CP violation in the secluded U(1)' extended MSSM

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Outline

- Motivation
- Higgs sector of the secluded U(1)' extended MSSM
- Explicit CP violation (ECPV)
- Spontaneous CP violation (SCPV)
- Summary

Motivation

The MSSM is a well-motivated model.

- Stabilization of the gauge hierarchy
- Gauge coupling unification
- Cold dark matter etc...

However, it has an unattractive feature.

μ problem: $\mathcal{W} \ni -\epsilon_{ij}\mu H_d^i H_u^j$

From the naturalness point of view, μ is supposed to be $M_{\rm pl}$ or $M_{\rm GUT}$.

However, the vacuum conditions gives

$$\mu^2 = \frac{\tilde{m}_u^2 \sin^2\beta - \tilde{m}_d^2 \cos^2\beta}{\cos 2\beta} - \frac{m_Z^2}{2}$$

Solution: Introduction of a gauge singlet field S.

$$\mathcal{W} \ni -\epsilon_{ij}\lambda S H_d^i H_u^j \qquad \mu_{\rm eff} = \lambda\langle S\rangle.$$

Singlet extended MSSMs

- So far, the several models have been proposed:
  - Next-to-MSSM (NMSSM)
  - Nearly MSSM (nMSSM)
  - U(1)' extended MSSM (UMSSM)
  - Secluded U(1)' extended MSSM (sMSSM/ S model)

No dimensional parameter in the superpotential.

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  - ❧ Secluded U(1)' extended MSSM (sMSSM/ S model)

No dimensional parameter in the superpotential.

- The GUTs predict an extra U(1) symmetry, and then the extra Z' boson exists.
- Once we have the Z' boson, the large Z-Z' mixing could appear. → It's easy to avoid this in the sMSSM.

Secluded U(1)' extended MSSM (sMSSM)

[J. Erler, P. Langacker, T. Li, PRD66,015002 (02)]
[T Han, P. Langacker, B. McElarah, PRD70,115006 (04)]
[J Kang, P. Langacker, T. Li, T. Liu, PRL94,061801 (05)]

Particle content of the Higgs sector:

Higgs	$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'_{Q'}$
H_d	$\left(\mathbf{1}, \mathbf{2}, -1/2, Q_{H_d}\right)$
H_u	$\left(\mathbf{1}, \mathbf{2}, 1/2, Q_{H_u}\right)$
S	$(\mathbf{1}, \mathbf{1}, 0, Q_S)$
S_1	$(\mathbf{1}, \mathbf{1}, 0, Q_{S_1})$
S_2	$(\mathbf{1}, \mathbf{1}, 0, Q_{S_2})$
S_3	$(\mathbf{1}, \mathbf{1}, 0, Q_{S_3})$

2 Higgs doublets
+ 4 Higgs singlets

Q's are $U(1)'$ charges

Superpotential:

secluded sector

$$\mathcal{W} \ni -\epsilon_{ij}\lambda S H_d^i H_u^j - \lambda_S S_1 S_2 S_3$$

solutions for μ propblem, for Z-Z' hierarchy

Tree-level Higgs potential

$$V_0 = V_F + V_D + V_{\rm soft},$$

$$\begin{aligned}
V_F &= |\lambda|^2\{|\epsilon_{ij}\Phi_d^i\Phi_u^j|^2 + |S|^2(\Phi_d^\dagger\Phi_d + \Phi_u^\dagger\Phi_u)\} \\
&\quad +|\lambda_S|^2(|S_1S_2|^2 + |S_2S_3|^2 + |S_3S_1|^2), \\
V_D &= \frac{g_2^2+g_1^2}{8}(\Phi_d^\dagger\Phi_d - \Phi_u^\dagger\Phi_u)^2 + \frac{g_2^2}{2}|\Phi_d^\dagger\Phi_u|^2 \\
&\quad +\frac{g_1'^2}{2}\left(Q_{H_d}\Phi_d^\dagger\Phi_d + Q_{H_u}\Phi_u^\dagger\Phi_u + Q_S|S|^2 + \sum_{i=1}^{3}Q_{S_i}|S_i|^2\right)^2, \\
V_{\rm soft} &= \tilde{m}_d^2\Phi_d^\dagger\Phi_d + \tilde{m}_u^2\Phi_u^\dagger\Phi_u + m_S^2|S|^2 + \sum_{i=1}^{3}m_{S_i}^2|S_i|^2 \\
&\quad -(\epsilon_{ij}\lambda A_\lambda S\Phi_d^i\Phi_u^j + \lambda_S A_{\lambda_S}S_1S_2S_3 \\
&\quad -m_{SS_1}^2 SS_1 - m_{SS_2}^2 SS_2 - m_{S_1S_2}^2 S_1^\dagger S_2 + \text{h.c.}).
\end{aligned}$$

Higgs VEVs:

$$\Phi_d = e^{i\theta_1}\begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + h_d + ia_d) \\ \phi_d^- \end{pmatrix}, \quad \Phi_u = e^{i\theta_2}\begin{pmatrix} \phi_u^+ \\ \frac{1}{\sqrt{2}}(v_u + h_u + ia_u) \end{pmatrix},$$

$$S = \frac{e^{i\theta_S}}{\sqrt{2}}(v_S + h_S + ia_S), \quad S_i = \frac{e^{i\theta_{S_i}}}{\sqrt{2}}(v_{S_i} + h_{S_i} + ia_{S_i}), \quad (i = 1-3).$$

Tree-level Higgs potential

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&\quad +|\lambda_S|^2(|S_1S_2|^2 + |S_2S_3|^2 + |S_3S_1|^2), \\
V_D &= \frac{g_2^2+g_1^2}{8}(\Phi_d^\dagger\Phi_d - \Phi_u^\dagger\Phi_u)^2 + \frac{g_2^2}{2}|\Phi_d^\dagger\Phi_u|^2 \\
&\quad +\frac{g_1'^2}{2}\left(Q_{H_d}\Phi_d^\dagger\Phi_d + Q_{H_u}\Phi_u^\dagger\Phi_u + Q_S|S|^2 + \sum_{i=1}^{3} Q_{S_i}|S_i|^2\right)^2, \\
V_{\rm soft} &= \tilde{m}_d^2\Phi_d^\dagger\Phi_d + \tilde{m}_u^2\Phi_u^\dagger\Phi_u + m_S^2|S|^2 + \sum_{i=1}^{3} m_{S_i}^2|S_i|^2 \\
&\quad -(\epsilon_{ij}\lambda A_\lambda S\Phi_d^i\Phi_u^j + \lambda_S A_{\lambda_S} S_1S_2S_3 \\
&\quad -m_{SS_1}^2 SS_1 - m_{SS_2}^2 SS_2 - m_{S_1S_2}^2 S_1^\dagger S_2 + \text{h.c.}).
\end{aligned}$$

complex parameters

5-4=1 physical explicit CPV

Higgs VEVs:

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$$S = \frac{e^{i\theta_S}}{\sqrt{2}}(v_S + h_S + ia_S), \quad S_i = \frac{e^{i\theta_{S_i}}}{\sqrt{2}}(v_{S_i} + h_{S_i} + ia_{S_i}), \quad (i = 1-3).$$

The tree-level effective potential:

$$
\begin{aligned}
\langle V_0 \rangle \;=\; & \frac{1}{2} m_1^2 v_d^2 + \frac{1}{2} m_2^2 v_u^2 + \frac{1}{2} m_S^2 v_S^2 + \sum_i \frac{1}{2} m_{S_i}^2 v_{S_i}^2 \\
& - \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1}) v_S v_{S_1} - \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2}) v_S v_{S_2} - \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}}) v_{S_1} v_{S_2}, \\
& - R_\lambda v_d v_u v_S - R_{\lambda_S} v_{S_1} v_{S_2} v_{S_3} + \frac{g_2^2 + g_1^2}{32} (v_d^2 - v_u^2)^2 \\
& + \frac{|\lambda|^2}{4} (v_d^2 v_u^2 + v_d^2 v_S^2 + v_u^2 v_S^2) + \frac{|\lambda_S|^2}{4} (v_{S_1}^2 v_{S_2}^2 + v_{S_2}^2 v_{S_3}^2 + v_{S_3}^2 v_{S_1}^2) + \frac{g_1'^2}{8} \Delta^2 .
\end{aligned}
$$

where

$$
\boxed{\varphi_1 = \theta_S + \theta_{S_1},\; \varphi_2 = \theta_S + \theta_{S_2},\; \varphi_3 = \theta_S + \theta_1 + \theta_2,\; \varphi_4 = \theta_{S_1} + \theta_{S_2} + \theta_{S_3}}
$$

→ SCPV

$$
R_\lambda = \frac{\mathrm{Re}(\lambda A_\lambda e^{i\varphi_3})}{\sqrt{2}},\; R_{\lambda_S} = \frac{\mathrm{Rm}(\lambda A_{\lambda_S} e^{i\varphi_4})}{\sqrt{2}},
$$

$$
\Delta = Q_{H_d} v_d^2 + Q_{H_u} v_u^2 + Q_S v_S^2 + \sum_{i=1}^{3} Q_{S_i} v_{S_i}^2 ,
$$

Vacuum stability: $v_S = v_{S_i}$ ($i = 1, 2$ other VEV's are zero), we require (unbounded from below)

$$
m_S^2 + m_{S_i}^2 + 2\mathrm{Re}(m_{SS_i}^2 e^{i\varphi_i}) > 0, \quad i = 1, 2.
$$

Effective potential @1-loop

- In our analysis, we use the 1-loop effective potential.

Here we consider the 1-loop corrections from top/bottom and stop/sbottom.

$$V_1 = \frac{N_C}{32\pi^2} \sum_{q=t,b} \left[\sum_{a=1,2} \bar{m}^4_{\tilde{q}_a} \left(\ln \frac{\bar{m}^2_{\tilde{q}_a}}{M^2} - \frac{3}{2} \right) - 2\bar{m}^4_q \left(\ln \frac{\bar{m}^2_q}{M^2} - \frac{3}{2} \right) \right],$$

which is regularized using the $\overline{\mathrm{DR}}$-scheme.
N_C denotes the number of color, and $\bar{m}$'s are the field dependent masses.

