Dispute on k_T factorization

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Ref: 0807.0296, 0808.1526

Outlines

- Introduction
- k_T factorization
- Dispute on gauge invariance
- Correct results
- Summary

Introduction

- Dispute on gauge invariance of k_T factorization:
- 0704.3790 (Nandi, Li)---computed one-loop correction to pion transition form factor in k_T factorization
- 0807.0296 (Feng, Ma, Wang)---the above contains gauge-dep light-cone singularity
- 0808.1526 (Li, Mishima)---FMW are wrong
- 0808.4017 (FMW)---LM are wrong
- 0907.0166 (LM)---final response

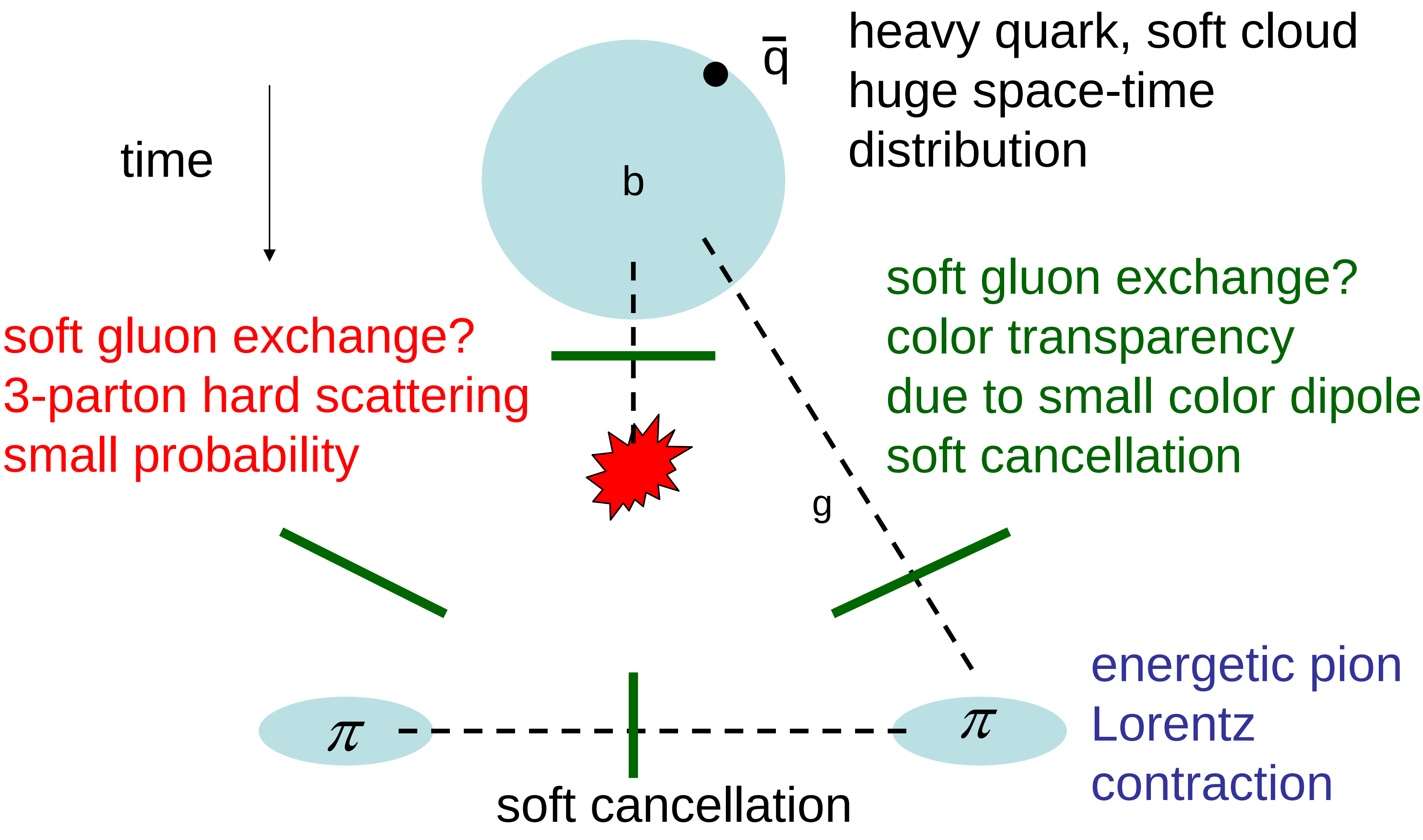
Factorization theorem

- QCD Lagrangian $\mathcal{L}=\overline{\psi}(iD^{\mu}\gamma_{\mu}-m)\psi-F^{\mu\nu}F_{\mu\nu}/4$
- Confinement at low energy, hadronic bound states: pion, proton, B meson,….
- Asymptotic freedom at high energy $\Rightarrow$ a small coupling constant $\Rightarrow$ perturbation
- Test QCD at hgih-energy scattering!
- Nontrivial due to involved hadrons
- A sophisticated prescription is necessary
- Dramatically different dynamics factorizes

