Static black holes with back reaction from vacuum energy

Yoshinori Matsuo

National Taiwan University

In collaboration with Pei-Ming Ho

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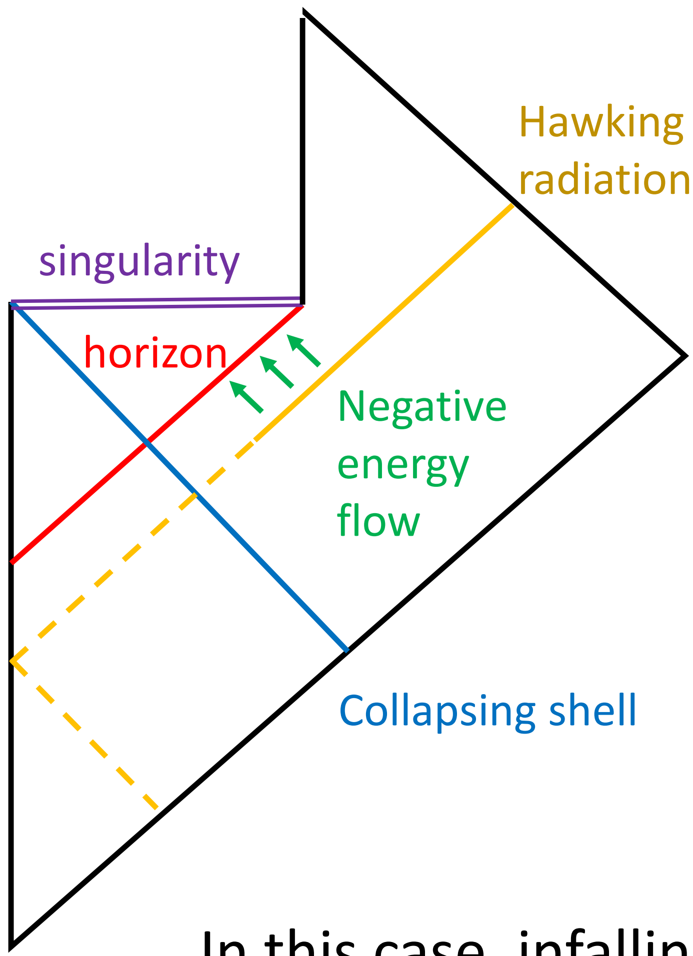
Motivation



Quantum effects of matter fields around black holes

↓

Hawking radiation ⇨ BH evaporation

Conservation of $\langle T_{\mu\nu} \rangle$

Hawking radiation $\langle T_{uu} \rangle > 0$

↓

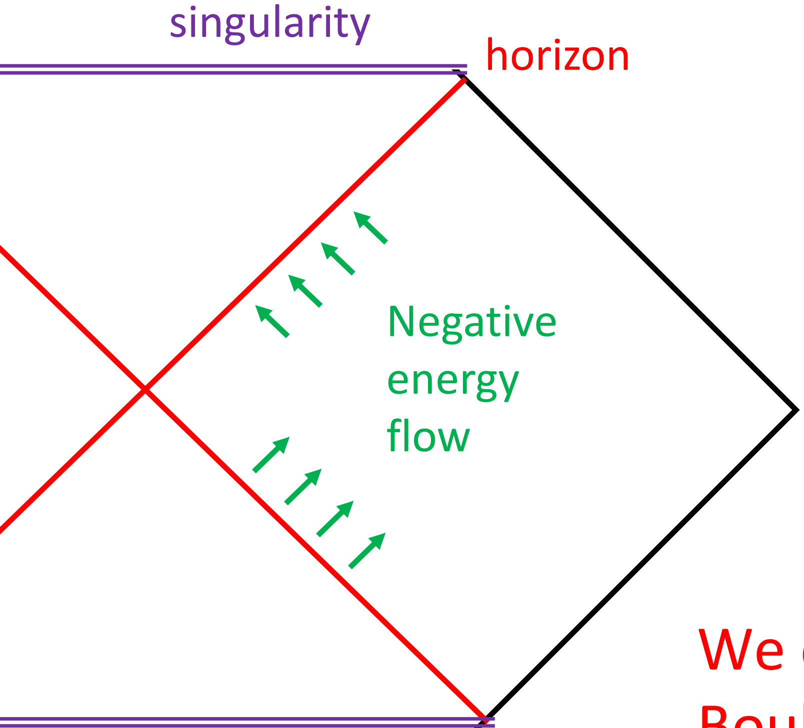
In-falling negative energy $\langle T_{vv} \rangle < 0$

It is sometimes considered that Hawking radiation appears in bulk

In this case, infalling negative energy cancels BH energy

However, negative energy appears even around static black holes

Introduction



Negative energy appears even around static black holes

For example, models with 2D matters

Conservation of $\langle T_{\mu\nu} \rangle$

Weyl anomaly $\langle T^{\mu}{}_{\mu} \rangle \neq 0$

negative energy $\langle T_{vv} \rangle < 0 \quad \langle T_{uu} \rangle < 0$

We consider effects of negative energy in Boulware vacuum $\langle T_{\mu\nu} \rangle \to 0$ in $r \to \infty$

Einstein equation with back reaction from $\langle T_{\mu\nu} \rangle$

$$G_{\mu\nu}^{(4D)} = 8\pi G \langle T_{\mu\nu}^{(4D)} \rangle$$

2D model for 4D black hole

Separate 4D metric to angular part and others

$$ds^2 = \sum_{\mu=0,1,2,3} g_{\mu\nu} dx^\mu dx^\nu = \sum_{\mu=0,1} g^{(2D)}_{\mu\nu} dx^\mu dx^\nu + r^2 d\Omega^2$$

We integrate out angular directions

The Einstein-Hilbert action gives dilaton action

$$S = \frac{1}{16\pi G} \int dx^4 \sqrt{-g} R \qquad \text{where} \quad e^{\phi} = \frac{\mu}{r}$$

$$= \frac{\mu^2}{16\pi G} \int dx^2 \sqrt{-g_{2D}}\, e^{-2\phi} [R_{2D} + 2(\partial\phi)^2 + 2e^{2\phi}]$$

2D curvature is non-zero even in the vacuum

$$R_{4D} = 0 \qquad R_{2D} \neq 0 \qquad \text{for} \quad \langle T_{\mu\nu} \rangle = 0$$

We consider scalar fields

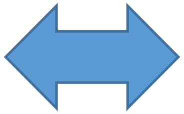
$$S = -\frac{1}{2}\int dx^4\sqrt{-g}[(\partial\chi)^2] = -2\pi\int dx^2\sqrt{-g_{2D}}\, r^2[(\partial\chi)^2]$$

Energy-momentum tensor in 4D and 2D are

$$T_{\mu\nu}^{(4D)} = -\frac{2}{\sqrt{-g_{4D}}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad T_{\mu\nu}^{(2D)} = -\frac{2}{\sqrt{-g_{2D}}}\frac{\delta S}{\delta g_{(2D)}^{\mu\nu}}$$

For $\mu, \nu = 0{,}1,$ $\langle T_{\mu\nu}^{(4D)}\rangle = \frac{1}{r^2}\langle T_{\mu\nu}^{(2D)}\rangle$

We treat dilaton $r = \mu e^{-\phi}$ as a background field

EM tensor for dilaton  4D Einstein tensor

Semi-classical Einstein equation

$$G_{\mu\nu}^{(4D)} = \frac{8\pi G}{r^2}\langle T_{\mu\nu}^{(2D)}\rangle$$

Toy model: 4D gravity with 2D scalar

We consider the 2D model as a toy model

2D scalar fields

We neglect factor of r^2

$$S = -\frac{1}{2}\int d^2x\,\sqrt{-g}\,(\partial\chi)^2$$

classically conformal, but has anomaly

$$\langle T_\mu^{(2D)\mu}\rangle = \frac{1}{24\pi}R_{2D}$$

Using anomaly, we can integrate conservation law;

$$\nabla^\mu\langle T_{\mu\nu}^{(2D)}\rangle = 0$$

and then, 2D energy-momentum tensor is completely fixed.