[N.B.]
The secluded sector (S_1, S_2, S_3) is not corrected by those of particles.
$\Delta\mathcal{M}^2_{\mathrm{secluded}} = 0.$

Mass matrix

Neutral Higgs bosons:

$$\frac{1}{2}\begin{pmatrix} \boldsymbol{H}^T & \boldsymbol{A}^T \end{pmatrix} \mathcal{M}_N^2 \begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{A} \end{pmatrix}, \quad \mathcal{M}_N^2 = \begin{pmatrix} \mathcal{M}_S^2 & \mathcal{M}_{SP}^2 \\ (\mathcal{M}_{SP}^2)^T & \mathcal{M}_P^2 \end{pmatrix}$$

where

$$\boldsymbol{H}^T \equiv (\boldsymbol{h}_O^T = (h_d\ h_u\ h_S)\ \boldsymbol{h}_S^T = (h_{S_1}\ h_{S_2}\ h_{S_3}))$$ CP-even Higgs

$$\boldsymbol{A}^T \equiv (\boldsymbol{a}_O^T = (a_d\ a_u\ a_S)\ \boldsymbol{a}_S^T = (a_{S_1}\ a_{S_2}\ a_{S_3}))$$ CP-odd Higgs

For CP conserving case, $\mathcal{M}_{SP}^2 = 0.\ \Rightarrow \mathcal{M}_S^2, \mathcal{M}_P^2 \in M(6, \boldsymbol{R})$.

decoupling of the 2 NG bosons ↓ G^0, G'^0

$\mathcal{M}_P^2 \in M(4, \boldsymbol{R})$

CP-even Higgs: $H_1, H_2, H_3, H_4, H_5, H_6$

CP-odd Higgs: A_1, A_2, A_3, A_4

For the CP violating case, $\mathcal{M}_N^2$ is a 10×10 symmetric matrix.

Charged Higgs boson mass

- The charged Higgs is the same as in the MSSM

Charged Higgs mass:

$$m_{H^\pm}^2 = \frac{1}{\sin\beta\cos\beta}\left\langle \frac{\partial^2 V_0}{\partial\phi_d^+ \partial\phi_u^-} \right\rangle = m_W^2 + \frac{2R_\lambda}{\sin 2\beta} v_S - \frac{|\lambda|^2}{2} v^2.$$

$\langle\cdots\rangle \equiv$ evaluation at the vacuum

Upper bound of the charged Higgs mass

After imposing the tadpole conditions in $\langle V_0 \rangle$, we obtain

$$\begin{aligned} \langle V_0 \rangle_{\rm vac} &= \frac{1}{2} R_\lambda v_d v_u v_S + \frac{1}{2} R_{\lambda_S} v_{S_1} v_{S_2} v_{S_3} - \frac{g_2^2 + g_1^2}{32} (v_d^2 - v_u^2)^2 \\ &\quad - \frac{|\lambda|^2}{4} (v_d^2 v_u^2 + v_d^2 v_S^2 + v_u^2 v_S^2) - \frac{|\lambda_S|^2}{4} (v_{S_1}^2 v_{S_2}^2 + v_{S_2}^2 v_{S_3}^2 + v_{S_3}^2 v_{S_1}^2) - \frac{g_1'^2}{8} \Delta^2. \end{aligned}$$

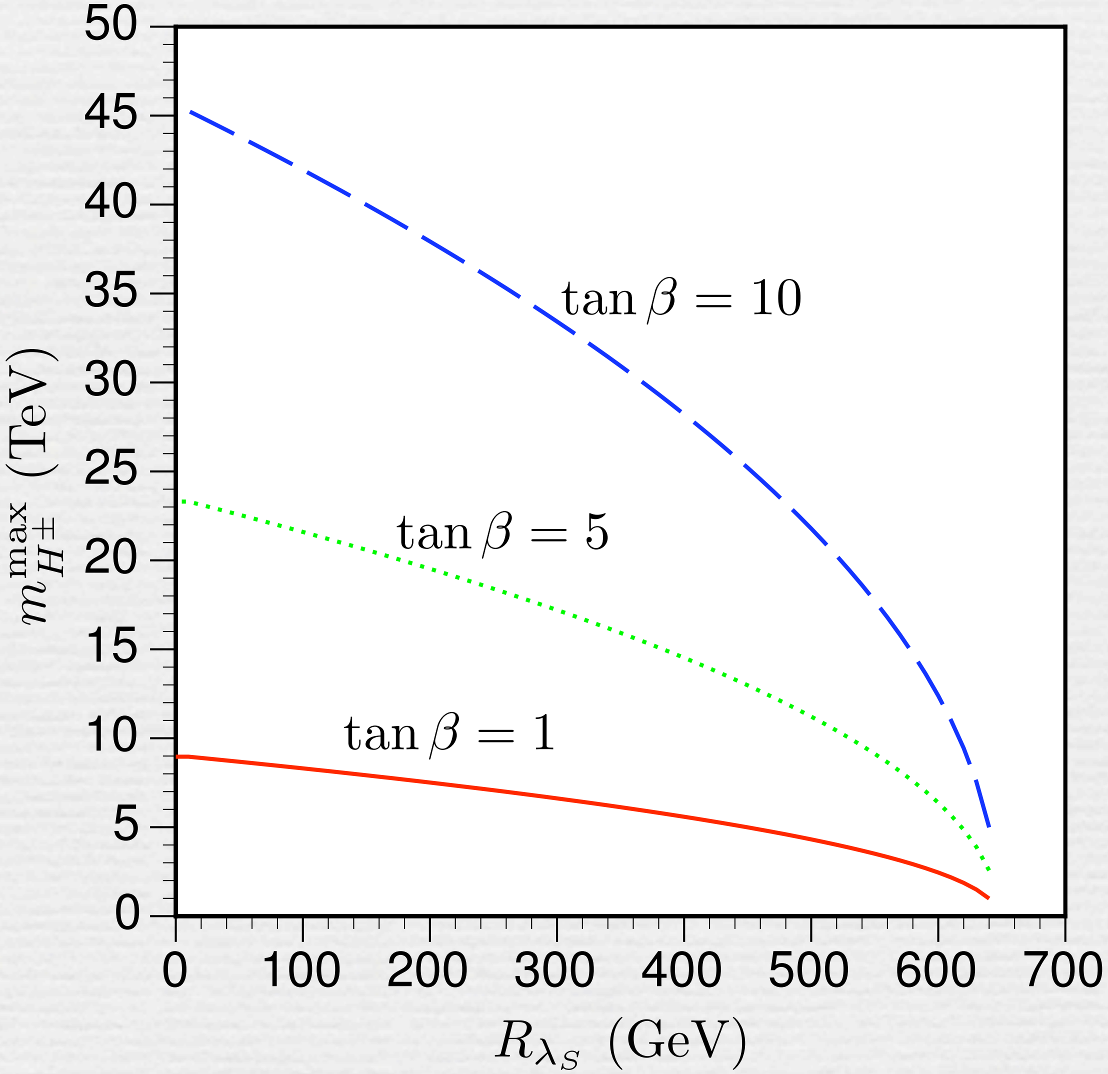
We can eliminate R_λ by using $m_{H^\pm}$:

$$\begin{aligned} \langle V_0 \rangle_{\rm vac} &= \frac{v^2}{8} (m_{H^\pm}^2 \sin^2 2\beta - m_W^2 \sin^2 2\beta - m_Z^2 \cos^2 2\beta) - \frac{|\lambda|^2}{4} v^2 v_S^2 + \frac{1}{2} R_{\lambda_S} v_{S_1} v_{S_2} v_{S_3} \\ &\quad - \frac{|\lambda_S|^2}{4} (v_{S_1}^2 v_{S_2}^2 + v_{S_2}^2 v_{S_3}^2 + v_{S_3}^2 v_{S_1}^2) - \frac{g_1'^2}{8} \Delta^2. < 0 \end{aligned}$$

Upper bound of the charged Higgs mass

$\langle V_0 \rangle_{\rm vac} < 0$ gives

$$m_{H^\pm}^2 < m_W^2 + \frac{2|\lambda|^2 v_S^2}{\sin^2 2\beta} + m_Z^2 \cot^2 2\beta - \frac{4R_{\lambda_S}}{v^2 \sin^2 2\beta} v_{S_1} v_{S_2} v_{S_3} + \frac{2|\lambda_S|^2}{v^2 \sin^2 2\beta}(v_{S_1}^2 v_{S_2}^2 + v_{S_2}^2 v_{S_3}^2 + v_{S_3}^2 v_{S_1}^2) + \frac{g_1'^2}{v^2 \sin^2 2\beta}\Delta^2 \equiv (m_{H^\pm}^{\rm max})^2 \ .$$



e.g.

$\tan\beta = 1, 3, 5,\ \lambda = -0.8,\ \lambda_S = 0.1,$
$v_S = 300$ GeV, $v_{S_1} = v_{S_2} = v_{S_3} = 3000$ GeV,
$Q_{H_d} = Q_{H_u} = 1,\ Q_S = -2,\ Q_{S_1} = Q_{S_2} = 2,$
$Q_{S_3} = -4,\ g_1' = \sqrt{5/3}\, g_1.$

R_{λ_S} is also constrained,

i.e., $R_{\lambda_S}^{\rm max} \simeq 640$ GeV.

Z-Z' mixing

- Once we have the U(1)' symmetry, the extra Z boson would appear and can mix with the ordinary Z boson.