$\Rightarrow$ Factorization theorem

An example: B decays

- With hard scattering (large energy release)



The concern is how to calculate hard kernel in a gauge-invariant way

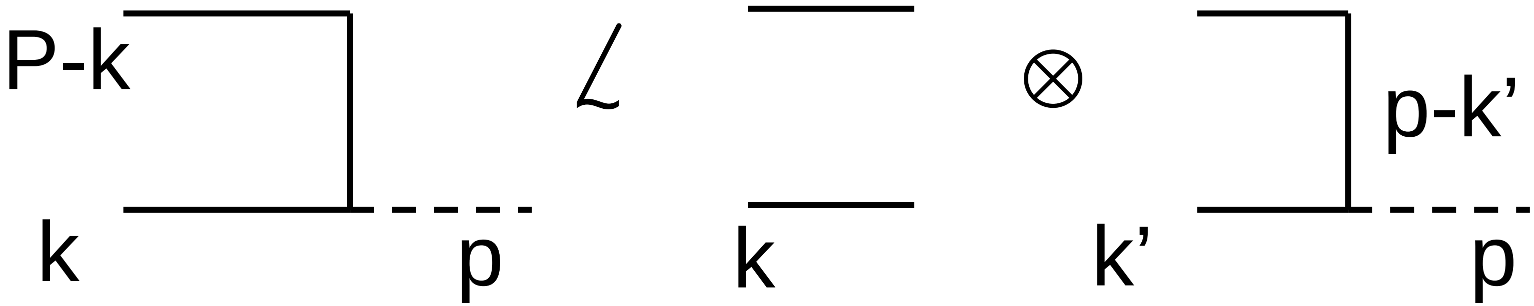
Hadron wave function describes probability of parton carrying fractional momentum.

Hard kernel is convoluted with model wave function, so it must be gauge invariant.

k_T factorization

Pion transition form factor

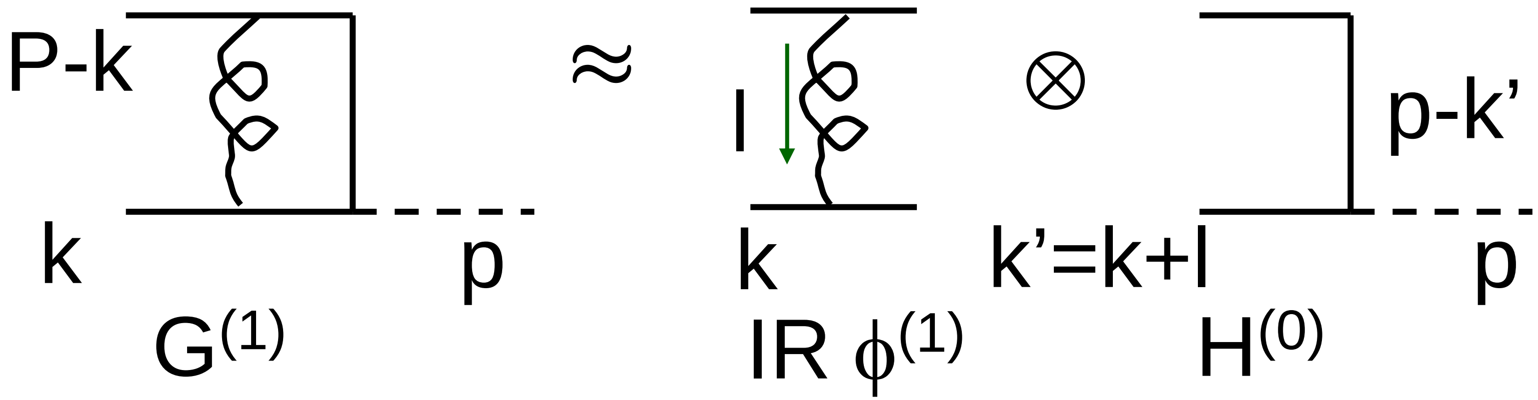
- $\pi^0(P)\gamma^* \to \gamma(p)$, $Q^2=2P.p$ at LO



Full QCD	Effective theory	Wilson coeff
$G^{(0)}$	Wave function $\phi^{(0)}$	Hard kernel $H^{(0)}$

- $G^{(0)}(x,Q^2)=\int dx'\phi^{(0)}(x;x')H^{(0)}(x',Q^2)$
- $H^{(0)} \propto 1/(p-k')^2 \propto 1/(x'Q^2)$, $k'=(x'P^+,0,0_T)$

NLO collinear factorization



- At $O(\alpha_s)$, collinear divergence is generated
- $l \parallel P \Rightarrow l^+ \sim P^+ \gg l_T \sim \Lambda \gg l^- \sim \Lambda^2/Q$,

  $\Rightarrow P^2 \sim l^2 \sim O(\Lambda^2) \Rightarrow$ On-shell gluon

- $(p-k-l)^2 = -xQ^2 - 2p^- l^+ + 2k^+ l^- + 2l^+ l^- - l_T^2$
- Drop l^- and l_T, $H^{(0)} \propto 1/(x + l^+/P^+)Q^2$

