Gravity & Beyond

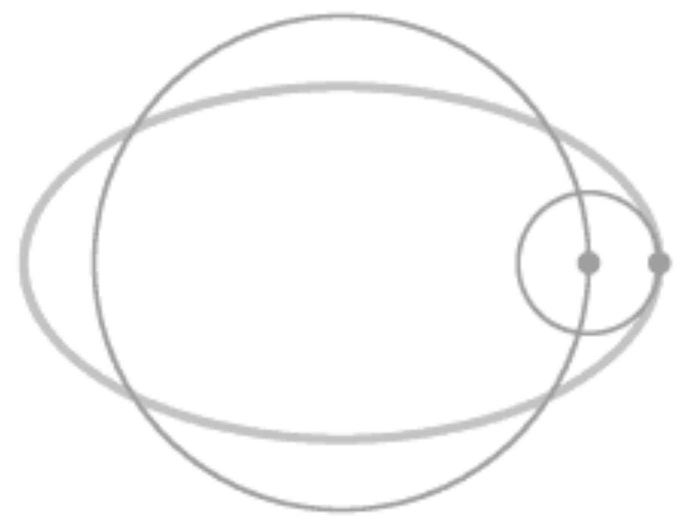
林豐利（台師大物理系）

What is gravity?

Pre-gravity: Aristole & Ptolemy

Earth: free fall

Heaven: perfect circle



Newton: gravity is a universal force!

Earth (Cavendish expt)

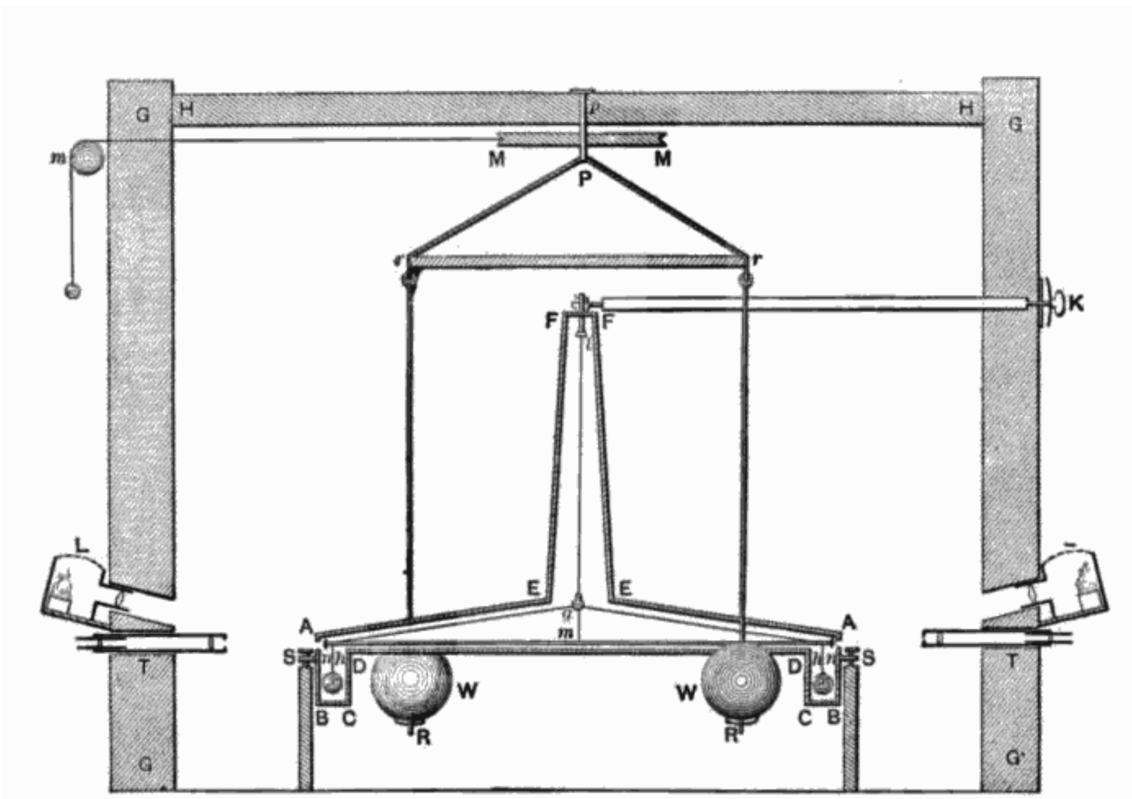
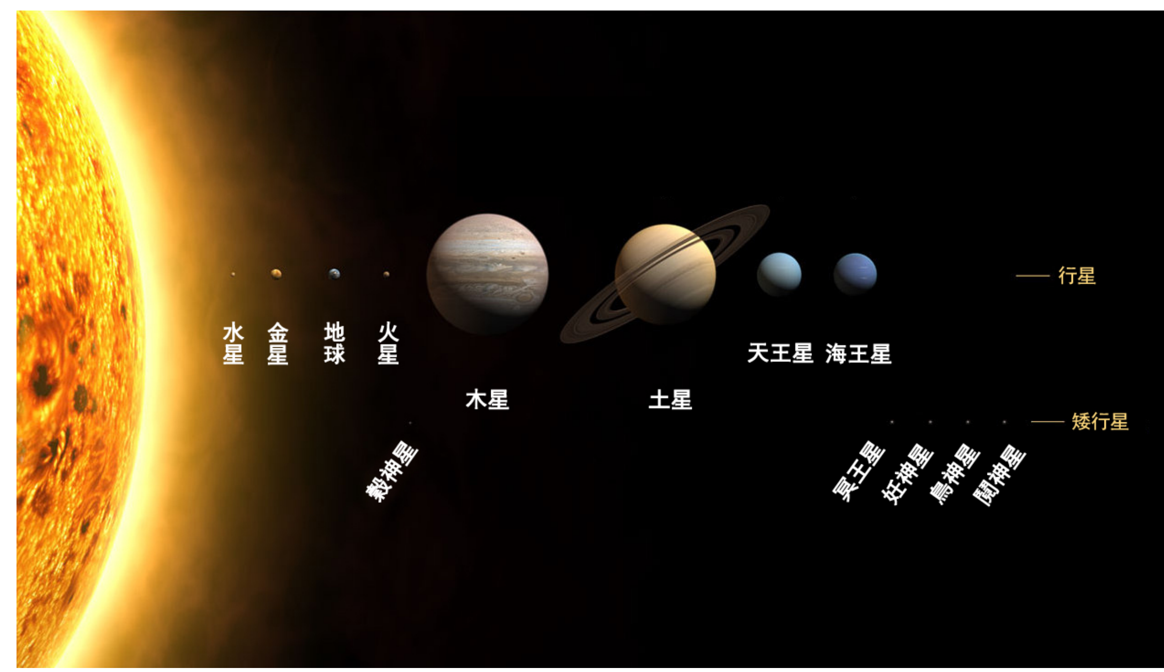


Fig. 1

Heaven (solar system)



$$\vec{F} = \frac{G m_1 m_2}{r^2}\hat{r}$$

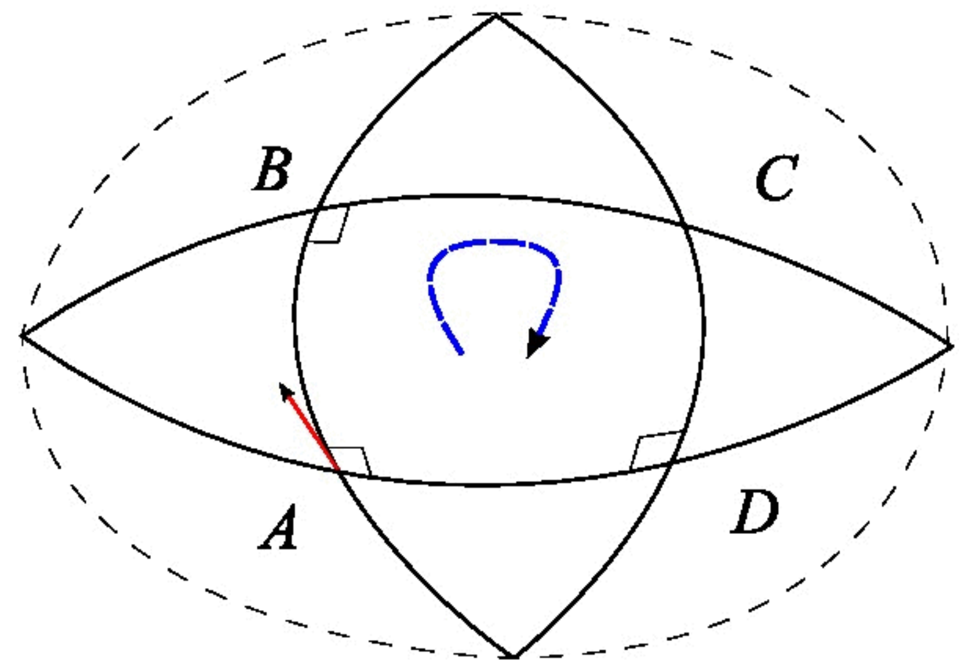
Einstein: gravity is geometry!

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

E.g., flat spacetime metric: $ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$

connection: $\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})$

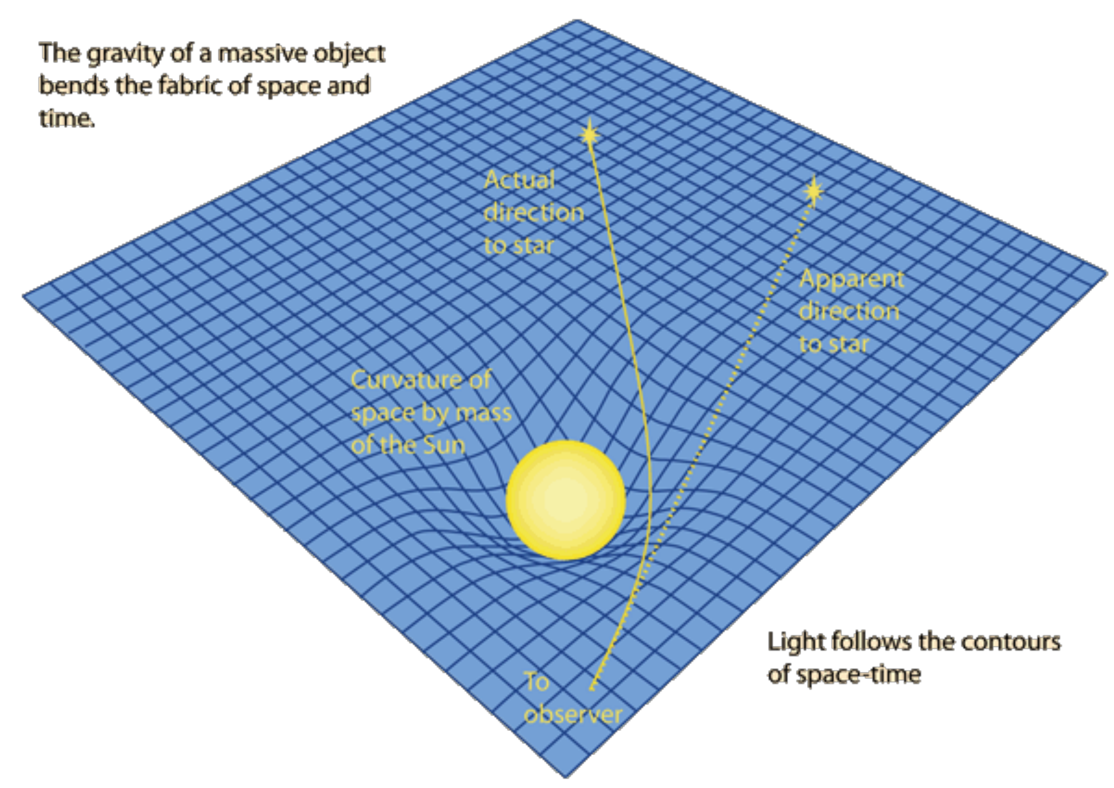
Riemann curvature: $R^\rho_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma}$



geodesic equation: $\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0$

Equivalence principle: $m_{inertial} = m_{gravitational}$

$$F_j = -m R_{j0k0} x^k$$



Gravity & Spacetime

- Matters curve spacetime.
- As matters move, the spacetime geometry becomes time-dependent.
- Thus, the spacetime has its own dynamics, i.e., the spacetime geometry evolves.
- This is a novel idea in contrast to Newtonian spacetime, which is fixed and absolute.

Einstein equation

- The spacetime dynamics is dictated by Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

spacetime curvature **cosmological constant** **energy-momentum tensor**

$R_{\mu\nu} := R^{\rho}{}_{\mu\rho\nu}$, $R := g^{\mu\nu} R_{\mu\nu}$

Accelerating Universe!

E.g. in comving frame of fluid:

$$(T^{\alpha\beta})_{\alpha,\beta=0,1,2,3} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

J. A. Wheeler:
Spacetime tells matter how to move; matter tells spacetime how to curve.

Gravitational waves

- Like the EM waves, the linear perturbation of the metric about flat space time can propagate in the wave form.

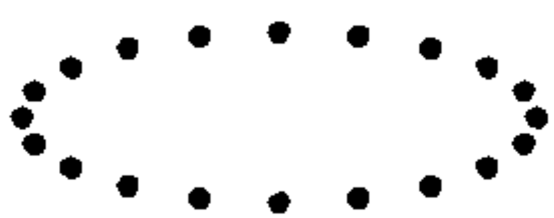
$$\Box \bar{h}_{\mu\nu} = 8\pi G T_{\mu\nu} \ , \qquad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$$

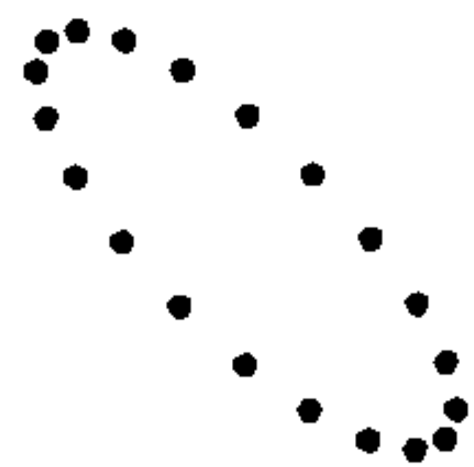
- There are two polarizations:

$$\bar{h}^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\bar{h}^{TT}_{ij} = \frac{2G}{c^4 r} \ddot{I}^{TT}_{ij}|_{ret} \qquad I_{ij} = \int d^3x \ (x_i x_j - \frac{x^2}{3}\delta_{ij}) \ T_{00}(t, x)$$

← **quadrupole moment**





Strong gravity

- Gravity is the weakest one among the known 4 fundamental forces, i.e., gravity, weak force, electromagnetism (EM), strong/nclear force.

- E.g. the gravity force produced by a proton is 23 order less than the associated EM force.

- However, gravity is always attractive, so we can feel it daily but not the electric force.

- Two arenas where gravity is really strong and dominates are (i) cosmology (ii) black holes

Cosmology

- Cosmological principle: At large scale, the Universe is homogeneous and isotropic.

