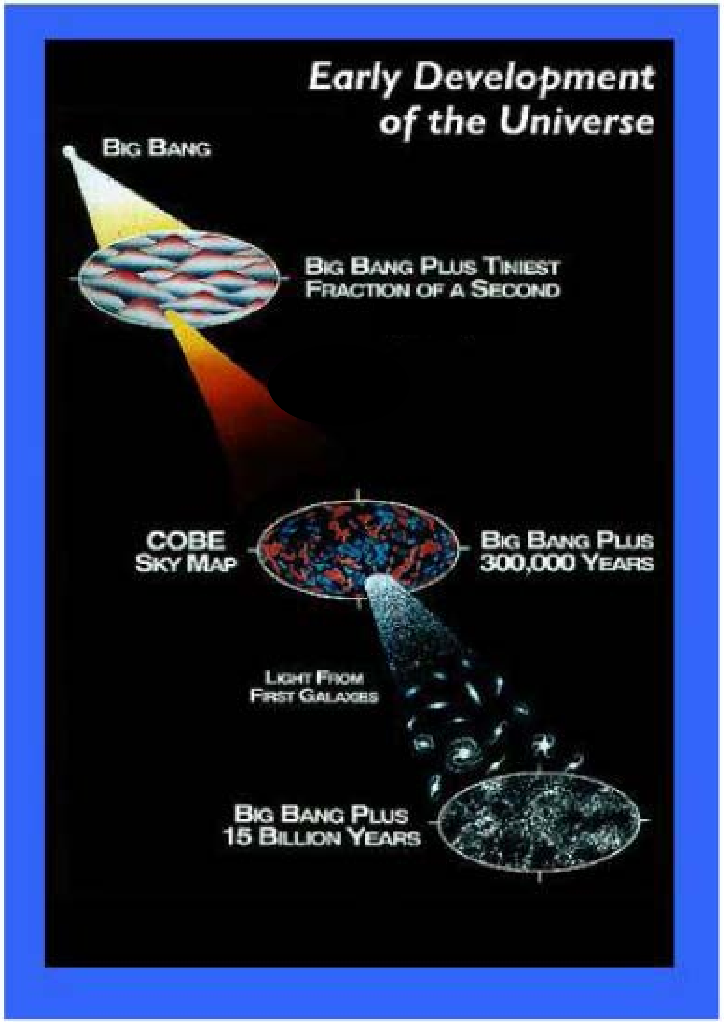
2.7k

10^{-4}k

Why do CMB photons have the same temperature?

Inflation

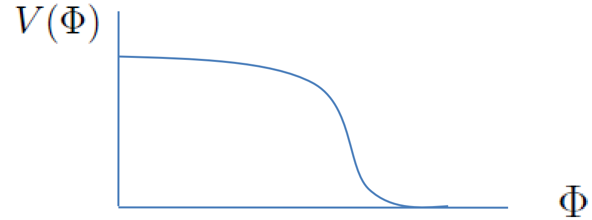
- A phase of exponential expansion can solve horizon, flatness and monopole problems



Inflation

$$ds^2 = g_{\mu\nu} dy^\mu dy^\nu = dt^2 - a^2(t) d\vec{y}^2$$

Slow rolling potential



$$\ddot{\phi}(t) + 3\frac{\dot{a}}{a}\dot{\phi}(t) + \frac{\partial V(\phi)}{\partial \phi} = 0 \quad , \quad \dot{\phi}(t) = \frac{d\phi(t)}{dt}$$

$$\frac{\dot{a}}{a} = H \qquad \rho = \frac{(\dot{\phi})^2}{2} + V \qquad p = \frac{(\dot{\phi})^2}{2} - V$$

Slow roll ⇒ $\ddot{\phi} \sim 0$, $\dot{\phi}$ small ⇒ $p = -\rho$, Λ de Sitter Space-time

during inflation $a(t) = e^{Ht}$ 60 e foldings

$$\left.\frac{\delta\rho}{\rho}\right| = \left.\frac{\delta\rho_\phi}{P_\phi + \rho_\phi}\right| = \left.\frac{\frac{\partial V}{\partial \phi}}{\dot{\phi}^2}\delta\phi\right| = \frac{-3H}{\dot{\phi}}\delta\phi$$