Mass matrix: $$\frac{1}{2}\begin{pmatrix} Z_\mu & Z'_\mu \end{pmatrix} \mathcal{M}^2_{ZZ'} \begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix},$$

$$\mathcal{M}^2_{ZZ'} = \begin{pmatrix} \frac{1}{4}(g_2^2+g_1^2)v^2 & \frac{g'_1}{2}\sqrt{g_2^2+g_1^2}(Q_{H_d}v_d^2 - Q_{H_u}v_u^2) \\ \frac{g'_1}{2}\sqrt{g_2^2+g_1^2}(Q_{H_d}v_d^2 - Q_{H_u}v_u^2) & g_1'^2(Q^2_{H_d}v_d^2 + Q^2_{H_u}v_u^2 + Q^2_S v_S^2 + \sum_i Q^2_{S_i}v^2_{S_i}) \end{pmatrix}$$

Masses and Mixing angle:

$$\begin{aligned} m_Z^2 &= \frac{g_2^2+g_1^2}{4}v^2, \\ m_{Z'}^2 &= g_1'^2(Q^2_{H_d}v_d^2 + Q^2_{H_u}v_u^2 + Q^2_S v_S^2 + \sum_i Q^2_{S_i}v^2_{S_i}). \end{aligned}$$

$$\alpha_{ZZ'} = \frac{1}{2}\arctan\left(\frac{2(\mathcal{M}_{ZZ'})_{12}}{m^2_{Z'} - m^2_Z}\right)$$

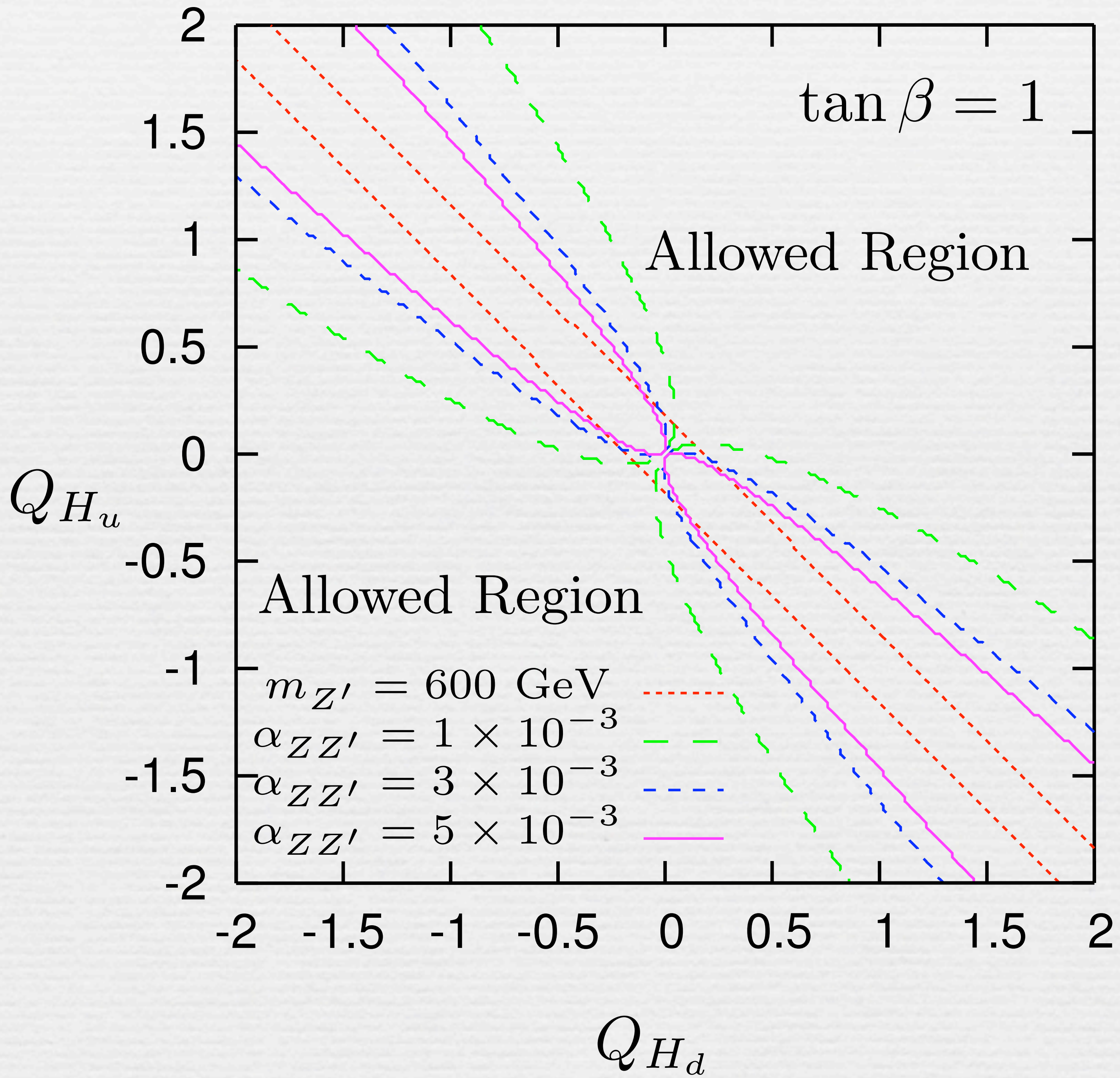
The experimental bounds are rather model dependent.
Here we adopt the typical values:

$$m_{Z'} > 600 \text{ GeV and } \alpha_{ZZ'} < \mathcal{O}(10^{-3})$$

To avoid these constraints, we should take $v_{S_i} = \mathcal{O}(\text{TeV})$.

Allowed region in the Q_{H_u}-Q_{H_d} plane

□ $\alpha_{ZZ'}$ and $m_{Z'}$ are plotted in the Q_{H_u}-Q_{H_d} plane.



$$Q_{H_d} + Q_{H_u} + Q_S = 0$$

$$Q_S = -Q_{S_1} = -Q_{S_2} = Q_{S_3}/2$$

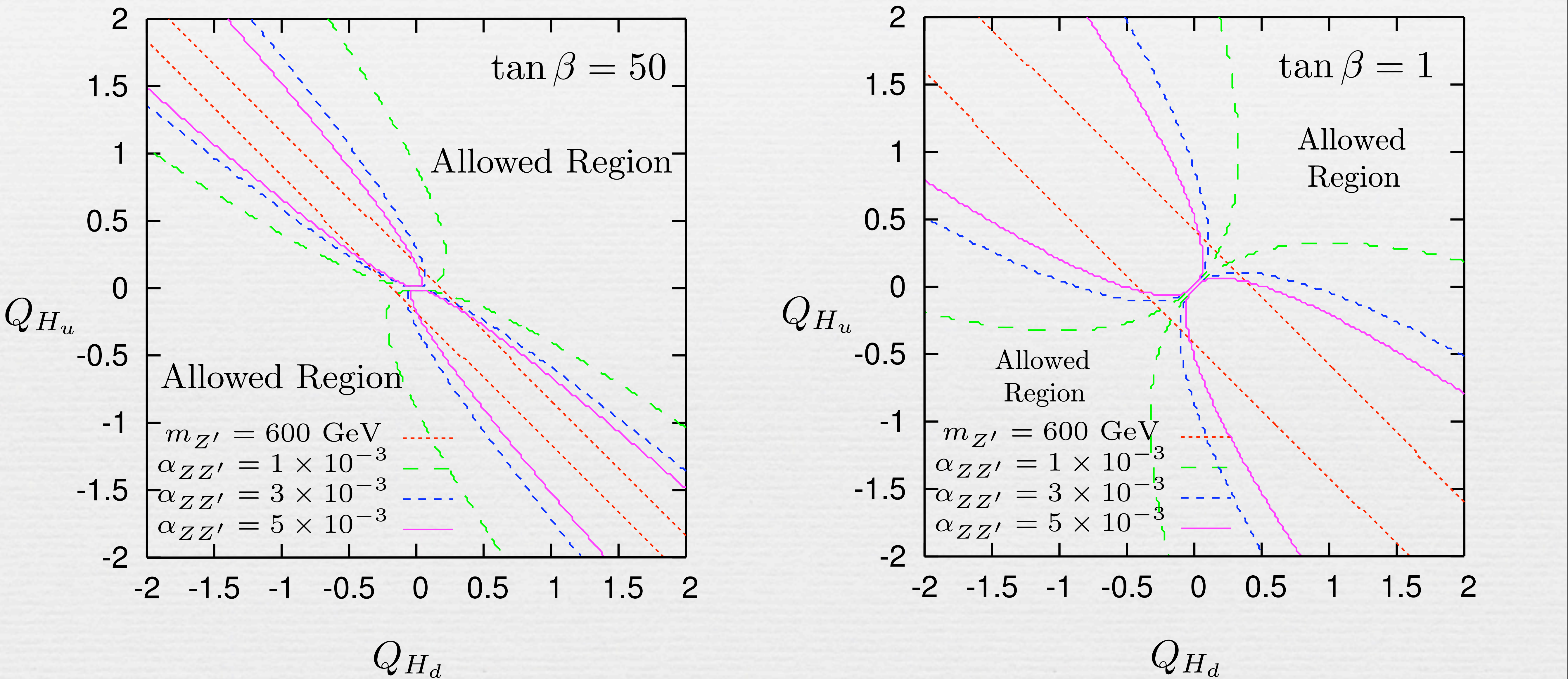
$\tan\beta = 1$, $v_S = 300$ GeV,
$v_{S_i} = 3000$ GeV $(i = 1-3)$

For $Q_{H_d} \simeq -Q_{H_u}$,
$\alpha_{ZZ'}$ becomes large.

Changing the input parameters as

$\tan\beta = 50,\ v_S = 300$ GeV, $v_{S_i} = 3000$ GeV $(i = 1-3)$

$\tan\beta = 1,\ v_{S_1} = v_{S_2} = 100$ GeV, $v_S = 500$ GeV, $v_{S_3} = 3000$ GeV.



For $Q_{H_u} \simeq Q_{H_d}$ and v_{S_i} $(i = 1-3) > \mathcal{O}(\text{TeV})$, no constraint on ZZ' mixing. This supports the original motivation of the sMSSM.

In the following, we take $Q_{H_d} = Q_{H_u} = 1$.

Searching the allowed regions

- To search a correct vacuum is a quite non-trivial task.

$\because$ presence of the trilinear terms in the potential.

c.f. In the MSSM at the tree-level, a global minimum is guaranteed by the tadpole conditions.

