Walking step by step in search of technicolour

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Outline

- Motivation
- The step-scaling method
- A new non-perturbative scheme

  $\longrightarrow$ The finite-volume Wilson-loop scheme

  $\longrightarrow$ Feasibility numerical study in quenched QCD
- Conclusion and outlook

Fundamental scalar Higgs in the SM

- Hierarchy problem

  $\longrightarrow m_H^2 = 2\lambda v^2 \sim \Lambda_{\rm UV}^2$.

- The theory is trivial

  $\longrightarrow$ Higgs self-coupling: $\lambda(\mu) = \frac{\lambda(\Lambda_{\rm UV})}{1+(24/16\pi^2)\lambda(\Lambda_{\rm UV})\log(\Lambda_{\rm UV}/\mu)}$

  $\longrightarrow$ The coupling vanishes for all μ when the cut-off $\Lambda \to \infty$.

- A solution: strong interactions involving fermions

  $\longrightarrow \Lambda_{\rm EW} \sim \Lambda_{\rm UV} e^{-g_c^2/g_{\rm UV}^2}$.

Motivation
Extended technicolour model

- Standard-model fermion masses

  $\rightarrow m_f = \frac{1}{\Lambda^2_{ETC}}\langle\bar{\psi}\psi\rangle$ via dim-6 operators $\frac{1}{\Lambda^2_{ETC}}\bar{\psi}\psi\bar{f}f$.

  $\rightarrow \Lambda_{ETC} \sim$ TeV via estimating $\langle\bar{\psi}\psi\rangle$ using M_W and slow running of $\bar{\psi}\psi$.

- FCNC processes

  $\rightarrow \frac{1}{\Lambda^2_{ETC}}\bar{f}f\bar{f}f$.

  $\rightarrow \Lambda_{ETC} \sim 10^2 \sim 10^3$ TeV from constraints imposed by K^0–$\bar{K}^0$ mixing.

- A solution: fast running of $\bar{\psi}\psi$ (large anomalous dimension).

Motivation
Slow and fast running of the condensate

- Slow running ("TeV-QCD")

  $\longrightarrow \langle\bar{\psi}\psi\rangle_{\rm ETC} \approx \langle\bar{\psi}\psi\rangle_{\rm EW}\ [1 + \alpha\ \log(\Lambda_{\rm ETC}/\Lambda_{\rm EW})] \sim \Lambda^3_{\rm EW}.$

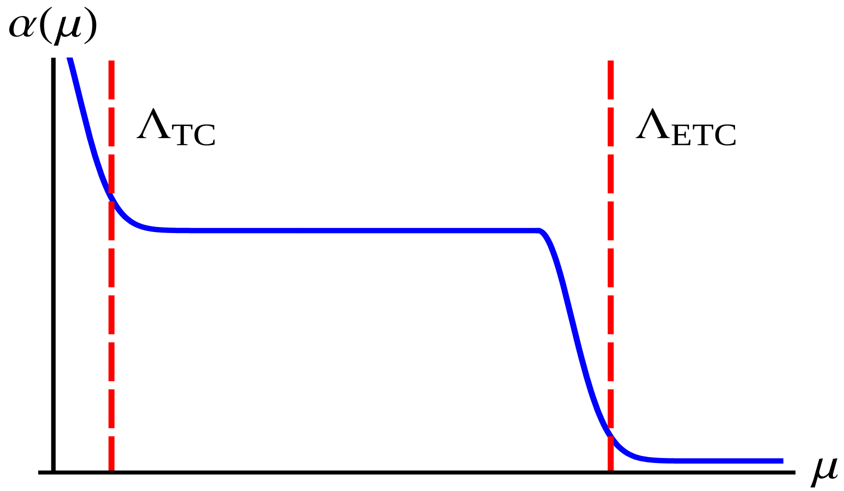
  $\longrightarrow \Lambda_{\rm ETC} \sim \Lambda_{\rm EW}(\Lambda_{\rm EW}/m_f)^{1/2}$

- Fast running (IR fixed point with $\gamma^* = 1$)

  $\longrightarrow \langle\bar{\psi}\psi\rangle_{\rm ETC} \approx \langle\bar{\psi}\psi\rangle_{\rm EW}\ (\Lambda_{\rm ETC}/\Lambda_{\rm EW}) \sim \Lambda^2_{\rm EW}\Lambda_{\rm ETC}.$

  $\longrightarrow \Lambda_{\rm ETC} \sim \Lambda_{\rm EW}(\Lambda_{\rm EW}/m_f)$

Motivation
Walking technicolour



- Generates large anomalous dimension for $\bar{\psi}\psi$ to solve the FCNC problem.

