Four-fermion operators in Heavy Meson χ Perturbation Theory

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D.Arndt and C.-J.D.L., PRD 70:014503 (2004).
W.Detmold and C.-J.D.L., PRD 76:14501 (2007).

Outline

- Introduction
- Heavy meson χPT (HMχPT)
- Finite volume effects

D.Arndt and C.-J.D.L., PRD 70:014503 (2004).

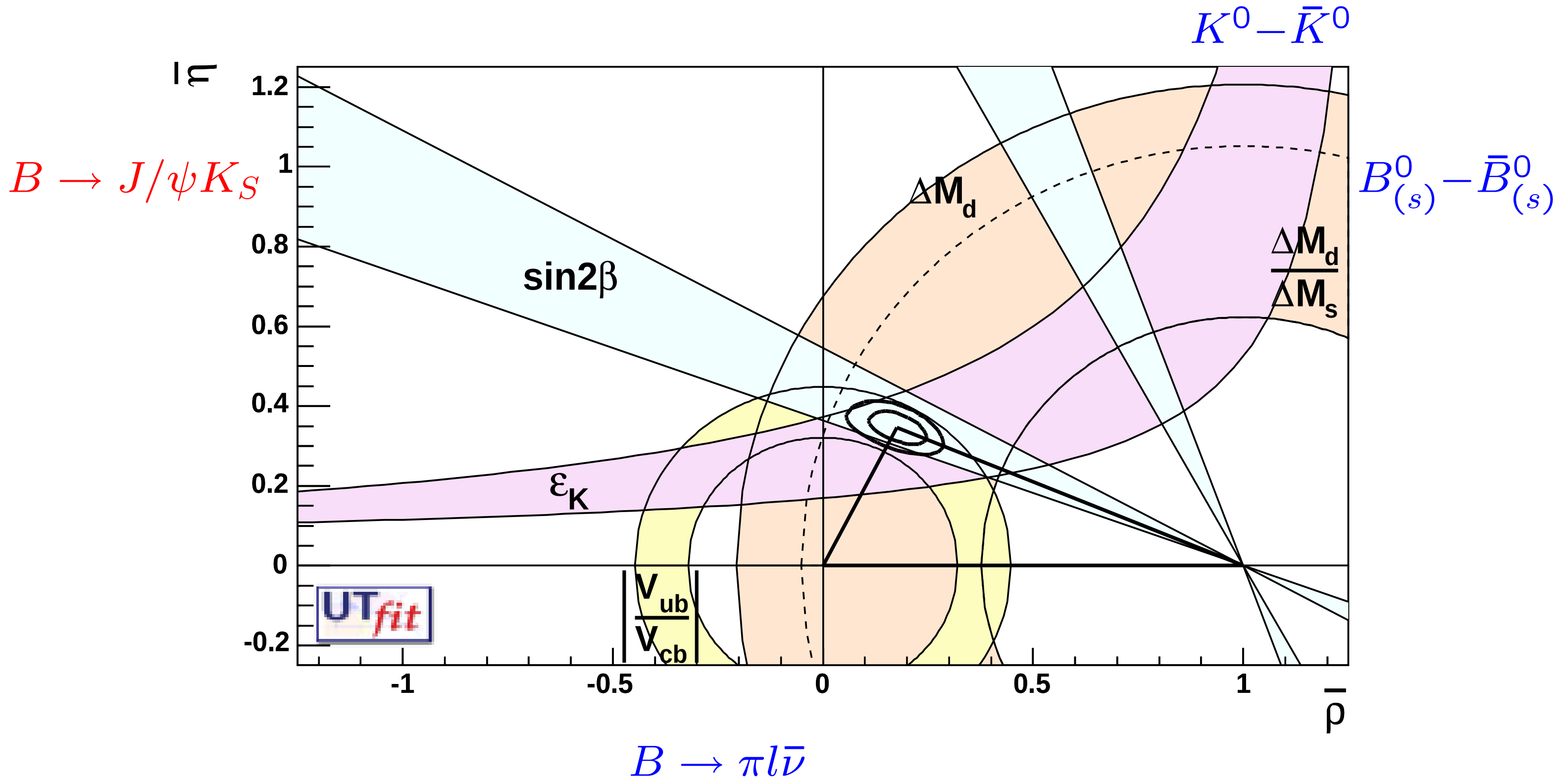
- Four-fermion operators in BSM physics

W.Detmold and C.-J.D.L., PRD 76:14501 (2007).