Horizon Cross-in — Horizon Cross-out — Horizon Cross-out

$$3H\dot{\phi} = -\frac{\partial V}{\partial \phi}$$

$$P_\phi = \left(\frac{\delta\phi}{\phi}\right)^2$$

$$P_R = \left(\frac{\delta\rho}{\rho}\right)^2$$

$$\delta\phi \propto \frac{H}{2\pi}$$ de Sitter quantum fluctuations

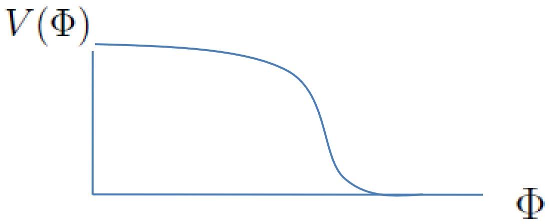
$$P_R = \frac{H^2}{\dot{\phi}^2} P_\phi$$

Basic idea

Slow roll

60 e foldings $a(t) = e^{Ht}$

- Flat potential



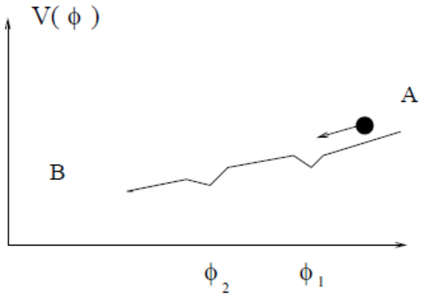
Slow rolling potential

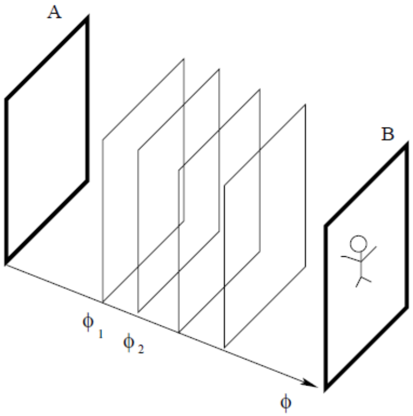
- Steep potential

Trapped inflation , potential is too steep for slow roll

JHEP 0405:030,2004 Kofman et al

Phys.Rev.D80:063533,2009 Green et al



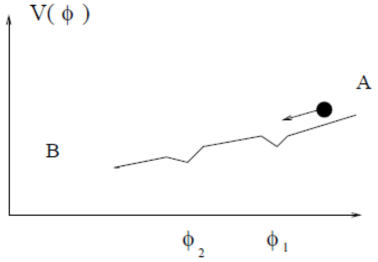


Trapped inflation

idea

- Particle production slow down the velocity while rolling down a steep potential

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{1}{2}\sum_i(\partial_\mu\chi_i\partial^\mu\chi_i - g^2(\phi-\phi_i)^2\chi_i^2)$$



Trapping effect

- Approximate the space-time by de Sitter metric

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$$

- A trapping point at

$$\phi = \phi_0$$

- Influence functional approach, trace out χ and then obtain the semiclassical Langevin equation

Ann. Phys. 24, 118(1963)
Feynman and Vernon

- Power spectrum

$$\langle \varphi_{\mathbf{k}}(\eta)\varphi^*_{\mathbf{k}'}(\eta)\rangle = \frac{2\pi^2}{k^3}\Delta^{\xi}_{k}(\eta)\delta(\mathbf{k}-\mathbf{k}')$$

P_ϕ

Our model

One trapped point

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \, \partial_\nu \Phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \, \partial_\nu \chi - V(\Phi) - \frac{g^2}{2} (\Phi - \Phi_0)^2 \chi^2$$

Shift the field $\phi = \Phi - \Phi_0$

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \, \partial_\nu \chi - V(\phi) - \frac{g^2}{2} \phi^2 \chi^2$$