Here, we use the following boundary condition

$$\langle T_{\mu\nu}^{(2D)}\rangle \to 0 \qquad \text{in} \qquad r\to\infty$$

Vacuum energy without back reaction

We consider the fixed background of Schwarzschild BH

$$ds^2 = -\left(1 - \frac{a_0}{r}\right) dt^2 + \frac{1}{1 - \frac{a_0}{r}} dr^2 + r^2 d\Omega^2$$

Quantum effects in energy-momentum tensor

$$\langle T_{uv}^{(2D)} \rangle = \frac{N}{48\pi}\left(\frac{a_0^2}{r^4} - \frac{a_0}{r^3}\right)$$

$$\langle T_{uu}^{(2D)} \rangle = \frac{N}{48\pi}\left(\frac{3a_0^2}{r^4} - \frac{a_0}{r^3}\right)$$

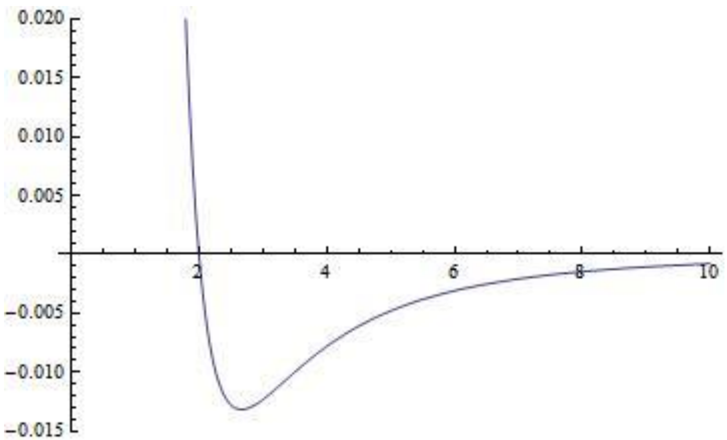
$$\langle T_{vv}^{(2D)} \rangle = \frac{N}{48\pi}\left(\frac{3a_0^2}{r^4} - \frac{a_0}{r^3}\right)$$

Vacuum energy without back reaction

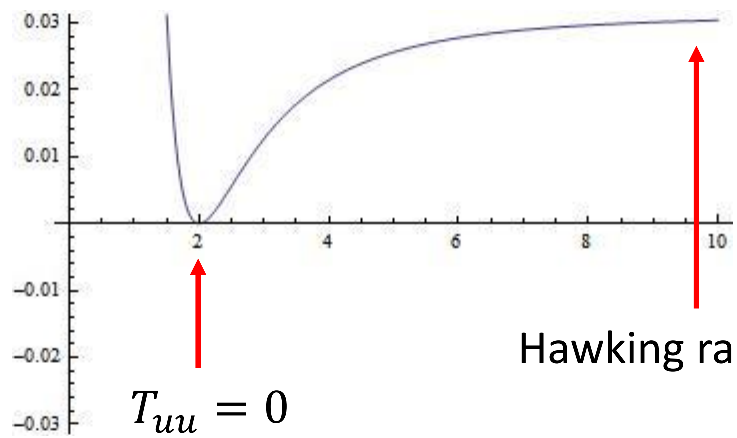
No incoming or outgoing energy at the horizon

Quantum effects give energy flow in $r \to \infty$ (Hawking radiation)

$\langle T_{uv}^{(2D)} \rangle$



$\langle T_{uu}^{(2D)} \rangle$



Hawking radiation

$T_{uu} = 0$
at horizon

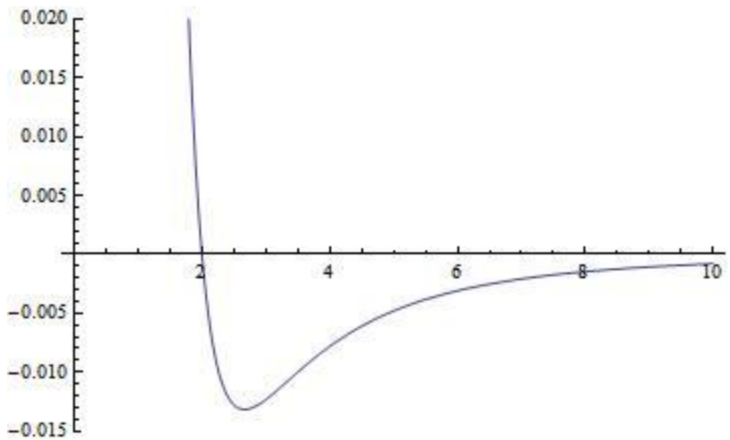
$M = 1$, the horizon is at $r = 2$

Vacuum energy without back reaction

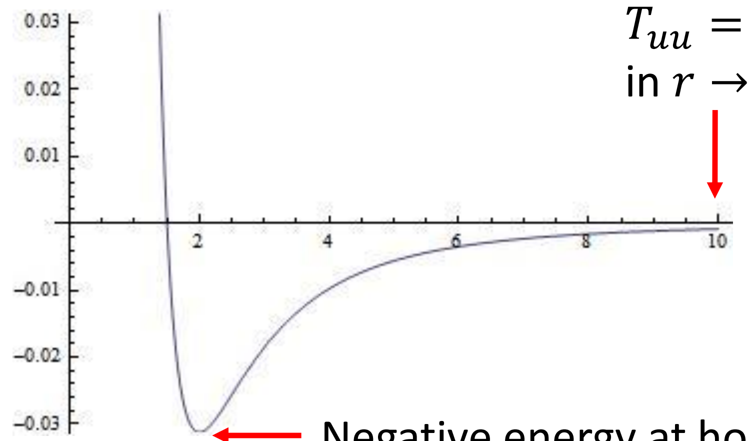
No incoming or outgoing energy in $r \to \infty$

Quantum effects give negative energy outside the horizon

$\langle T_{uv}^{(2D)} \rangle$



$\langle T_{uu}^{(2D)} \rangle$



$T_{uu} = 0$
in $r \to \infty$

Negative energy at horizon

The horizon is at $a_0 = 2$

Breakdown of perturbative expansion

Perturbative expansion around classical solution

$$ds^2 = -C(r)dt^2 + \frac{C(r)}{F^2(r)}dr^2 + r^2 d\Omega^2 \qquad \alpha = \frac{GN}{3}$$

$$C(r) = C_0(r) + \alpha C_1(r) + \cdots$$

The leading term $C_0(r)$ is classical solution $\quad C_0 = 1 - \frac{a_0}{r}$

$C_1(r)$ is the correction at $\mathcal{O}(\alpha)$ from $\langle T_{\mu\nu} \rangle$

$$C_1(r) = \frac{4r^2 + a_0^2 + 4a_0 r(2c_1 r - 1)}{4a_0 r^2 (r - a_0)} - \frac{2r - 3a_0}{2a_0^2 (r - a_0)} \log(1 - \frac{a_0}{r})$$

Perturbative correction $C_1(r)$ diverges at the horizon $r = a_0$.

⇨ We cannot use α-expansion near $r = a_0$.

We solve the Einstein equation without using α-expansion.

Self-consistent Einstein equation

We solve semi-classical Einstein equation for $g_{\mu\nu}$ and $\langle T_{\mu\nu}\rangle$

$$G_{\mu\nu}^{(4D)} = \frac{8\pi G}{r^2}\langle T_{\mu\nu}^{(2D)}\rangle \quad \mu,\nu = 0,1 \qquad G_{\theta\theta} = 0$$

where metric and $\langle T_{\mu\nu}^{(2D)}\rangle$ are given by

$$ds^2 = -C(r)dudv + r^2 d\Omega^2$$

$$\langle T_{uv}^{(2D)}\rangle = -\frac{1}{12\pi}(C\partial_u\partial_v C - \partial_u C\partial_v C)$$

$$\langle T_{uu}^{(2D)}\rangle = \langle T_{vv}^{(2D)}\rangle = -\frac{1}{12\pi}C^{1/2}\partial_u^2 C^{-1/2}$$