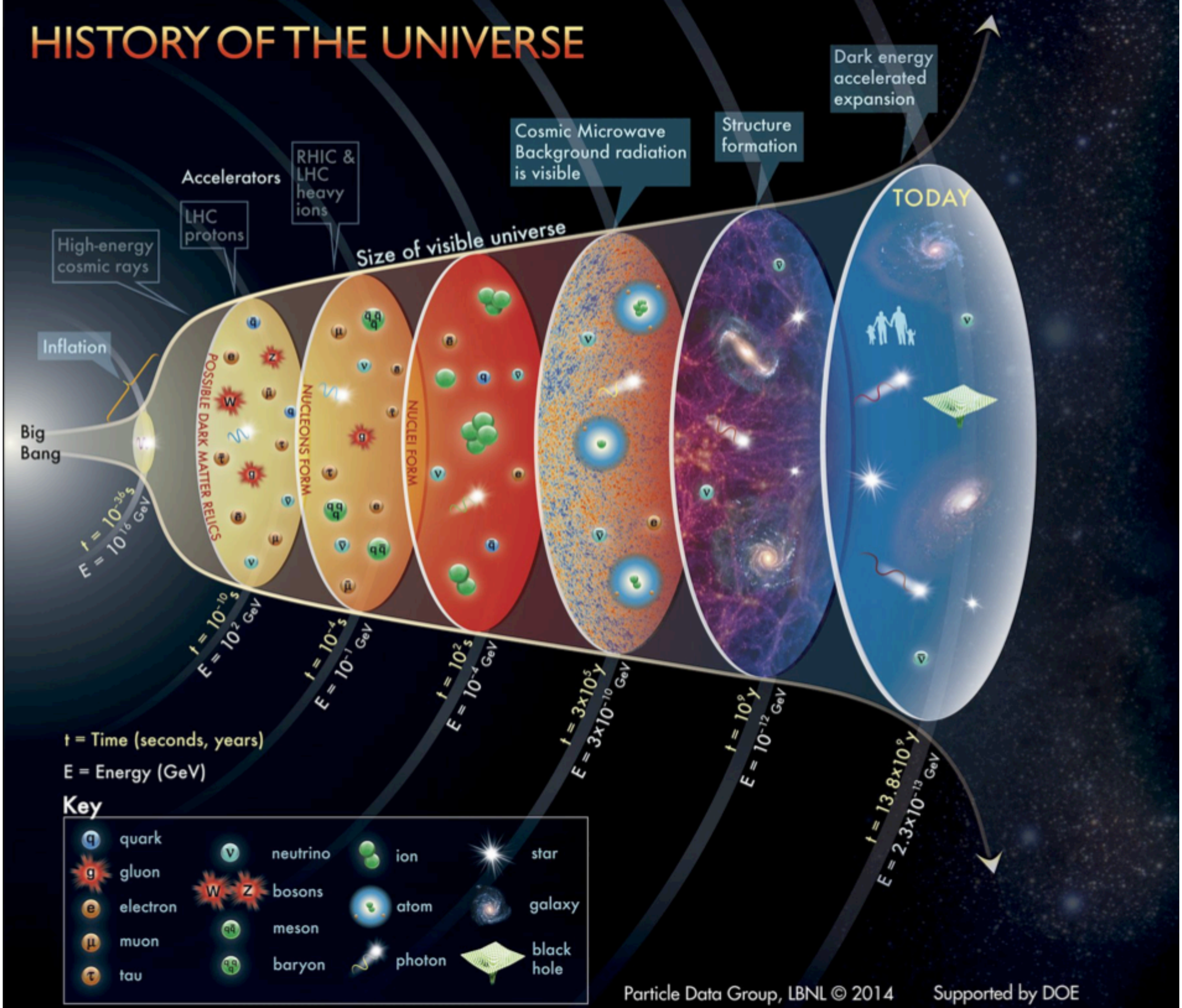
- Based on this principle, the large-scale is a simple system controlled the scale factor.

$$\text{FRW metric: } ds^2 = -c^2dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

$$(\frac{\dot{a}}{a})^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3p}{c^2}) + \frac{\Lambda c^2}{3}$$

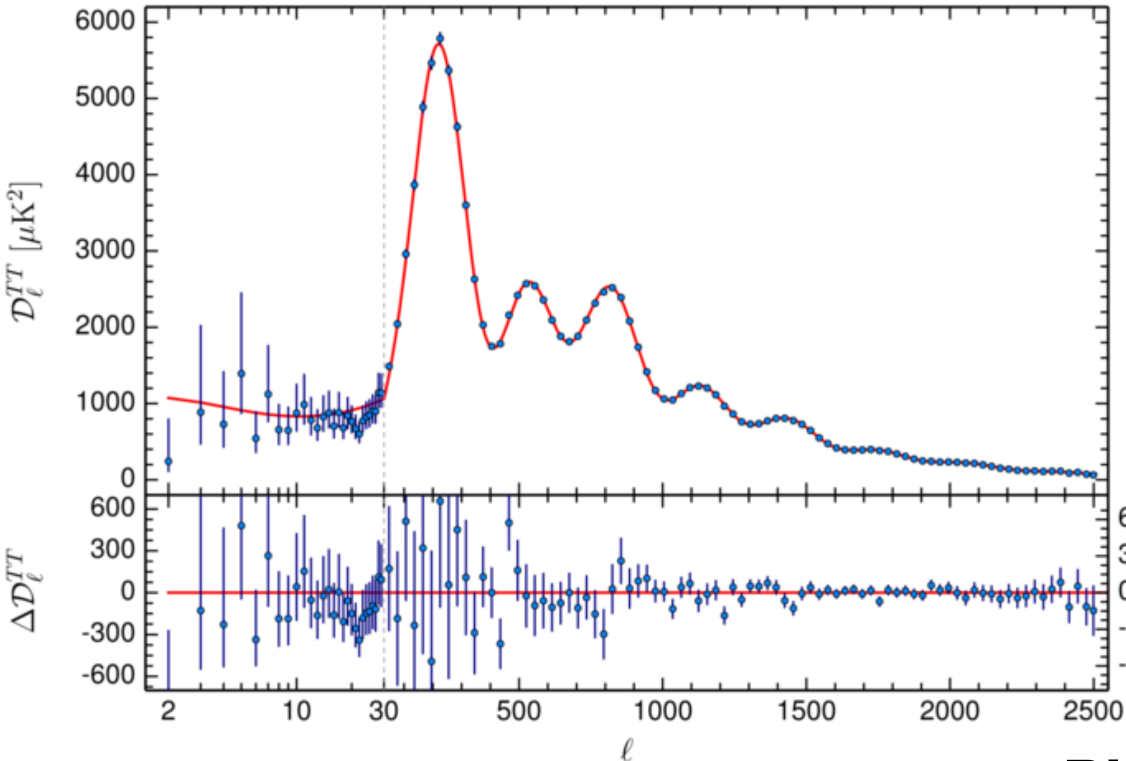
- This yield big-bang Universe, confirmed by the discovery of 2.73K cosmic microwave background (CMB).

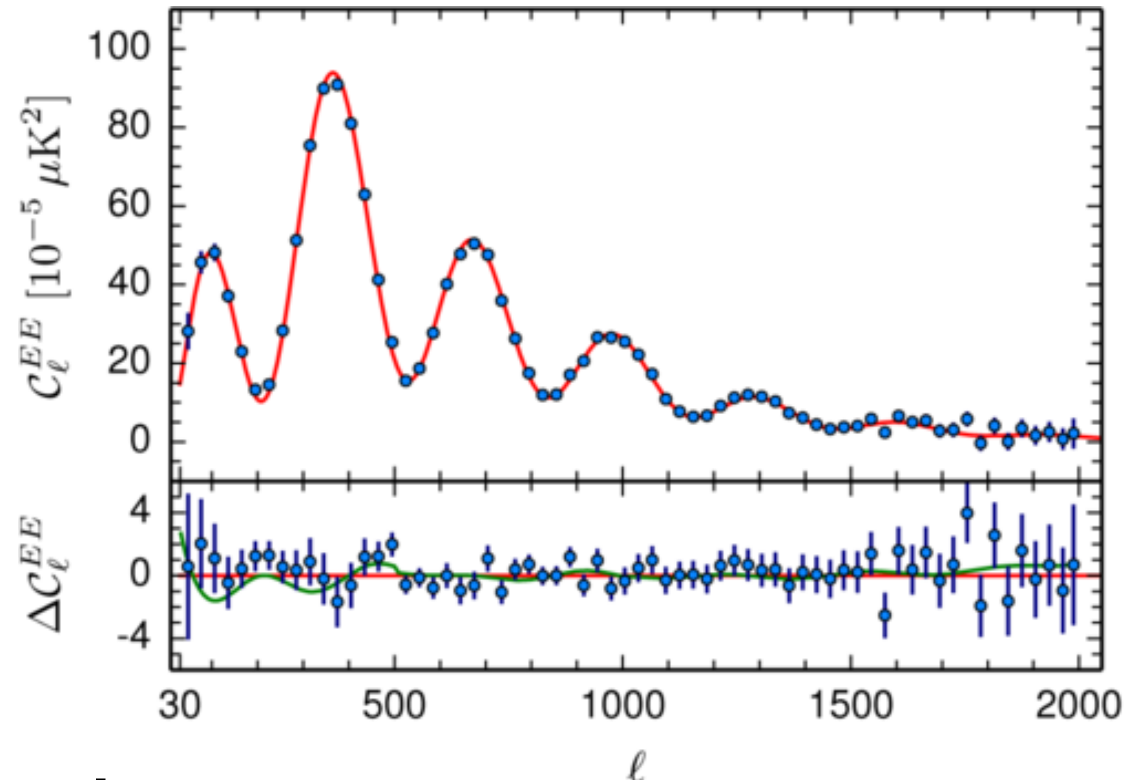

HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Inflation
Size of visible universe
TODAY
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3x10^{5}$ y
$E = 3x10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8x10^{9}$ y
$E = 2.3x10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
Supported by DOE

Accelerating Universe

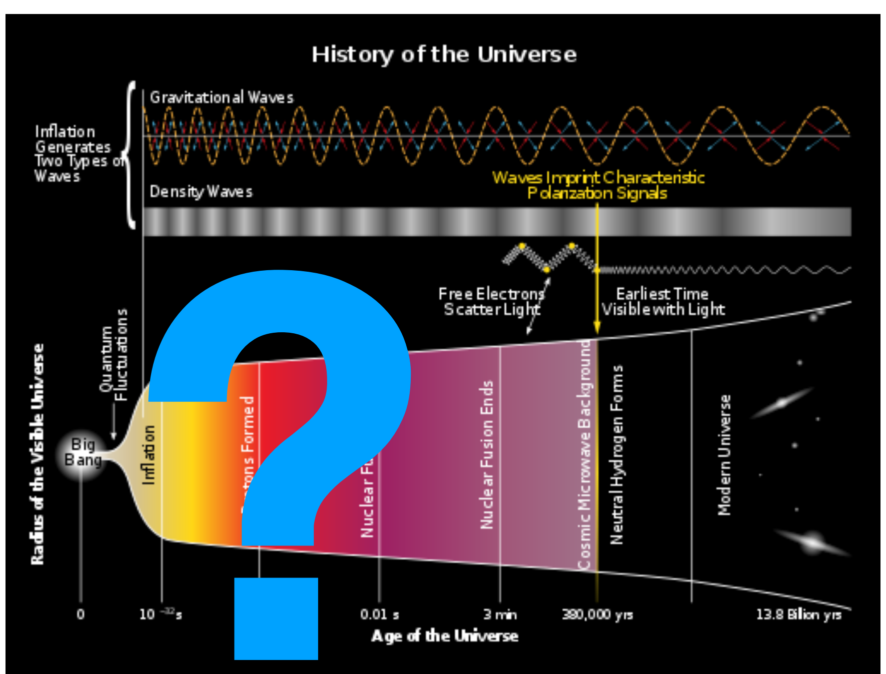
- It is interesting to note that our universe is accelerating inflated now and in the very early stage.
- The acceleration is observed in 1997 by the deviation from the Hubble's law: $v = H_0 D$, and confirmed by other observations such as power spectrum of CMB.
- The earlier time acceleration, called inflation is conjectured to solve the so-called horizon problem and flatness problem.
- It waits for the observation of primordial gravitational waves for confirmation.

CMB: Quantum Fields+Classical Gravity

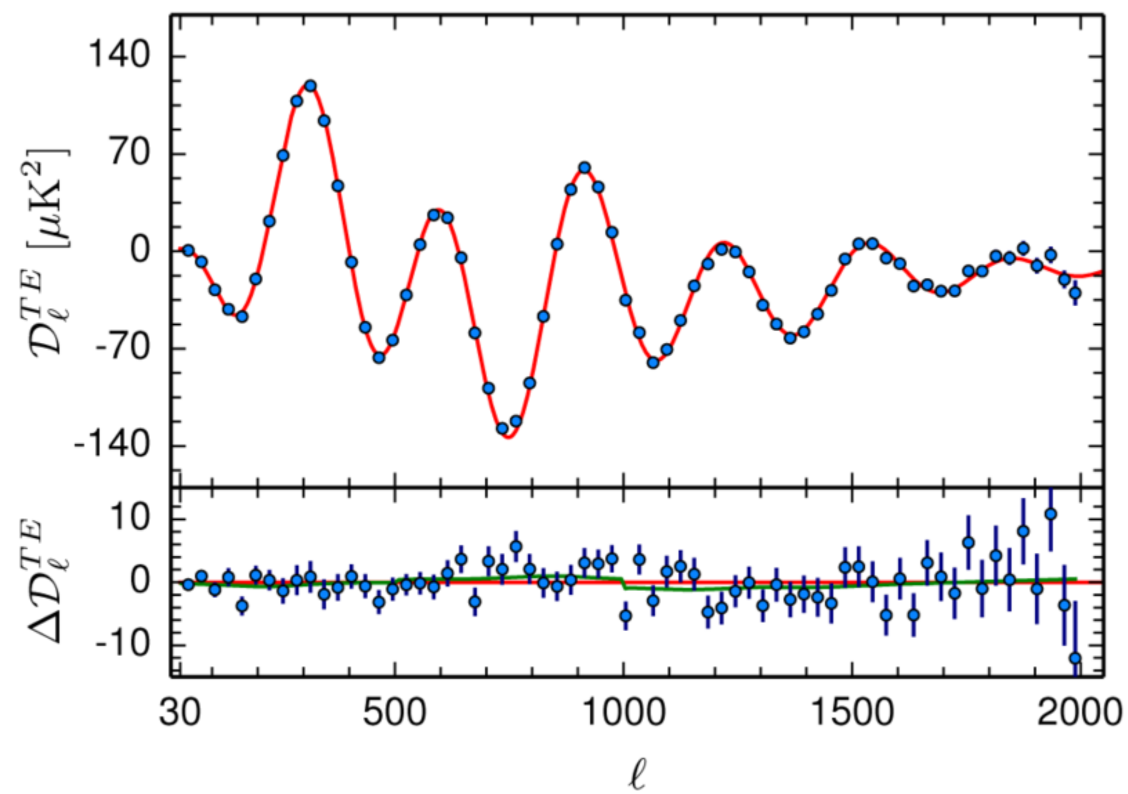




Planck results



B mode??



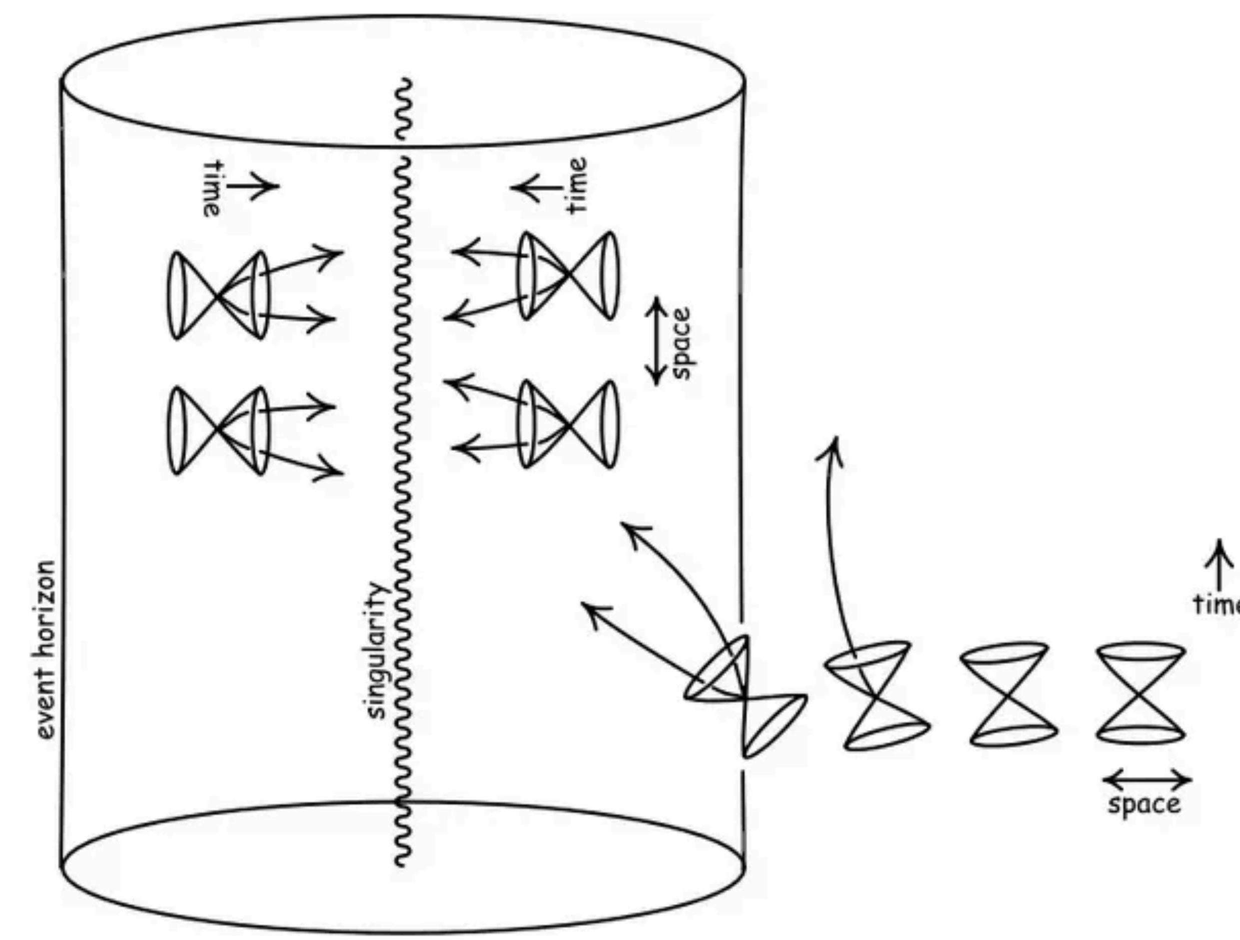
Black holes

- Black hole is a vacuum solution of Einstein gravity, e.g., the Schwarzschild metric:

$$ds^2 = -(1 - \frac{r_s}{r})c^2 dt^2 + (1 - \frac{r_s}{r})^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

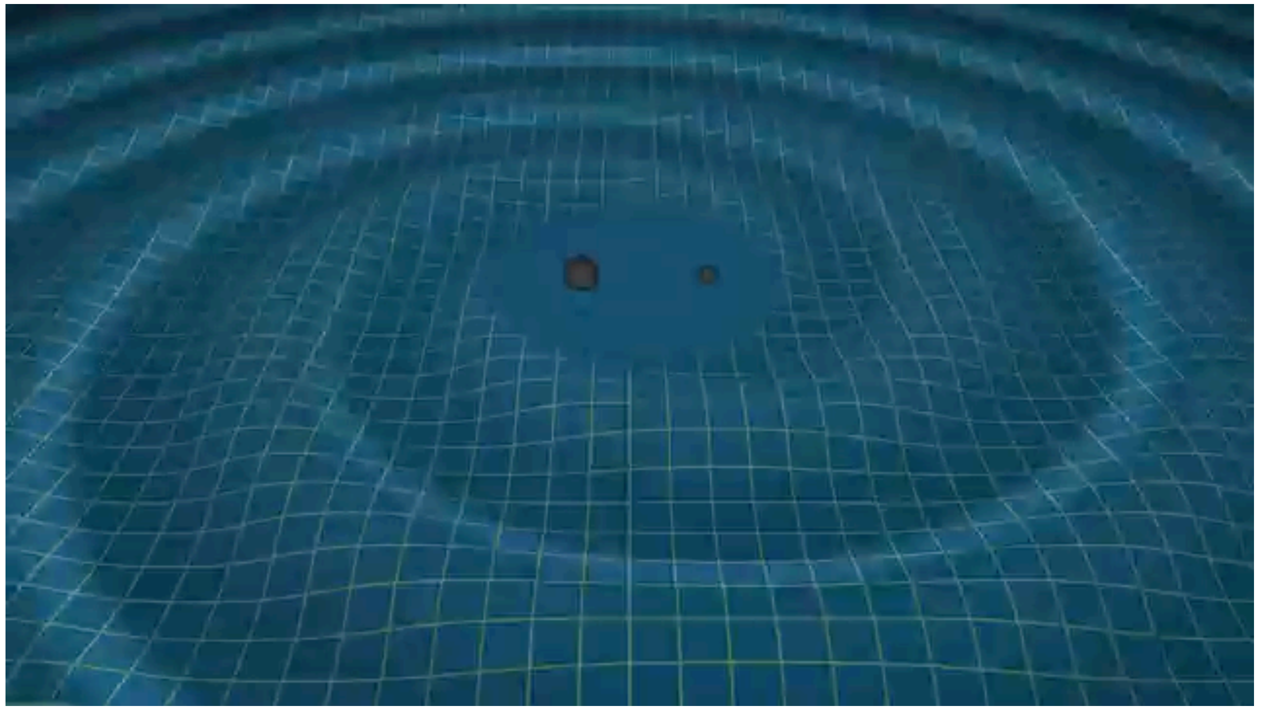
- The neutron stars of larger than 2 solar masses or white dwarfs of larger than 1.4 solar masses will collapse into black holes.