- Conclusions and outlook

The global CKM fit

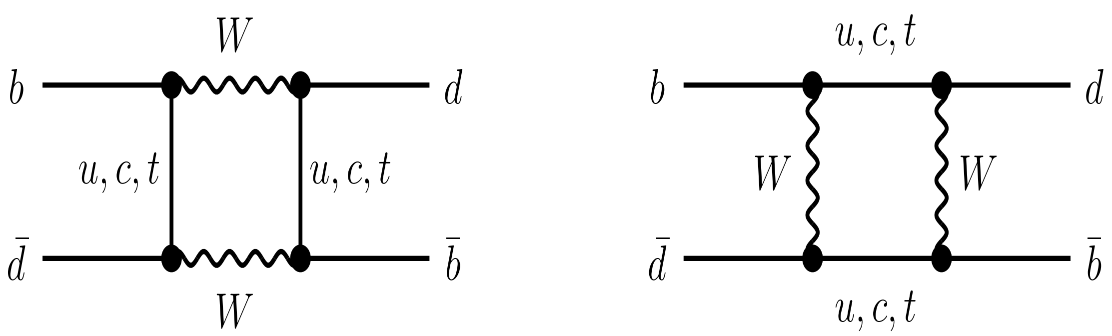
Exp $= V_{ij}$ (short-distance effects)$\otimes$(long-distance QCD matrix element)



α & γ : $B \to M_1 M_2$. Need ϕ_{B,M_1,M_2}.

B^0–$\bar{B}^0$ mixing

- The Standard Model contribution:



$$\mathcal{O}_1 = \overline{b}_\alpha \gamma^\mu (1-\gamma_5) d_\alpha \overline{b}_\beta \gamma^\mu (1-\gamma_5) d_\beta .$$

- BSM

$$\mathcal{O}_2 = \overline{b}_\alpha (1-\gamma_5) d_\alpha \overline{b}_\beta (1-\gamma_5) d_\beta, \ \ \mathcal{O}_3 = \overline{b}_\alpha (1-\gamma_5) d_\beta \overline{b}_\beta (1-\gamma_5) d_\alpha .$$

$$\mathcal{O}_4 = \overline{b}_\alpha \gamma^\mu (1-\gamma_5) d_\alpha \overline{b}_\beta \gamma^\mu (1+\gamma_5) d_\beta, \ \ \mathcal{O}_5 = \overline{b}_\alpha \gamma^\mu (1-\gamma_5) d_\beta \overline{b}_\beta \gamma^\mu (1+\gamma_5) d_\alpha .$$

Lattice QCD on one slide

- Partial quenching (limited by algorithm):

$$\begin{aligned}\langle \mathcal{O} \rangle &= \frac{1}{Z}\int [dA]\,[d\psi]\,\left[d\bar{\psi}\right]\,\mathcal{O}\ \mathrm{e}^{-S_{\mathrm{QCD}}} \\ &= \frac{1}{Z}\int [dA] \prod_i \det\left(\not{D}+m_i\right) \mathrm{e}^{-S_g[A]} \mathcal{F}\left(\frac{1}{\not{D}+m_i}\right) \\ &\xrightarrow{\mathrm{PQ}} \frac{1}{Z_{\mathrm{PQ}}}\int [dA] \prod_{i,gi,si} \left[\frac{\det\left(\not{D}+m_i\right)}{\det\left(\not{D}+m_{gi}\right)}\right]_{m_{gi}=m_i} \det\left(\not{D}+m_{si}\right) \mathrm{e}^{-S_g[A]} \mathcal{F}\left(\frac{1}{\not{D}+m_i}\right).\end{aligned}$$