Using in-in formalism, we trace out χ to obtain the effective action

$$\begin{aligned} S_{\text{eff}}[\phi, \phi_\Delta, \xi] \quad &= \int d^4x \, a^2(\eta) \, \phi_\Delta(x) \Big\{ -\ddot{\phi}(x) - 2aH\dot{\phi}(x) + \nabla^2 \phi(x) - a^2 \left[V'(\phi) + g^2 \langle \chi^2 \rangle \phi(x) \right] \\ &\quad - g^4 a^2 \phi(x) \int d^4x' \, a^4(\eta') \, \theta(\eta - \eta') \, iG_-(x, x') \phi^2(x') + g^2 a^2 \phi(x) \xi(x) \Big\} \end{aligned}$$

Main result

$$\ddot{\phi} + 2aH\dot{\phi} - \nabla^2\phi + a^2\left[V'(\phi) + g^2\langle\chi^2\rangle\phi\right] + g^4a^2\phi\int d^4x'a^4(\eta') \times$$
$$\theta(\eta - \eta')\, i\, G_-(x, x')\phi^2(x') = g^2\, a^2\, \phi\, \xi$$

where

$$G_-(x, x') = \langle\chi(x)\chi(x')\rangle^2 - \langle\chi(x')\chi(x)\rangle^2$$
$$\langle\xi(x)\xi(x')\rangle = G_+(x, x') = \langle\chi(x)\chi(x')\rangle^2 + \langle\chi(x')\chi(x)\rangle^2$$

The effect of χ on inflaton are presented by dissipation term and noise term.

Approximate solution

- Drop the dissipation term $g^2 \ll 1$
- Decompose $\phi(\eta, \mathbf{x}) = \bar{\phi}(\eta) + \varphi(\eta, \mathbf{x})$ mean field + classical perturbation

$$\ddot{\bar{\phi}} + 2aH\dot{\bar{\phi}} + a^2\left[V'(\bar{\phi}) + g^2\langle\chi^2\rangle\bar{\phi}\right] = 0$$

Langevin equation $$\ddot{\varphi} + 2aH\dot{\varphi} - \nabla^2\varphi + a^2 m_{\varphi\mathrm{eff}}^2\varphi = g^2\, a^2\, \bar{\phi}\,\xi$$

where $$m_{\varphi\mathrm{eff}}^2 = V''(\bar{\phi}) + g^2\langle\chi^2\rangle$$

$$\ddot{\chi} + 2aH\dot{\chi} - \nabla^2\chi + a^2 m_{\chi\mathrm{eff}}^2\chi = 0$$

where $$m_{\chi\mathrm{eff}}^2 = g^2\bar{\phi}^2$$

decompose

$$Y(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} Y_\mathbf{k}(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}}, \quad \text{where } Y = \varphi, \xi,$$

$$\chi(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \left[b_\mathbf{k} \chi_k(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right], \qquad [b_\mathbf{k}, b^\dagger_{\mathbf{k}'}] = \delta(\mathbf{k}-\mathbf{k}')$$

Solution for classical perturbation

$$\varphi_\mathbf{k}(\eta) = g^2 \int_{\eta_i}^{\eta} d\eta' a^2(\eta') \bar{\phi}(\eta') \xi_\mathbf{k}(\eta') G_r(\eta', \eta)$$

$$\chi_k(\eta) = \frac{1}{2a} (\pi|\eta|)^{\frac{1}{2}} \left[c_1 H^{(1)}_\mu(k\eta) + c_2 H^{(2)}_\mu(k\eta) \right] \quad , \quad \mu^2 = 9/4 - m^2_{\chi\text{eff}}/H^2$$

weak coupling limit $g^2 \ll 1$, $\mu = 3/2$

We can select Bunch-Davies vacuum to calculate the noise driven power spectrum

$$\langle \varphi_\mathbf{k}(\eta) \varphi^*_{\mathbf{k}'}(\eta) \rangle = \frac{2\pi^2}{k^3} \Delta^\xi_k(\eta) \delta(\mathbf{k} - \mathbf{k}'),$$