We search the allowed region under the theoretical and experimental constrains:

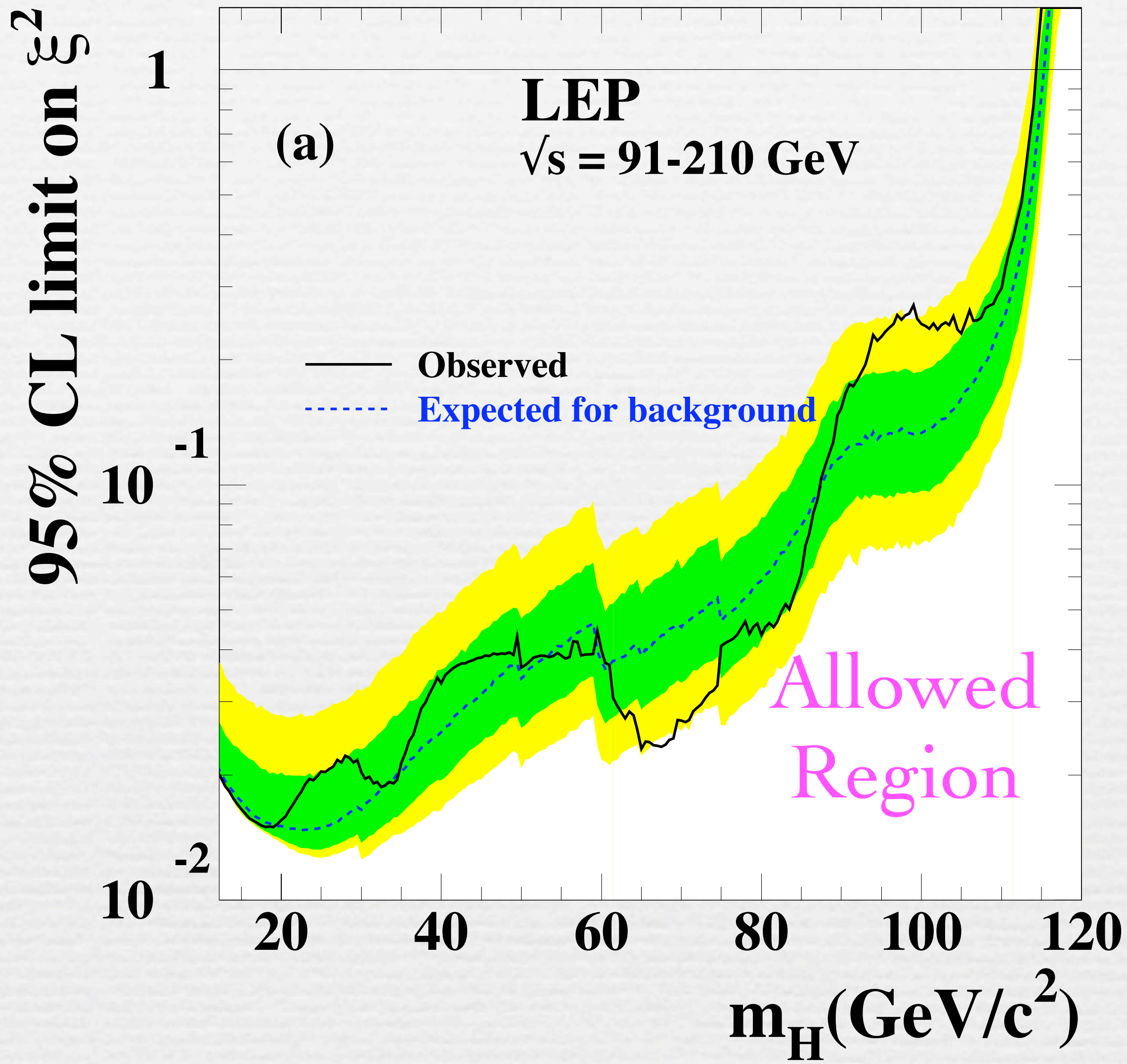
Theoretical constraints:

- Vacuum stability (unbounded from below)
- The EW vacuum should be a global minimum.

Experimental constraints:

Experimental constraints

- Higgs bounds@LEP [PLB565, 61 (2003)]
  Higgsstrahlung process constraints the Higgs mass m_{H_i} and coupling $\xi = g_{HVV}/g^{\rm SM}_{HVV}$.



- Lower bounds for SUSY particles:
  e.g., chargino mass > 104 GeV
  However, neutralino mass bounds are rather model dependent.
- ρ-parameter:

$$\Delta\rho \equiv \frac{\Pi^T_{ZZ}(0)}{m_Z^2} - \frac{\Pi^T_{WW}(0)}{m_W^2} < 0.002$$

- Z boson decays: $Z \to H_i H_j$ and $Z \to H_i l^+ l^-$

$$\sum_{i,j}\Gamma(Z\to H_iH_j) + \sum_i \Gamma(Z\to H_i l^+ l^-) < \Delta\Gamma_Z$$

where $\Delta\Gamma_Z = 2.0$ MeV @95%CL

- For simplicity, we here consider the following 2 cases:

  - Large VEV scenario (Case I)

    All secluded singlets VEVs are larger than 1 TeV.

$$\text{Case I}: \quad m^2_{SS_1} = m^2_{SS_2} = (500\ \text{GeV})^2,\ m^2_{S_1S_2} = -(50\ \text{GeV})^2,$$
$$v_S = 300\ \text{GeV},\ \boxed{v_{S_1} = v_{S_2} = v_{S_3} = 3000\ \text{GeV}.}$$

  - Small VEV scenario (Case II)

    The 2 secluded singlet VEVs are smaller than 1 TeV.

$$\text{Case II}: \quad m^2_{SS_1} = (306\ \text{GeV})^2,\ m^2_{SS_2} = (56\ \text{GeV})^2,\ m^2_{S_1S_2} = (100\ \text{GeV})^2,$$
$$v_S = 500\ \text{GeV},\ \boxed{v_{S_1} = v_{S_3} = 100\ \text{GeV},\ v_{S_2} = 3000\ \text{GeV}.}$$