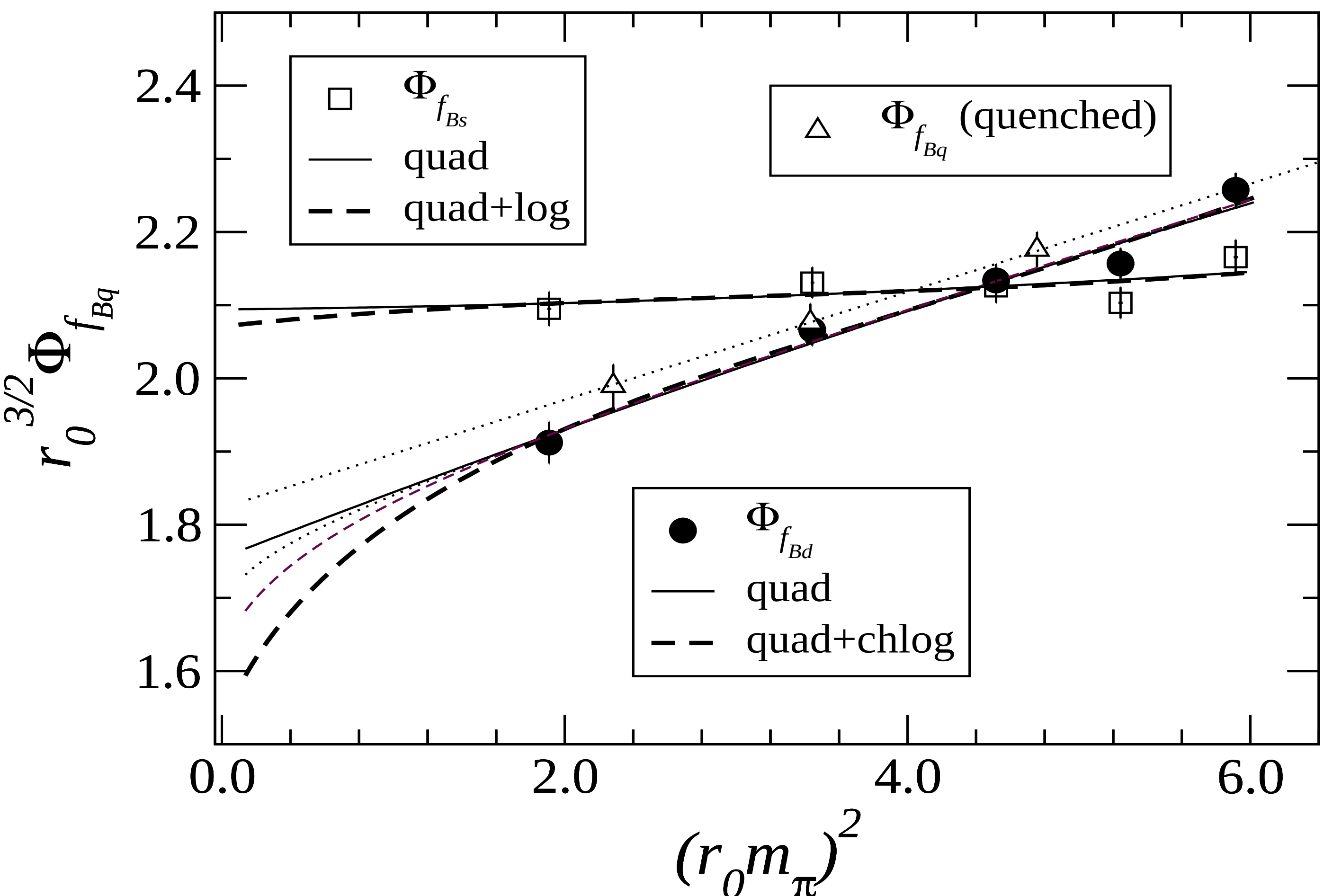
- Finite lattice spacing: $1/a \sim 3$ GeV $< m_b$.

- Finite volume: $L \sim 3$ fm.

- Extrapolate to physical quark masses and volume using EFT.