- Collinear factorization: $k' = (k^+ + l^+, 0, 0_T)$

NLO k_T factorization

- k_T factorization works for small x region
- At small x, xQ^2 is small ~ k_T
- Drop l^- only, $H^{(0)} \propto 1/[(x+l^+/P^+)Q^2+l_T^2]$
- k_T factorization: $k'=(k^+ + l^+,0,l_T)$
- $G^{(1)}(x,Q^2)=\int dx'dk'_T \phi^{(1)}(x;x',k'_T)H^{(0)}(x',k'_T,Q^2)$

  $+H^{(1)}(x,Q^2)$
- Radiative gluon modifies both parton longitudinal and transverse momenta.
- Need wave function $\phi_\pi(\xi,k_T)$ to describe the probability.

Wave function vs. distribution amplitude

- Neglect k- in hard kernel. k- can be integrated out in wave function

$$\phi(x,k_T)=\int dk^-\psi(k^+\equiv xp^+,k^-,k_T)$$

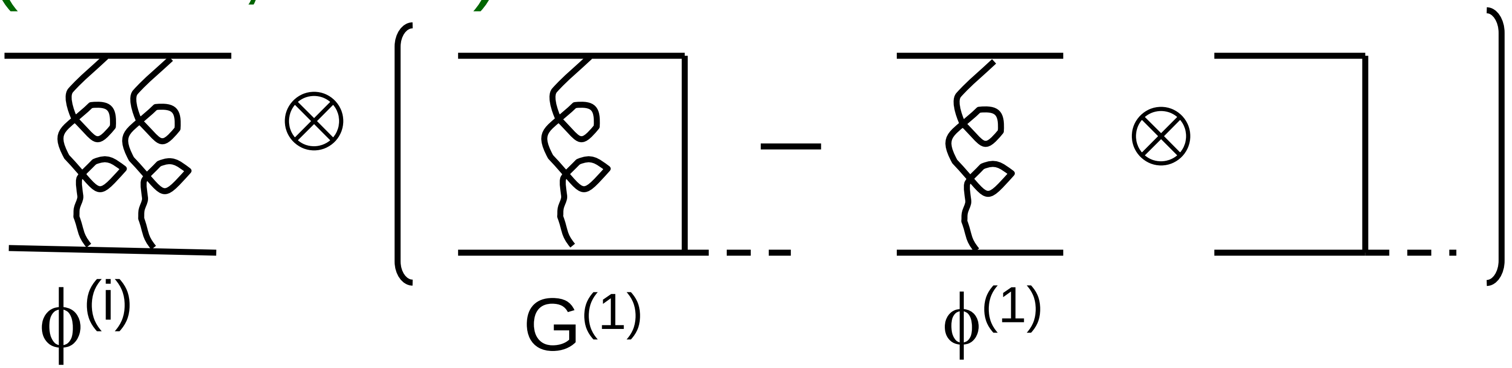
- Parton in hard kernel carries momentum (k+, 0, k_T). It is off-shell in k_T factorization.
- Further neglect k_T in hard kernel. Define DA $\phi(x)$ with k_T integrated out,

$$\phi(x)=\int d^2k_T\int dk^-\psi(k^+\equiv xp^+,k^-,k_T)$$

- Parton carries (k+,0,0). It is on-shell in collinear factorization.

$H^{(1)}$ in k_T factorization

- Beyond NLO, partons in $H^{(1)}$ are off-shell (Nandi, Li 07)