Results

$$ds^2 = -C(r)dt^2 + \frac{C(r)}{F^2(r)}dr^2 + r^2 d\Omega^2$$

Define $C(r) = e^{2\rho}$

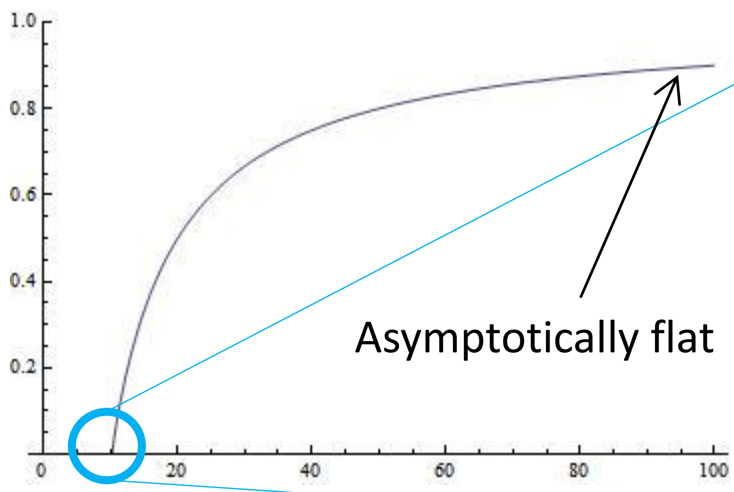
ρ satisfies

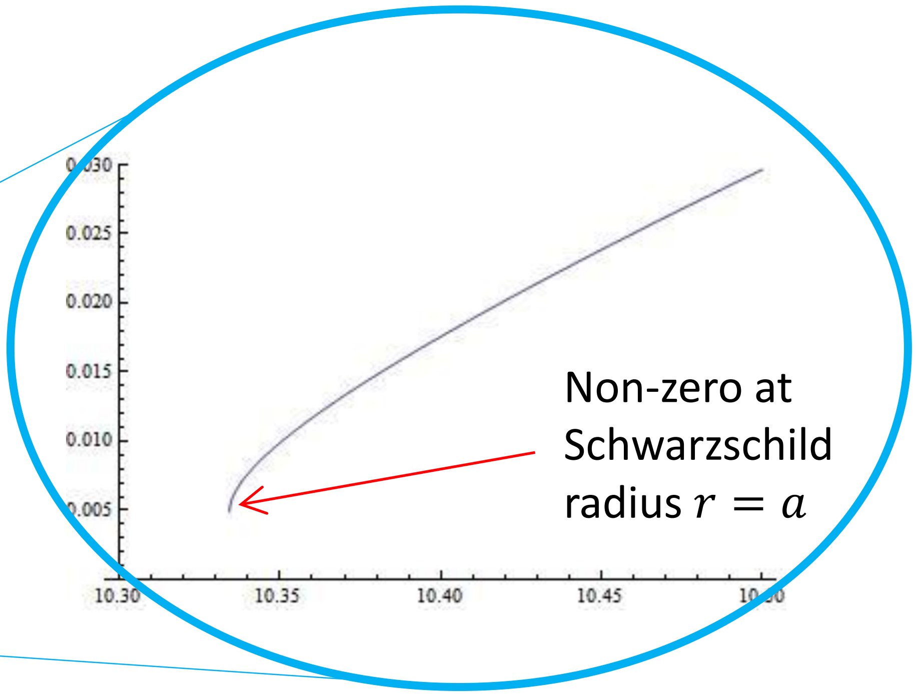
$$\alpha = \frac{GN}{3}$$

N:Number of DoF (scalar)

$$r\rho' + (2r^2 + \alpha)\rho'^2 + \alpha r \rho'^3 + (r^2 - \alpha)\rho'' = 0$$

Numerical result for $C(r)$





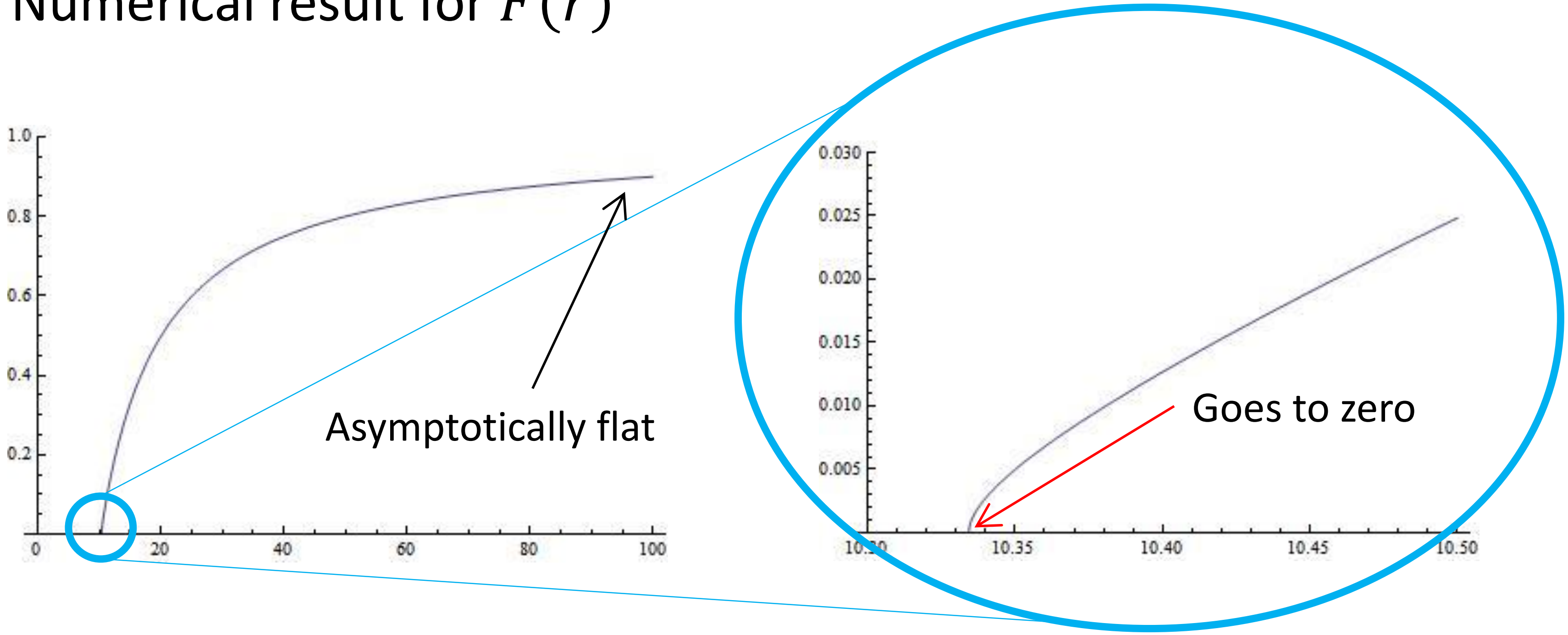
Results

$$ds^2 = -C(r)dt^2 + \frac{C(r)}{F^2(r)}dr^2 + r^2 d\Omega^2$$

$F(r)$ is related to $C(r)$ as

$$F(r) = \frac{C^{3/2}(r)}{\sqrt{4C^2(r) + 4rC(r)C'(r) + \alpha C'^2}}$$

Numerical result for $F(r)$



Near "horizon" behavior

horizon ⇒ $C(r) = 0$ ⇒ $\rho \to -\infty$ ⇒ $\rho' \to \infty$

$(C(r) = e^{2\rho})$

Assume $\rho' \xrightarrow{r \to a} \infty$, at some point $r = a$,

Differential equation for ρ is approximated as

$$r\rho' + (2r^2 + \alpha)\rho'^2 + \alpha r \rho'^3 + (r^2 - \alpha)\rho'' = 0$$

⇒

$$\alpha a \rho'^3 + (a^2 - \alpha)\rho'' = 0$$

Then, ρ' behaves as $\rho' \sim \dfrac{k}{\sqrt{r-a}}$ where $k \sim \left(\dfrac{2a}{\alpha}\right)^2$

$C(r), F(r)$ behaves near $r = a$ as

$$C(r) = c_0 e^{2k\sqrt{r-a}} \qquad F(r) = \frac{1}{k}\sqrt{4c_0(r-a)}$$

Near "horizon" geometry

Metric is given by

$$ds^2 = -C(r)dt^2 + \frac{C(r)}{F^2(r)}dr^2 + r^2 d\Omega^2$$

Assuming that $C(r = a) = 0,\ C(r), F(r)$ behaves near $r = a$ as

$$C(r) = c_0 e^{2k\sqrt{r-a}} \qquad F(r) = \frac{1}{k}\sqrt{4c_0(r-a)}$$

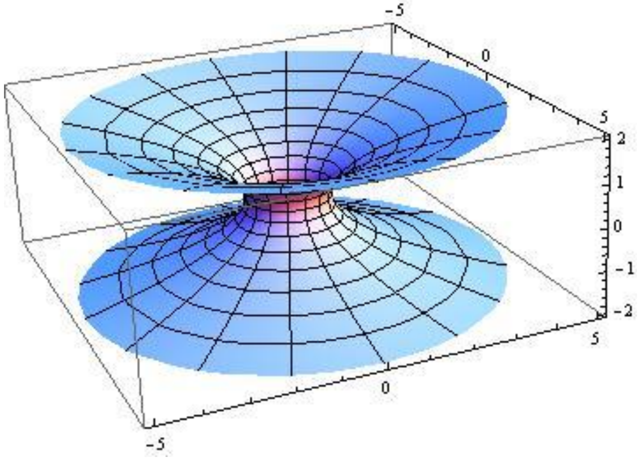
metric near $r = a$

$$ds^2 \sim -c_0 dt^2 + \frac{k\alpha\, dr^2}{4(r-a)} + r^2 d\Omega^2$$

Define x by $r = a + \frac{c_0}{\alpha k}x^2$

$$ds^2 \sim -c_0(dt^2 + dx^2) + a^2 d\Omega^2$$

This is wormhole metric



Generic energy-momentum tensor

Consider the semi-classical Einstein equation for generic $\langle T_{\mu\nu}\rangle$

$$G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$$

Regularity of curvature is related to energy-momentum tensor

For static and spherically symmetric metric

$g^{uv}R_{uv}$

$R_{\theta\theta}$ are regular ⇒ $\langle T_{\theta\theta}\rangle$

$g^{uv}\langle T_{uv}\rangle$ are regular

$\langle T_{\mu\nu}\rangle$ diverges at the horizon in Boulware vacuum

⇓

Geometry must be modified for regularity

Condition for (Killing) horizon

(u, v) coordinate has coordinate singularity at (future) horizon

⇨ Take another coordinate $(\tilde{u}, v)$ to avoid singularity

$$ds^2 = -C\, du\, dv = -\tilde{C}\, d\tilde{u}\, dv$$

where $C = 0$ but $\tilde{C} \neq 0$ at the horizon.