Light-quark mass extrapolation

$$\Phi_{f_B} = \alpha_0[1 + \mathrm{loop}(L, m_\pi, \mu)] + \alpha_2(\mu) m_\pi^2.$$



EFT for LQCD

- Partially quenched χPT, based on $SU(6|3)_{\mathrm{L}} \otimes SU(6|3)_{\mathrm{R}} \to SU(6|3)_V$.

$$\Sigma = \mathrm{e}^{2i\phi/f} \quad , \quad \langle \Sigma \rangle = 1 \ , \ \Sigma \stackrel{SU(6|3)_{\mathrm{L}} \otimes SU(6|3)_{\mathrm{R}}}{\longrightarrow} L \Sigma R^\dagger ,$$

  where ϕ is a 9×9 matrix containing Goldstone meson fields.

  - Same χ symmetry breaking pattern as "full QCD".

- Combining HQ spin symmetry and χ symmetry $\to$ HMχPT (HL mesons).
  - Heavy mesons are almost on-shell $p_\mu = M_B v_\mu + k_\mu$.
  - The velocity superselection rule.
  - HQET-like procedure leading to $1/M_B$ expansion.

- An important issue is how to couple HL mesons to Goldstone mesons.
  $\Rightarrow$ Normally assume $M_\pi \ll \Lambda_{\mathrm{QCD}} \sim \Lambda_\chi \ll M_B$ and discard $\mathcal{O}(M_\pi/M_B)$.

- $\int \to \sum$ for Goldstone loops to obtain FV effects.

HMχPT
Basic ingredients

- Heavy-quark spin (HQS) symmetry at $M_B \to \infty$
  - Covariant field (bosonisation of interpolating operators $\bar{q}\Gamma^\mu Q$)

$$H_a = \frac{1+\not{v}}{2}\left[B_a^{*\mu}\gamma_\mu + iB_a\gamma_5\right],$$

$$H_a \xrightarrow{\text{HQS}} SH_a, \;\; H_a \xrightarrow{SU(6|3)_{\text{L}}\otimes SU(6|3)_{\text{R}}} H_b U_{ba}^\dagger.$$

- Coupled to Goldstone fields in:

$$\xi = \sqrt{\Sigma}, \;\; \xi \xrightarrow{SU(6|3)_{\text{L}}\otimes SU(6|3)_{\text{R}}} L\xi U^\dagger = U\xi R^\dagger.$$

  - Combine H and ξ to cancel the effect of U.
  - While L and R are global, U is local $\Rightarrow$ need to be gauged.

HMχPT
The HM Lagrangian up to $\mathcal{O}(1/M_B)$

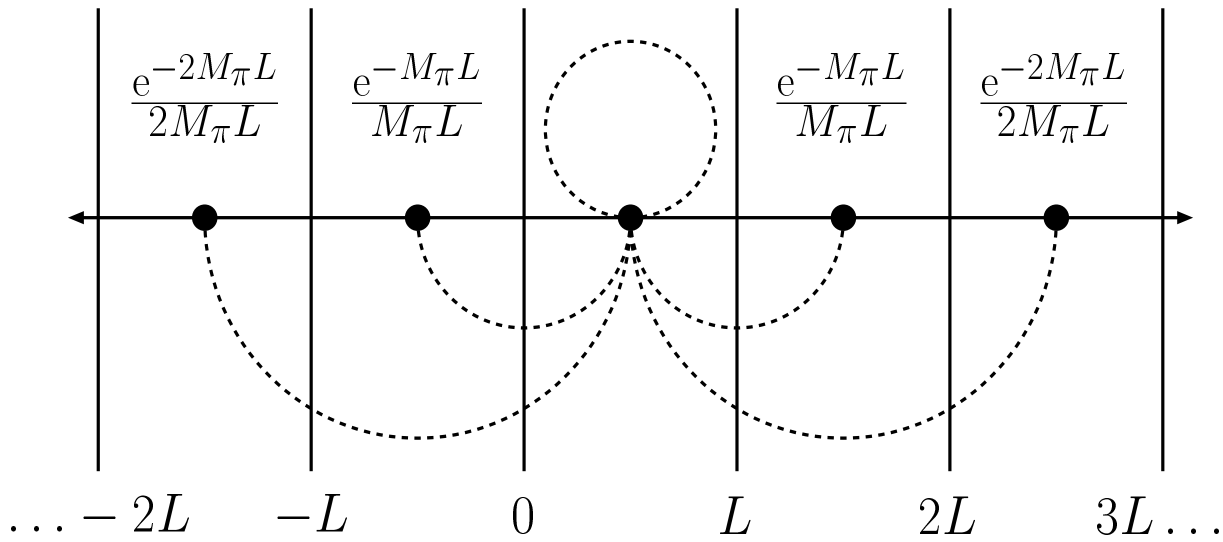
- Invariant under chiral and HQS transformation