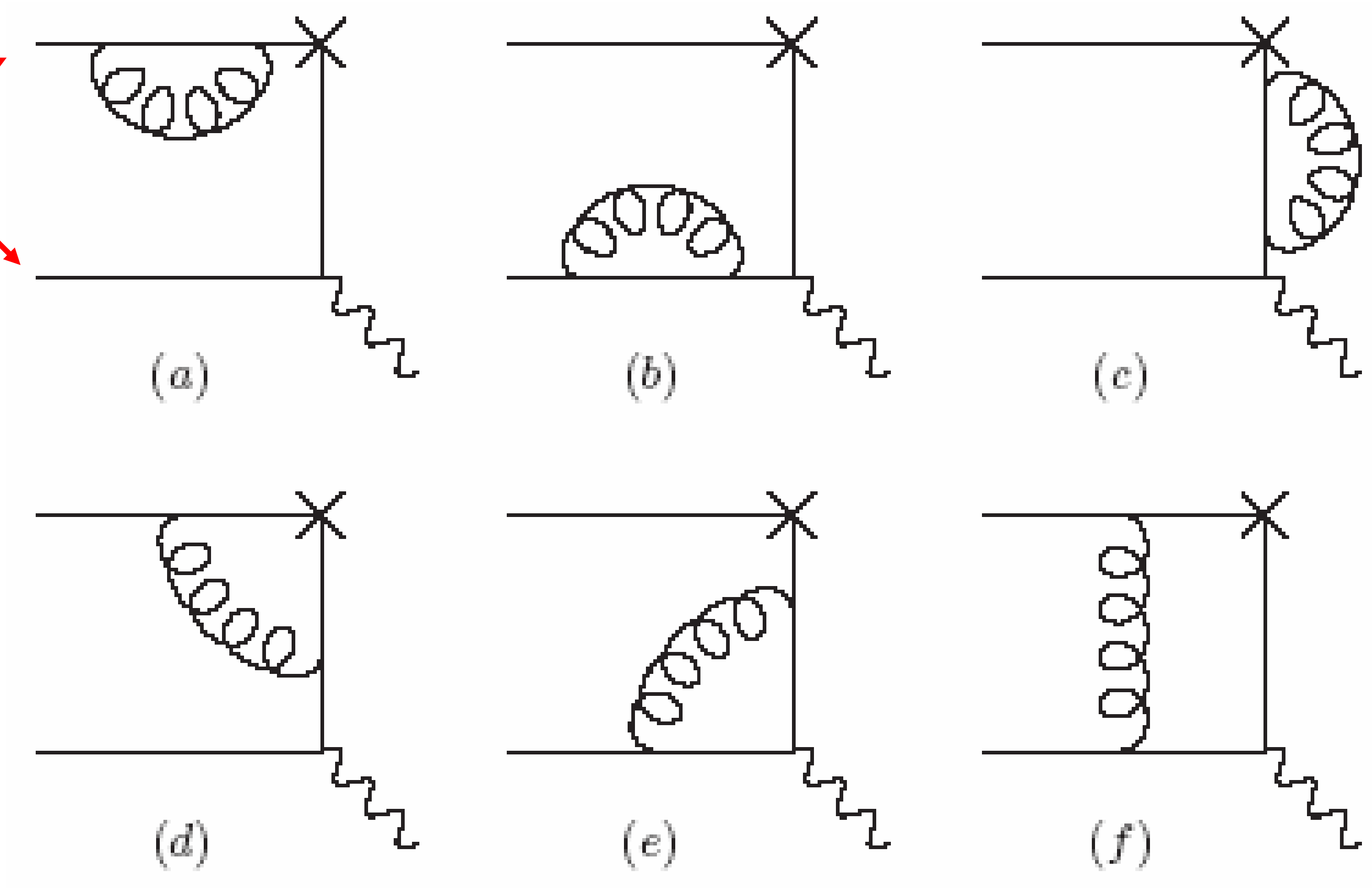
Initial parton $k=(xP^+,0,k_T)$ in $G^{(1)}$ and $\phi^{(1)}$

- $H^{(1)}(x,k_T,Q^2)=G^{(1)}(x,k_T,Q^2) - \int dx' dk'_T \phi^{(1)}(x,k_T;x',k'_T) H^{(0)}(x',k'_T,Q^2)$

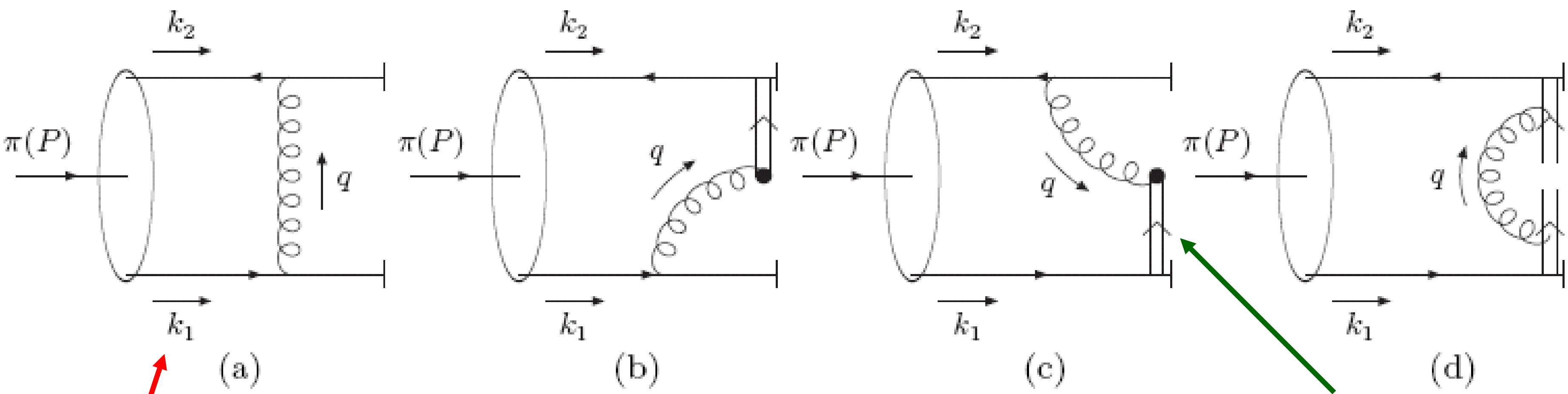
- Have shown IR divergences cancel between $G^{(1)}$ and $\phi^{(1)}$