- The blackness is due to the coordinate singularity at the event horizon, which is a causal boundary and nothing inside can come out.



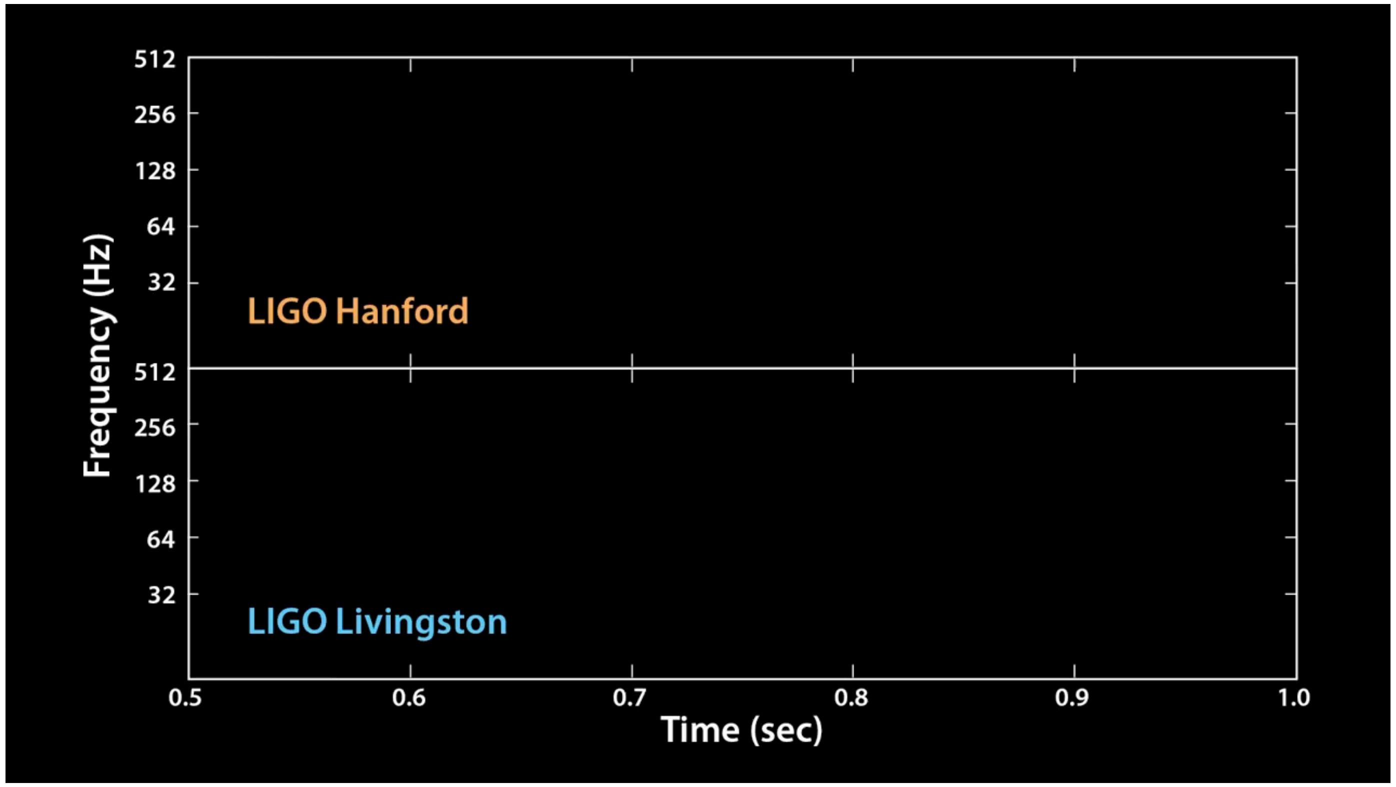
“Seeing” the black hole?

- As the black hole is black, how can we “see” it?
- We cannot “see” a stationary black hole, however, we can “see” the merger of two black holes by their emission of gravitational waves.
- However, how do we know the observed gravitational wave comes from the merger of two black hole?

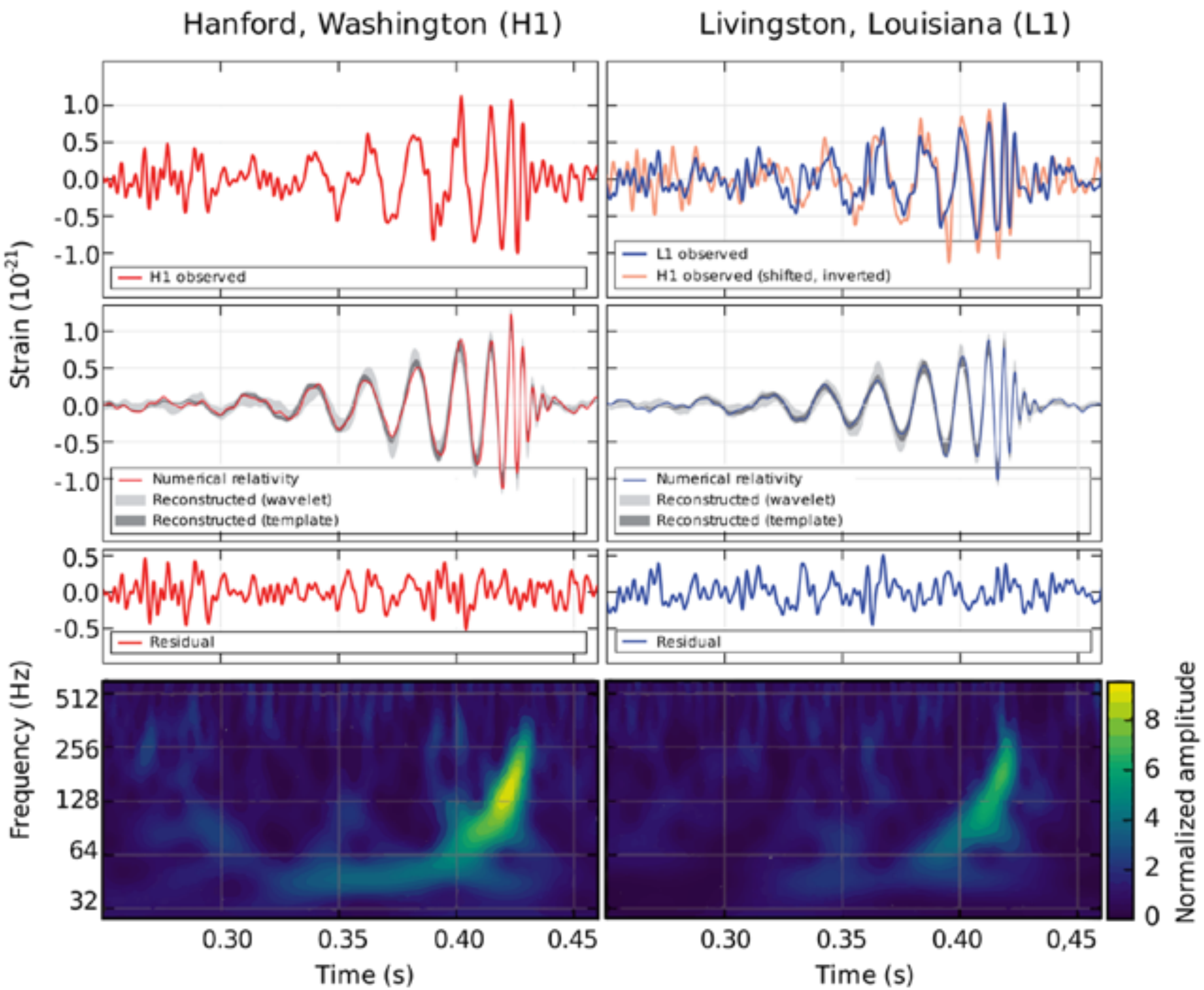


No-hair theorem

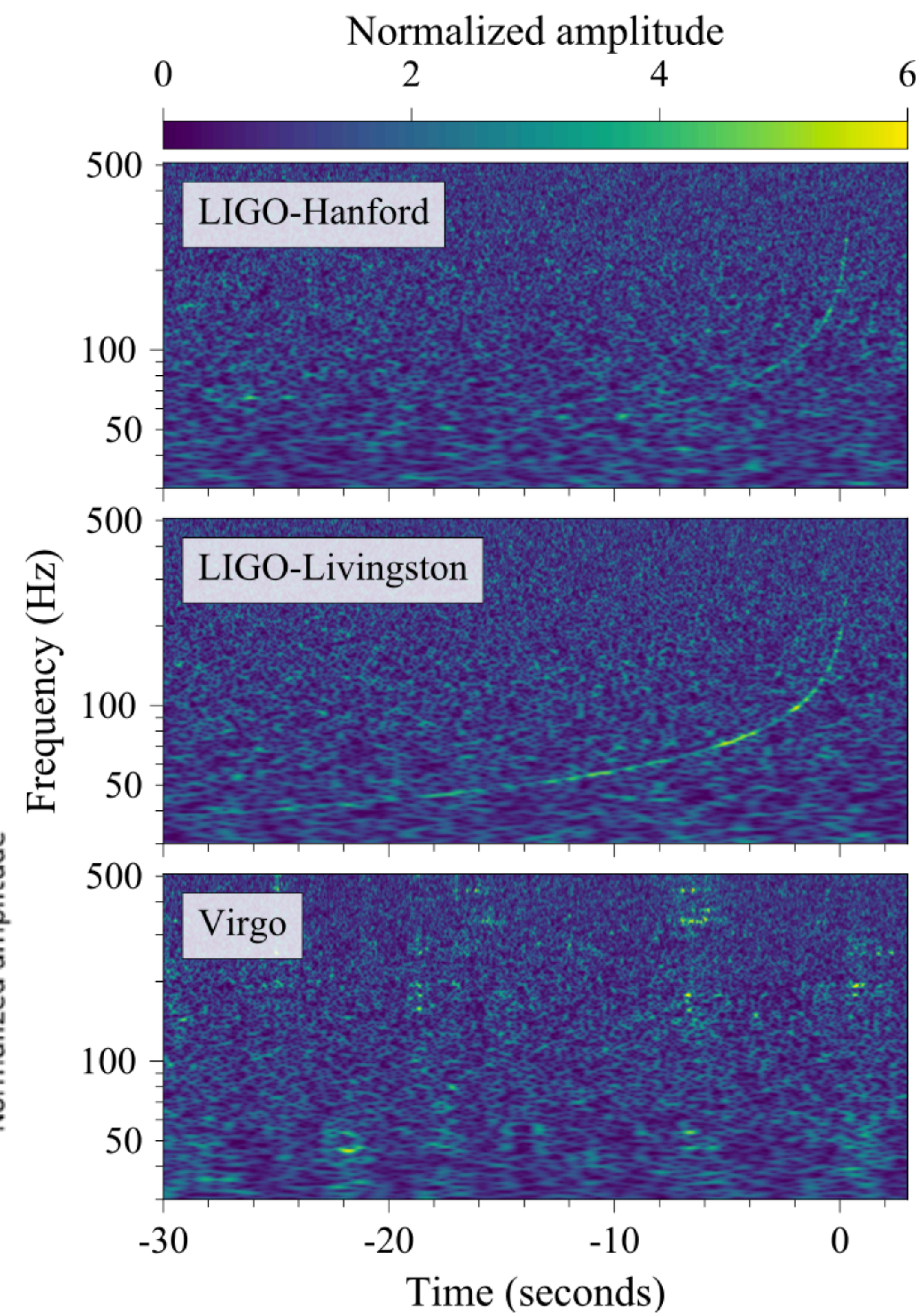
- The black hole is the vacuum solutions of Einstein equation.
- Each black hole is only characterized by its mass, charge & angular momentum. This is known as no-hair theorem of black hole.
- Thus, other characteristics will be decayed away very quickly, i.e., black hole ring-down. The emitted gravitational wave of black hole merger should look/sound like this:

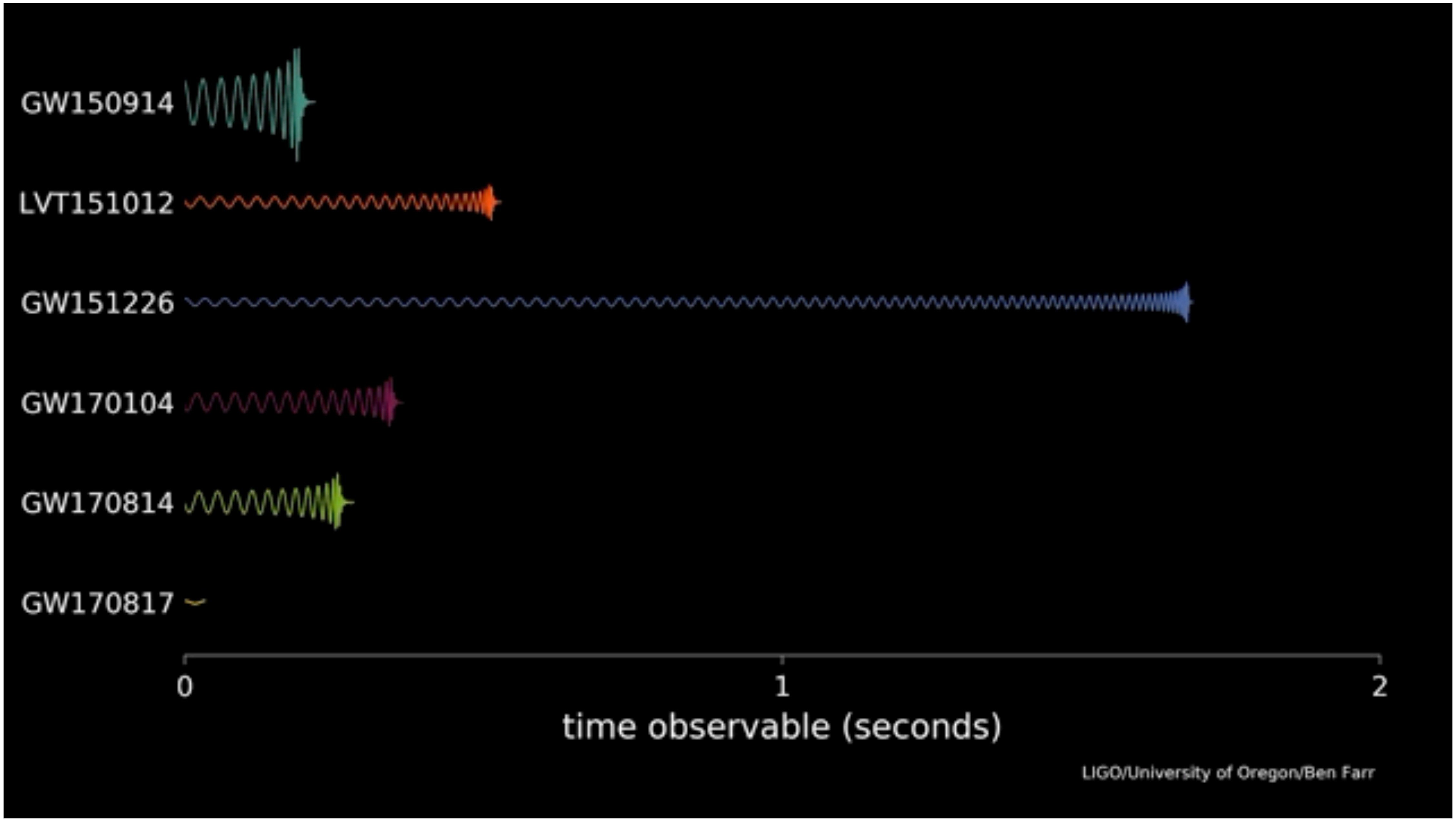


GW150914: Binary BH



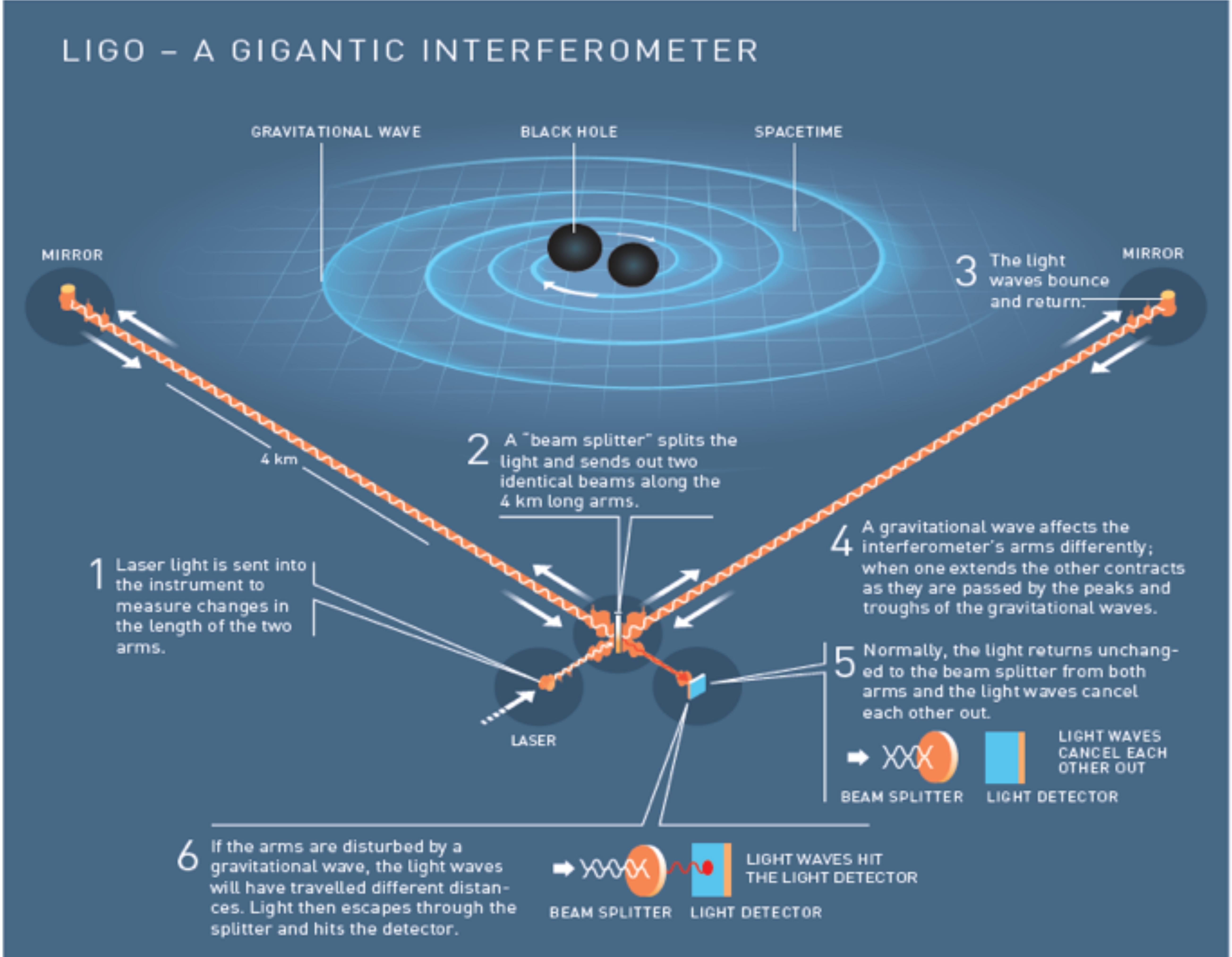
GW170817: Binary NS




GW150914
LVT151012
GW151226
GW170104
GW170814
GW170817
0
1
2
time observable (seconds)
LIGO/University of Oregon/Ben Farr

How to detect gravitational wave?

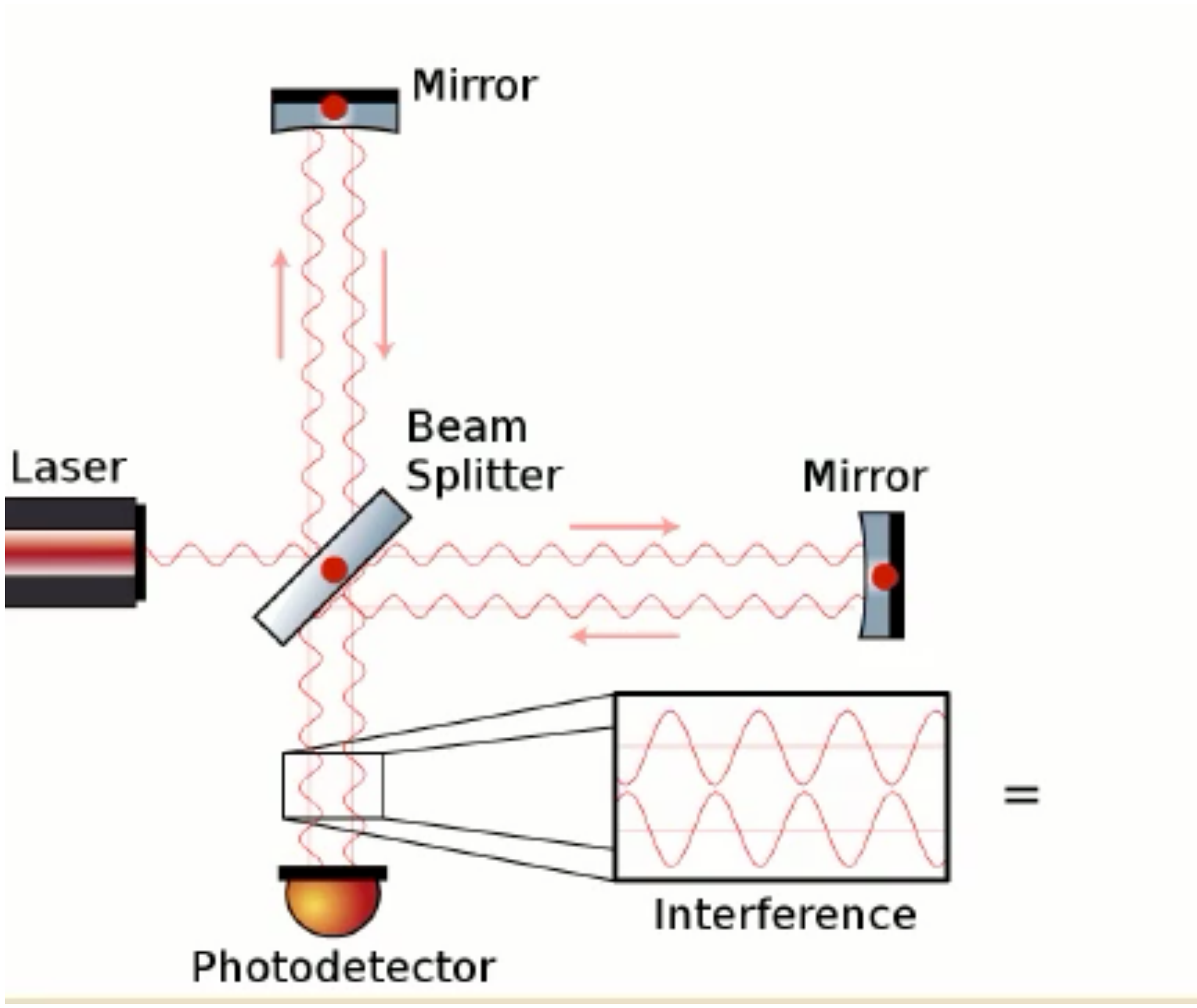
2017 Physics Laureates. Ill: N. Elmehed. © Nobel Media 2017


LIGO – A GIGANTIC INTERFEROMETER
GRAVITATIONAL WAVE
BLACK HOLE
SPACETIME
MIRROR
MIRROR
3 The light waves bounce and return.
4 km
2 A "beam splitter" splits the light and sends out two identical beams along the 4 km long arms.
1 Laser light is sent into the instrument to measure changes in the length of the two arms.
4 A gravitational wave affects the interferometer's arms differently; when one extends the other contracts as they are passed by the peaks and troughs of the gravitational waves.
5 Normally, the light returns unchanged to the beam splitter from both arms and the light waves cancel each other out.
LASER
LIGHT WAVES CANCEL EACH OTHER OUT
BEAM SPLITTER
LIGHT DETECTOR
6 If the arms are disturbed by a gravitational wave, the light waves will have travelled different distances. Light then escapes through the splitter and hits the detector.
LIGHT WAVES HIT THE LIGHT DETECTOR
BEAM SPLITTER
LIGHT DETECTOR

Strain fro GW

- Gravity force: $F_j = -mR_{j0k0}x^k$
- Curvature of GW: $R^{GW}_{j0k0} = -\frac{1}{2}\ddot{h}^{TT}_k$
- Gravity for by GW: $F^{GW}_j = -mR^{GW}_{j0k0}x^k$
- Geodesic deviation: $m\delta\ddot{x}^j = F^{GW}_j$
- Thus, the strain due to GW:

$$h^{TT}_{jk} = 2\frac{\delta x^j}{x^k}$$



How small a typical strain?

- There are different length scales for the binary black hole inspiral: black hole radius r_s, inspiral radius R, gravitational wave length λ, and the source distance r .
- At Newtonian order, $v^2 \sim \frac{Gm}{R} \sim \frac{r_s}{R}\,,\qquad f \sim \frac{1}{\lambda} \sim \frac{v}{R}$ $\qquad$ set light speed $c = 1$
- As merger proceeds, the gravity force becomes stronger, and the inspiral frequency get higher. This results ***chirp*** gravitational waveform.
- Typical GW frequency of BBH: $f_{GW} = \frac{1}{\pi}\Big(\frac{Gm}{R^3}\Big)^{\frac{1}{2}} \sim 19\ \mathrm{Hz}\Big(\frac{m}{2.8 m_\odot}\Big)^{\frac{1}{2}}\big(\frac{R}{427\mathrm{km}}\big)^{-\frac{3}{2}}$
- Typical strain size of BBH: $h \sim \epsilon \frac{r_s}{r}\frac{r_s}{R} \sim 10^{-23}\big(\frac{\epsilon}{0.1}\big)\big(\frac{m}{m_\odot}\big)\big(\frac{r}{Gpc}\big)^{-1}$ $\qquad \epsilon = \mathrm{eccentricity}$
- Typical GW luminosity of BBH: $L_{GW} = \frac{G}{5c^5}\dddot{I}_{ij}\dddot{I}^{ij} \sim \epsilon^2 \frac{c^5}{G}\big(\frac{r_s}{R}\big)^5 \sim 10^{57}\ \frac{\mathrm{erg}}{\mathrm{s}}\ \big(\frac{\epsilon}{0.1}\big)^2\big(\frac{r_s}{R}\big)^5$

$1\ \mathrm{erg} = 10^{-7}$ joule, Sun's burning rate: 10^{26} Watt

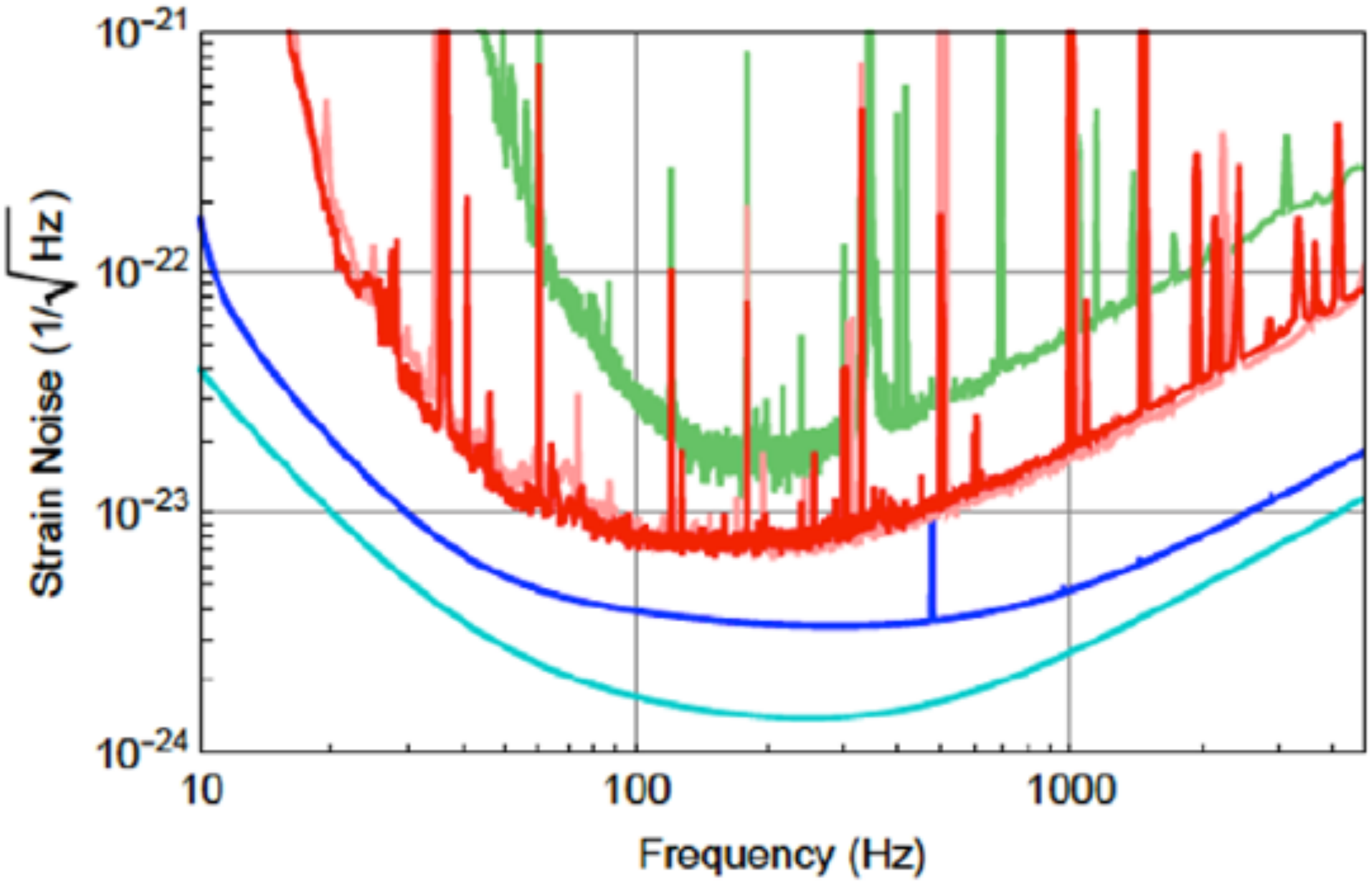
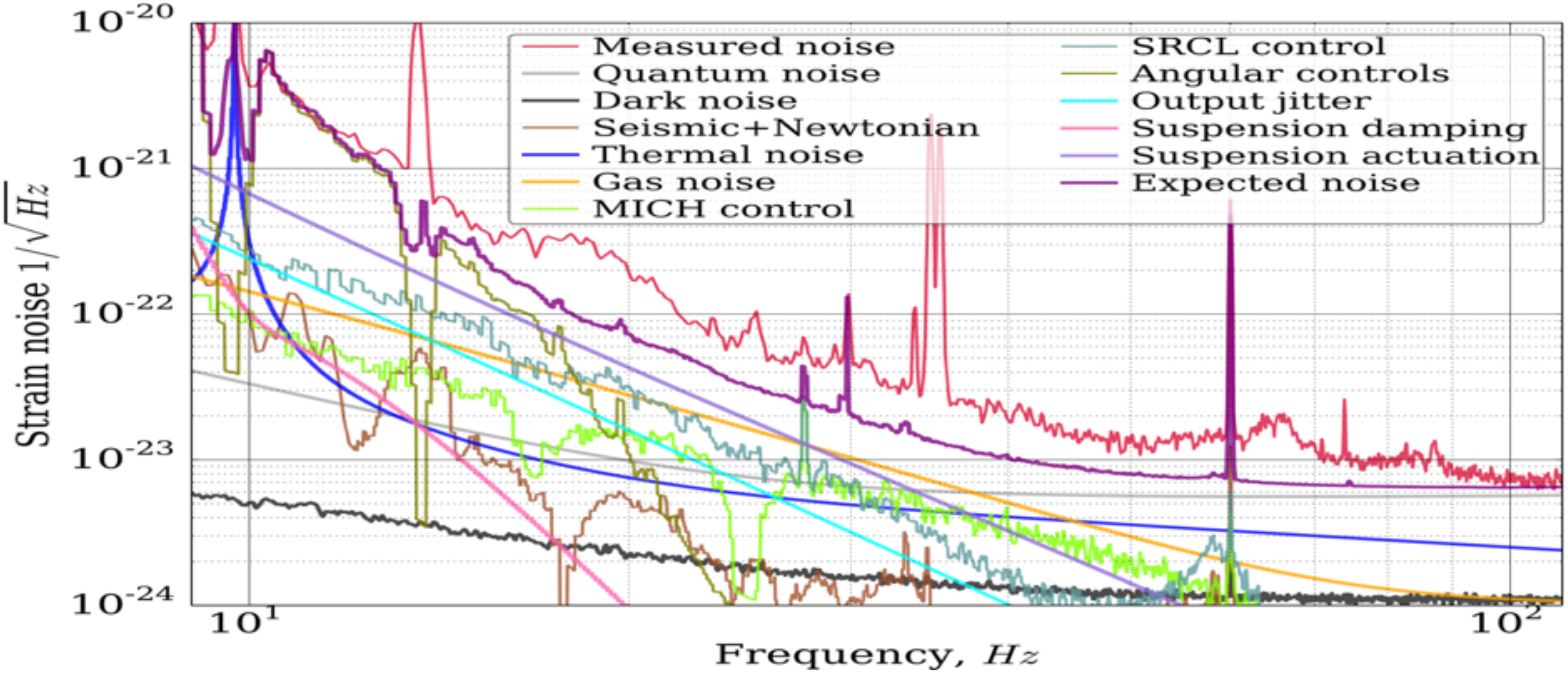
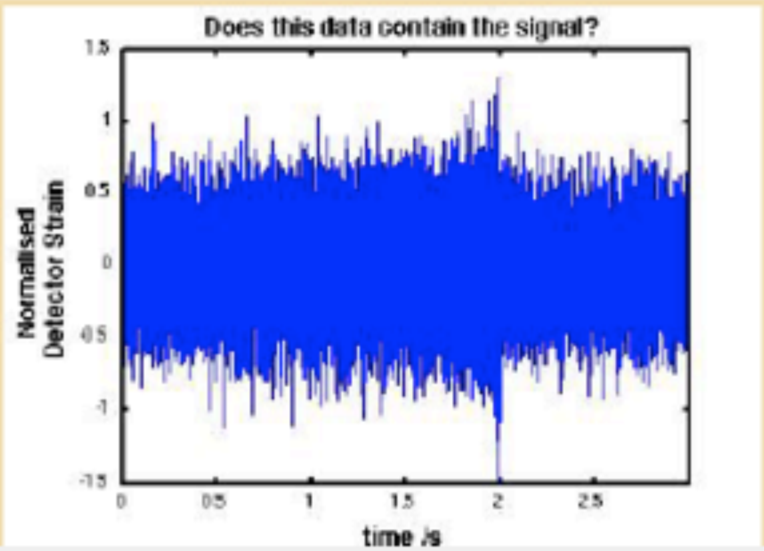