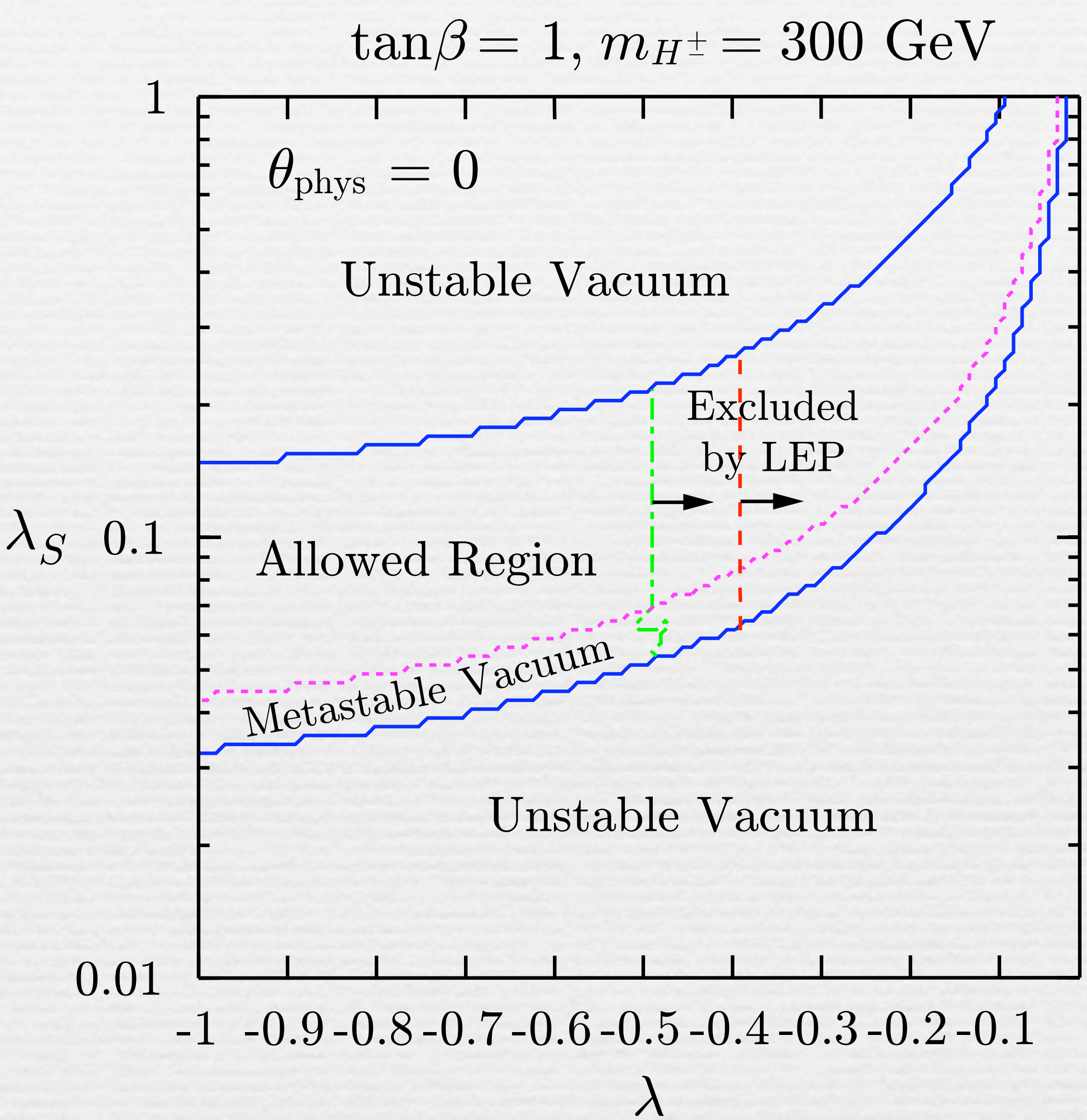
The remaining parameters are fixed as

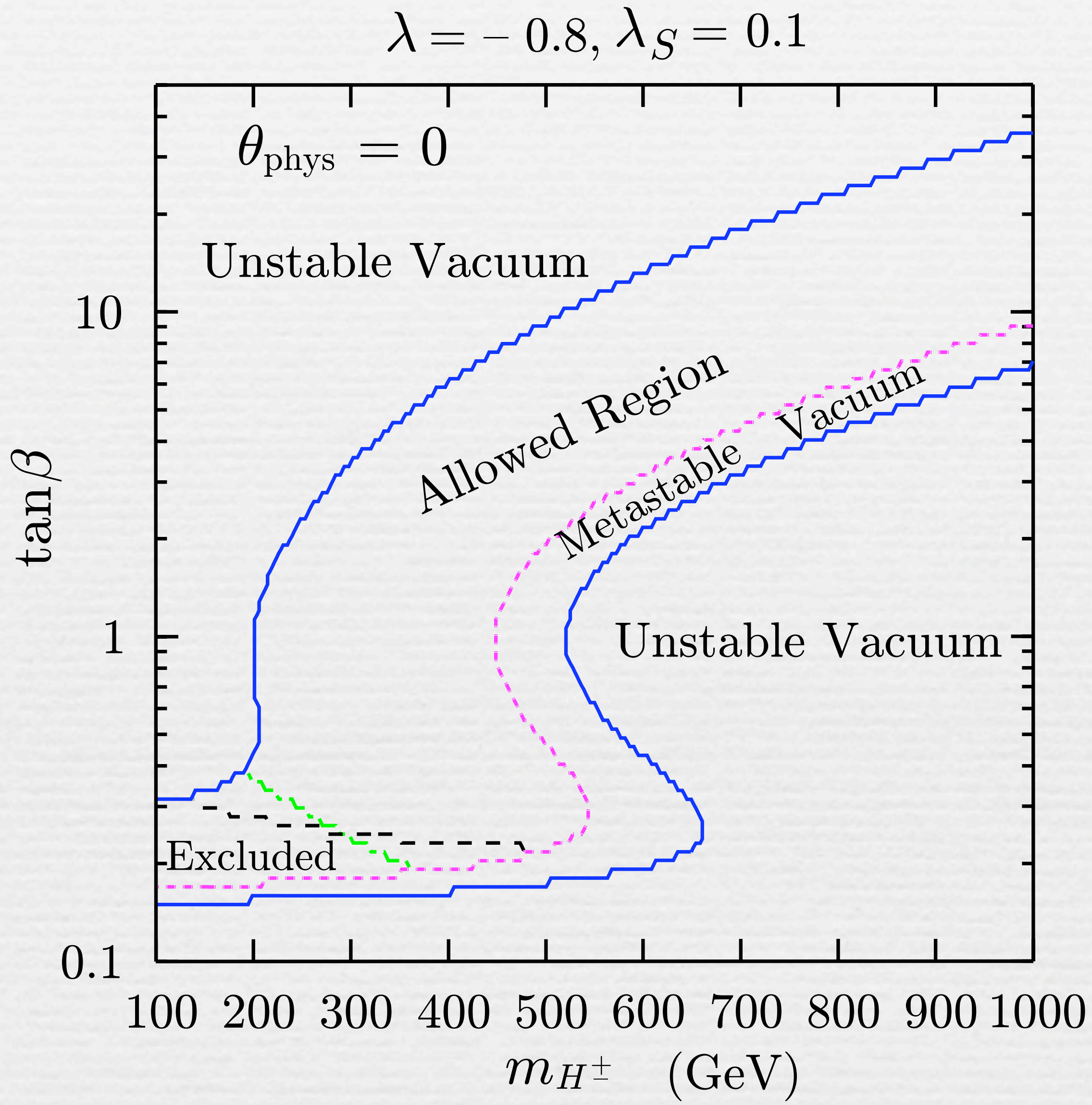
$$Q_{H_d} = Q_{H_u} = 1, \quad A_{\lambda_S} = A_\lambda(m_{H^\pm}), \quad A_t = A_b = \mu_{\text{eff}}/\tan\beta,$$
$$m_{\tilde{q}} = 1000\ \text{GeV}, \quad m_{\tilde{t}_R} = m_{\tilde{b}_R} = 500\ \text{GeV}, \quad M_2 = 200\ \text{GeV},$$

N.B. A_λ is a function of $m_{H^\pm}$.

For the moment, we consider the CP conserving case.

Allowed region for Case I

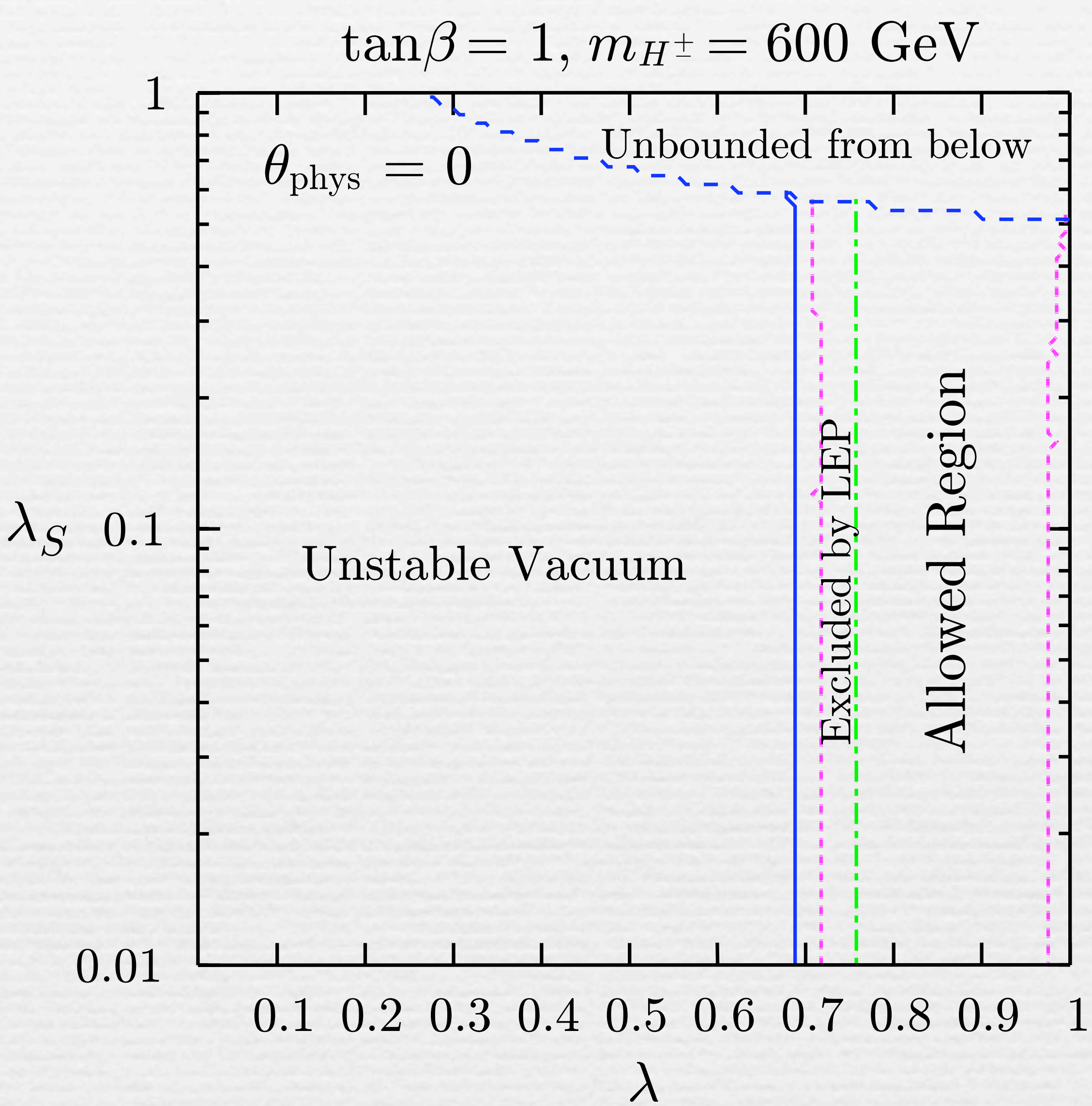


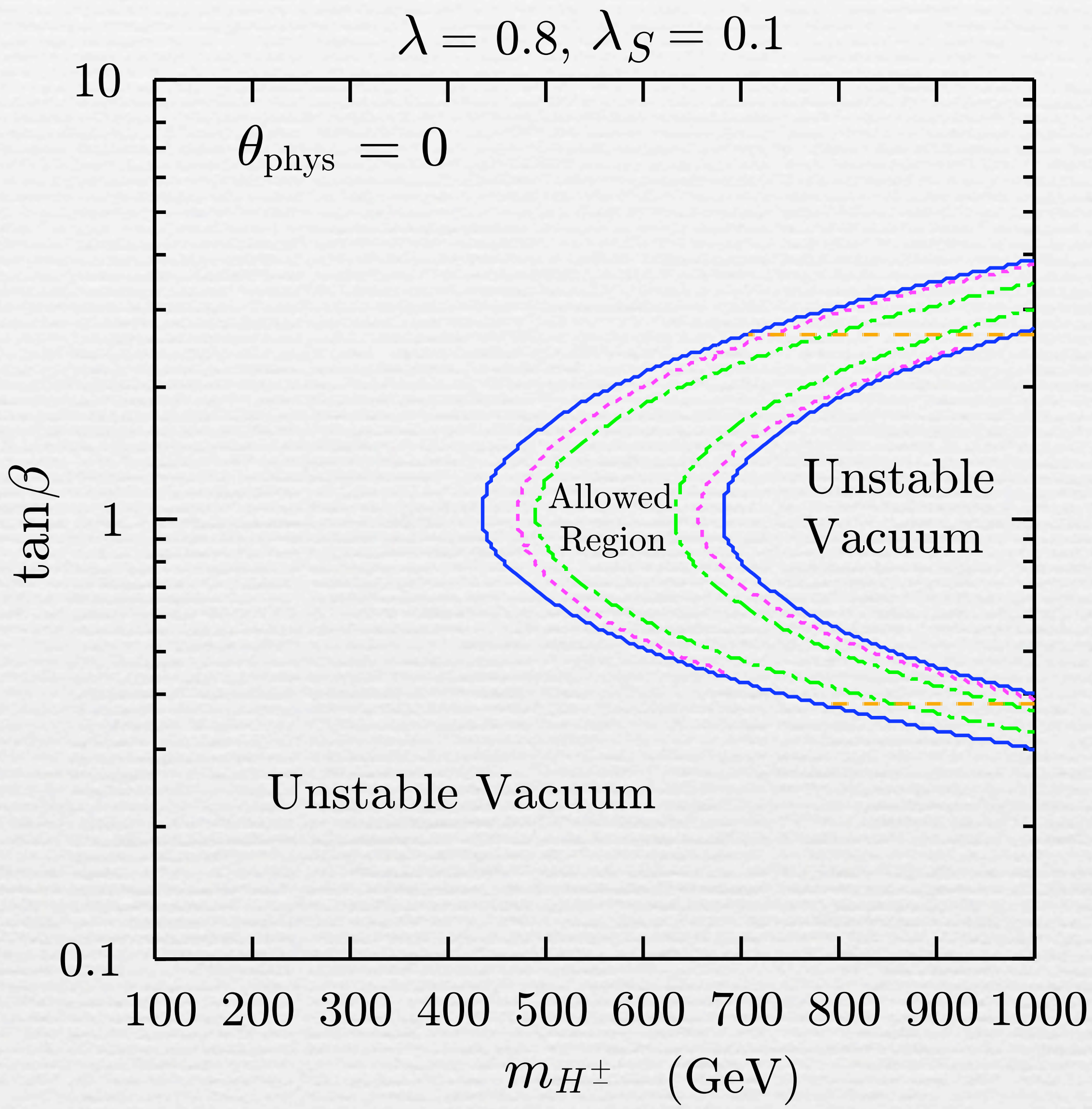


- $\lambda \simeq 10 \times \lambda_S$ is favored.
- Unlike the MSSM, $\tan\beta = 1$ is allowed.