- Modifies the relevant spectral function to elude the S-paramter criticism *a'la* Peskin and Takeuchi.

- $\Lambda_{\rm ETC}/\Lambda_{\rm TC} \sim 10^2$.

  $\longrightarrow$ Compared to the typical lattice size $L/a \sim 20$ in each direction.

Motivation
Technicolour in the twenty-first century

- Except for "TeV-QCD", technicolour has not been ruled out.
- Serious lattice calculations to support/kill technicolour
  - Hadron spectrum.

    L. Del Debbio, A. Patella and C. Pica
    S. Catterall and F. Sannino
    A. Hietanen *et al.*

  - Pheses of candidate walking theories.

    S. Catterall *at al.*
    T. DeGrand, B. Svetitsky and Y. Shamir

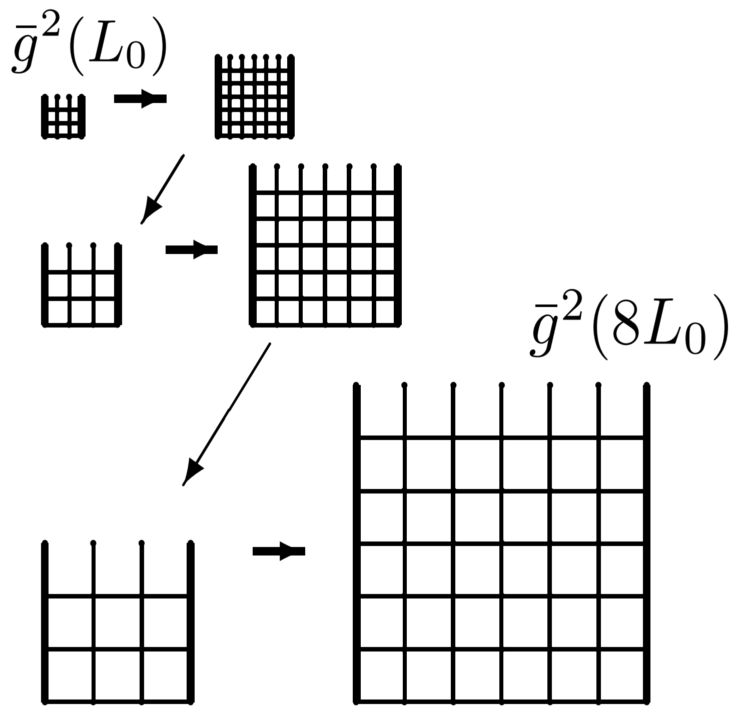
  - Spectrum of the Dirac operator.

    Z. Fodor *et al.*

  - The step-scaling method for the coupling constant.

    T. Appelquist, G. Fleming and E. Neil
    This work

The step-scaling method
The idea



- Extrapolate to the continuum limit at every step.
- Vary the scale by changing the dimensionful lattice size L_0.

The step-scaling method
Implementation: An example

1. Prepare lattices of size $\widehat{L_0}^4$:

$$\widehat{L_0} = L_0/a = 6, 8, 10, 12,$$

   and tune the lattice spacing (via varying the bare coupling).

   $\longrightarrow$ The renormalised coupling $g(L_0)$ is the same on these lattices.

2. Double the lattice size to be

$$\widehat{L_0} = L_0/a = 12, 16, 20, 24,$$

   and calculate $g(2L_0, a/L_0)$.