$$\tilde{C} = \frac{du}{d\tilde{u}} C \quad \Rightarrow \quad \frac{du}{d\tilde{u}} \propto C^{-1} \to \infty \qquad \text{at horizon}$$

Energy-momentum tensor in this coordinate must be regular

$$T_{\tilde{u}\tilde{u}} = \left(\frac{du}{d\tilde{u}}\right)^2 T_{uu} \qquad T_{\tilde{u}v} = \left(\frac{du}{d\tilde{u}}\right) T_{uv}$$

$$\Rightarrow \quad T_{uv} = T_{uu} (= T_{vv}) = 0 \qquad \text{at the horizon}$$

Classification by energy-momentum tensor

I. $\langle T_{uu} \rangle = 0 \qquad \langle T_{uv} \rangle = 0 \qquad \langle T^u{}_u \rangle > -\frac{1}{8\pi G a^2}$

⇨ Near horizon geometry is Rindler space

II. $\langle T_{uu} \rangle = 0 \qquad \langle T_{uv} \rangle = 0 \qquad \langle T^u{}_u \rangle = -\frac{1}{8\pi G a^2} \qquad \langle T_{\theta\theta} \rangle > 0$

⇨ Near horizon geometry is Rindler or AdS_2

III. $\langle T_{uu} \rangle < 0 \qquad \langle T_{uu} \rangle - \langle T_{uv} \rangle = -\frac{C(a)}{16\pi G a^2}$

⇨ Near horizon geometry is wormhole

VI. $\langle T_{uu} \rangle > 0$

⇨ No special structure such as horizon or wormhole

Discussions

Black hole geometry is modified by quantum effects in $\langle T_{\mu\nu} \rangle$

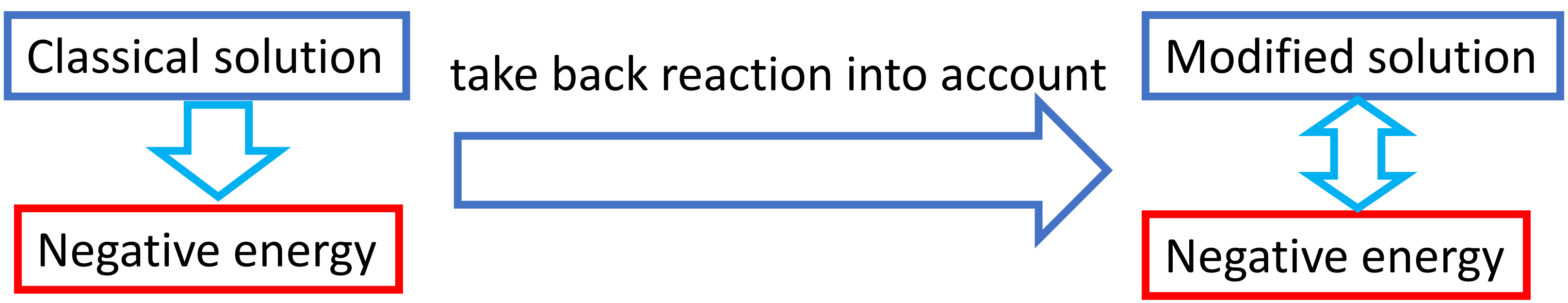
There are various vacua with positive, negative or zero energy

Negative energy ➡ Wormhole geometry

Positive energy ➡ No horizon or wormhole

For general $\langle T_{\mu\nu} \rangle$, we have not calculated any solution explicitly.

However, negative energy will still appear even with back reaction



If vacuum energy around a black hole is negative,
the black hole is modified to wormhole by the negative energy.

Interior of wormhole

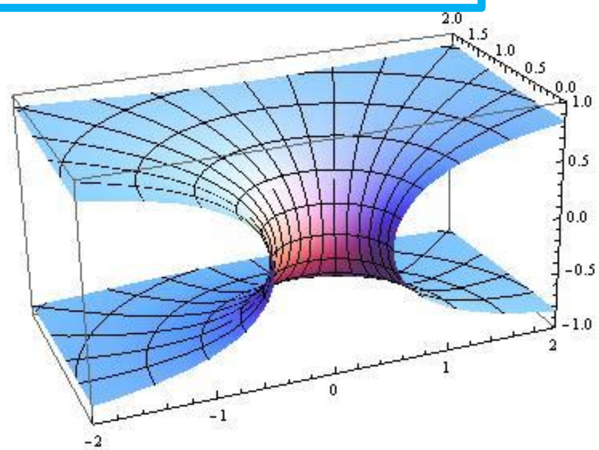
In the other side of wormhole radius r decreases as it goes inside

"Vacuum" solution: without matters ⇨ Singularity in $r \to \infty$

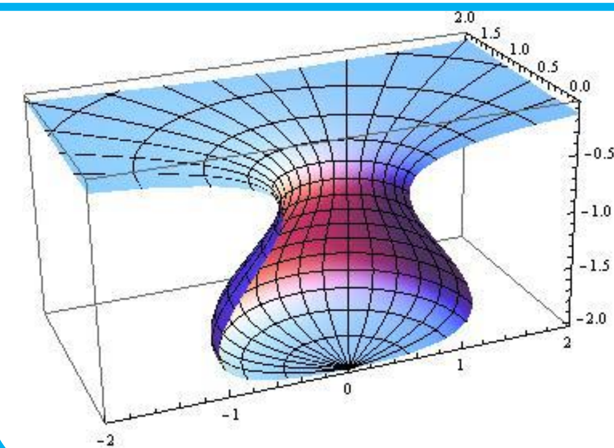
Physical model: there are matters which form the black hole

⇨ the radius r starts decrease as it goes inside matters

"Vacuum" solution



With matters inside wormhole



Geometry of interior of black hole

We put the surface of the star at $r = r_s$

Outside the surface ⇨ Vacuum solution (wormhole)

Inside the surface ⇨ Geometry with matter distribution

Energy-momentum tensor

$$\langle T_{\mu\nu} \rangle = T^{\Omega}_{\mu\nu} + T^{m}_{\mu\nu}$$

Energy-momentum tensor of matters ($T^{m}_{\mu\nu}$)

Energy-momentum tensor of vacuum ($T^{\Omega}_{\mu\nu}$)

$$T^{\Omega}_{\mu\nu} = \frac{1}{r^2} \langle T^{(2D)}_{\mu\nu} \rangle$$

We consider incompressible fluid

$$T^{m}_{\mu\nu} = (m_0 + P) u_\mu u_\nu + P g_{\mu\nu}$$

m_0: Density (constant)

P: Pressure

Classical star of incompressible fluid

Relation between a and m_0

$$\frac{a_0}{2} = \frac{4\pi}{3} m_0 r_s^3$$

Pressure in classical limit

$$P(r) = 8\pi G \frac{\sqrt{3 - 8\pi G m_0 r^2} - \sqrt{3 - 8\pi G m_0 r_s^2}}{3\sqrt{3 - 8\pi G m_0 r_s^2} - \sqrt{3 - 8\pi G m_0 r^2}}$$

Condition for non-singular pressure

$$m_0 < \frac{1}{3\pi G r_s^2} \quad \Longleftrightarrow \quad r_s > \frac{9}{8} a$$

Semi-classical geometry of interior

Assumption: $T^{\Omega}_{\mu\nu}$ and $T^{m}_{\mu\nu}$ are conserved independently.

Vacuum energy-momentum tensor (approx. by 2D scalar)

$$T^{\Omega}_{uv} = -\frac{N}{12\pi r^2}(C\partial_u\partial_v C - \partial_u C\partial_v C)$$

$$T^{\Omega}_{uu} = T^{\Omega}_{vv} = -\frac{N}{12\pi r^2}C^{1/2}\partial_u^2 C^{-1/2}$$

Energy-momentum tensor for incompressible fluid

Conservation law ⇨ $P = -m_0 + P_0\left(\frac{C(r_s)}{C(r)}\right)^{\frac{1}{2}}$

Tortoise coordinate r_* is convenient to see interior

$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$

Numerical analysis and free parameters

We solve the semi-classical Einstein equation numerically.