Full QCD diagrams $G^{(1)}$

Off-shell by k_T^2



Some effective diagrams $\Phi^{(1)}$



Off-shell by k_T^2

l flows through hard kernel

Wilson line $1/n \cdot l$

Gauge Invariance

- Hard kernel must be gauge invariant.
- In collinear factorization, partons entering H are on-shell. Gauge invariant!
- Partons off-shell by k_T^2. Quark diagrams (full QCD) and effective diagrams (wave function) depend on gauge.
- It was proved using induction by Nandi and Li that gauge dependences in $G^{(1)}$ and $\phi^{(1)}$ cancel, and $H^{(1)}$ is gauge-invariant.
- No explicit check at NLO

Dispute on gauge invariance

Story started during KITPC 4-week flavor program at Beijing in July 2008

On-shell partons in k_T?

- Ma postulated that partons in H should be on shell in order to have explicit gauge invariance, $k=(k^+, k^-, k_T)$, $k^2 = 0$.
- How to define this wave function?

$$\phi(x,k_T) = \int dk^- \psi(k^+,k^-,k_T)\delta(k^2)\delta((P_1-k)^2)?$$

- Not make sense
- Another on-shell parton $(P\text{-}k)^2 = 0 \Rightarrow P.k=0 \Rightarrow k^- = 0$, and then $k^2 = 0 \Rightarrow k_T = 0$
- back to collinear factorization!

Gauge-dependent IR singularity

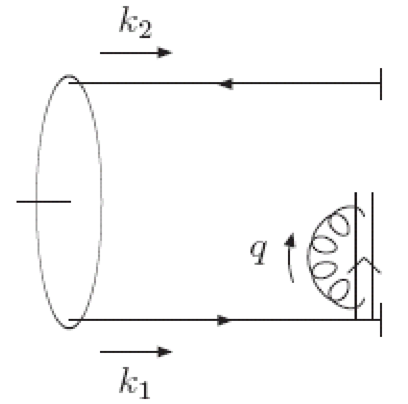
- FMW identified gauge-dependent IR singularity in NL's off-shell formalism
- Gluon propagator in covariant gauge

$$\frac{-i}{q^2 - \lambda_L^2 + i\varepsilon}\left[g^{\mu\nu} - \alpha\frac{q^\mu q^\nu}{q^2 - (1-\alpha)\lambda_L^2 + i\varepsilon}\right]$$

Gauge parameter

- Effective diagrams have gauge-dependent IR singularity, not cancelled by full diagrams. Gauge-dependent hard kernel.

FMW's calculation

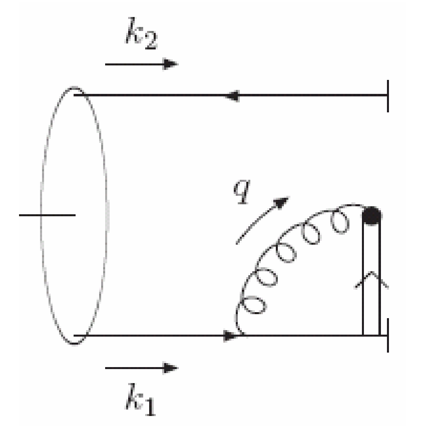


- Feynman parametrization for 2(d)

$$I = 16i\alpha g_s^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - \lambda_L^2 + i\varepsilon)[q^2 - (1-\alpha)\lambda_L^2 + i\varepsilon]}$$

$$I = \frac{4\alpha\alpha_s}{\pi} \ln \frac{\lambda_L^2}{\mu^2} + \text{UV pole}$$

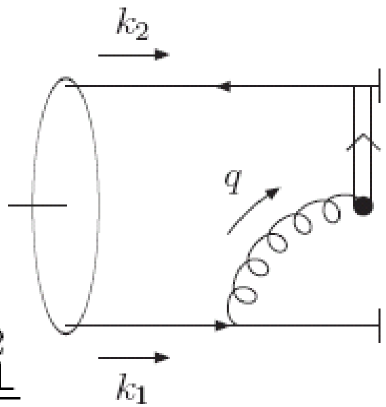
- Feynman parametrization for 3(c)



vanish in light-cone region $(q^+, q^-, q_\perp) \sim (\delta^2, 1, \delta)$

$$\phi_\alpha|_{3c} \otimes H^{(0)} = -\frac{16i\alpha g_s^2}{P^+} \int \frac{d^4q}{(2\pi)^4} \frac{2(k_1^+ - q^+)q^- - \vec{k}_{1\perp} \cdot \vec{q}_\perp + q_\perp^2}{[(k_1 - q)^2 + i\varepsilon](q^2 + i\varepsilon)^2(x_0 Q^2 + k_{1\perp}^2)}$$