$$\mathcal{L}_{\rm HM} = -i\,{\rm Tr}\bar{H}_a v_\mu \left(\partial^\mu \delta_{ab} + i V^\mu_{ab}\right) H_b + g\,{\rm Tr}\bar{H}_a H_b \gamma_\nu \gamma_5 A^\nu_{ba} + \underbrace{\frac{\lambda_2}{M_B}{\rm Tr}\bar{H}_a \sigma_{\mu\nu} H_a \sigma^{\mu\nu}}_{\mathcal{O}(1/M_B)},$$

$$V_\mu, A_\mu = \frac{i}{2}\left(\xi^\dagger \partial_\mu \xi \pm \xi \partial_\mu \xi^\dagger\right).$$

- Propagators :
  - B : $\frac{i}{2(v\cdot k + i\epsilon)} \xrightarrow{x-\text{space, rest } B} \theta(t)\,\delta^{(3)}(\vec{x})$.
  - B^* : $\frac{-i(g_{\mu\nu} - v_\mu v_\nu)}{2(v\cdot k - \Delta + i\epsilon)}$.
- Unknown parameters (Low Energy Constants):
  - g : $B^{(*)}$–B^*–(π, K, η) coupling.
  - λ_2: $\Delta = M_{B^*} - M_B = -8\frac{\lambda_2}{M_B} \sim 50$ MeV at physical M_B.

HMχPT in finite spatial volume ($T \to \infty$)
Physical picture: pions wrapping the world



- B always stays as B $\Rightarrow$ pion propagator $\sim$ Exp$(-M_\pi L)/M_\pi L$.
- $B \to B^* \to B$ $\Rightarrow$ B^* brought off-shell with virtuality Δ $\Rightarrow$ $\delta t \sim 1/\Delta$:
  - FV effects decrease as Δ increases.
  - The alteration is controlled by M_π/Δ. $\Rightarrow$ $\sim \mathcal{A}(M_\pi/\Delta)\times$Exp$(-M_\pi L)$.

HMχPT in finite spatial volume ($T \to \infty$)
How to perform a one-loop calculation?

- $T \to \infty$ means performing $\int dk_0$ to pick up energy poles first.

- Periodic BC in L^3 means

$$\int d^3k \quad \longrightarrow \quad \left(\frac{2\pi}{L}\right)^3 \sum_{\vec{i}} .$$

- The Poisson summation formula:

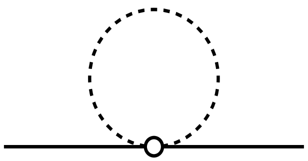
$$\sum_{\vec{i}} \delta^{(3)}(\vec{y} - \vec{i}) \quad = \quad \sum_{\vec{n}} \exp(2\pi i \vec{n} \cdot \vec{y}) .$$

- Let $\vec{y} = \vec{k}\left(\frac{L}{2\pi}\right)$ and multiply both sides by $\int f(\vec{k}) d^3k$:

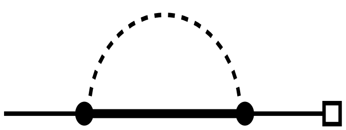
$$\left(\frac{2\pi}{L}\right)^3 \sum_{\vec{i}} f\left(\frac{2\pi}{L}\vec{i}\right) \quad = \quad \underbrace{\int f(\vec{k})\ d^3k}_{V \to \infty} + \underbrace{\sum_{\vec{n} \neq \vec{0}} \int f(\vec{k})\ \exp(i\vec{n} \cdot \vec{k}L)\ d^3k}_{\text{FV}} .$$

- FV effect exp-like if no multi-particle on-shell state in loop.