Initial condition:

Metric is approx. by classical Schwarzschild metric in $r \to \infty$

Junction condition:

- Pressure $P = 0$ at $r = r_s$ ⇨ $P_0 = m_0$
- Metric is smooth at the surface $r = r_s$

Parameters of the system (3 parameters)

Classical Schwarzschild radius: a_0 ⇨ Total mass

⇕ must be same

density: m_0 Surface radius: r_s ⇨ Total mass

Only 2 of 3 are independent parameters: e.g. $m_0 = \hat{m}_0(a_0, r_s)$

difficult to find exact relation by numerical calculation

Numerical analysis and free parameters

Appropriate density $\hat{m}_0(a_0, r_s)$

However, numerical calculation can be done with 3 parameters as free parameters. What happens for $m_0 \neq \hat{m}_0$?

Case I: Too small density ($m_0 \ll \hat{m}_0$)

- There is a singularity with positive mass in "center" ($r \to \infty$).
- Geometry is similar to "vacuum": $r \to \infty$ at $r_* \to -\infty$.

Case II: Too large density ($m_0 \gg \hat{m}_0$)

- There is a singularity with negative mass at center ($r \sim 0$).
- There is another $P = 0$ at $r < r_s$, inside surface.

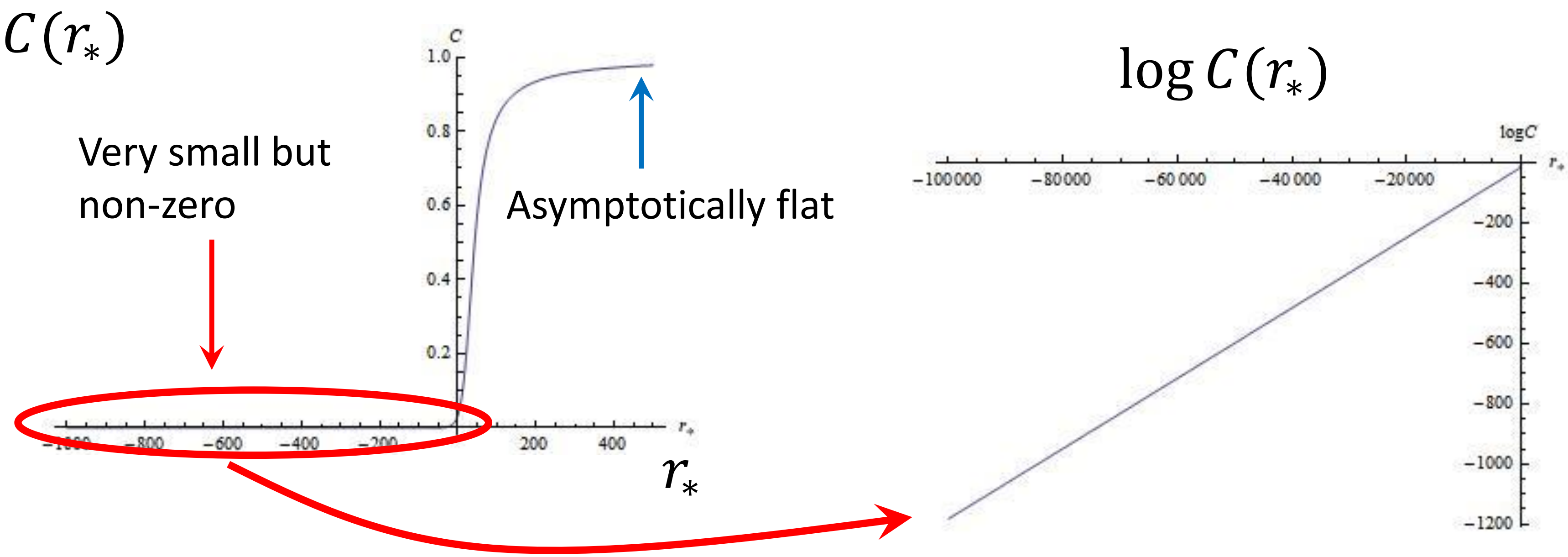
Case III: approximately appropriate density ($m_0 \sim \hat{m}_0$)

- continues to $r \sim 0$ with $P > 0$ (physical fluid).
- would have no singularity if $m_0 = \hat{m}_0$ exactly.

Case I: Too small density ($m_0 \ll \hat{m}_0$)