FMW's calculation



- Contour integration for 2(b)

$$\phi_\alpha|_{2b} = 16i\alpha g_s^2 \int \frac{d^4q}{(2\pi)^4} \frac{2(k_1^+ - q^+)q^- - \vec{k}_{1\perp}\cdot\vec{q}_\perp + q_\perp^2}{[(k_1-q)^2 + i\varepsilon](q^2+i\varepsilon)^2}$$

$$\delta(k^+ - (k_1^+ - q^+))\delta^2(\vec{k}_\perp - (\vec{k}_{1\perp} - \vec{q}_\perp))$$

$$\phi_\alpha|_{2b} \otimes H^{(0)} = \int_0^1 dx \int d^2k_\perp \frac{1}{xQ^2 + k_\perp^2}\phi_\alpha|_{2b}$$

$$\phi_\alpha^{\mathrm{FMW}}|_{2b} \otimes H^{(0)} = \frac{4\alpha\alpha_s}{P^+\pi}\frac{\ln\lambda_L^2}{x_0Q^2 + k_{1\perp}^2}$$

loop momentum flows through hard kernel

- gauge-dependent light-cone singularity exists. But full diagram (e) gauge invariant

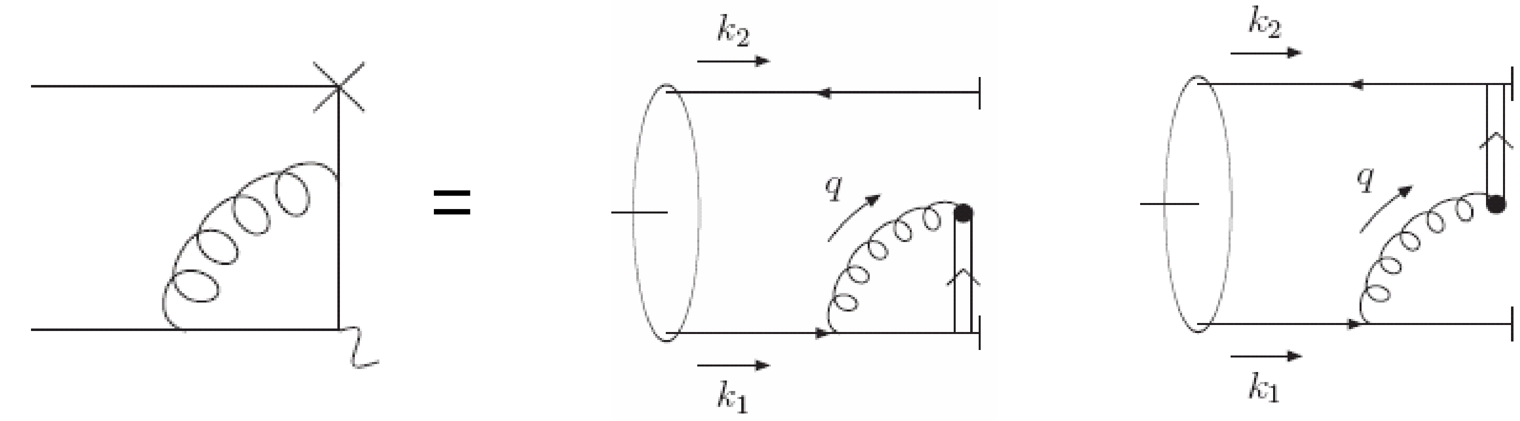
Repeated their calculation in first two weeks. Could not find any mistake. But…

Ward identity

- q^\nu in the gauge-dependent term hints Ward identity

LO hard kernel

$$\frac{\not p - \not k_1}{(p-k_1)^2}\,\not q\,\frac{\not p - \not k_1 + \not q}{(p-k_1+q)^2} = \frac{\not p - \not k_1}{(p-k_1)^2} - \frac{\not p - \not k_1 + \not q}{(p-k_1+q)^2}$$



no light-cone = no light-cone light-cone

- How could it be possible?