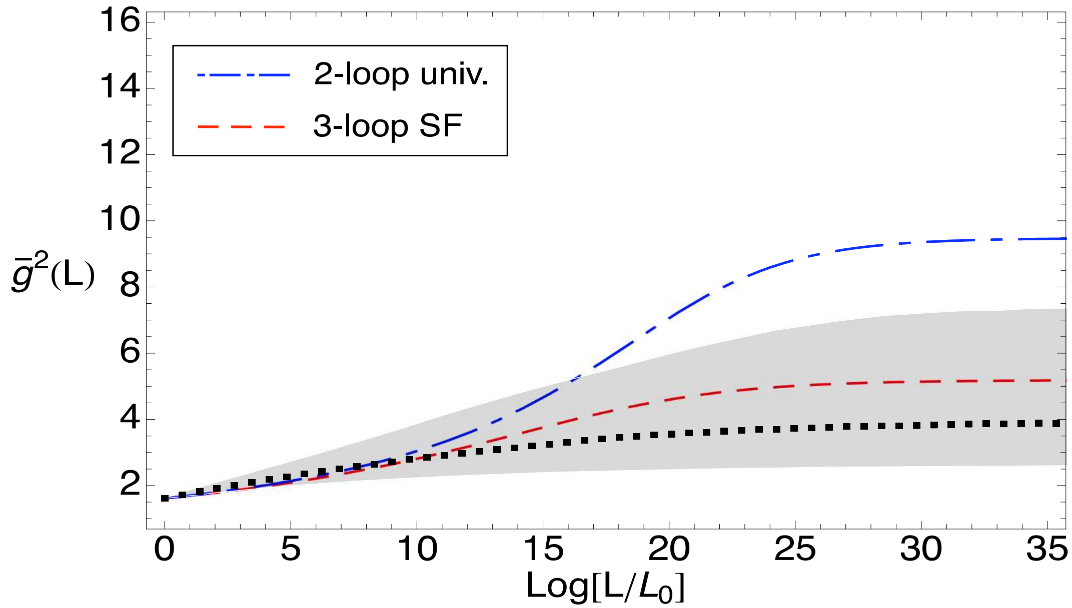
   $\longrightarrow$ Extrapolate to the continuum limit to get $g(2L_0)$.

3. Go back to the lattices in 1., and tune the lattice spacing.

   $\longrightarrow$ The renormalised coupling equals $g(2L_0)$ on all these lattices.

   $\longrightarrow$ Repeat 2. to obtain $g(4L_0)$.

The step-scaling method
Results for the Schödinger Functional scheme



T.Appelquist, G.Fleming and E.Neil, PRL100, 171607 (2008)

- $N_f = 12$.
- Scheme dependence?
- Controversy from the study of the Dirac operator eigenvalue spectrum?

A new non-perturbative scheme
General considreations

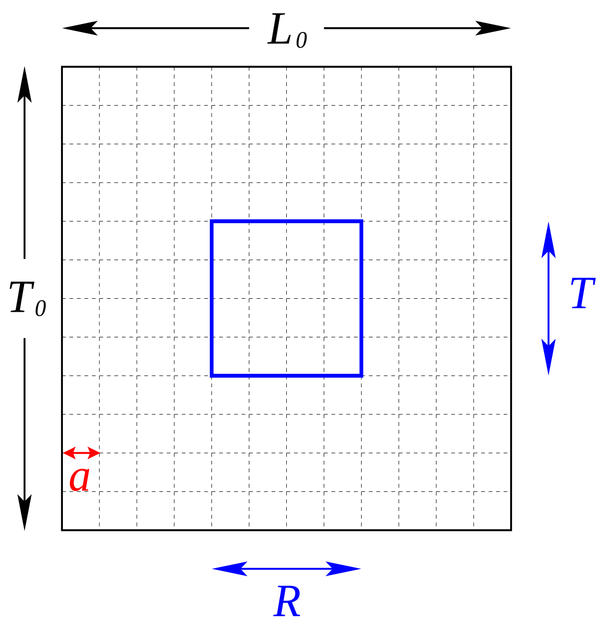
$$A_{\mathrm{LO}} = k\ g_0^2$$

$$A_{\mathrm{NP}}(\mu) = k\ g^2(\mu)$$

$$\Big\downarrow \quad \text{Lattice}$$

$$A_{\mathrm{NP}}(L,a) = k(a)\ g^2(L,a)$$

The finite-volume Wilson-loop scheme
The idea



- Set up $T_0 = L_0$ and $T = R$.
- When $a \to 0$, with fixed $\hat{r} \equiv (R + a/2)/L_0$, there is only one scale.

  $\longrightarrow$ Varying $\hat{r}$ corresponds to changing scheme.