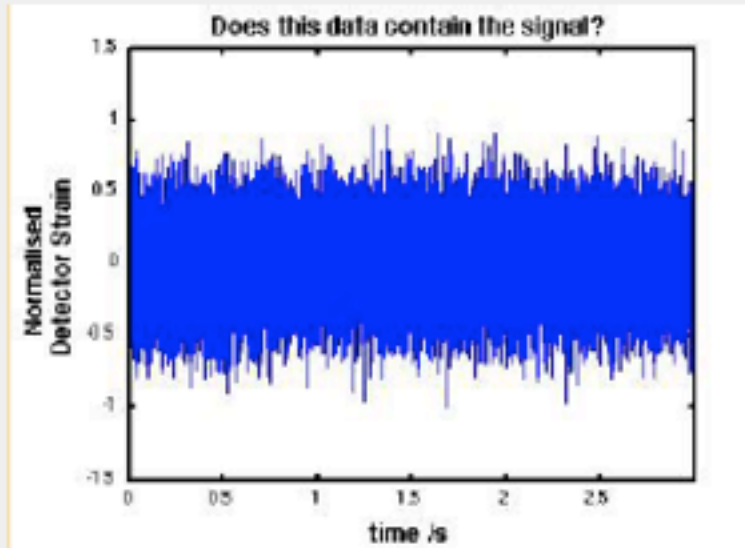
Figure 3: The strain sensitivity of the Advanced LIGO detectors during the first observation run in 2015 (red) and the last science run of the Initial LIGO detectors (green). The improvement in sensitivity is a factor of 3-4 in the most sensitive frequency band 100-300 Hz, and nearly a factor 100 at 50 Hz. The Advanced LIGO design sensitivity, which has not been reached yet, is shown in dark blue, and a possible future upgrade – in light blue. The narrow features include calibration lines, vibrational modes of suspension fibres and 60-Hz power grid harmonics (figure from [37]).

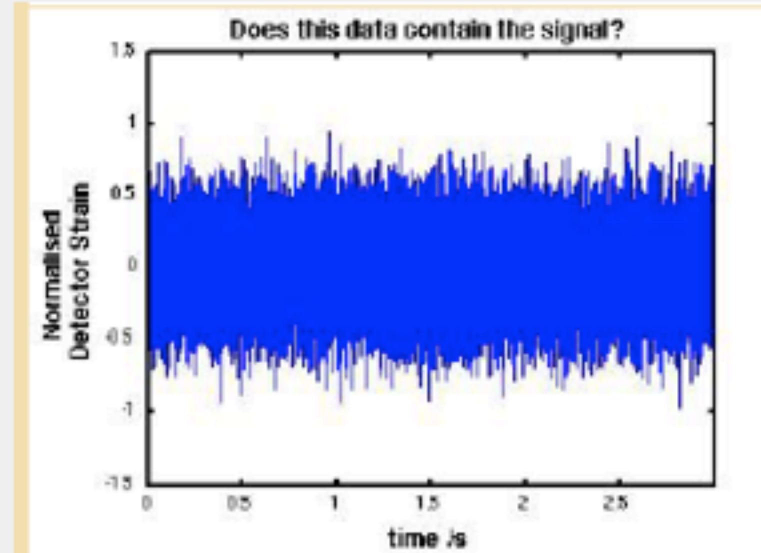


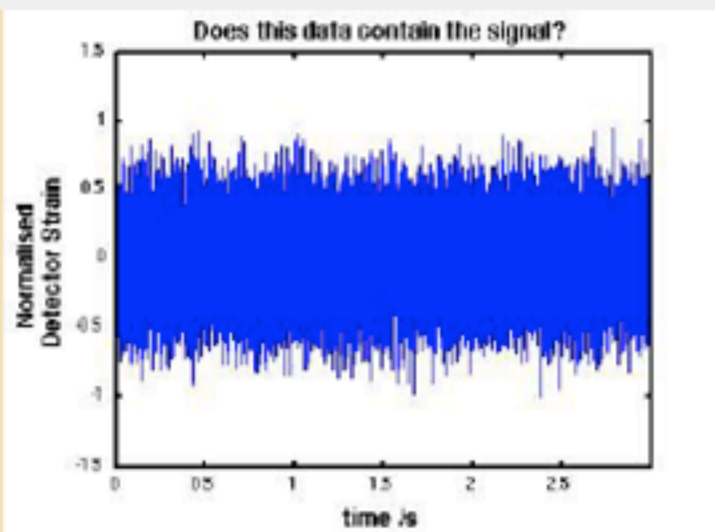
Measured noise
Quantum noise
Dark noise
Seismic+Newtonian
Thermal noise
Gas noise
MICH control
SRCL control
Angular controls
Output jitter
Suspension damping
Suspension actuation
Expected noise
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{1}
10^{2}
Strain noise $1/\sqrt{Hz}$
Frequency, Hz

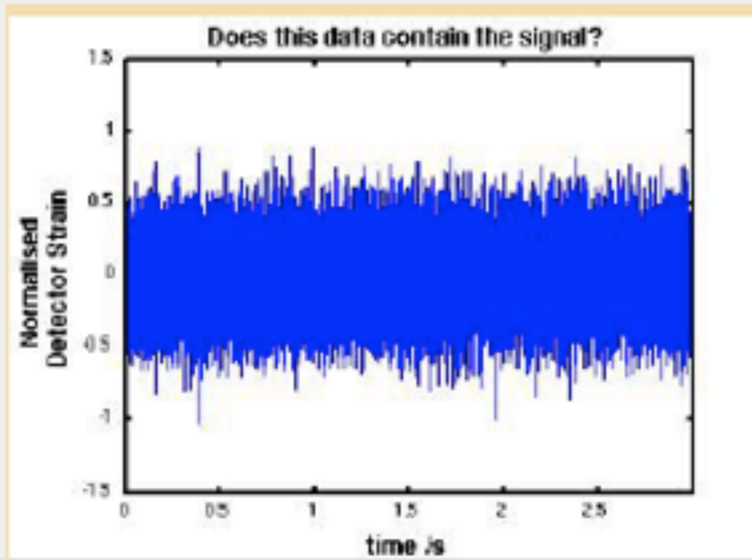
CAN YOU TELL WHICH ONES CONTAIN GW SIGNAL?

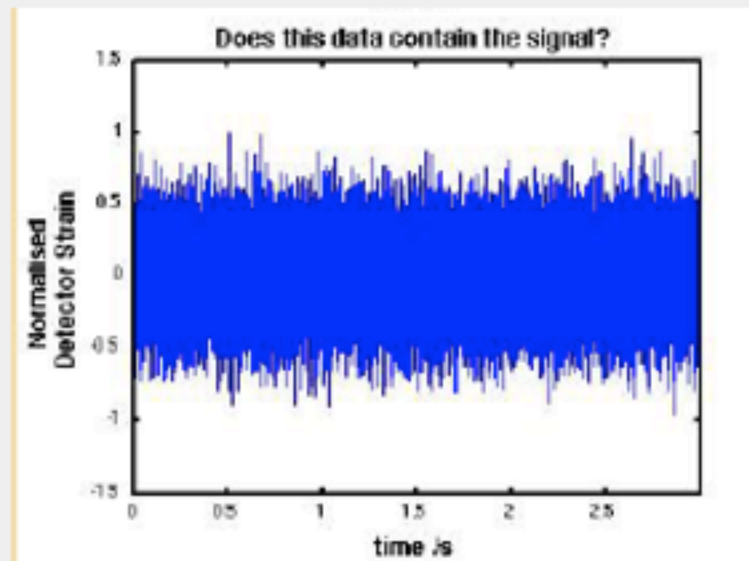




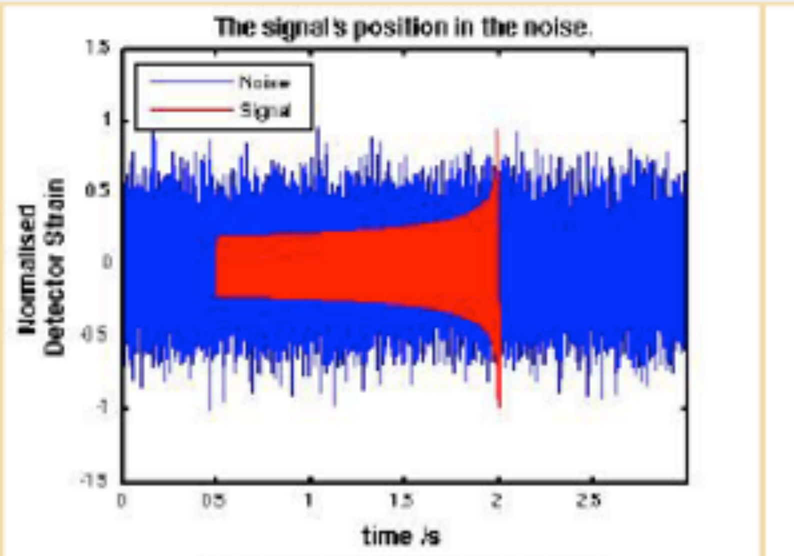


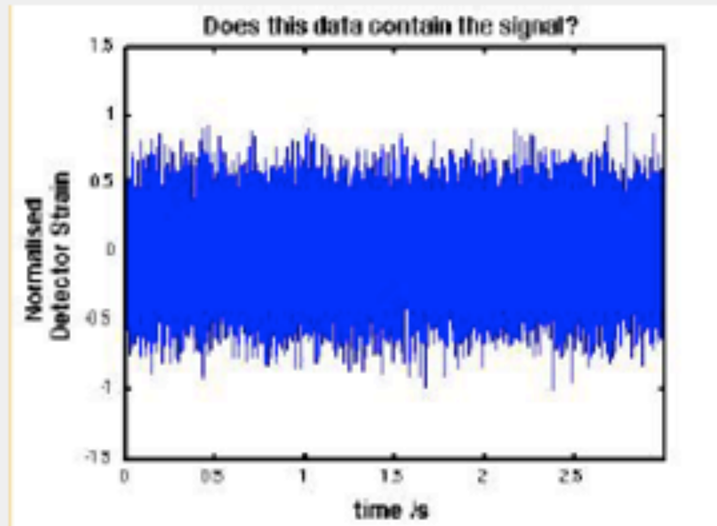


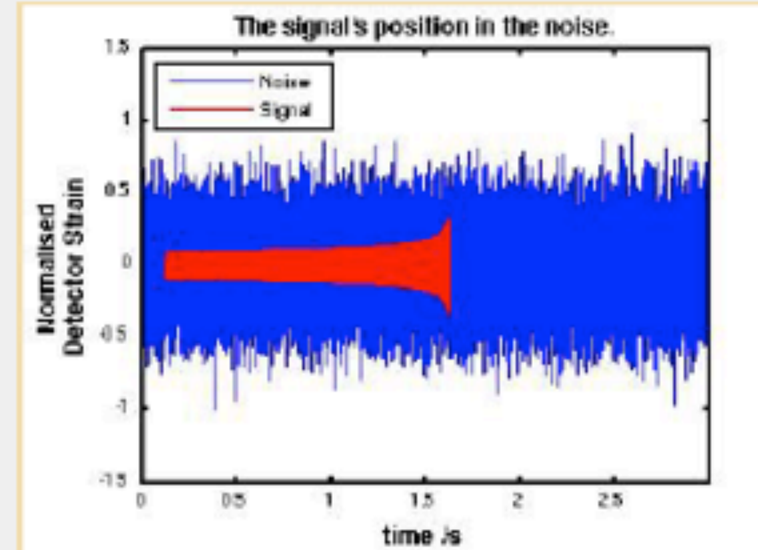


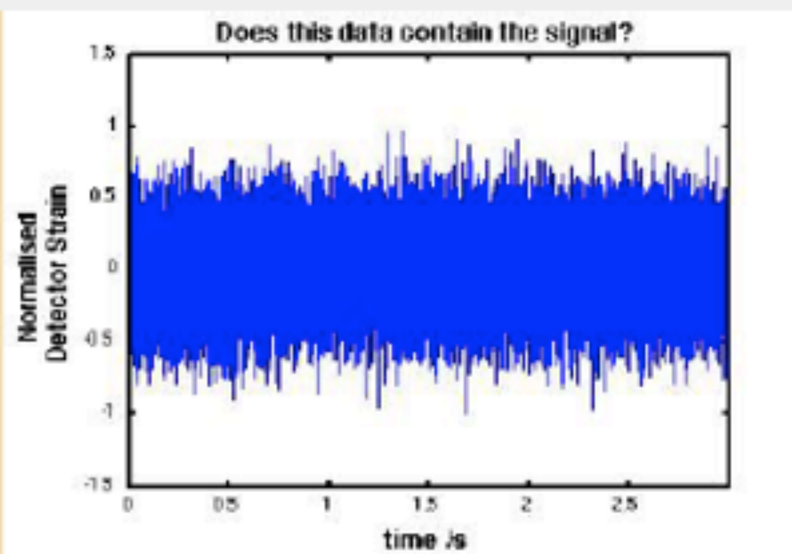


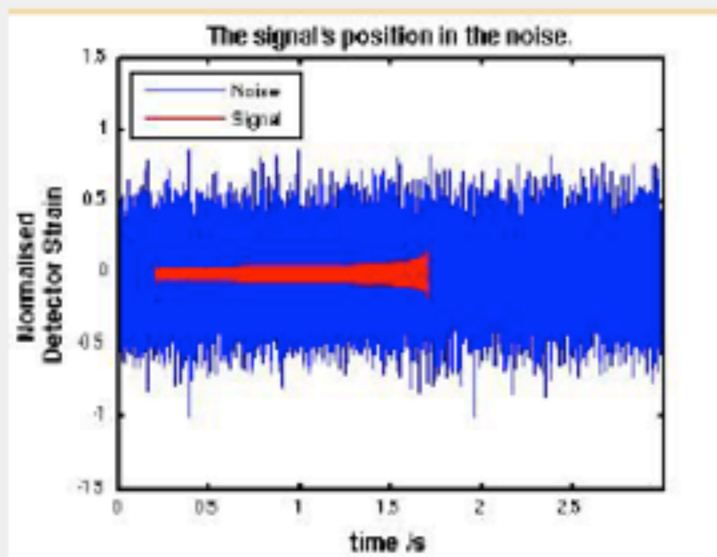
THE ANSWER

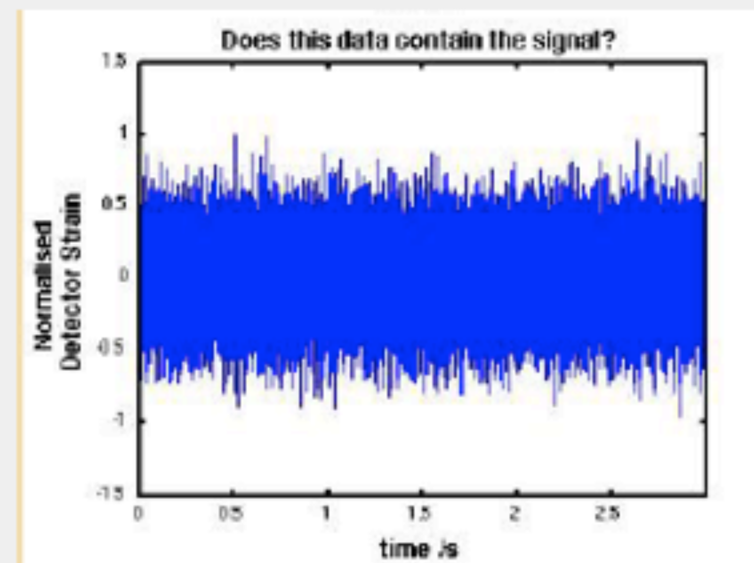

The signal's position in the noise.
Noise
Signal
Normalised Detector Strain
time /s