$$P_\phi$$

The noise-driven power spectrum

$$\Delta_k^{\xi}(\eta) = \frac{g^4 z^2}{8\pi^4} \int_{z_i}^{z} dz_1 \int_{z_i}^{z} dz_2\, \bar{\phi}(\eta_1)\bar{\phi}(\eta_2) F(z_1) F(z_2) \left\{ \frac{\sin z_-}{z_1 z_2 z_-} \left[\sin(2\Lambda z_-/k)/z_- - 1 \right] + G(z_1, z_2) \right\}$$

where

$$F(y) = \left(1 + \frac{1}{yz}\right) \sin(y - z) + \left(\frac{1}{y} - \frac{1}{z}\right) \cos(y - z)$$

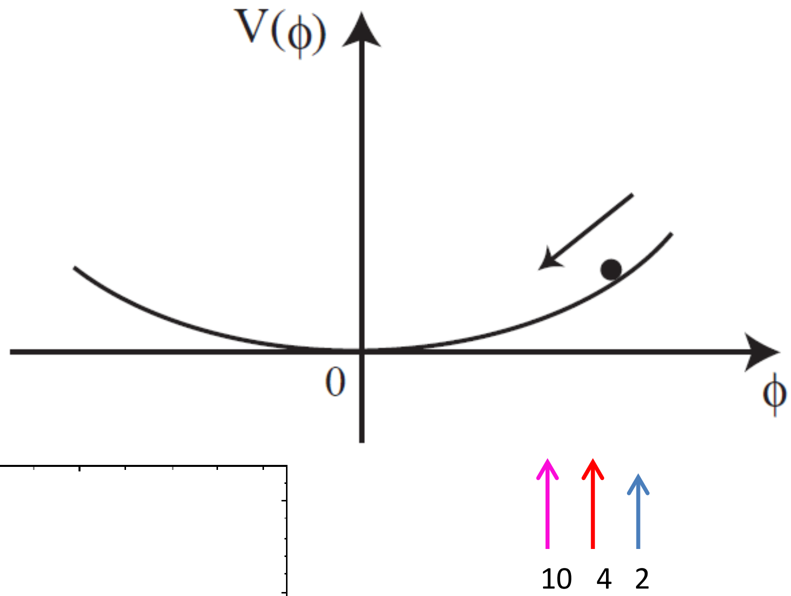
$$\begin{aligned}
& G(z_1, z_2) \\
&= \int_0^{\Lambda} dk_1 \int_{|k-k_1|}^{k+k_1} dq \left\{ \left[\frac{2}{z_1 z_2 q k_1} \left(\frac{k_1}{q} - \frac{z_1}{z_2} - \frac{z_2}{z_1} + 2 + \frac{k^2}{z_1 z_2 k_1 q} \right) \right] \cos\left[\left(\frac{k_1 + q}{k} \right) (z_2 - z_1) \right] \right. \\
&\quad \left. + \frac{2}{kq} \left(\frac{1}{z_1} - \frac{1}{z_2} \right) \left[1 + \frac{k^2}{z_1 z_2 k_1} \left(\frac{1}{q} + \frac{1}{k_1} \right) \right] \sin\left[\left(\frac{k_1 + q}{k} \right) (z_2 - z_1) \right] \right\} \\
&\quad + \frac{2}{z_1 z_2} \int_0^{\Lambda} \frac{dk_1}{k_1} \left\{ \cos(z_2 - z_1) - \cos\left[(z_2 - z_1) \left(\frac{2k_1}{k} - 1 \right) \right] \right\} \\
&\quad + \frac{2k}{z_1 z_2 (z_2 - z_1)} \left\{ \int_0^{\Lambda} \frac{dk_1}{k_1^2} \sin\left[(z_2 - z_1) \left(1 + \frac{2k_1}{k} \right) \right] \right. \\
&\quad \left. - \int_0^{k} \frac{dk_1}{k_1^2} \sin(z_2 - z_1) - \int_k^{\Lambda} \frac{dk_1}{k_1^2} \sin\left[(z_2 - z_1) \left(\frac{2k_1}{k} - 1 \right) \right] \right\}
\end{aligned}$$