The finite-volume Wilson-loop scheme
More details

- The Creutz Ratio (CR):

$$-R^2\frac{\partial^2}{\partial R\partial T}\ln\langle W(R,T;L_0,T_0)\rangle|_{T=R,T_0=L_0} \xrightarrow{\text{LO}} kg_0^2.$$

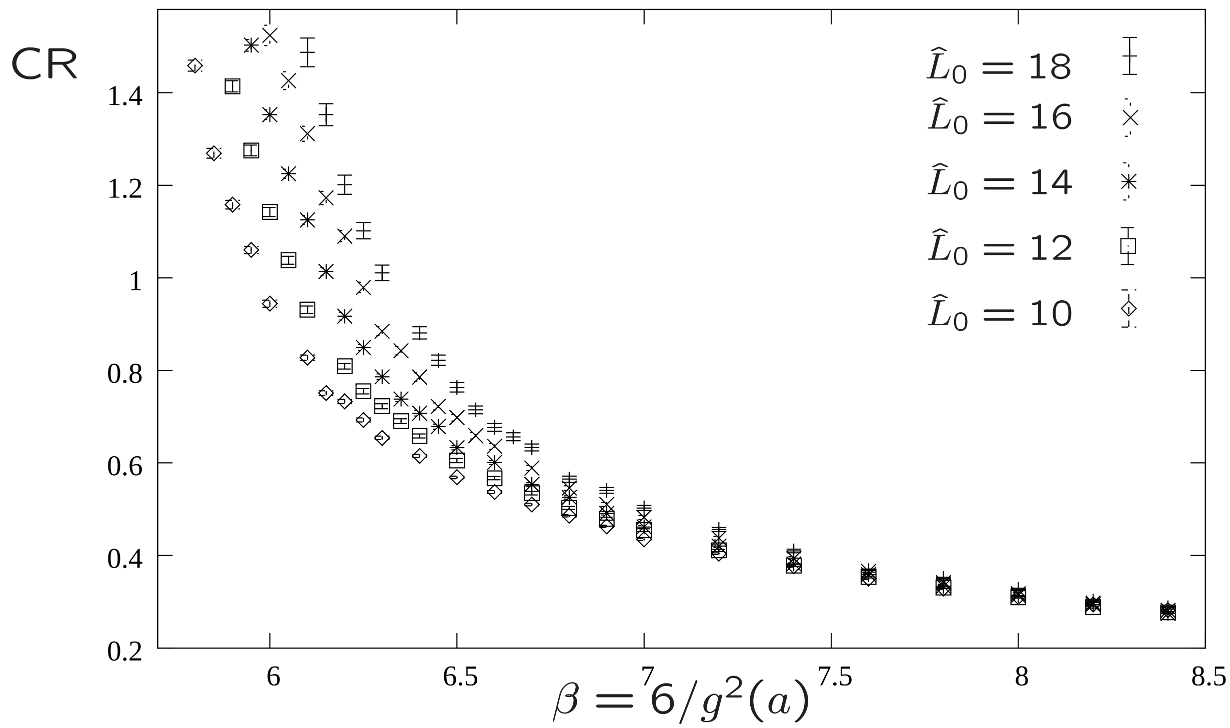
- On the lattice ($\hat{R}=R/a$ and $\hat{L}_0=L/a$):

$$-(\hat{R}+1/2)\ \chi(\hat{R}+1/2,\hat{T}+1/2;\hat{L}_0) = -(\hat{R}+1/2)\ \ln\left[\frac{W(\hat{R}+1,\hat{T}+1;\hat{L}_0)W(\hat{R},\hat{T};\hat{L}_0)}{W(\hat{R}+1,\hat{T};\hat{L}_0)W(\hat{R},\hat{T}+1;\hat{L}_0)}\right].$$

- The factor k can be calculated analytically (periodic BC)