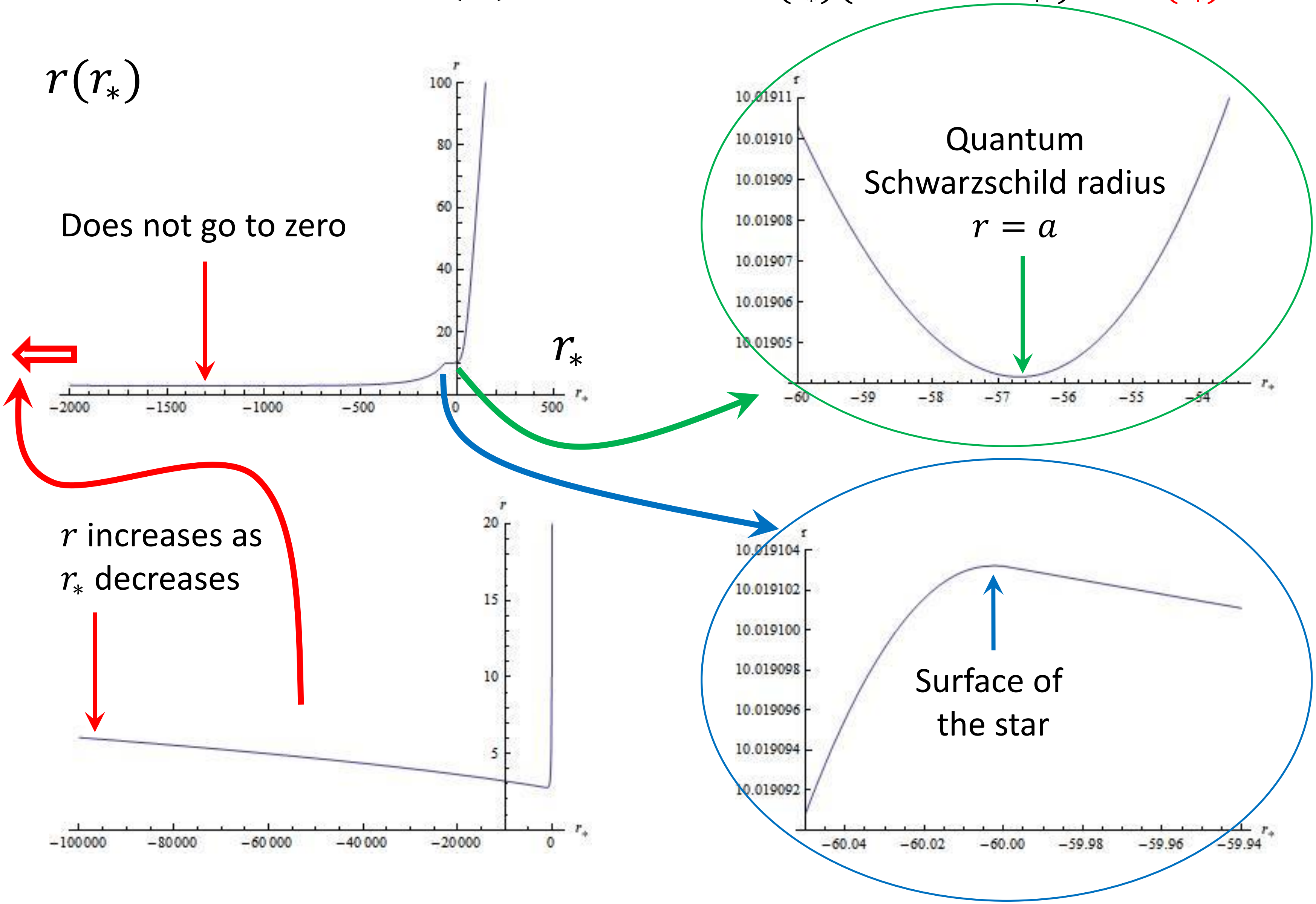
Numerical result for $C(r_*)$

$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$



Numerical result for $r(r_*)$

$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$

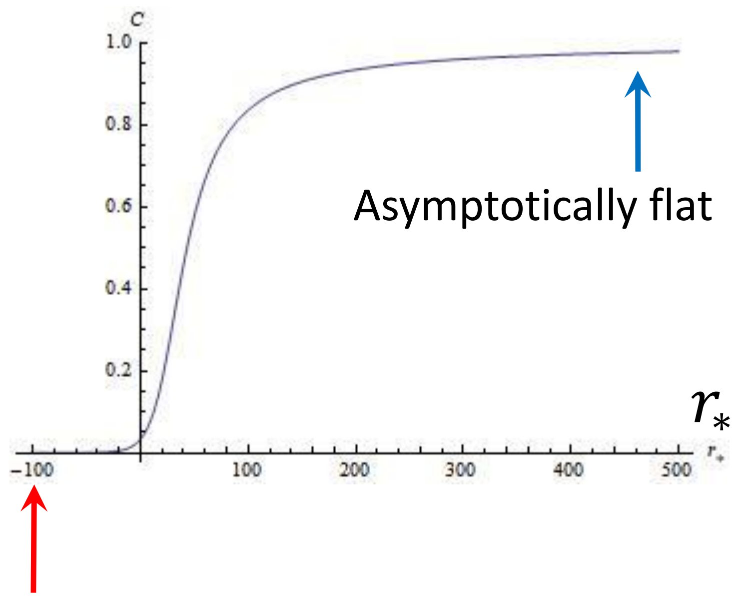


Case II: Too large density ($m_0 \gg \widehat{m}_0$)

Numerical result for $C(r_*)$

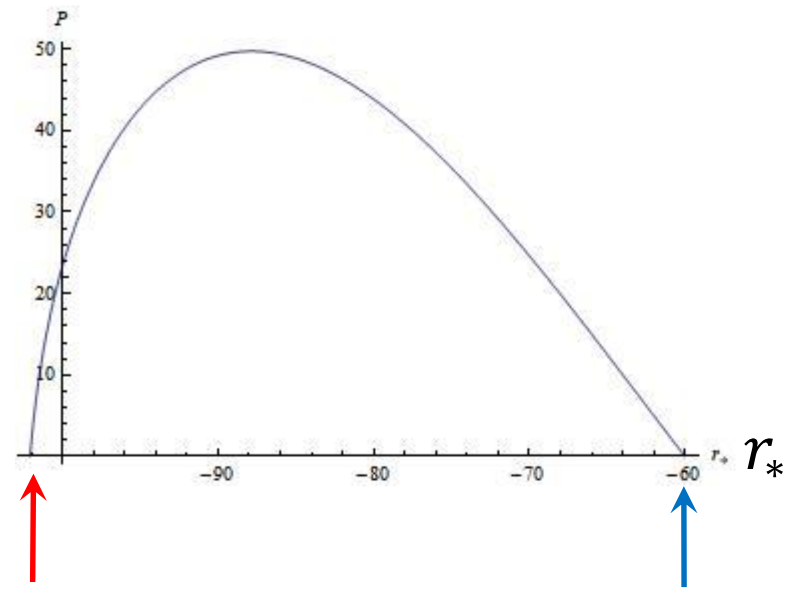
$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$