Does this data contain the signal?
Normalised Detector Strain
time /s


The signal's position in the noise.
Noise
Signal
Normalised Detector Strain
time /s

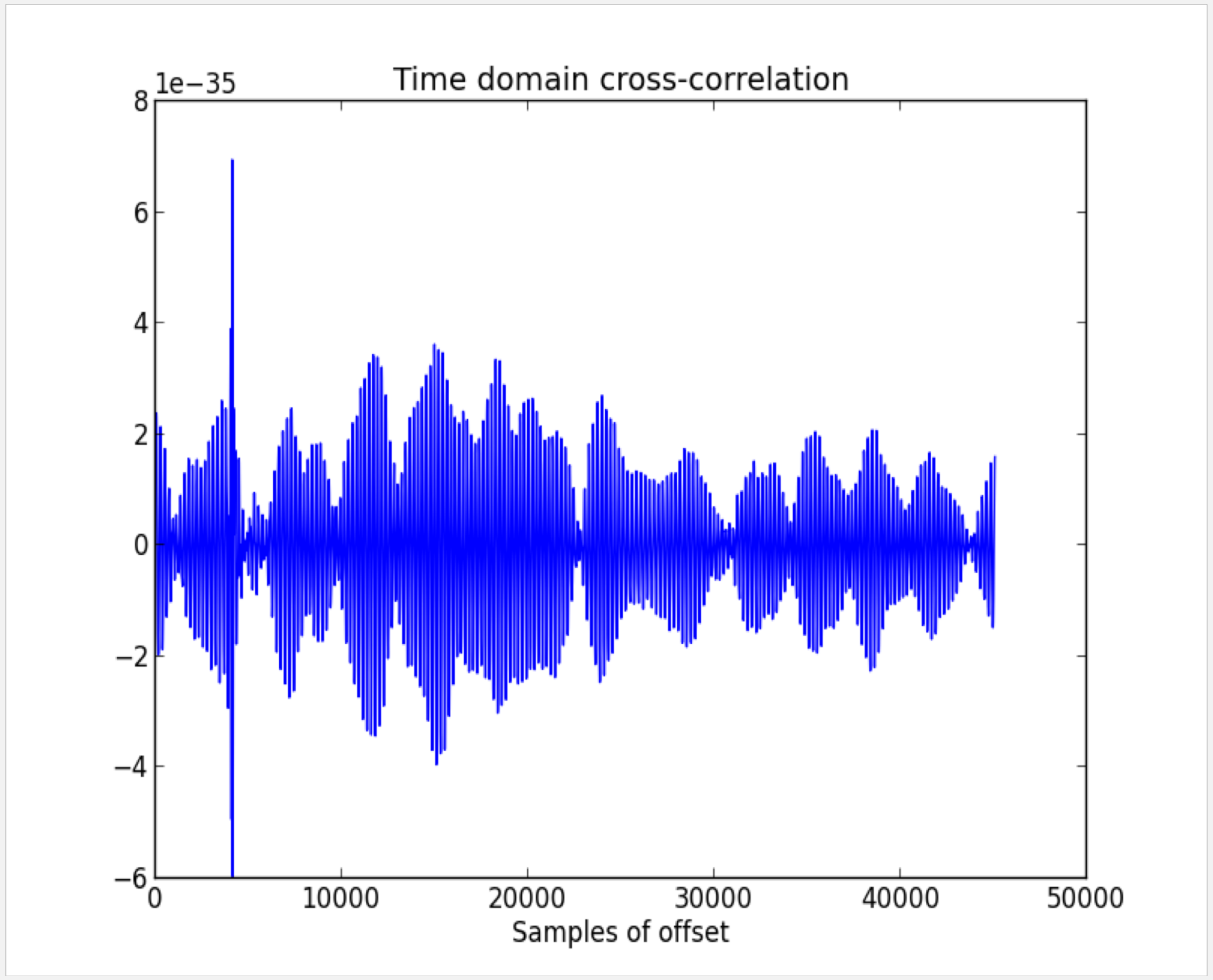

Does this data contain the signal?
Normalised Detector Strain
time /s

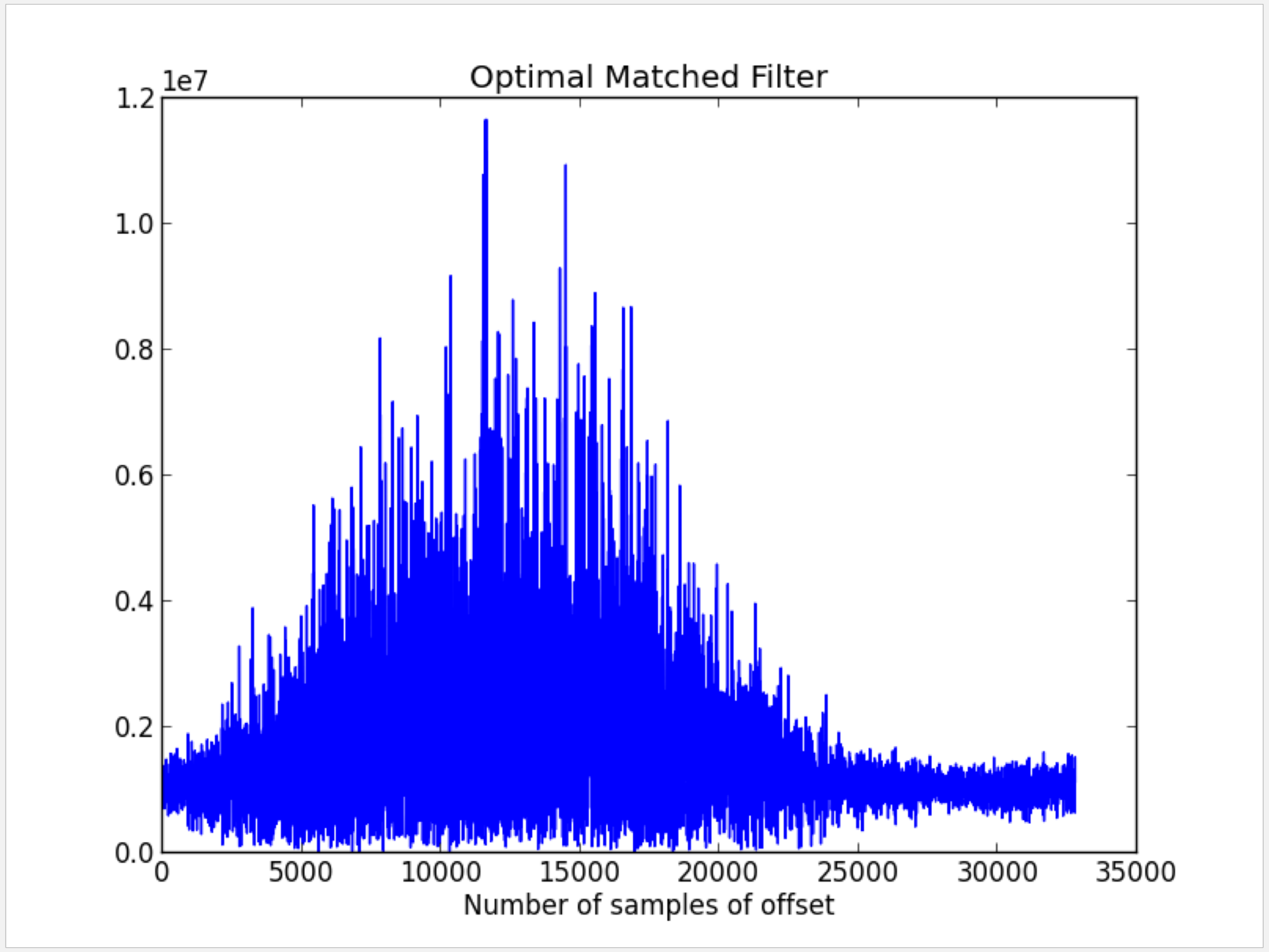

The signal's position in the noise.
Noise
Signal
Normalised Detector Strain
time /s


Does this data contain the signal?
Normalised Detector Strain
time /s

Data analysis

- The GW data analysis is a state of art.
- It is similar to pattern recognition, and potentially machine learning can be applied.
- The standard way of GW data analysis is the matching filtering. This requires the preparation of theoretical GW templates, and compares them with the whitening data by the PSD.
- However, there are non-Gaussian and non-stationary noise, i.e., glitches, which cannot be removed by whitening.



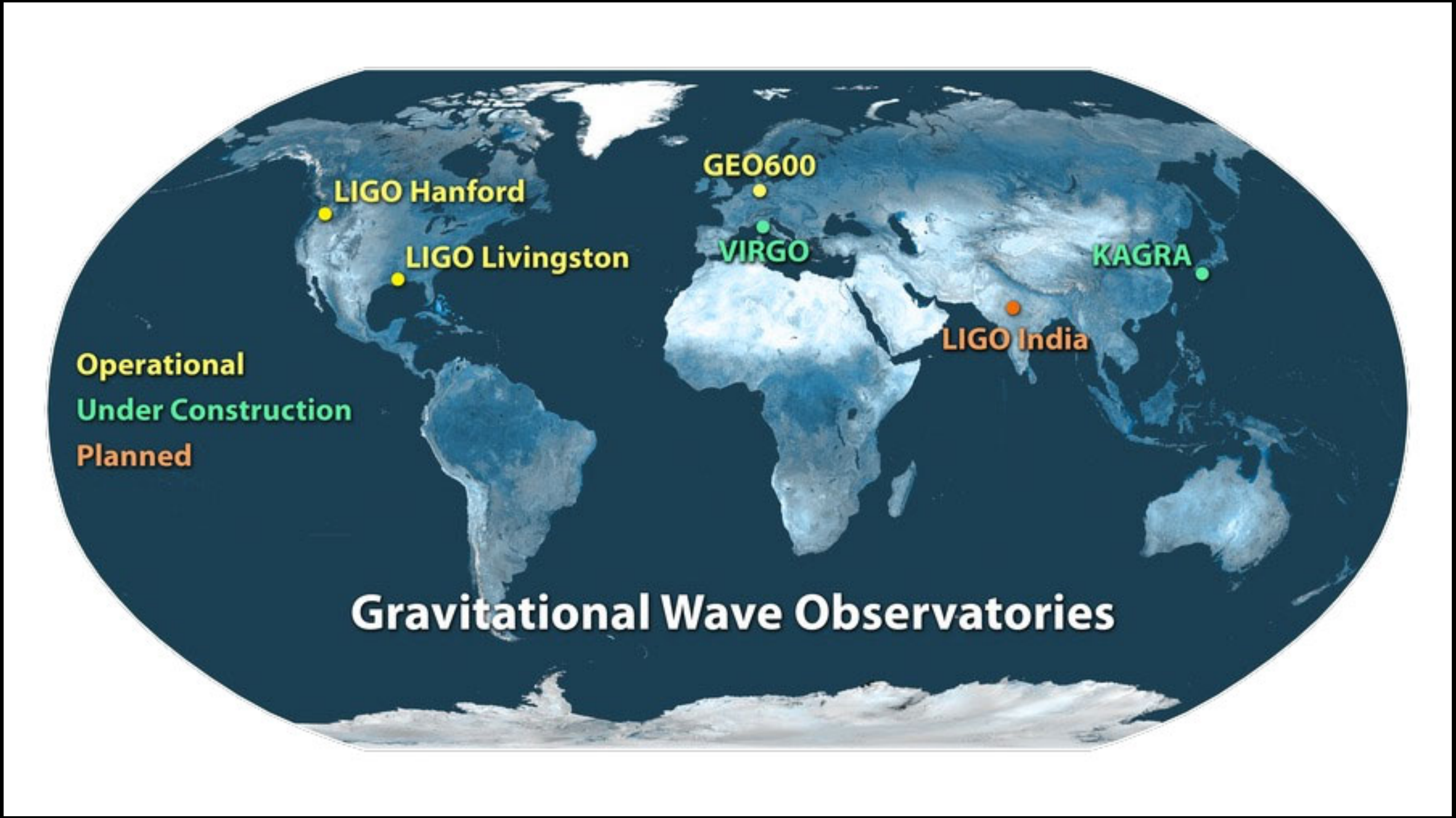


A good match!

A bad match!