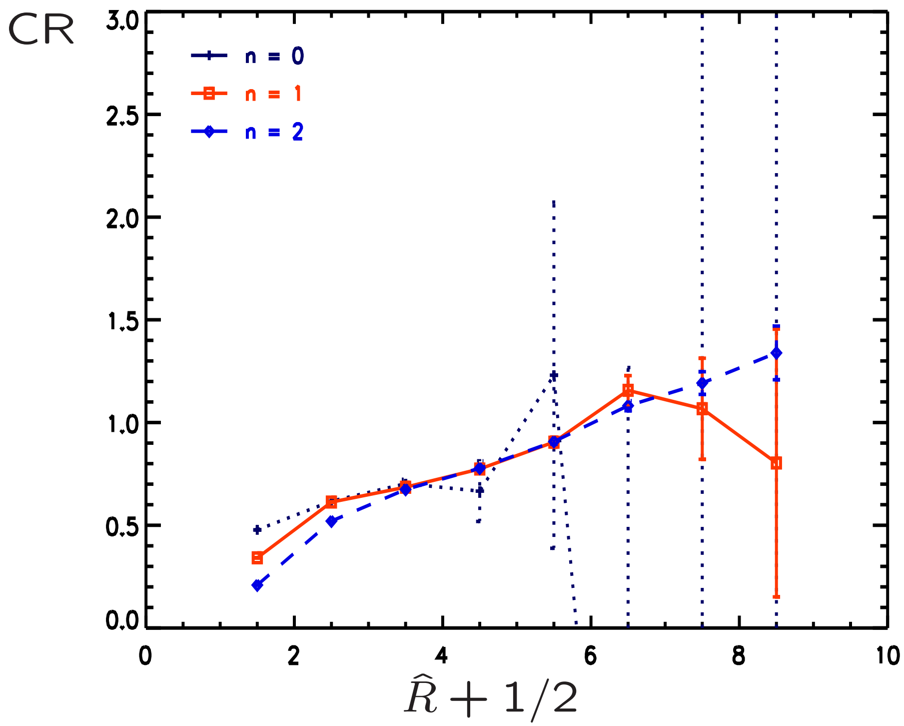
$$k=-R^2\frac{\partial^2}{\partial R\partial T}\left[\frac{4}{(2\pi)^4}\sum_{n_\mu\neq 0}\left(\frac{\sin\left(\frac{\pi n_0 T}{L_0}\right)}{n_0}\right)^2\frac{e^{i\frac{2\pi n_1 R}{L_0}}}{n_0^2+\vec{n}^2}\right]_{T=R} + \text{zero}-\text{mode contrib.}$$

Numerical test of the FV Wilson-loop scheme
Simulation parameters in quenched QCD



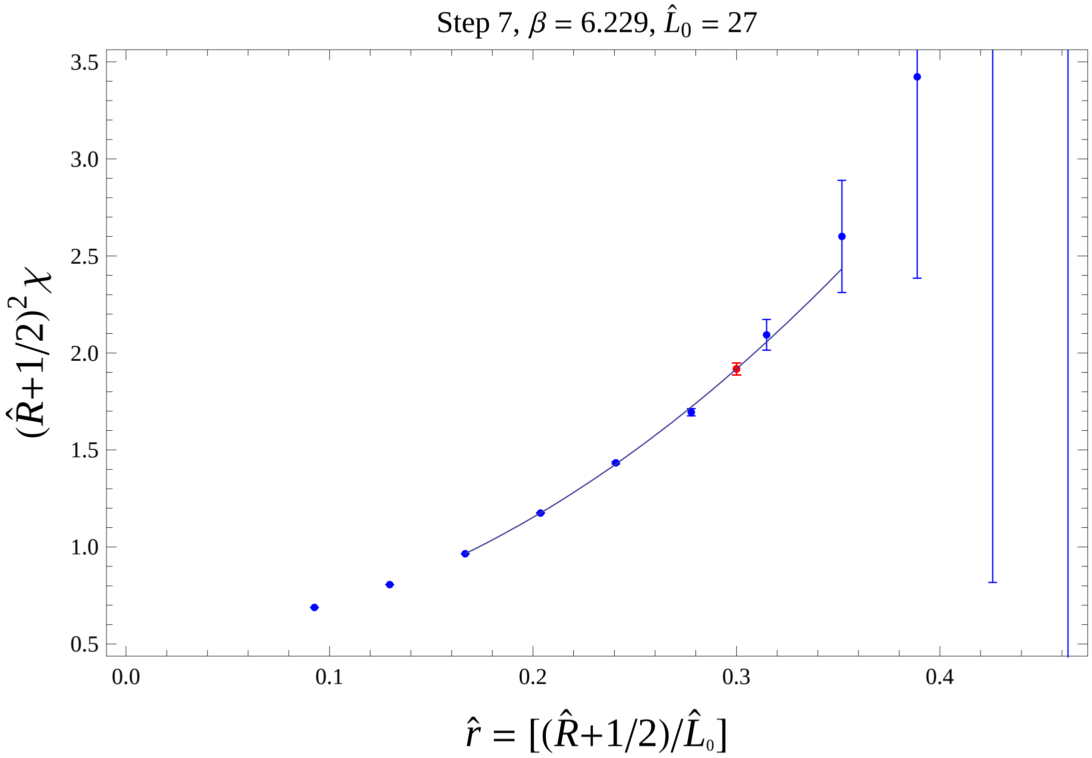
- Take CR $= 0.2871$ as the starting point (step 0, scale $\tilde{L}_0$).
- Take the step size $s = 1.5$.