  For $\tan\beta \simeq 1$, $m_{H_1} \simeq 140$ GeV. $\because$ Singlet contributions

Allowed region for Case II





- The conditions for the stable vacuum make the allowed region quite limited.

- Basically, the low $\tan\beta$ region is favored for the small VEV scenarios.

Explicit CP violation (ECPV)

CPV phases @1-loop

After solving the tadpole conditions for the CP-odd fields, we find

$$I_\lambda = -\frac{N_C}{8\pi^2 v^2}\left[\frac{m_t^2 I_t}{\sin^2\beta} f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \frac{m_b^2 I_b}{\cos^2\beta} f(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2)\right],$$

$$I_{\lambda_S} = 0,$$

$$\mathrm{Im}(m_{SS_1}^2 e^{i\varphi_1}) = \mathrm{Im}(m_{S_1 S_2}^2 e^{i\varphi_{12}})\frac{v_{S_2}}{v_S},$$

$$\mathrm{Im}(m_{SS_2}^2 e^{i\varphi_2}) = -\mathrm{Im}(m_{S_1 S_2}^2 e^{i\varphi_{12}})\frac{v_{S_1}}{v_S},$$

1-loop induced CPV

where $I_{t,b} = \mathrm{Im}(\lambda A_{t,b} e^{i\varphi_3})/\sqrt{2}$.

We take $I_{t,b} = 0$.

From the above eqns, it follows that

$$\theta_{SS_1} = \sin^{-1}\left[\left|\frac{m_{S_1 S_2}^2}{m_{SS_1}^2}\right|\frac{v_{S_2}}{v_S}\sin(\theta_{S_1 S_2} + \varphi_{12})\right] - \varphi_1,$$

$$\theta_{SS_2} = \sin^{-1}\left[-\left|\frac{m_{S_1 S_2}^2}{m_{SS_2}^2}\right|\frac{v_{S_1}}{v_S}\sin(\theta_{S_1 S_2} + \varphi_{12})\right] - \varphi_2.$$

where

$$\theta_{SS_1} = \mathrm{Arg}(m_{SS_1}^2),$$

$$\theta_{SS_2} = \mathrm{Arg}(m_{SS_2}^2)$$

$$\theta_{S_1 S_2} = \mathrm{Arg}(m_{S_1 S_2}^2)$$

N.B. The arguments in the arcsines should be smaller than 1.

$\longrightarrow$ Additional constraints on the input parameters

Scalar-pseudoscalar mixing

- The scalar-pseudoscalar mixing terms are proportional to the imaginary part of the CPV phase.

$$\frac{1}{2}\begin{pmatrix} \boldsymbol{h}_O^T \ \boldsymbol{h}_S^T \end{pmatrix} \mathcal{M}_{SP}^2 \begin{pmatrix} \boldsymbol{a}_O \\ \boldsymbol{a}_S \end{pmatrix}, \quad \mathcal{M}_{SP}^2 = \begin{pmatrix} \mathcal{M}_{SP}^{(O)} & \mathcal{M}_{SP}^{(OS)} \\ \left(\mathcal{M}_{SP}^{(OS)}\right)^T & \mathcal{M}_{SP}^{(S)} \end{pmatrix}.$$

$$\mathcal{M}_{SP}^{(O)} = \mathbf{0}_{3\times 3}, \quad \mathcal{M}_{SP}^{(OS)} = \boxed{\mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}})} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{v_{S_2}}{v_S} & -\frac{v_{S_1}}{v_S} & 0 \end{pmatrix},$$

$$\mathcal{M}_{SP}^{(S)} = \boxed{\mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}})} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

To enhance the effect of CP violation, we take the large

$$\mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}})\frac{v_{S_1}}{v_S}, \quad \mathrm{Im}(m_{S_1S_2}^2 e^{i\varphi_{12}})\frac{v_{S_2}}{v_S}$$

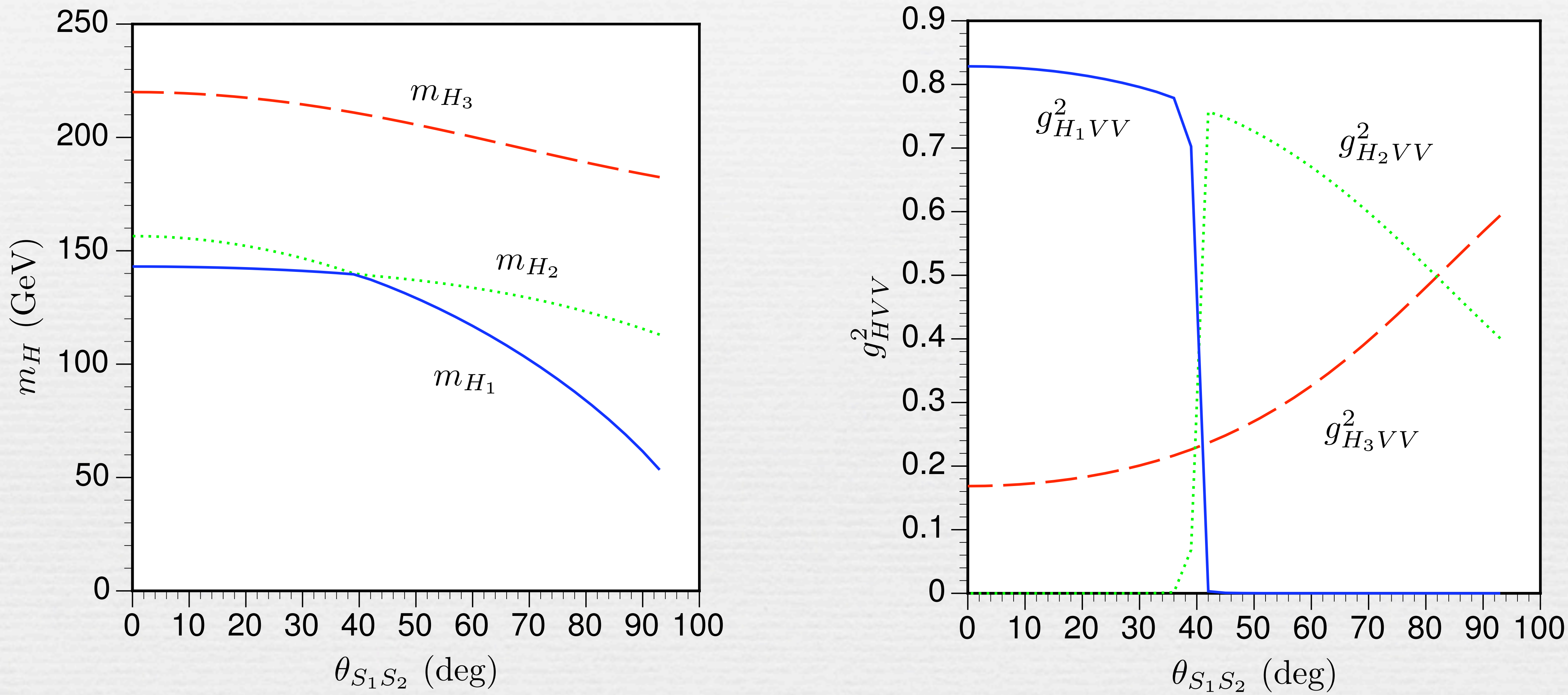
which gives

$$|m_{SS_1}^2| \simeq |m_{S_1S_2}^2|\frac{v_{S_2}}{v_S}, \qquad |m_{SS_2}^2| \simeq |m_{S_1S_2}^2|\frac{v_{S_1}}{v_S}, \quad \text{for } \sin(\theta_{S_1S_2}+\varphi_{12}) \simeq 1$$

Higgs boson masses and couplings

Actually, Case II is one of realization for the large CPV.

$\tan\beta = 1,\ m_{H^\pm} = 600$ GeV, $\lambda = 0.8,\ \lambda_S = 0.1.$



⋆ Similar to the CPX scenario in the MSSM, m_{H_1} can be as light as 50 GeV while $g^2_{H_1VV} \simeq 0$.

⋆ However, large A and μ are not necessarily required.

⟶ Different patters of the SUSY particle spectrum

c.f. It seems that this is not typical in the NMSSM.

[Funakubo and Tao, PTP113 (05)]

Spontaneous CP violation (SCPV)

Spontaneous CP violation

- Unlike the MSSM, the tree-level SCPV is possible.

The phase dependent terms in $\langle V_0 \rangle$ are

$$\begin{aligned}\langle V_0 \rangle &= -m^2_{SS_1} v_S v_{S_1} \cos\varphi_1 - m^2_{SS_2} v_S v_{S_2} \cos\varphi_2 - m^2_{S_1S_1} v_{S_1} v_{S_2} \cos(\varphi_1 - \varphi_2) \\ &\quad - \frac{\lambda A_\lambda}{\sqrt{2}} v_d v_u v_S \cos\varphi_3 - \frac{\lambda_S A_{\lambda_S}}{\sqrt{2}} v_{S_1} v_{S_2} v_{S_3} \cos\varphi_4.\end{aligned}$$

For simplicity, $m^2_{SS_1}, m^2_{SS_2}, m^2_{S_1S_2}, \lambda A_\lambda, \lambda_S A_{\lambda_S} \in \boldsymbol{R}$.