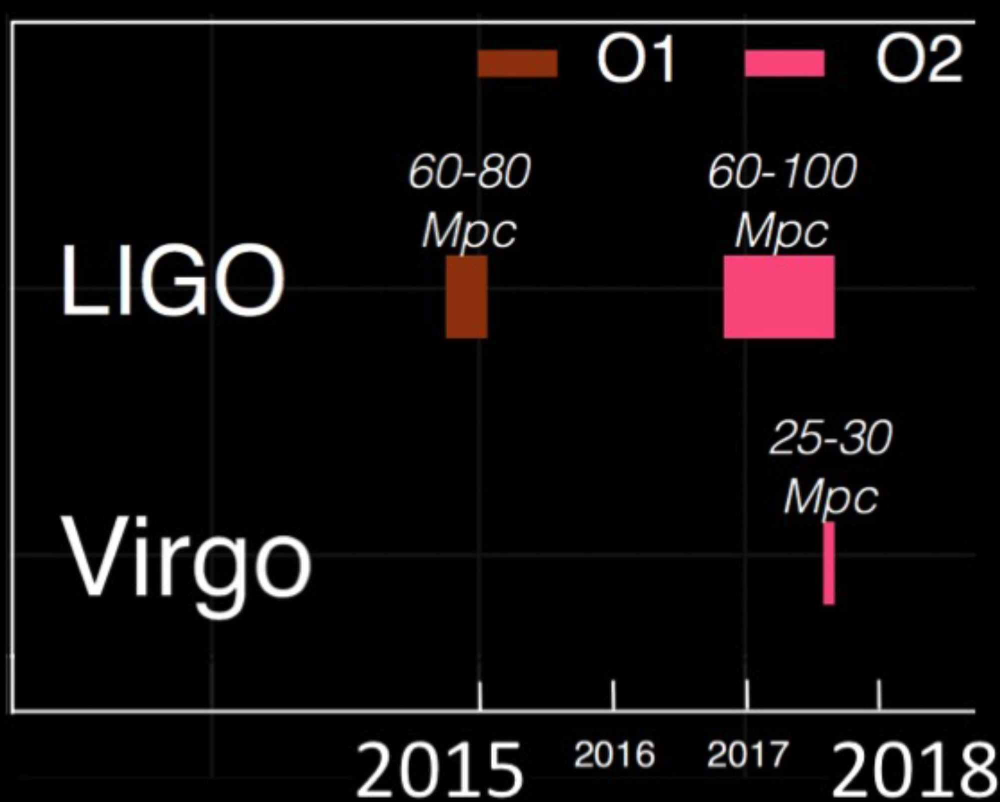
Future perspective

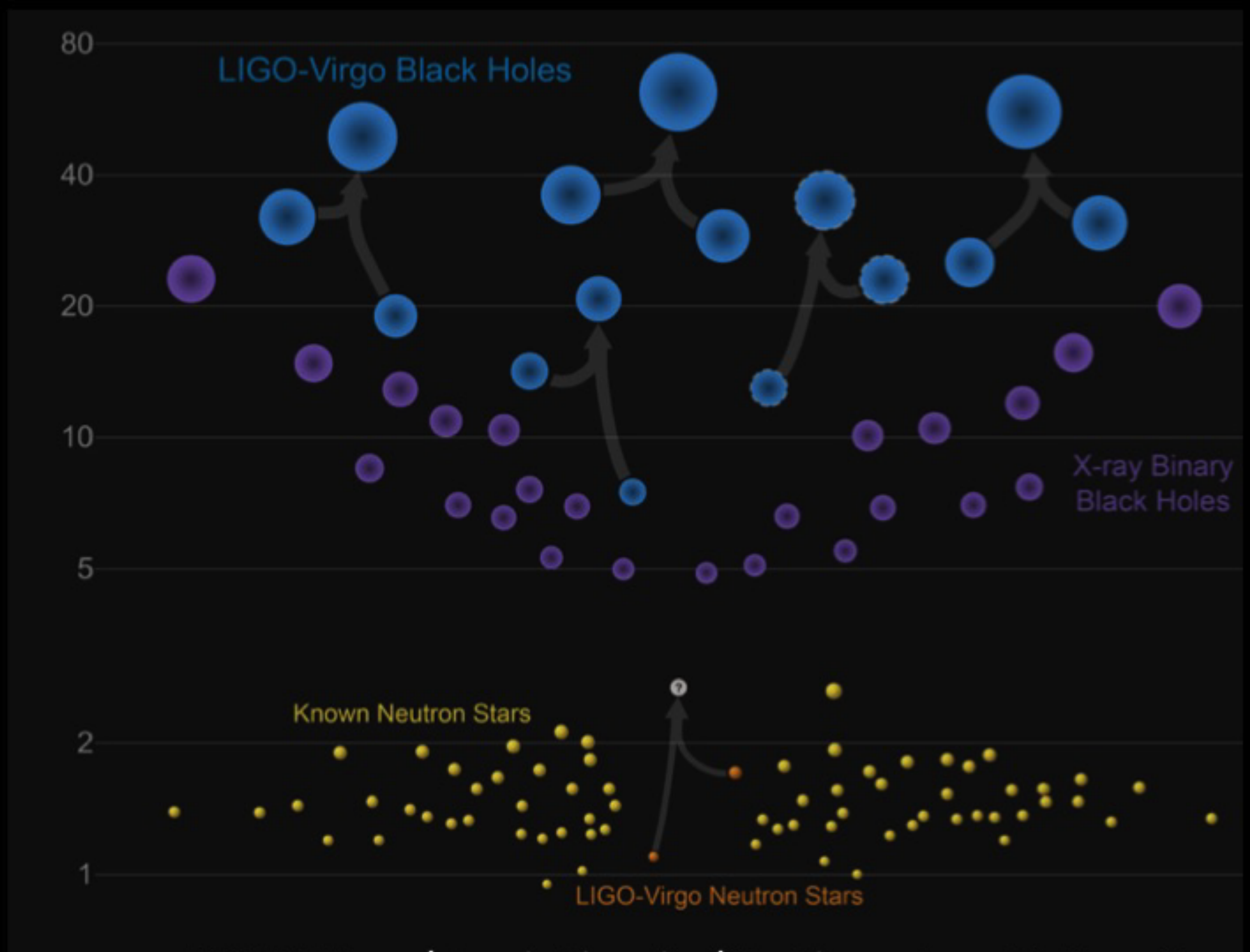
Worldwide GW detector network



Recent discoveries

- About 10 BBH & 1 BNS detected by LIGO/Virgo in O1 & O2

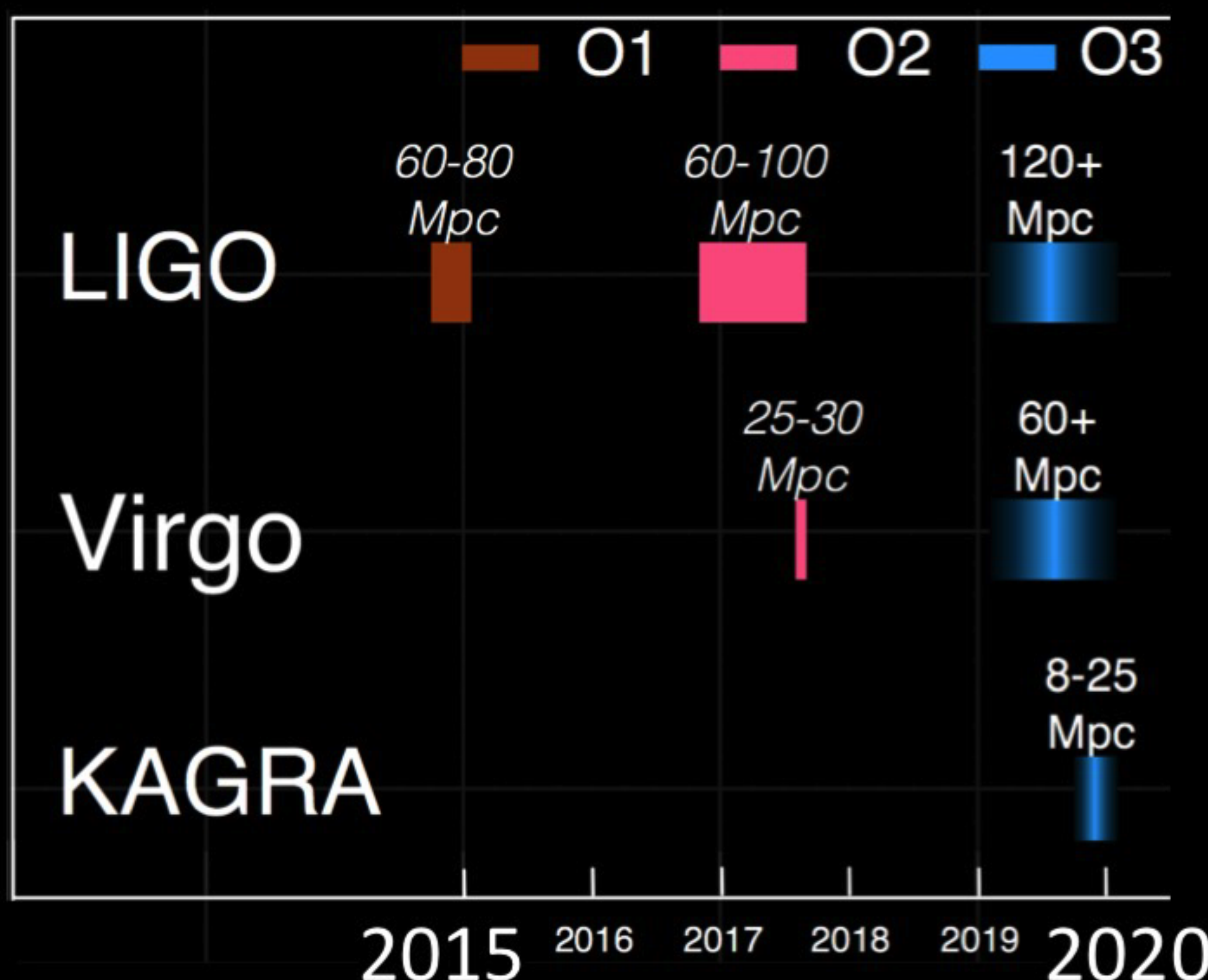




LIGO-Virgo/Frank Elavsky/Northwestern University

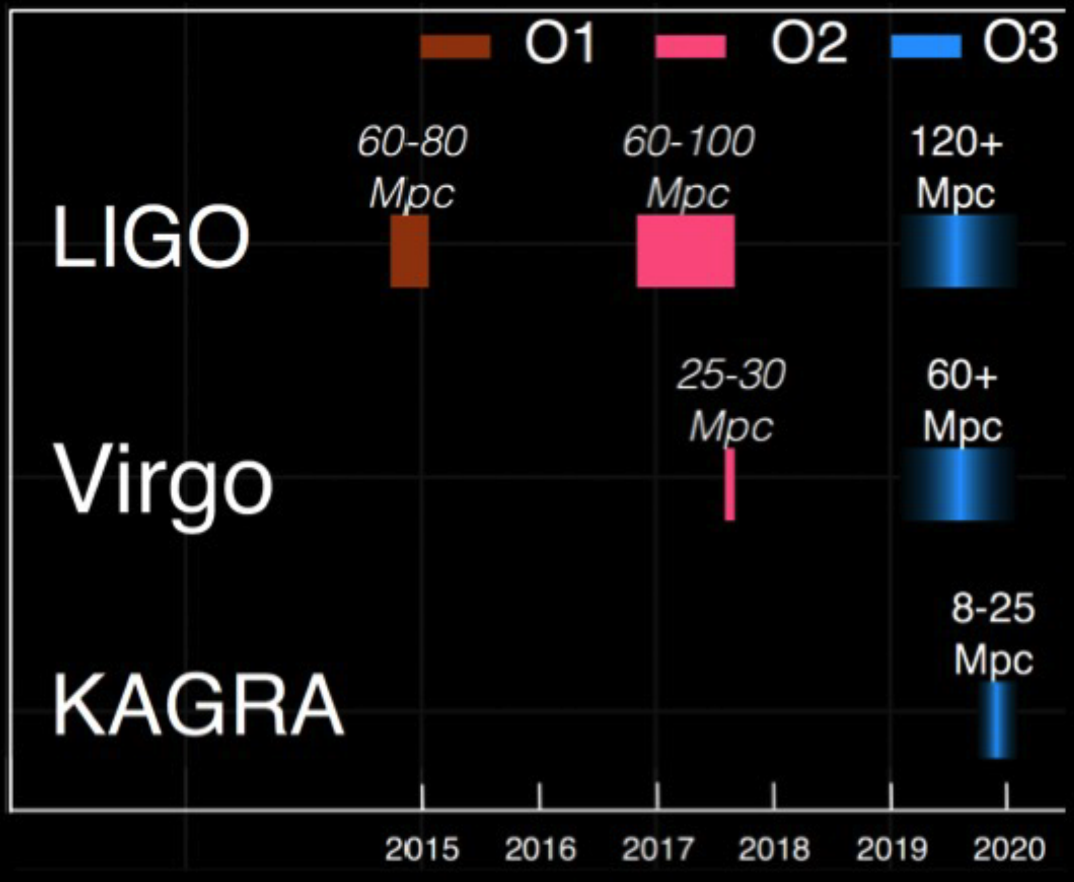
O3 (2019-2020)

- ⋆ LIGO/Virgo start O3 in early 2019
- ⋆ KAGRA tries to join O3 in late 2019



In O3, ….

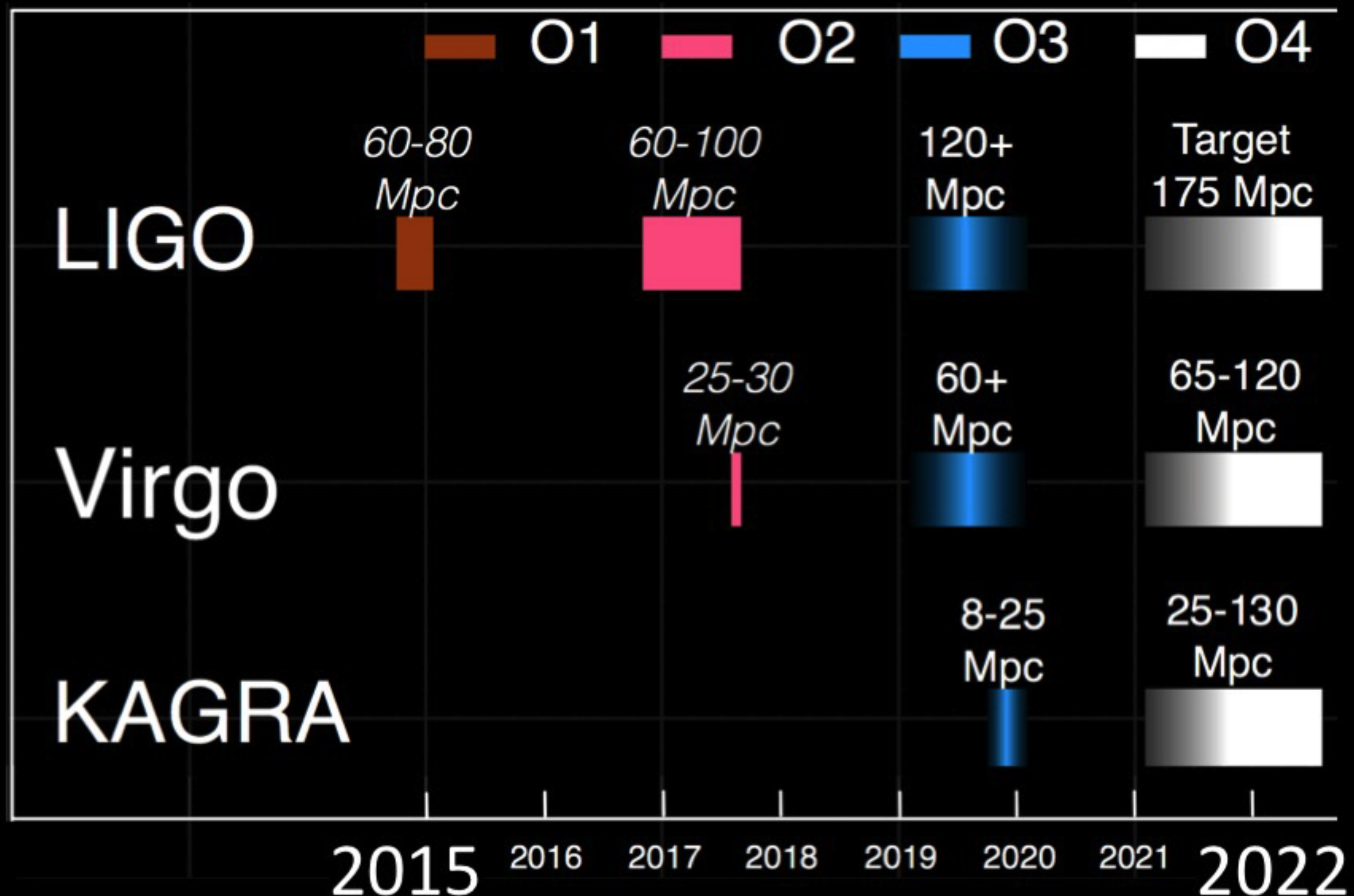
- ⋆ Sensitivity improvement (LIGO:600-100 -> 120+ Mpc)
    1. Expect 3 or 4 times events
        - A. 30-40 BBH, 1-10 BNS
        - B. New sources. e.g. NS-BH
- ⋆ Open Public Alert (OPA)
- ⋆ LIGO/Virgo/KAGRA data sharing