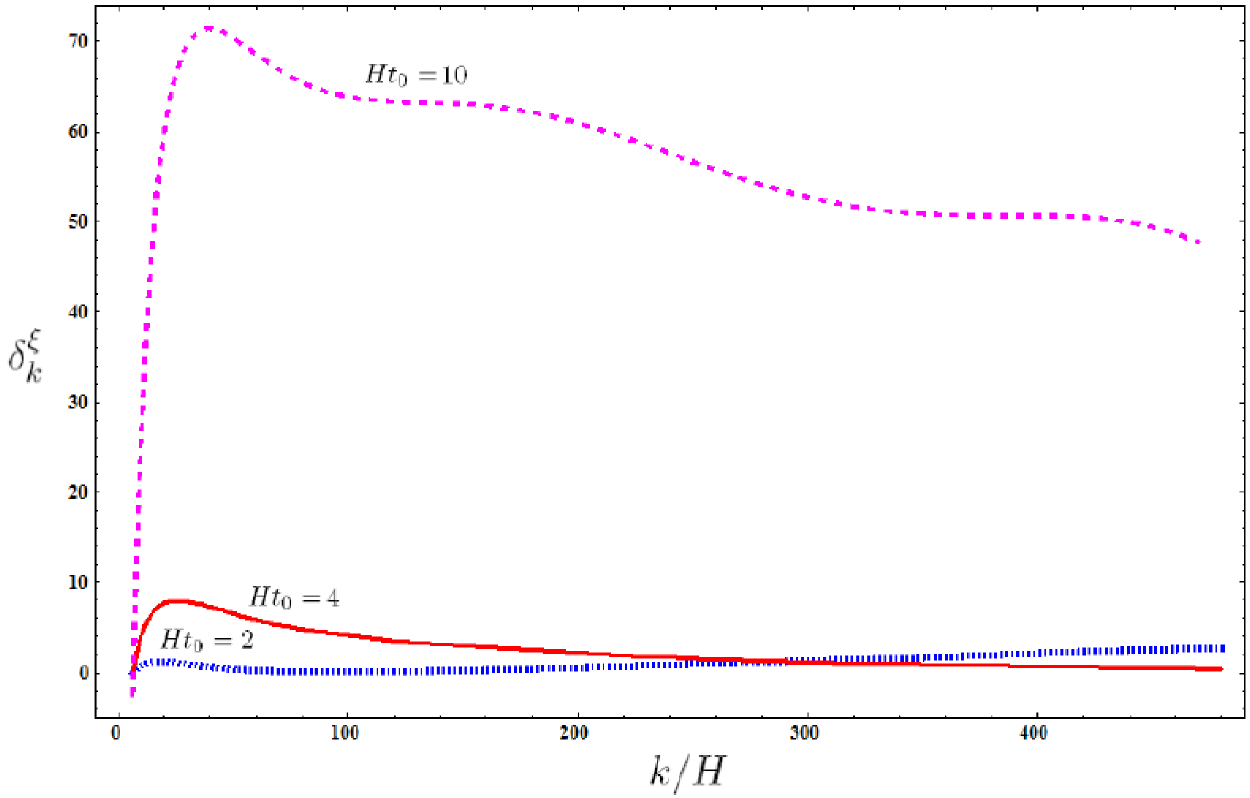
$$\bar{\phi}(\eta) = v(t_0 - t) = \frac{v}{H} \ln \frac{\eta}{\eta_0}$$