Tadpole conditions for the CPV phases:

$$\begin{aligned}\frac{\partial \langle V_0 \rangle}{\partial \varphi_1} &= m^2_{SS_1} v_S v_{S_1} \sin\varphi_1 + m^2_{S_1S_2} v_{S_1} v_{S_2} \sin(\varphi_1 - \varphi_2) = 0, \\ \frac{\partial \langle V_0 \rangle}{\partial \varphi_2} &= m^2_{SS_2} v_S v_{S_2} \sin\varphi_2 - m^2_{S_1S_2} v_{S_1} v_{S_2} \sin(\varphi_1 - \varphi_2) = 0, \\ \frac{\partial \langle V_0 \rangle}{\partial \varphi_3} &= \frac{\lambda A_\lambda}{\sqrt{2}} v_d v_u v_S \sin\varphi_3 = 0, \\ \frac{\partial \langle V_0 \rangle}{\partial \varphi_4} &= \frac{\lambda_S A_{\lambda_S}}{\sqrt{2}} v_{S_1} v_{S_2} v_{S_3} \sin\varphi_4 = 0, \qquad \therefore \quad \boxed{\varphi_3 = \varphi_4 = 0.}\end{aligned}$$

The CP violating solutions satisfy

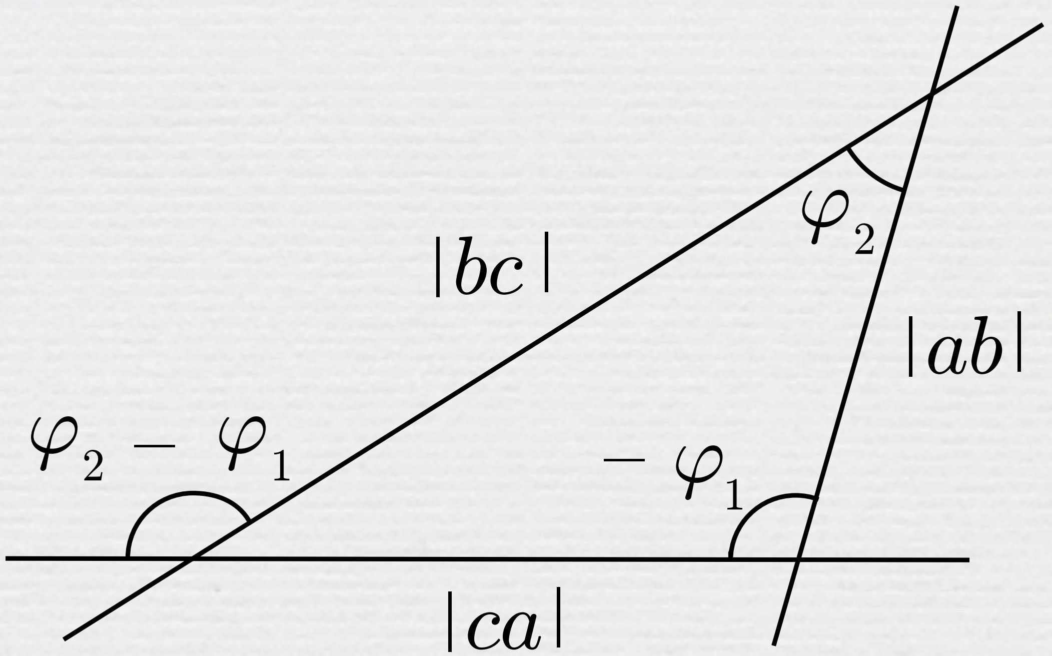
$$a \sin\varphi_1 + b\sin\varphi_2 = 0,$$
$$a\cos\varphi_1 + b\cos\varphi_2 = -\frac{ab}{c},$$

where $a = m^2_{SS_1} v_S v_{S_1},\ b = m^2_{SS_2} v_S v_{S_2},\ c = m^2_{S_1 S_2} v_{S_1} v_{S_2}$

If the solutions exist, the following triangle can be formed.

Solutions:

$$\cos\varphi_1 = \frac{1}{2}\left(\frac{bc}{a^2} - \frac{c}{b} - \frac{b}{c}\right),$$
$$\cos\varphi_2 = \frac{1}{2}\left(\frac{ac}{b^2} - \frac{a}{c} - \frac{c}{a}\right),$$
$$\cos(\varphi_1 - \varphi_2) = \frac{1}{2}\left(\frac{ab}{c^2} - \frac{b}{a} - \frac{a}{b}\right),$$

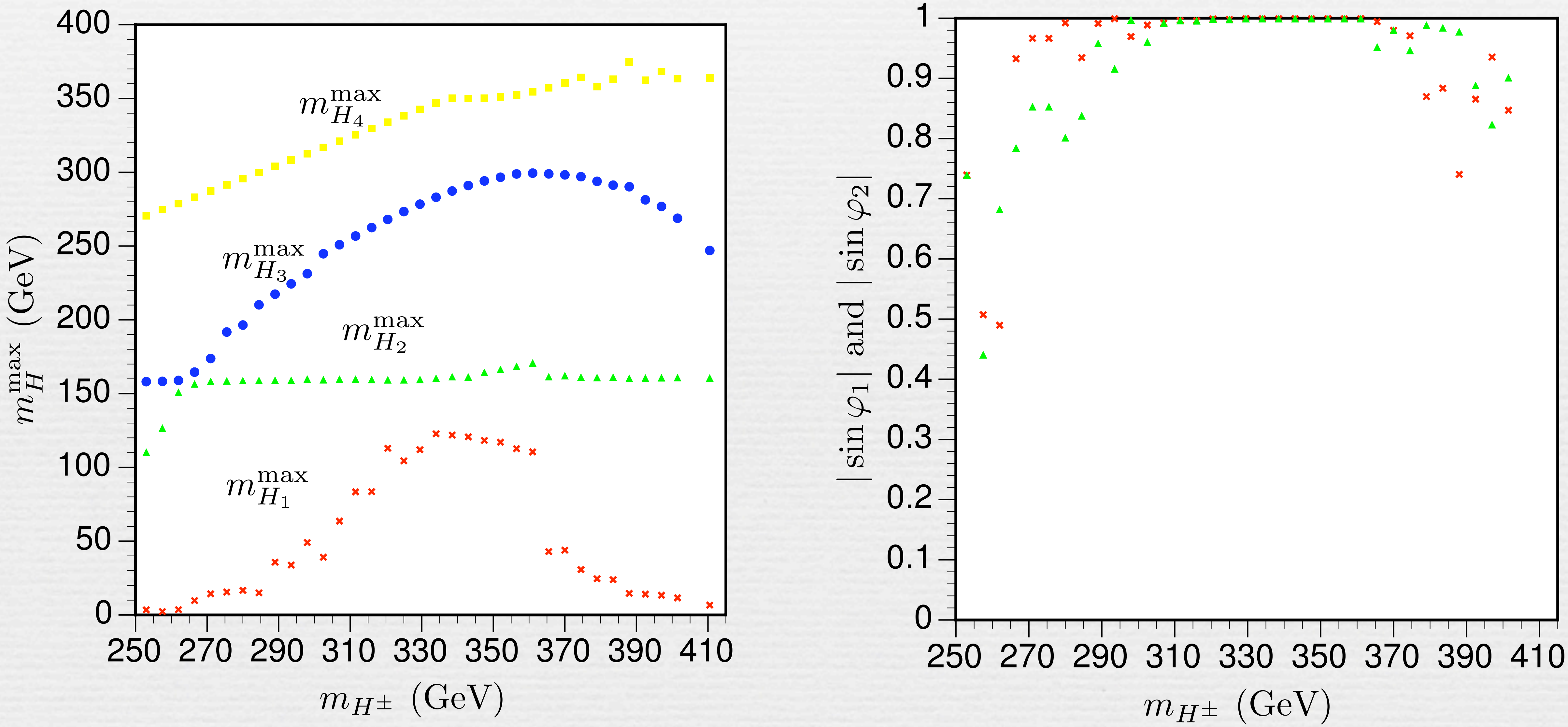


And the potential has the CPV minimum if $ac/b < 0$.

Scanning the following parameters,

$$m_{SS_1}^2 = m_{SS_2}^2 = (10\ \text{GeV})^2 - (1000\ \text{GeV})^2,$$
$$-m_{S_1S_2}^2 = (1000\ \text{GeV})^2 - (10\ \text{GeV})^2,$$

- We obtain the maximal values of $m_{H_i} (i = 1-4)$.



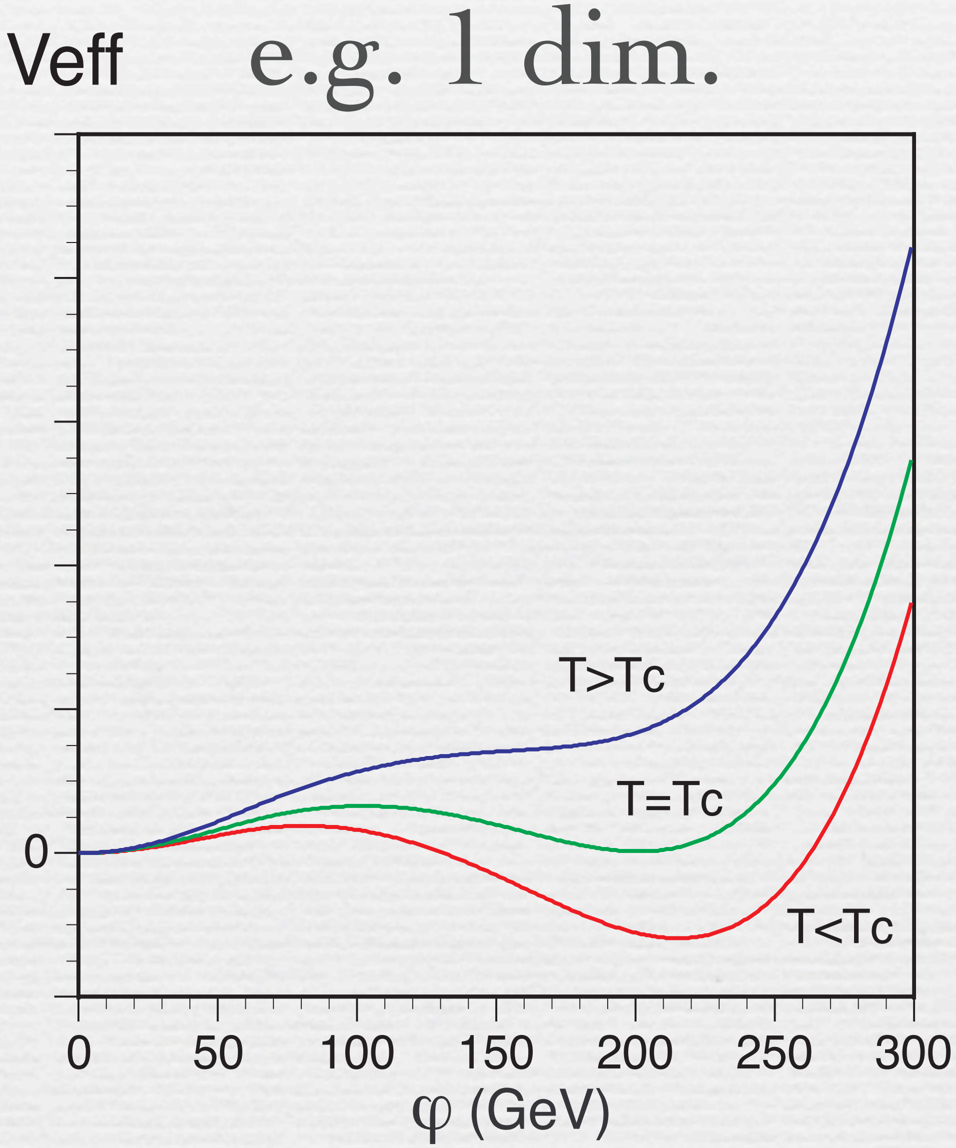
⋆ Typically, the light Higgs boson exists depending $m_{H^\pm}$.