Finally, found that the puzzle came from contour integration in the 3rd week

when I jogged on CAS campus

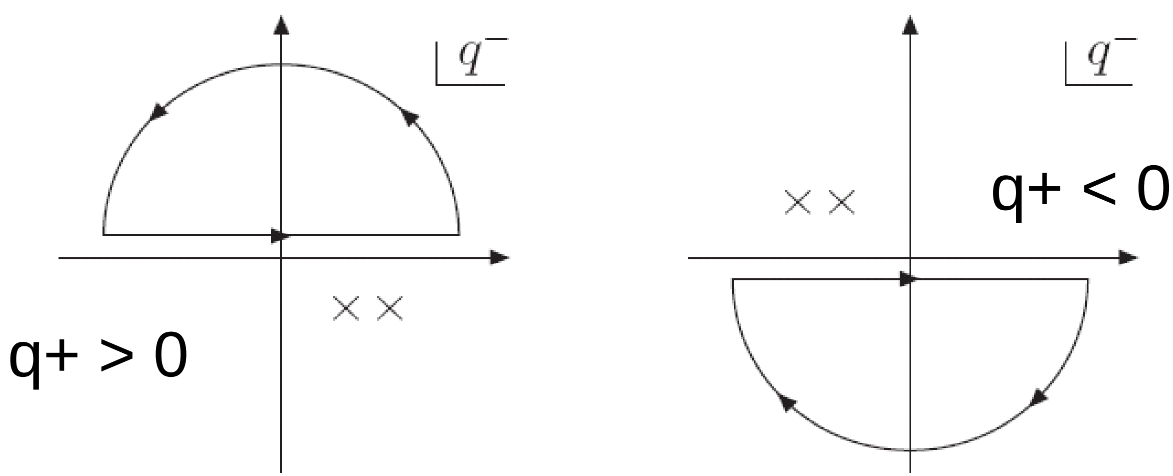
at midnight

Key integral

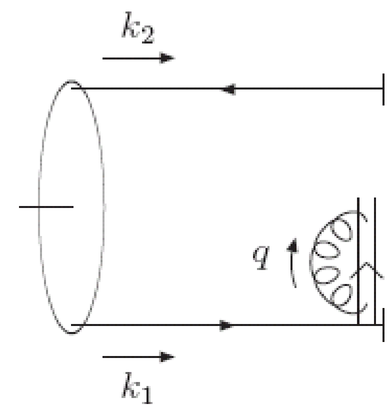
- How to calculate in the contour integration

$$I = \int d^4q \frac{1}{(q^2 - \lambda^2 + i\varepsilon)^2} = \int d^4q \frac{1}{(2q^+q^- - q_T^2 - \lambda^2 + i\varepsilon)^2}$$

- Feynman parametrization, I=IR+UV
- Contour integration, I=0. Why?



Ambiguity at infinity

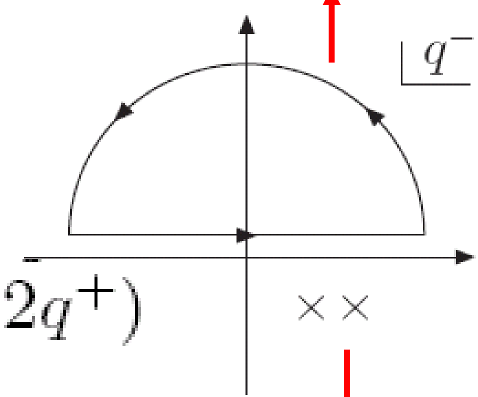


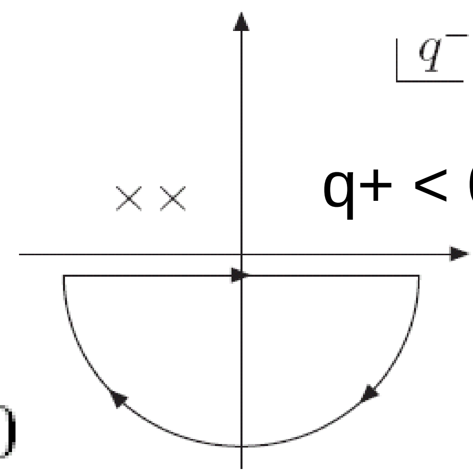
- Apply contour integration to 2(d)

$$I = 16 i \alpha g_s^2 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(2q^+q^- - q_\perp^2 - \lambda_L^2 + i\varepsilon)[2q^+q^- - q_\perp^2 - (1-\alpha)\lambda_L^2 + i\varepsilon]} = 0$$

infinity

q+ > 0





q+ < 0

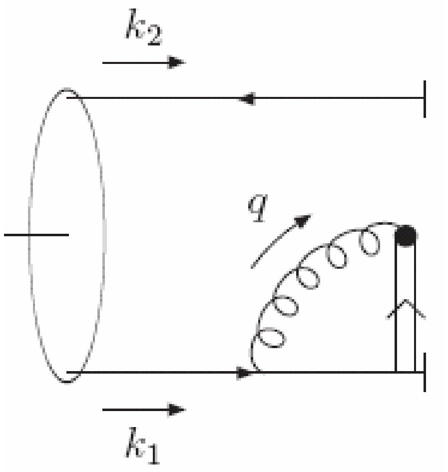
$$q^- = (q_\perp^2 + \lambda_L^2)/(2q^+)$$

$$q^- = [q_\perp^2 + (1-\alpha)\lambda_L^2]/(2q^+)$$

$$q^+ \rightarrow 0$$

Infinity, poles enter semicircle?