Future …

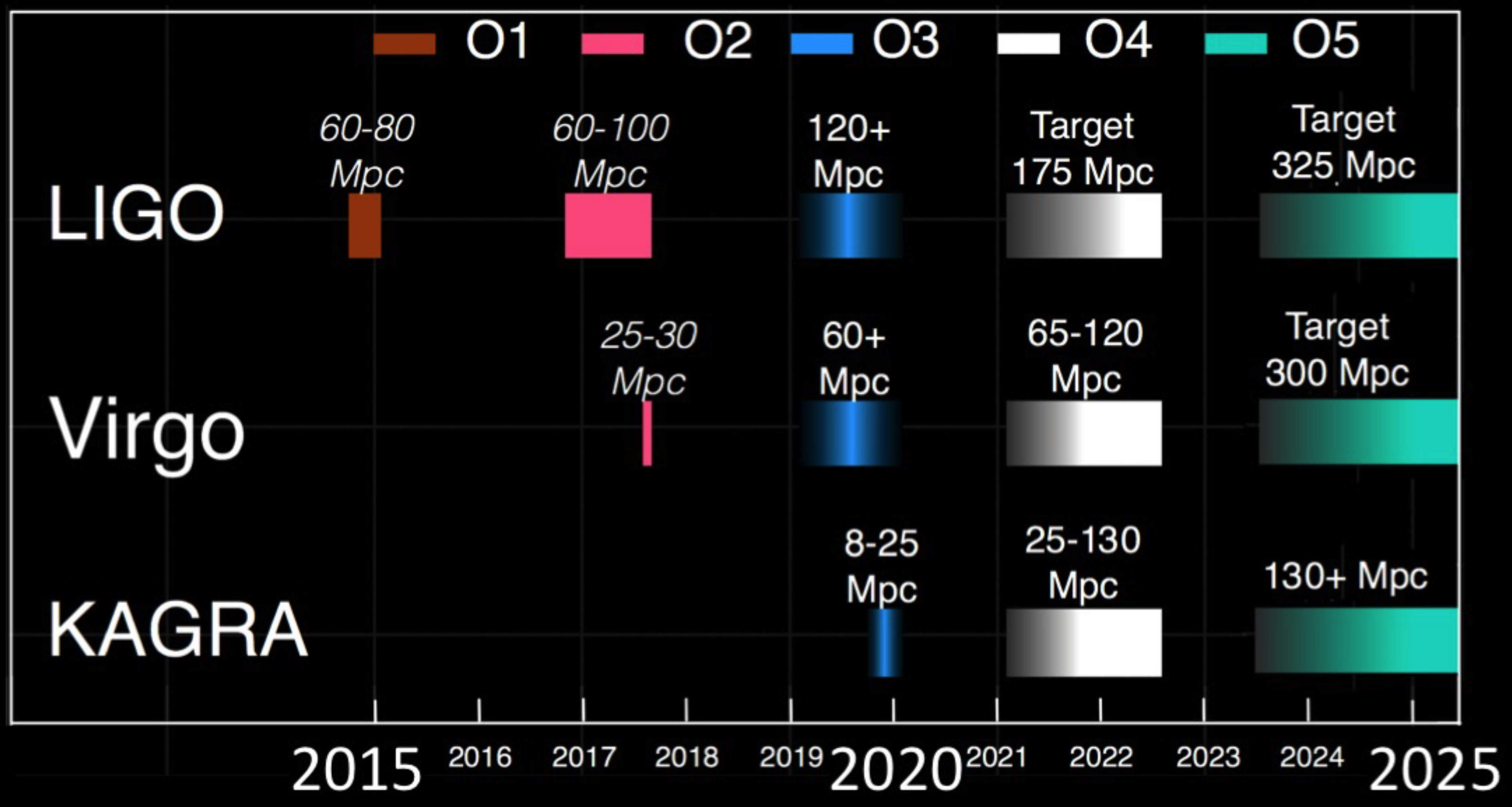
- LVK will run O4 in 2021-2022

LIGO: G1801329
Virgo: E. Majorana, KIW4
KAGRA: JGW-T1809078



Future upgrade

- LIGO/Virgo upgrade plan approved for O5
- KAGRA upgrade plan still on-going



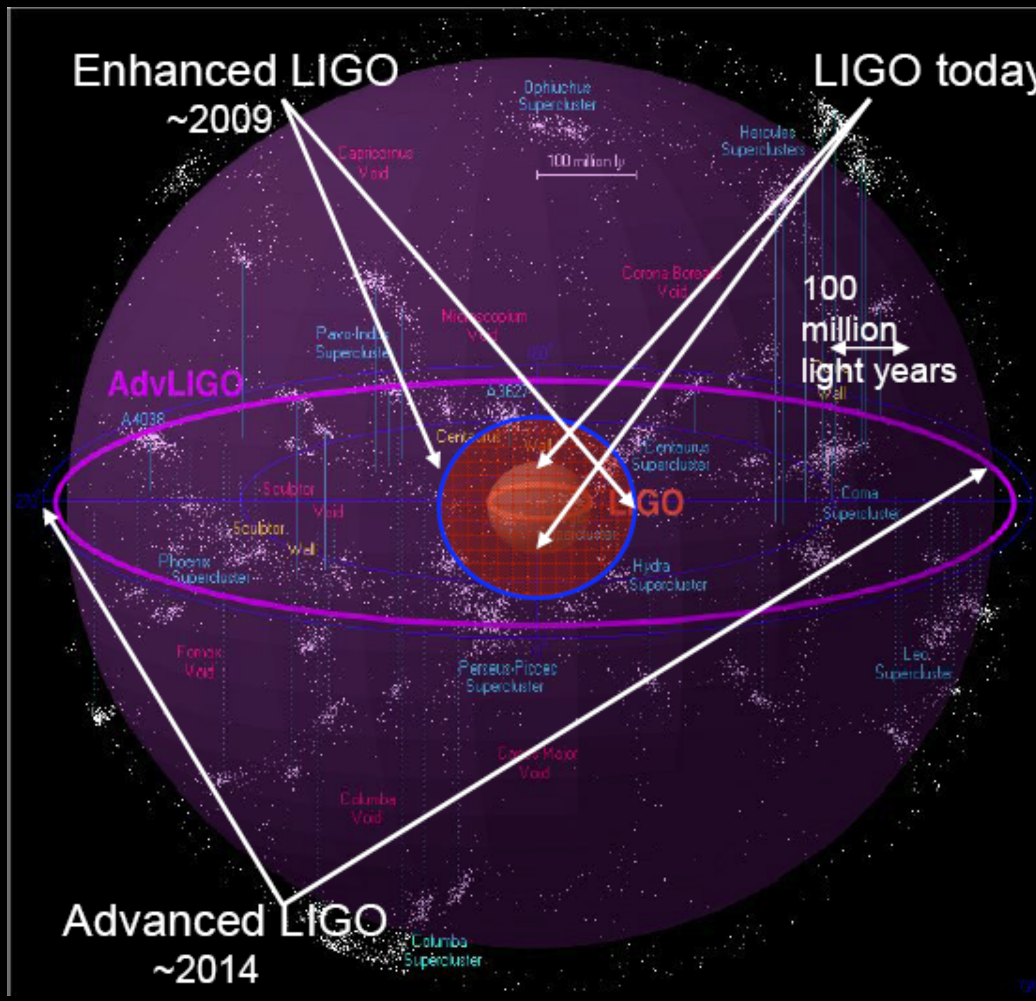
Open era of GW Astronomy

- GW amplitude is promotional to 1/Distance
—>

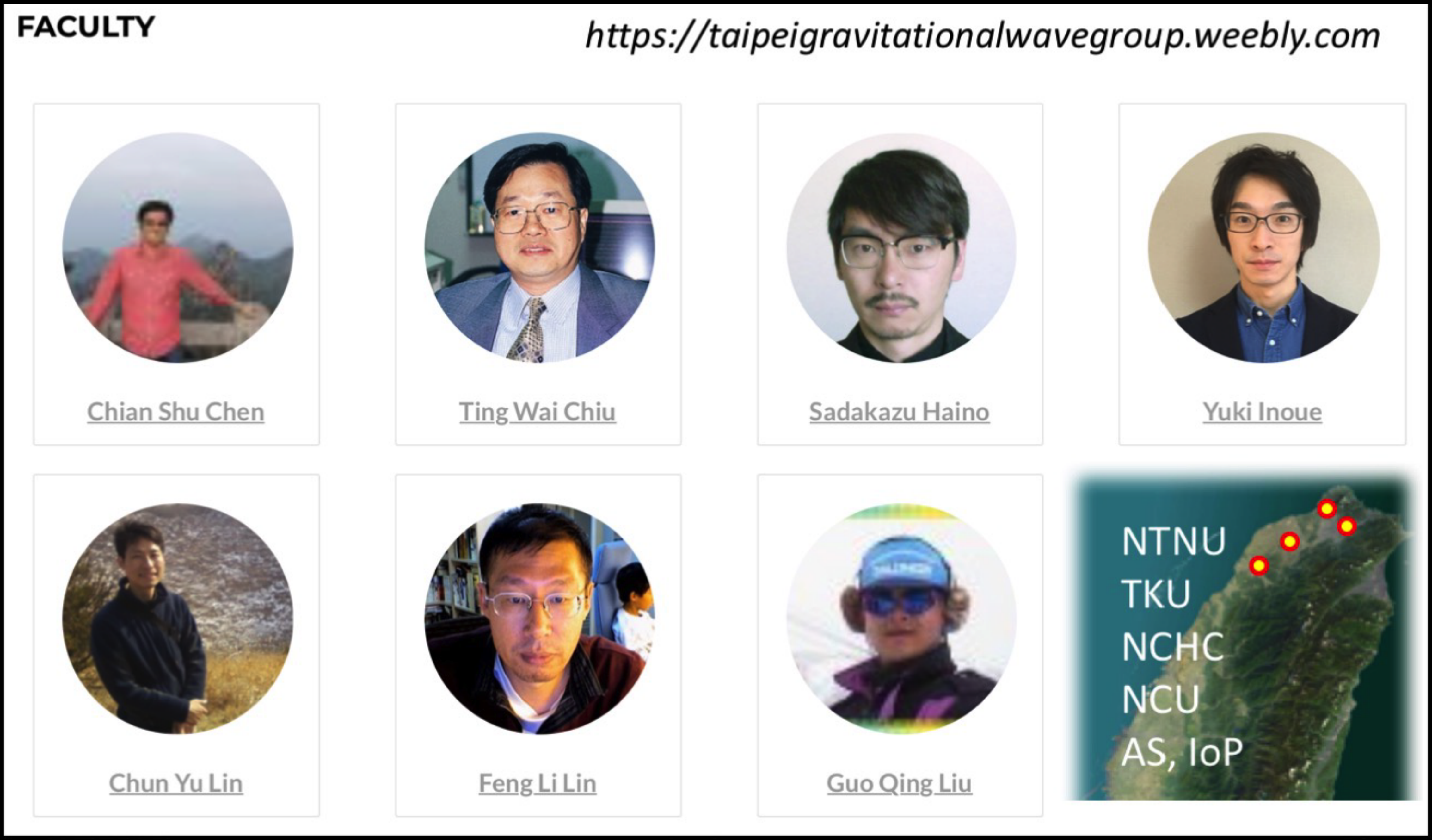
$$\text{Event Rate} \propto \text{Sensitivity}^3$$

- Increasing 10 times sensitivity will then enhance detection rate by 1000 times!

It will then open a new era of gravitational wave astronomy!



Taipei Gravitational Wave group



Tasks of TGWG

- Data analysis for LIGO/Virgo/KAGRA
  - GC Liu, Haino, JY Lin, FL Lin, SH Ho (PD) & 6 students
  - GPU acceleration & machine learning
- Theoretical study
  - FL Lin, CS Chen, A. Parisi (PD), LW Luo (PD), H Zhang (PD) & 2 students
  - EOS & Tidal effect for neutron stars & boson stars, GW waveform for modified gravity, optomechanics.

Data Analysis at TGWG

- Take advantage of GPU at NCHC and ASGC, …
- GW acceleration of GW event Parameter Estimation
  - testing GR & look for modified gravity
  - accelerating sky location
  - accelerating de-glitching procedure
  - accelerating waveform generations
- Machine learning for GW detection, PE & waveform generation

Beyond

Quantum Black Hole

- Classical black hole obey the law of mechanics:

$$dM = \frac{\kappa}{8\pi G} dA + \Omega dJ + \Phi d\Phi$$

- This relates the change of its hairs to the change of its area.

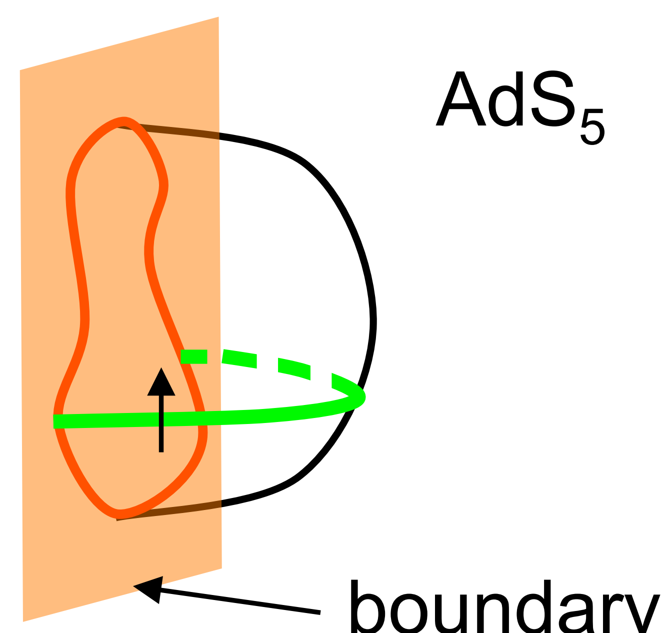
- Hawking and Bekenstein reformulate the above into 1st law of thermodynamics by (i) Hawking radiation and (ii) Bekenstein-Hawking entropy: $$S = \frac{A}{4G}$$

- Quantum black hole then becomes a thermal system with area-law entropy, not the volume law.

Holographic Principle

- This inspired ’t Hooft and Susskind to propose holographic principle: dynamics of gravity can be encoded on surface degrees of freedom.

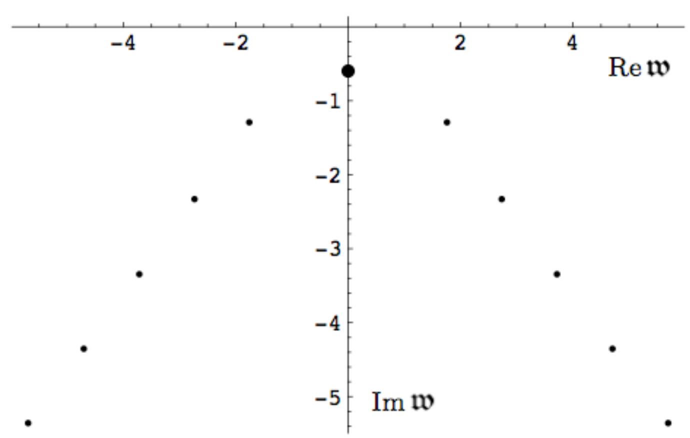
- This principle is manifested by Maldacena’s conjecture of AdS/CFT correspondence: The dynamics of classical gravity in anti-de Sitter space is dual to the dynamics of strongly coupled conformal field theory living on its boundary.

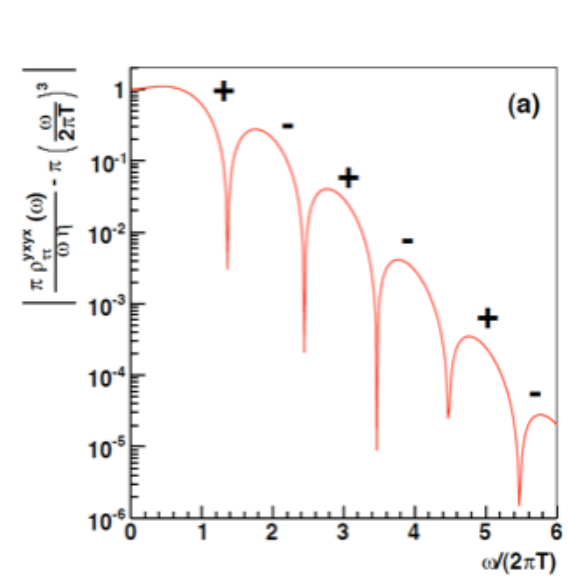


AdS/CFT duality

- AdS/CFT duality has been proposed over 20 years, and has yielded numerous new insights on our understanding of both gravity and quantum field theory.

- For example, the ring-downs of AdS black hole yields quasi-quasi-particle spectrum of boundary CFT.



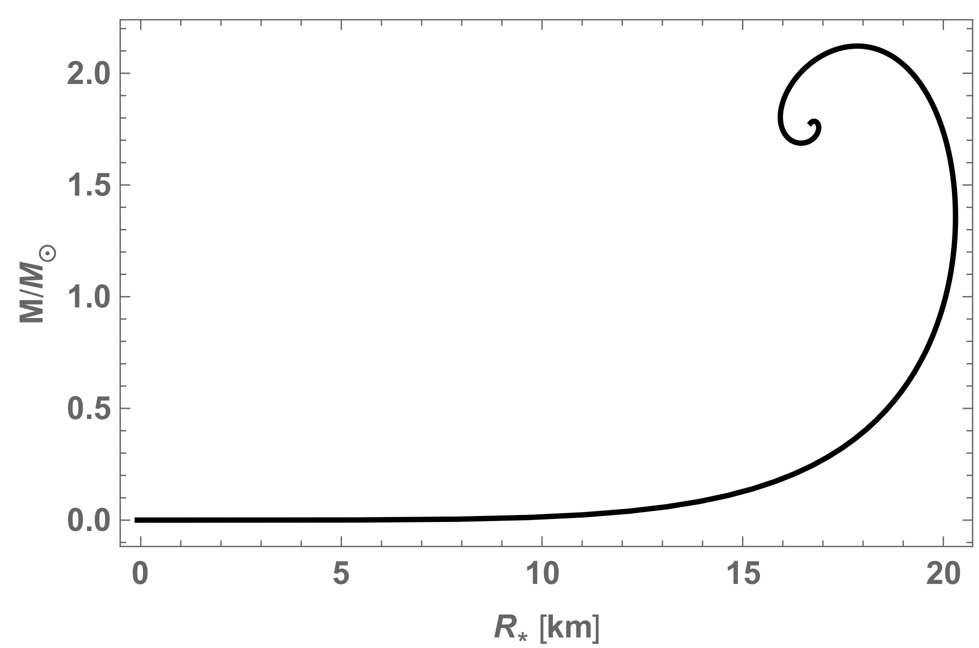


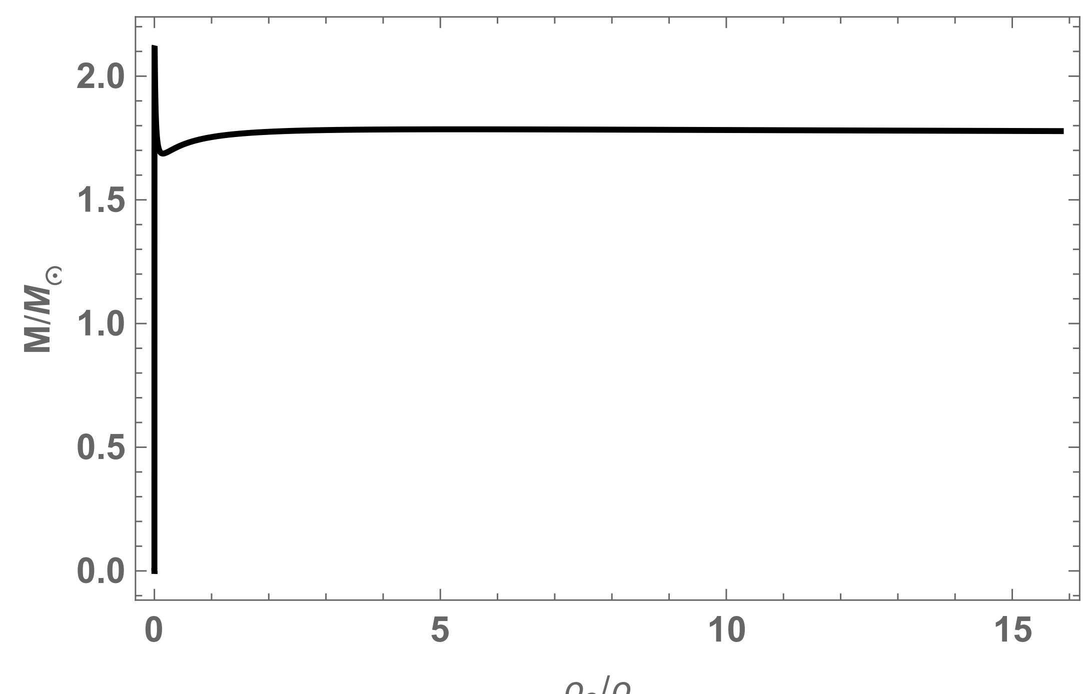
Holographic Neutron stars

- We recently use AdS/CFT to extract the equation of state for the nuclear matters, which has no first principle way of derivation.
- We then use this EOS to study mass-radius relation of neutron stars, which can be tested in future GW detection.

$$\epsilon_{TOV} = \mathcal{A} c_0 + \mathcal{A}^{1-\gamma}\,\kappa\, p_{TOV}^{\gamma}, \qquad \text{with } \gamma \simeq 1.018, \qquad \kappa \simeq 2.11, \qquad c_0 \simeq -0.01$$

$$\mathcal{A} = 0.04$$





Conclusion

- A new era of GW astronomy will arrive soon in the next 5 years.

- The expecting numerous events will test Einstein’s gravity, shed lights on our understandings about black holes and neutron stars.

- In 2035, LISA will start the missions, and we will have the events for supermassive black holes, and detect primordial GWs, and hope to understand their formation mechanism.

- Hope these studies will help us to uncover the mysteries of the quantum black hole and its holographic nature.