HMχPT in finite spatial volume ($T \to \infty$)
Typical one-loop diagrams

- 
  B stays as B $\Rightarrow$

$$\mathsf{FV}_{\mathsf{tadpole}} \sim \sum_{\vec{n}\neq\vec{0}} \frac{1}{nL} \int_0^\infty dk \frac{k\ \sin(nkL)}{\sqrt{k^2+M_\pi^2}} \overset{M_\pi L \gg 1}{\sim} \sum_{\vec{n}\neq\vec{0}} \frac{\mathrm{e}^{-nM_\pi L}}{nM_\pi L}$$

- 
  $B \to B^* \to B$ $\Rightarrow$

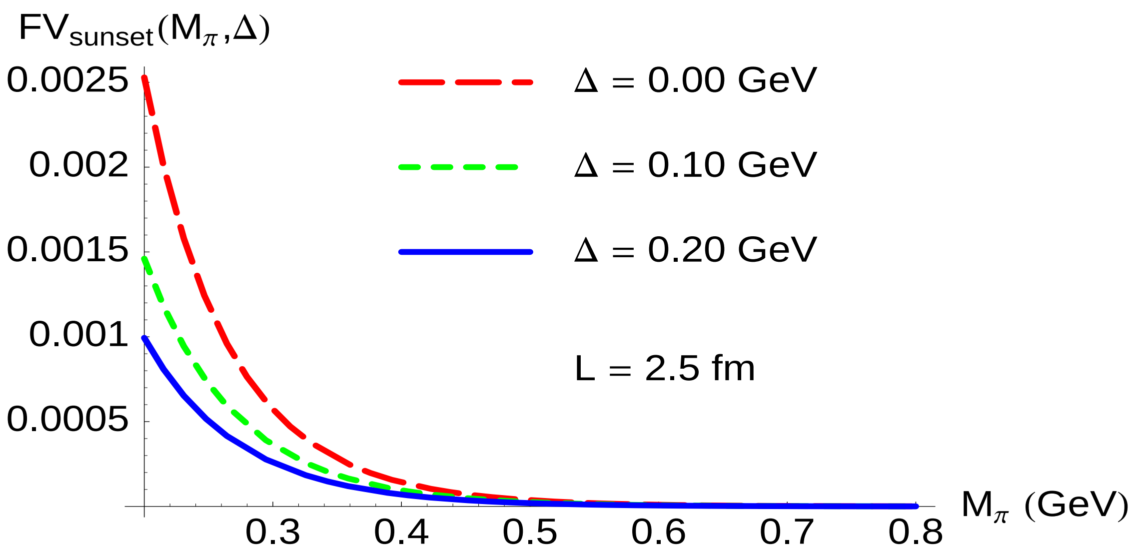
$$\mathsf{FV}_{\mathsf{sunset}} \sim \sum_{\vec{n}\neq\vec{0}} \frac{1}{nL} \int_0^\infty dk \frac{k\ \sin(nkL)}{\sqrt{k^2+M_\pi^2}(\sqrt{k^2+M_\pi^2}+\Delta)} \overset{M_\pi L \gg 1}{\sim} \sum_{\vec{n}\neq\vec{0}} \frac{\mathrm{e}^{-nM_\pi L}}{nM_\pi L}\mathcal{A},$$

$\mathcal{A}$ is the alteration of volume effects due to $M_{B^*} - M_B$.

HMχPT in finite volume
Reproducing the physical picture

$$\mathcal{A} = \exp(z^2)\,[1 - \mathrm{Erf}(z)] + \sum_{j=1}^{\infty} \mathcal{A}_j(z) \left(\frac{1}{nM_\pi L}\right)^j , \quad z = \frac{\Delta}{M_\pi}\sqrt{\frac{nM_\pi L}{2}} .$$

$$\mathcal{A} = 1 \text{ when } \Delta = 0 .$$



Leading-order HMχPT $\Delta B = 2$ operators

$$\begin{aligned}\mathcal{O}_{LL} &= \bar{b}\,\Gamma_{LL}\,d_L\;\;\bar{b}\,\Gamma_{LL}\,d_L,\\ \mathcal{O}_{LR} &= \bar{b}\,\Gamma^{(1)}_{LR}\,d_L\;\;\bar{b}\,\Gamma^{(2)}_{LR}\,d_R,\end{aligned}$$

invariant under the spurion transformation

$$\Gamma_{LL} \longrightarrow S\;\Gamma_{LL}\;L^\dagger,\;\Gamma^{(1)}_{LR} \longrightarrow S\;\Gamma^{(1)}_{LR}\;L^\dagger,\;\Gamma^{(2)}_{LR} \longrightarrow S\;\Gamma^{(2)}_{LR}\;R^\dagger\;.$$