⋆ For $m_{H^\pm} \simeq 334$ GeV, $m_{H_1} \simeq 125$ GeV with maximal CPV.

Application

- Electroweak phase transition

e.g. 1 dim.



10 order parameters

$$v_d,\ v_u e^{i\theta_2}, v_S e^{i\theta_S},\ v_{S_1} e^{i\theta_{S_1}},\ v_{S_2} e^{i\theta_{S_2}},\ v_{S_3}$$

Minimum search in the 10 dim. space.

In general, the calculation of the critical temperature is a time consuming task.

work in progress

Summary

- We have studied the Higgs sector of the secluded U(1)' extended MSSM (sMSSM).
- The upper bound of the charged Higgs boson can be obtained from the condition for the vacuum meta-stability.
- Similar to the CPX scenario in the MSSM, the CPV effect on the Higgs boson masses and couplings can be large. However, the large A and μ are not necessarily required.
  → The different patterns of the SUSY particle spectrum.
- SCPV can be possible at the tree level. In such case, the Higgs bosons are typically light. e.g. $m_{H_1} \simeq 125$ GeV with the maximal CPV.

work in progress

- Electroweak phase transition with/without CP violation

Backup

Tadpole conditions

- **Tadpole conditions**

$$
\begin{aligned}
\frac{1}{v_d}\left\langle\frac{\partial V_0}{\partial h_d}\right\rangle &= m_1^2+\frac{g_2^2+g_1^2}{8}(v_d^2-v_u^2)-R_\lambda\frac{v_u v_S}{v_d}+\frac{|\lambda|^2}{2}(v_u^2+v_S^2)+\frac{g_1'^2}{2}Q_{H_d}\Delta=0,\\
\frac{1}{v_u}\left\langle\frac{\partial V_0}{\partial h_u}\right\rangle &= m_2^2-\frac{g_2^2+g_1^2}{8}(v_d^2-v_u^2)-R_\lambda\frac{v_d v_S}{v_u}+\frac{|\lambda|^2}{2}(v_d^2+v_S^2)+\frac{g_1'^2}{2}Q_{H_u}\Delta=0,\\
\frac{1}{v_S}\left\langle\frac{\partial V_0}{\partial h_S}\right\rangle &= m_S^2+\mathrm{Re}(m_{SS_1}^2e^{i\varphi_1})\frac{v_{S_1}}{v_S}+\mathrm{Re}(m_{SS_2}^2e^{i\varphi_2})\frac{v_{S_2}}{v_S}-R_\lambda\frac{v_d v_u}{v_S}\\
&\quad+\frac{|\lambda|^2}{2}(v_d^2+v_u^2)+\frac{g_1'^2}{2}Q_S\Delta=0,\\
\frac{1}{v_{S_1}}\left\langle\frac{\partial V_0}{\partial h_{S_1}}\right\rangle &= m_{S_1}^2+\mathrm{Re}(m_{SS_1}^2e^{i\varphi_1})\frac{v_S}{v_{S_1}}+\mathrm{Re}(m_{S_1S_2}^2e^{i\varphi_5})\frac{v_{S_2}}{v_{S_1}}-R_{\lambda_S}\frac{v_{S_2}v_{S_3}}{v_{S_1}}\\
&\quad+\frac{|\lambda_S|^2}{2}(v_{S_2}^2+v_{S_3}^2)+\frac{g_1'^2}{2}Q_{S_1}\Delta=0,\\
\frac{1}{v_{S_2}}\left\langle\frac{\partial V_0}{\partial h_{S_2}}\right\rangle &= m_{S_2}^2+\mathrm{Re}(m_{SS_2}^2e^{i\varphi_2})\frac{v_S}{v_{S_2}}+\mathrm{Re}(m_{S_1S_2}^2e^{i\varphi_5})\frac{v_{S_1}}{v_{S_2}}-R_{\lambda_S}\frac{v_{S_1}v_{S_3}}{v_{S_2}}\\
&\quad+\frac{|\lambda_S|^2}{2}(v_{S_1}^2+v_{S_3}^2)+\frac{g_1'^2}{2}Q_{S_2}\Delta=0,\\
\frac{1}{v_{S_3}}\left\langle\frac{\partial V_0}{\partial h_{S_3}}\right\rangle &= m_{S_3}^2-R_{\lambda_S}\frac{v_{S_1}v_{S_2}}{v_{S_3}}+\frac{|\lambda_S|^2}{2}(v_{S_1}^2+v_{S_2}^2)+\frac{g_1'^2}{2}Q_{S_3}\Delta=0,\\
\frac{1}{v_u}\left\langle\frac{\partial V_0}{\partial a_d}\right\rangle &= \frac{1}{v_d}\left\langle\frac{\partial V_0}{\partial a_u}\right\rangle=I_\lambda v_S=0,\\
\left\langle\frac{\partial V_0}{\partial a_S}\right\rangle &= -\mathrm{Im}(m_{SS_1}^2e^{i\varphi_1})v_{S_1}-\mathrm{Im}(m_{SS_2}^2e^{i\varphi_2})v_{S_2}+I_\lambda v_d v_u=0,\\
\left\langle\frac{\partial V_0}{\partial a_{S_1}}\right\rangle &= -\mathrm{Im}(m_{SS_1}^2e^{i\varphi_1})v_S+\mathrm{Im}(m_{S_1S_2}^2e^{i\varphi_5})v_{S_2}+I_{\lambda_S}v_{S_2}v_{S_3}=0,\\
\left\langle\frac{\partial V_0}{\partial a_{S_2}}\right\rangle &= -\mathrm{Im}(m_{SS_2}^2e^{i\varphi_2})v_S-\mathrm{Im}(m_{S_1S_2}^2e^{i\varphi_5})v_{S_1}+I_{\lambda_S}v_{S_1}v_{S_3}=0,\\
\left\langle\frac{\partial V_0}{\partial a_{S_3}}\right\rangle &= I_{\lambda_S}v_{S_1}v_{S_2}=0,
\end{aligned}
$$

Mass matrix of the pseudoscalar

$$\frac{1}{2}\begin{pmatrix} \boldsymbol{a}_O^T & \boldsymbol{a}_S^T \end{pmatrix} \mathcal{M}_P^2 \begin{pmatrix} \boldsymbol{a}_O \\ \boldsymbol{a}_S \end{pmatrix}, \quad \mathcal{M}_P^2 = \begin{pmatrix} \mathcal{M}_P^{(O)} & \mathcal{M}_P^{(OS)} \\ \left(\mathcal{M}_P^{(OS)}\right)^T & \mathcal{M}_P^{(S)} \end{pmatrix}, \tag{A.23}$$

where

$$\mathcal{M}_P^{(O)} = \begin{pmatrix} R_\lambda \frac{v_u v_S}{v_d} & R_\lambda v_S & R_\lambda v_u \\ R_\lambda v_S & R_\lambda \frac{v_d v_S}{v_u} & R_\lambda v_d \\ R_\lambda v_u & R_\lambda v_d & (\mathcal{M}_P^{(O)})_{33} \end{pmatrix}, \quad \mathcal{M}_P^{(OS)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1}) & \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2}) & 0 \end{pmatrix},$$

$$\mathcal{M}_P^{(S)} = \begin{pmatrix} (\mathcal{M}_P^{(S)})_{11} & -\mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}}) + R_{\lambda_S} v_{S_3} & R_{\lambda_S} v_{S_2} \\ -\mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}}) + R_{\lambda_S} v_{S_3} & (\mathcal{M}_P^{(S)})_{22} & R_{\lambda_S} v_{S_1} \\ R_{\lambda_S} v_{S_2} & R_{\lambda_S} v_{S_1} & R_{\lambda_S} \frac{v_{S_1} v_{S_2}}{v_{S_3}} \end{pmatrix}, \tag{A.24}$$

with

$$(\mathcal{M}_P^{(O)})_{33} = \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1}) \frac{v_{S_1}}{v_S} + \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2}) \frac{v_{S_2}}{v_S} + R_\lambda \frac{v_d v_u}{v_S}, \tag{A.25}$$

$$(\mathcal{M}_P^{(S)})_{11} = \mathrm{Re}(m_{SS_1}^2 e^{i\varphi_1}) \frac{v_S}{v_{S_1}} + \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}}) \frac{v_{S_2}}{v_{S_1}} + R_{\lambda_S} \frac{v_{S_2} v_{S_3}}{v_{S_1}}, \tag{A.26}$$

$$(\mathcal{M}_P^{(S)})_{22} = \mathrm{Re}(m_{SS_2}^2 e^{i\varphi_2}) \frac{v_S}{v_{S_2}} + \mathrm{Re}(m_{S_1S_2}^2 e^{i\varphi_{12}}) \frac{v_{S_1}}{v_{S_2}} + R_{\lambda_S} \frac{v_{S_1} v_{S_3}}{v_{S_2}}. \tag{A.27}$$