$C(r_*)$



$P = 0$, here

Pressure $P(r_*)$

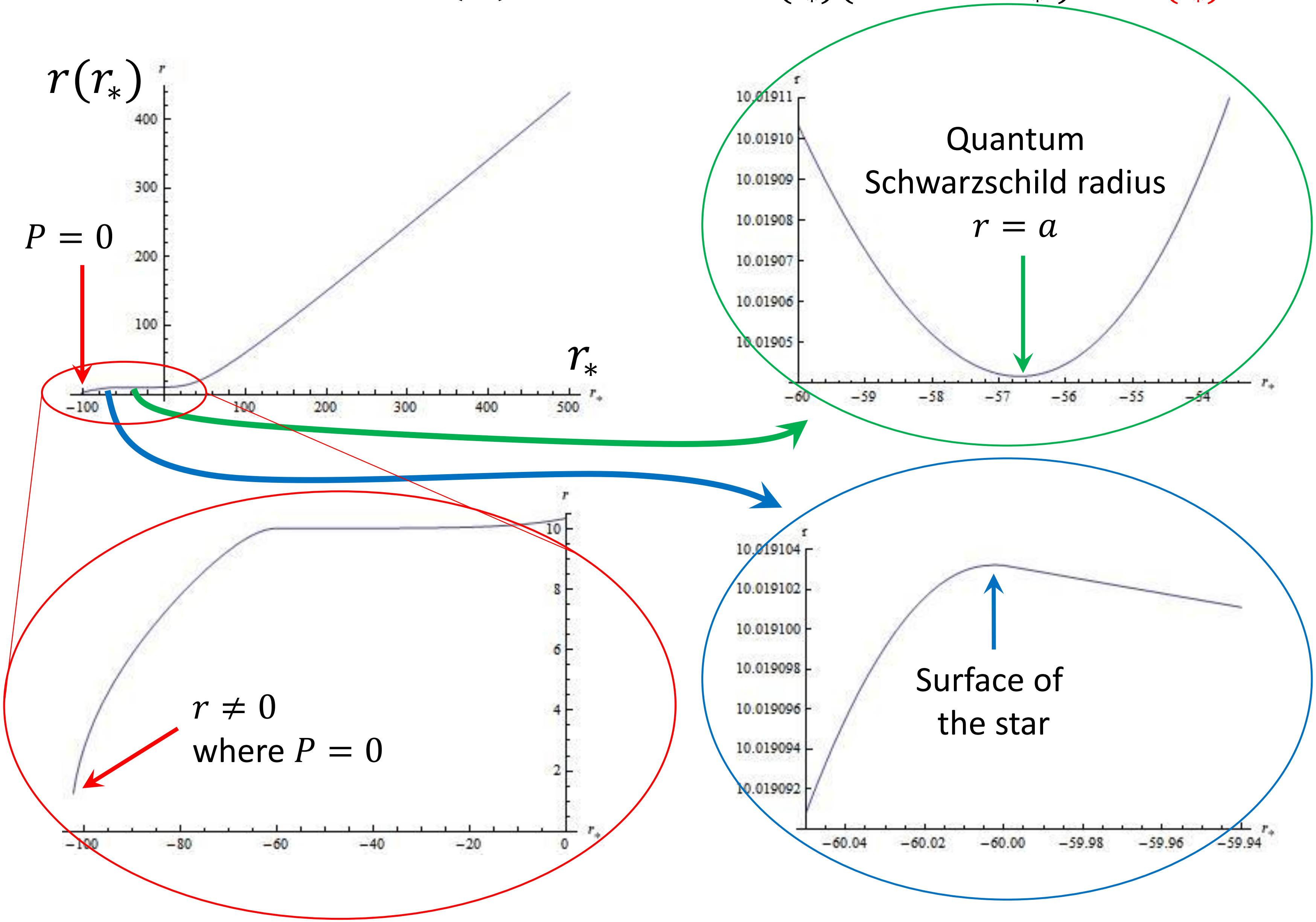


Another endpoint of matter distribution

Pressure is zero at the surface $r = r_s$

Numerical result for $r(r_*)$

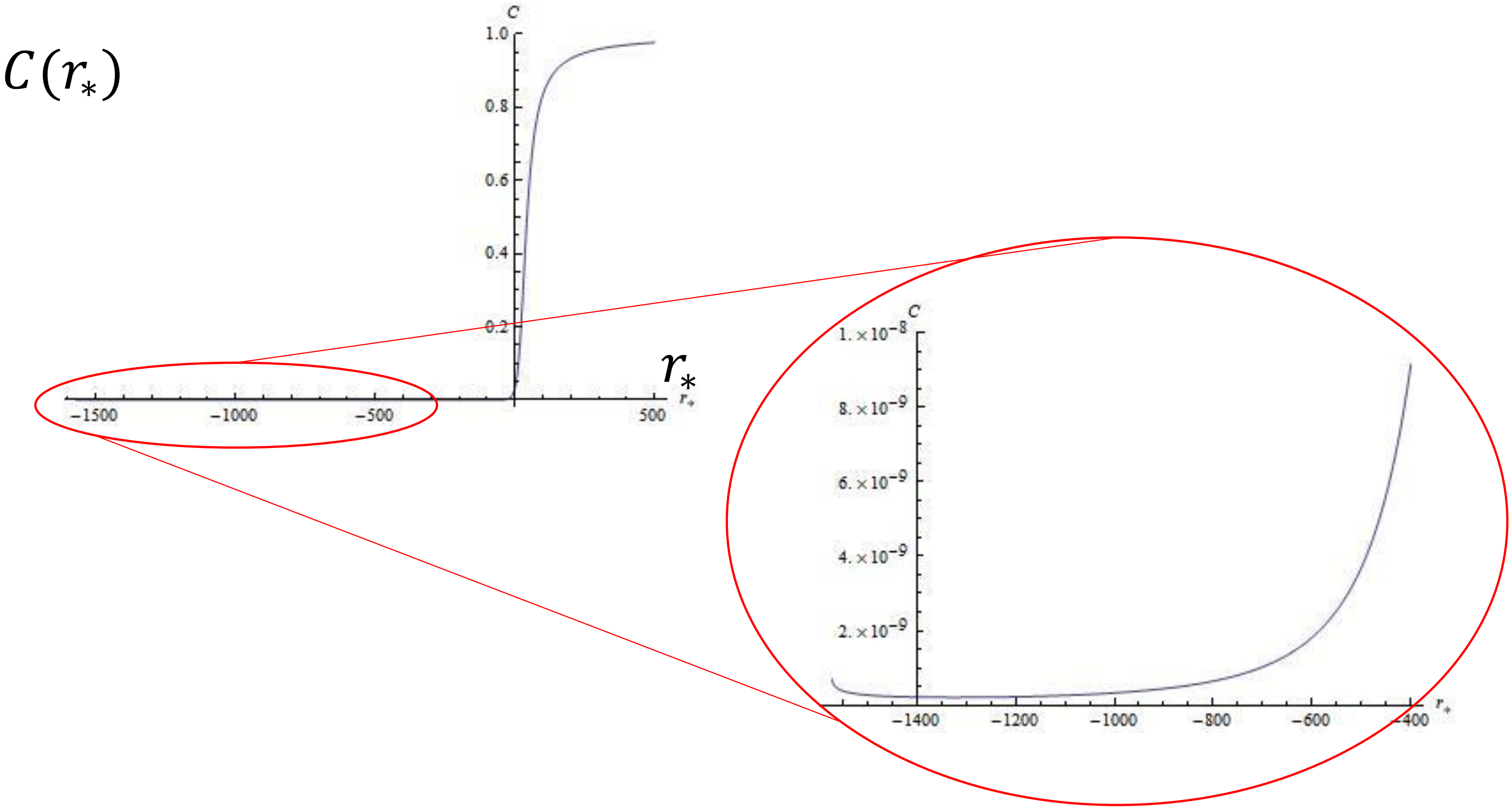
$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$



Case III: approx. appropriate density ($m_0 \sim \hat{m}_0$)