Numerical test of the FV Wilson-loop scheme
Smearing the gauge links



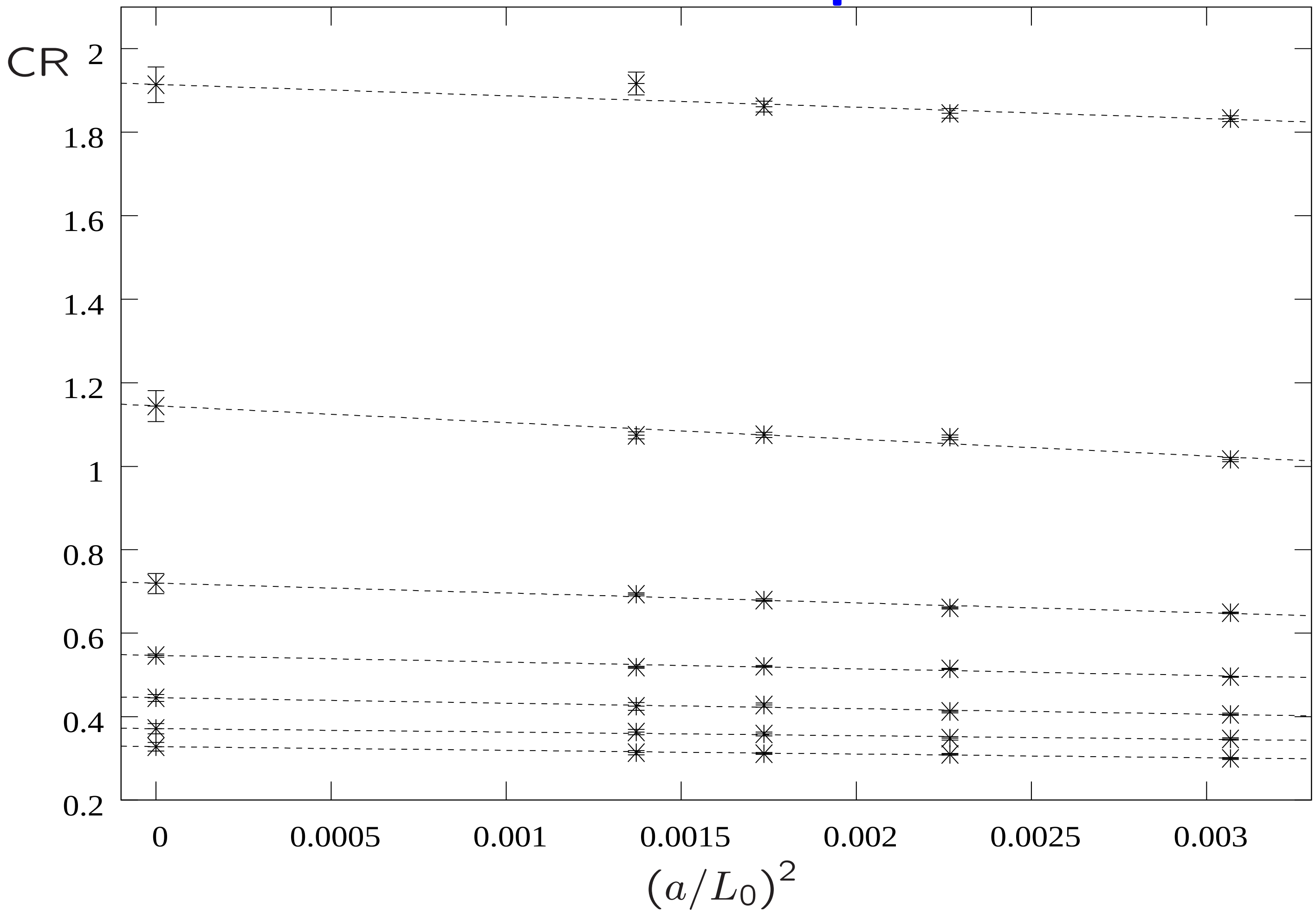
- Large Wilson loops: Significant statistical errors.
- Small Wilson loops: The onset of over-smearing.