Something wrong



- Also apply contour integration to 3(c)
- I=IR+UV, different from Feynman parametrization, which is zero.
- Found what is wrong!
- Asked Ma to check contour integrations of 2(d) and 3(c)

Ma's responses

- Ma ignored my request for 2(d)
- He got zero for 3(c)!!!!!
- He dropped IR regulator, and got two light-cone singularities, which cancel
- one from $(q^+, q^-, q_\perp) \sim (\delta^2, 1, \delta)$
- another from $(1, \Lambda^2, \Lambda) \quad \Lambda \to \infty$
- Why $q^2 \sim \Lambda^2 \to \infty$ on light cone?
- He ignored this question completely!
- He used the trick of IR + UV cancellation for scaleless integral

Correct contour integration

Finite semicircle

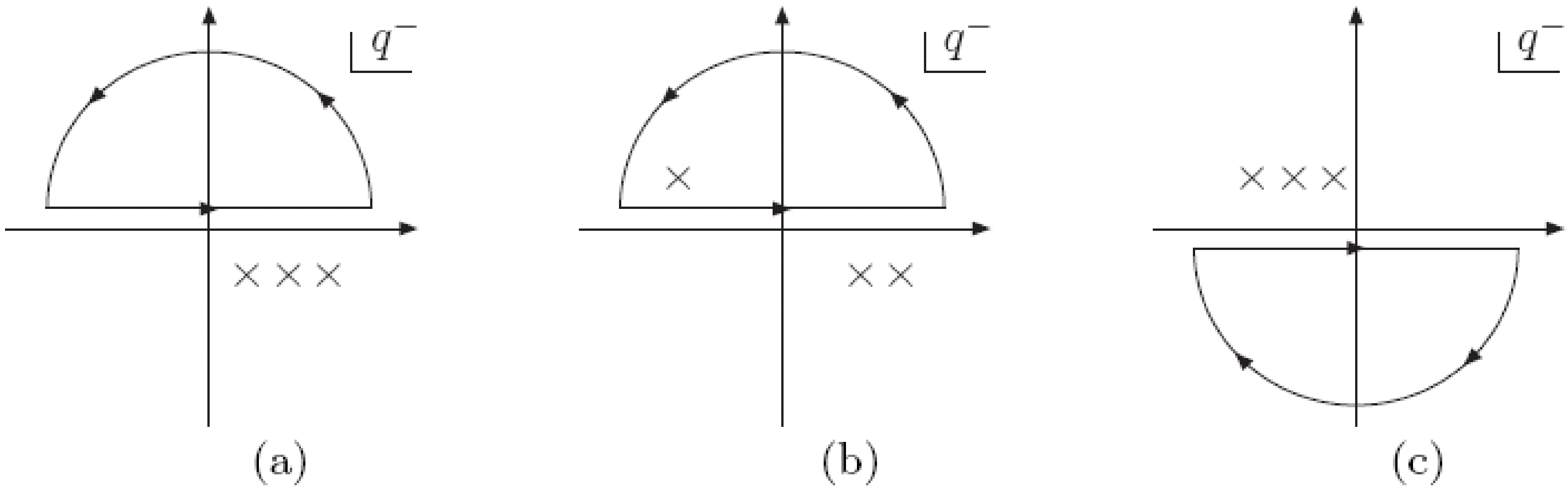
- Keep finite radius for semicircles first (Ma's idea actually)
- Poles never enter semicircle
- Ambiguity at infinity avoided
- 2(d) as an example

$$I = 16i\alpha g_s^2 \lim_{R\to\infty} \int \frac{d^2q_\perp}{(2\pi)^4} \left[i\int_\pi^0 d\theta \int_0^\infty dq^+ + i\int_{-\pi}^0 d\theta \int_{-\infty}^0 dq^+ \right] \times \frac{Re^{i\theta}}{(2q^+Re^{i\theta} - q_\perp^2 - \lambda_L^2 + i\varepsilon)[2q^+Re^{i\theta} - q_\perp^2 - (1-\alpha)\lambda_L^2 + i\varepsilon]}$$

- Result the same as from Feynman parametrization

Correct result for 2(b)

- Consider finite semicircles for 2(b)



(a) $q^+ > k_1^+$, (b) $0 < q^+ < k_1^+$, and (c) $q^+ < 0$

- Contribution from semicircles cancels FMW's light-cone singularity!

$$\phi'_\alpha|_{2b} \otimes H^{(0)} = -\frac{4\alpha\alpha_s}{P^+\pi} \frac{\ln \lambda_L^2}{x_0 Q^2 + k_{1\perp}^2}$$

Ma’s recent reply

- I said this dispute should be settled down between us. Other people won’t check calculation. Don’t waste resource of the community.
- He said “just send your comment to the archive”. We did.
- In their reply to our comment, Ma denied his idea of including semicircle…
- Result is semicircle-dependent? They missed the point.

Summary

- Pay attention to ambiguity from infinity when applying contour integration
- Effective diagrams have no gauge-dependent light-cone singularity. Ward identity is satisfied
- k_T-dependent hard kernel is gauge invariant and free of light-cone singularity
- I learned a lot from this dispute, especially about definition of k_T-dependent wave function

Status

- Ma still ignored the following simple questions:
- Why the simple integral 2(d)=0 in contour integration? $I = \int d^4q \frac{1}{(q^2-\lambda^2+i\varepsilon)^2} = 0 \quad ?$
- Why dropped IR regulator in 3(c), using IR+UV cancellation intentionally?
- Why off-shell gluon $q^2 \sim \Lambda^2 \rightarrow \infty$ on light cone?