Background field ~ Chaotic inflation

Power spectrum for a trapping point

Trapping point at $Ht_0 = 2, 4, 10$

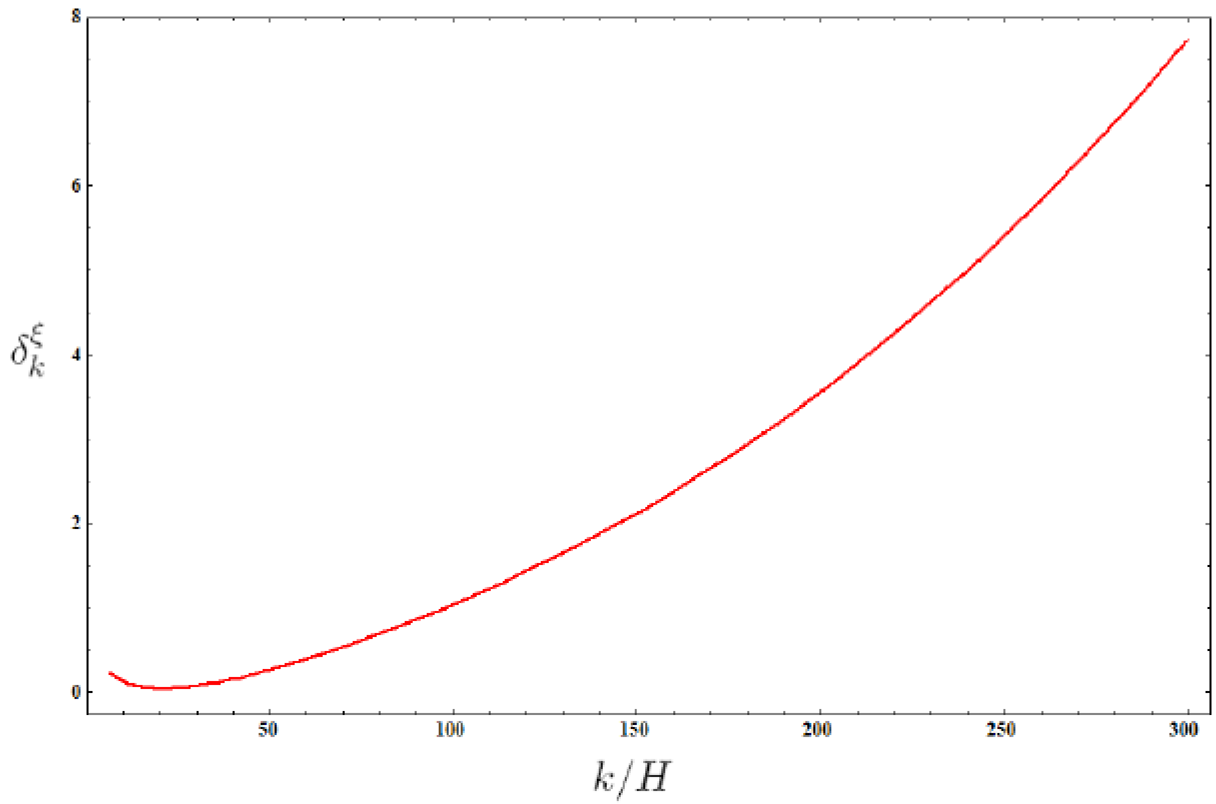


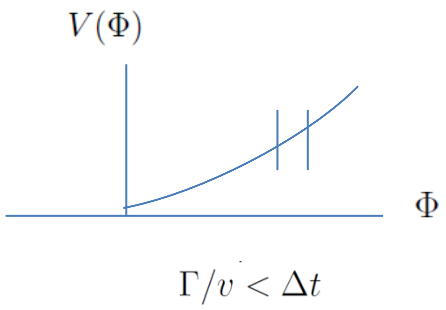


Closely spaced trapping points

$$g^2 \simeq 10^{-7}$$

$$\Delta_k^\xi(\eta) = \frac{g^4\Gamma^4 H^2}{8\pi^4 v^2}\int_{z_i}^{z} dz_1 \int_{z_i}^{z} dz_2\, z_1 z_2 \left\{ \frac{\sin z_-}{z_-}\left[\sin(2\Lambda z_-/k)/z_- - 1\right] + G(z_1, z_2)\right\}$$





Smaller than de Sitter quantum fluctuation by factor about 400

Summary

- By influence functional method, the trapping effects contains two terms

  the noise term - the particle number density fluctuation

  the dissipation term