Numerical test of the FV Wilson-loop scheme
Fixing the scheme

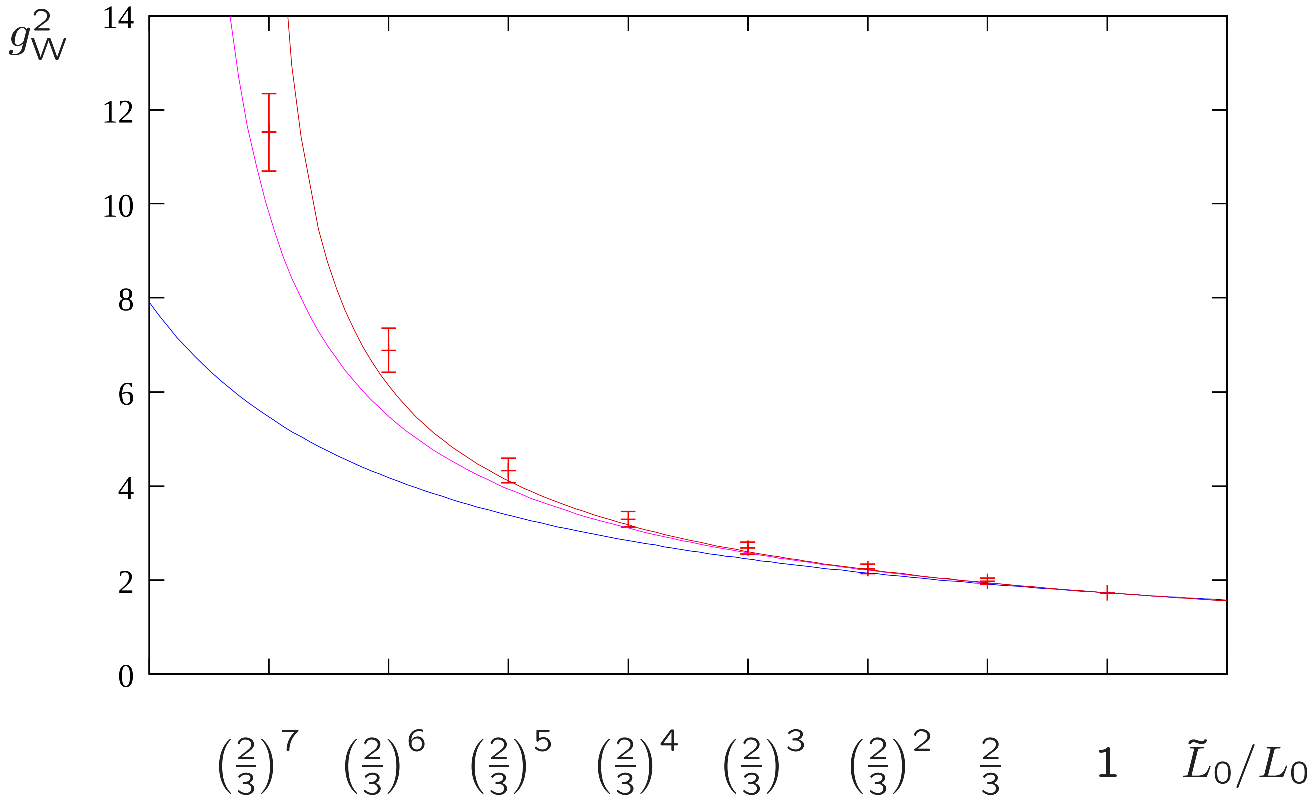


- Fit function: Polynomial in $\hat{r} = (R + a/2)/L_0$.
- Interpolating to $\hat{r} = 0.3$.

Numerical test of the FV Wilson-loop scheme
Continuum extrapolation



Numerical test of the FV Wilson-loop scheme
Result in quenched QCD



Concluding remarks and outlook

- We have designed a new non-perturbative scheme for calculating the running coupling constant.

- We have demonstrated it is valid through numerical tests in quenched QCD.

- Setting the (quenched) scale, our preliminary result is:

$$\frac{\Lambda_{\mathrm{SF}}^{2-\mathrm{loop}}}{\Lambda_{\mathrm{W}}^{2-\mathrm{loop}}} \sim 1.78.$$

- Study of large-N_f gauge theories and the $\bar{\psi}\psi$ anomalous dimension is under way.

It is a new lattice research avenue.