The operators are

$$\begin{aligned}\mathcal{O}_1^{\mathrm{HM}\chi\mathrm{PT}} &= \beta_1\left[\left(\xi B^\dagger\right)\left(\xi\bar{B}\right)+\left(\xi B^{*\dagger}_\mu\right)\left(\xi\bar{B}^{*,\mu}\right)\right]\;,\\ \mathcal{O}_{2(3)}^{\mathrm{HM}\chi\mathrm{PT}} &= \beta_{2(3)}\left(\xi B^\dagger\right)\left(\xi\bar{B}\right)+\beta'_{2(3)}\left(\xi B^{*\dagger}_\mu\right)\left(\xi\bar{B}^{*,\mu}\right)\;,\\ \mathcal{O}_{4(5)}^{\mathrm{HM}\chi\mathrm{PT}} &= \beta_{4(5)}\left(\xi B^\dagger\right)\left(\xi^\dagger\bar{B}\right)+\widehat{\beta}_{4(5)}\left(\xi^\dagger B^\dagger\right)\left(\xi\bar{B}\right)\\ &\quad+\beta'_{4(5)}\left(\xi B^{*\dagger}_\mu\right)\left(\xi^\dagger\bar{B}^{*,\mu}\right)+\widehat{\beta}'_{4(5)}\left(\xi^\dagger B^{*\dagger}_\mu\right)\left(\xi\bar{B}^{*,\mu}\right)\;.\end{aligned}$$

HQ symmetry breaking from the operators

- The $\Delta B = 2$ operators:

$$\mathcal{O}_i^{\text{HQET}} = \tilde{b}\Gamma_1 d\, b^\dagger \Gamma_2 d + b^\dagger \Gamma_1 q_a\, \tilde{b}\Gamma_2 d.$$

- The HQ spin operator (flip HQ spin):

$$S_b^3 = \epsilon^{ij3}[b^\dagger \sigma_{ij} b - \tilde{b}\sigma_{ij}\tilde{b}^\dagger].$$

- Spin relations:

$$\left\{S_b^3, \mathcal{O}_1^{\text{HQET}}\right\}|B\rangle = \left\{S_b^3, \mathcal{O}_1^{\text{HQET}}\right\}|\bar{B}\rangle = 0,$$

$$\left\{S_b^3, \mathcal{O}_{2,3,4,5}^{\text{HQET}}\right\}|B\rangle \neq 0,\ \left\{S_b^3, \mathcal{O}_{2,3,4,5}^{\text{HQET}}\right\}|\bar{B}\rangle \neq 0,$$

$$\left[S_b^3, \mathcal{O}_{2,3,4,5}^{\text{HQET}}\right]|B\rangle \neq 0,\ \left[S_b^3, \mathcal{O}_{2,3,4,5}^{\text{HQET}}\right]|\bar{B}\rangle \neq 0.$$

Chiral extrapolation for B^0–$\bar{B}^0$ matrix elements

- One-loop diagrams

- Extrapolation formulae

$$\langle \bar{B}^0 | \mathcal{O}_1 | B^0 \rangle = \beta_1 \left[1 + \mathrm{Log}(m_\pi, L, \mu)\right] + \gamma_1(\mu) m_\pi^2 \,,$$

$$\langle \bar{B}^0 | \mathcal{O}_{2(3)} | B^0 \rangle = \beta_{2(3)} \left[1 + \mathrm{Log}(m_\pi, L, \mu)\right] + \beta'_{2(3)} \mathrm{Log}'(m_\pi, L, \mu) + \gamma_{2(3)}(\mu) m_\pi^2 \,,$$

$$\langle \bar{B}^0 | \mathcal{O}_{4(5)} | B^0 \rangle = \beta_{4(5)} \left[1 + \mathrm{Log}(m_\pi, L, \mu)\right] + \beta'_{4(5)} \mathrm{Log}'(m_\pi, L, \mu) + \gamma_{4(5)}(\mu) m_\pi^2 \,,$$

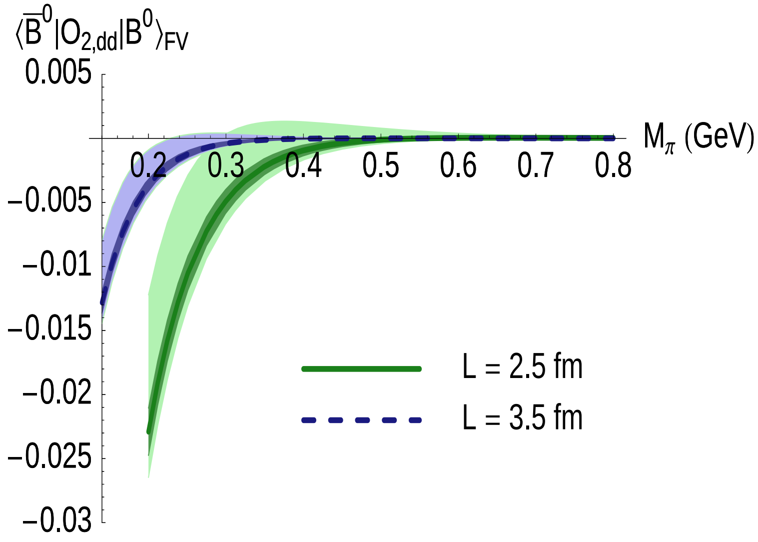
- Conventional wisdom is not applicable to the BSM operators.

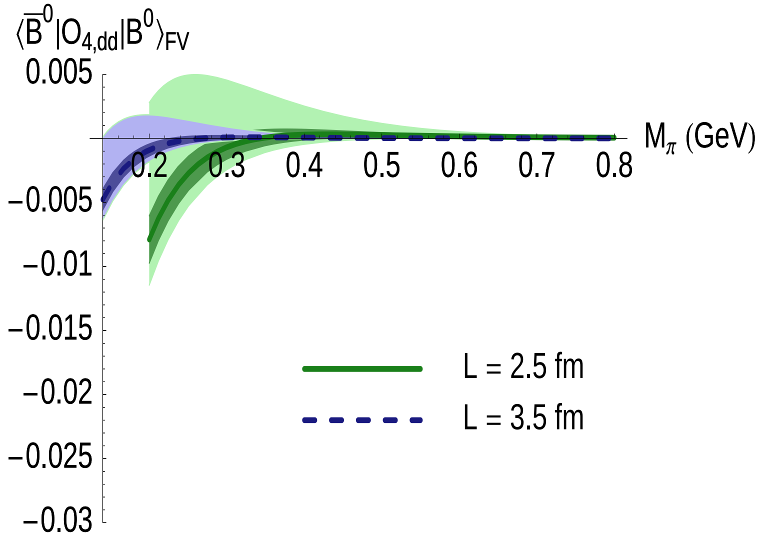
Plots for FV effects

- FV effects in B_B are around 5%.

D.Arndt and C.-J.D.L, PRD70:014503 (2004).

- BSM matrix elements:





Conclusions and outlook

- Conventional wisdom in chiral extrapolation:

$$\beta(1 + \text{Log}) + \gamma m_\pi^2.$$

- Chiral extrapolation for BSM B^0–$\bar{B}^0$ mixing matrix elements:

$$\beta(1 + \text{Log}) + \beta'\text{Log}' + \gamma m_\pi^2.$$

- The origin of the complication:

  $\Rightarrow$ HQ symmetry breaking effects.

  $\Rightarrow$ LO B^*–$\bar{B}^*$ matrix element is accompanied by a different LEC.

- On-going work:

  $\Rightarrow$ $\Delta B = 0$ operators for spectator effects in B and Λ_b lifetimes.