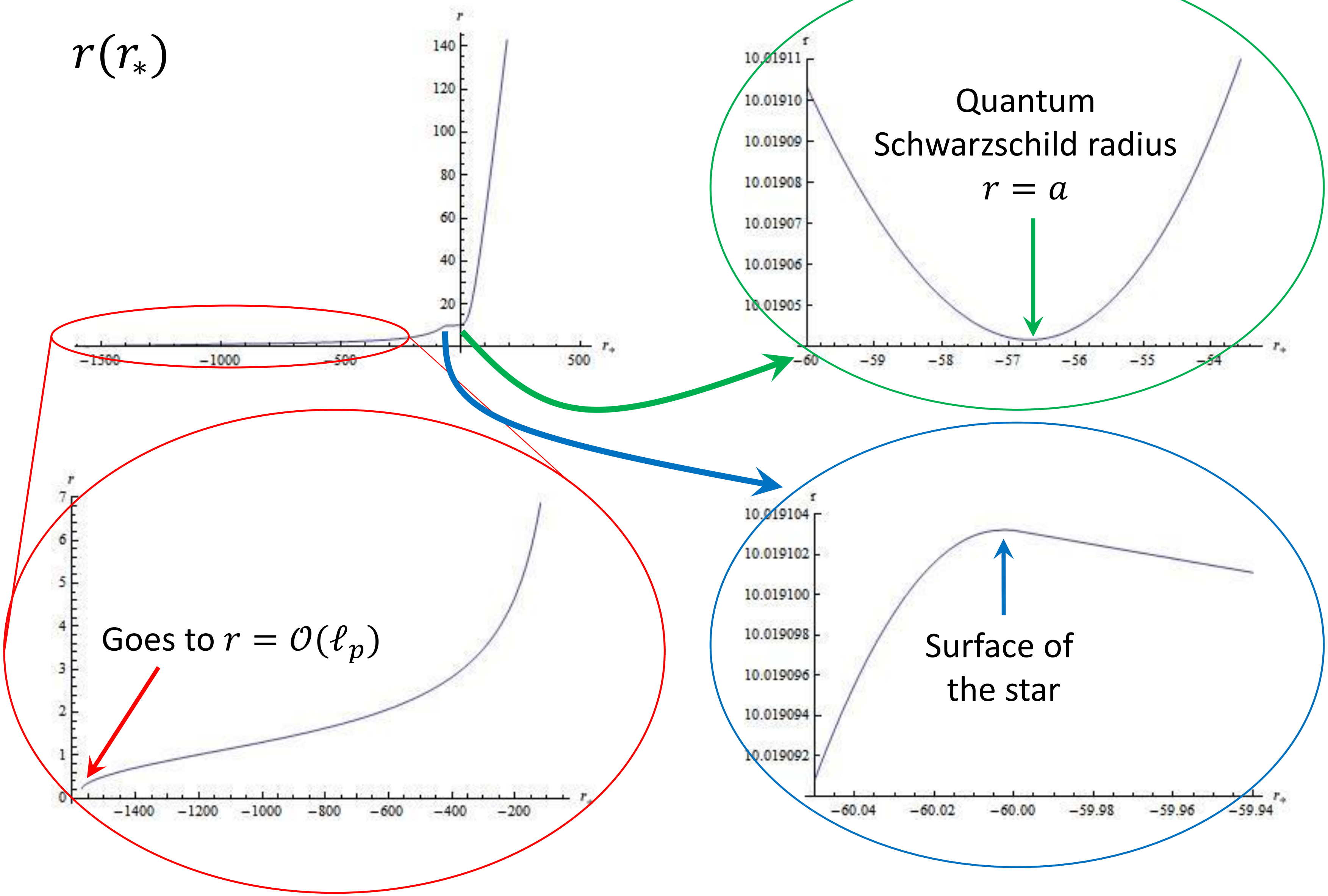
Numerical result for $C(r_*)$

$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$



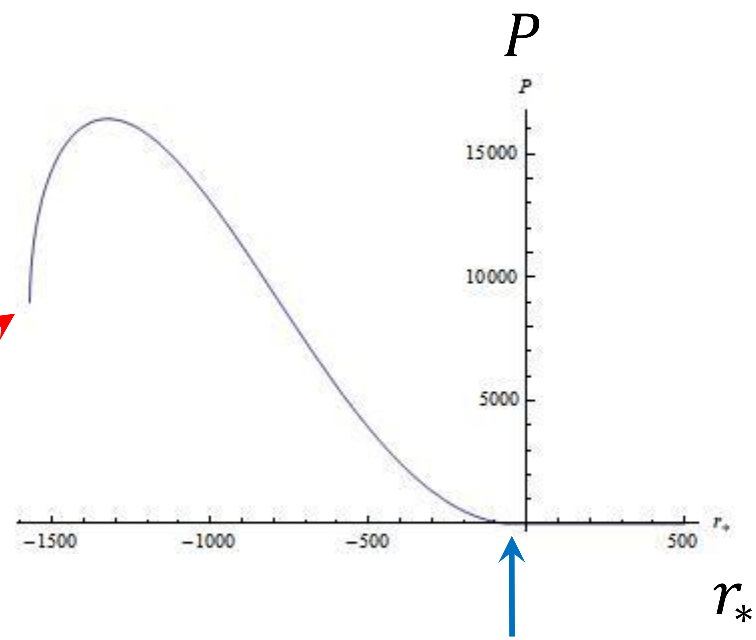
Numerical result for $r(r_*)$

$$ds^2 = C(r_*)(-dt^2 + dr_*^2) + r^2(r_*)d\Omega^2$$



Pressure $P(r_*)$

$P \neq 0$
at $r \sim \mathcal{O}(\ell_p)$

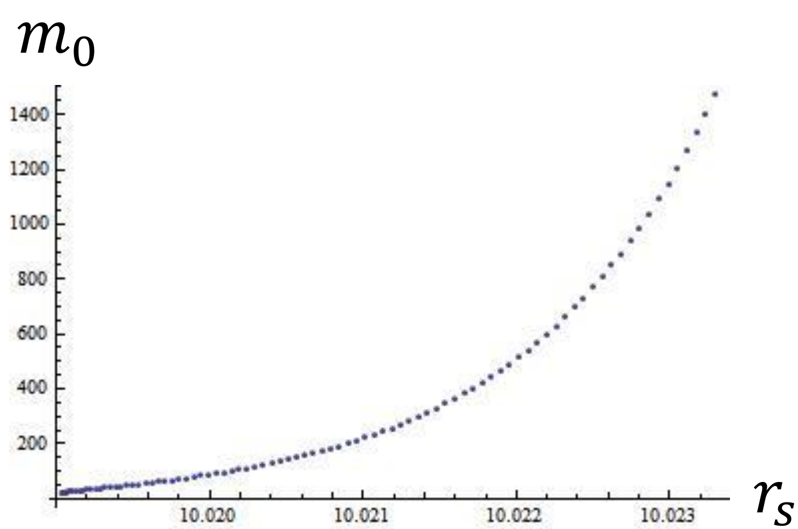


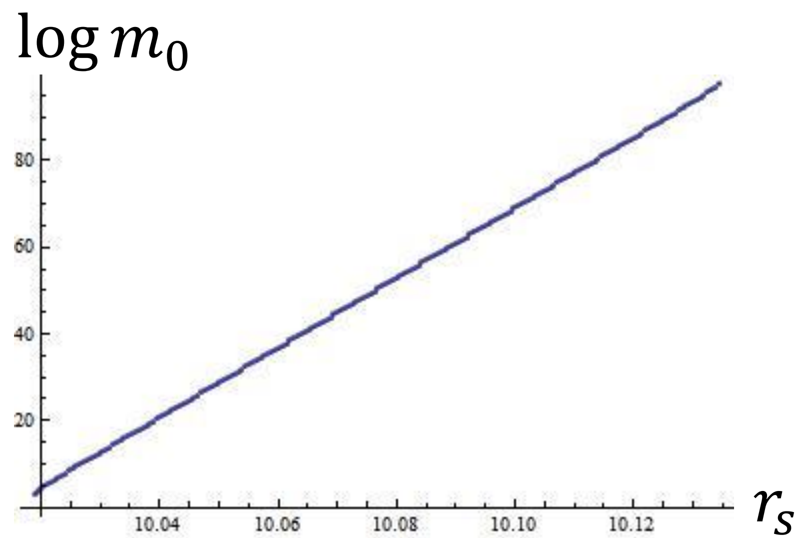
Pressure is zero at
the surface $r = r_s$

Surface at deeper place

Relation between m_0 and r_s for $a = 10$

Surface is inside of $r = a$



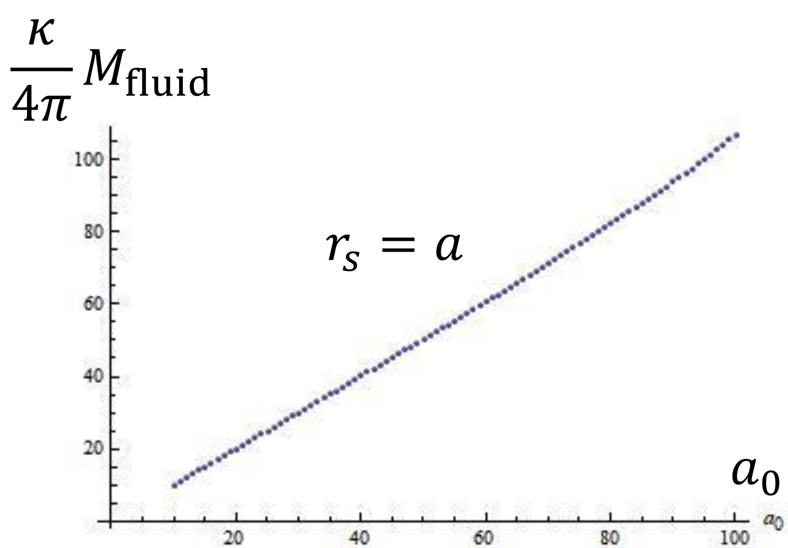


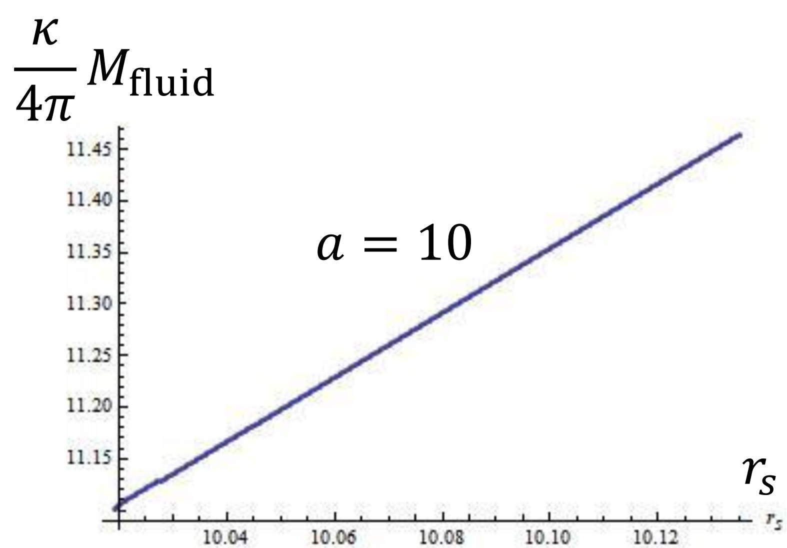
- Density m_0 increases exponentially as surface moves inside
- Difference between local minimum and local maximum of r would be of Planck scale.

Mass of fluid and black hole

Komar mass calculated from fluid density and pressure

$$M_{\text{fluid}} = -\int d^3x\sqrt{-g}\left(2T_0^0 - T_\mu^\mu\right) = 4\pi \int dr_*\, r^2 C(m_0 + 3P)$$

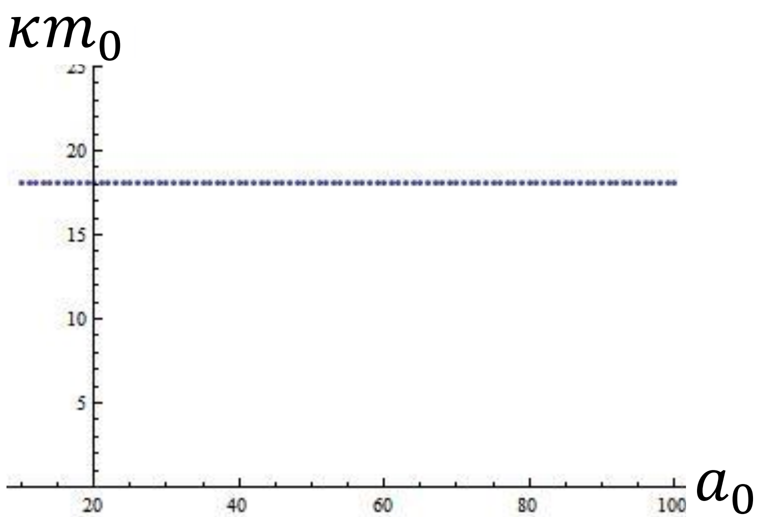


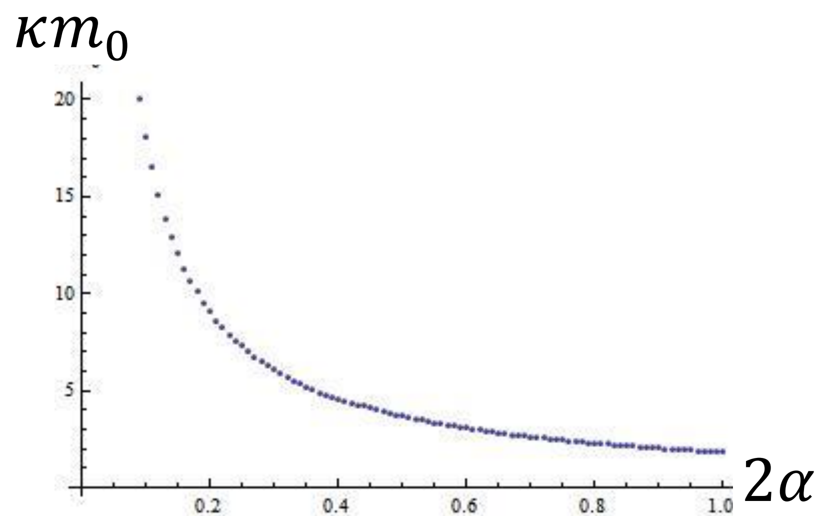


- Komar mass of fluid almost reproduce black hole mass
- Fluid mass is slightly larger than BH mass because of negative vacuum energy

Density for $r_s = a$

Density m_0 for the star with surface at neck of "wormhole"





- Density m_0 is independent of mass of black hole a_0
- Density is very large: $m_0 \sim \mathcal{O}(\kappa^{-1}\alpha^{-1}) \sim \mathcal{O}\left(\ell_p^{-4}\right)$

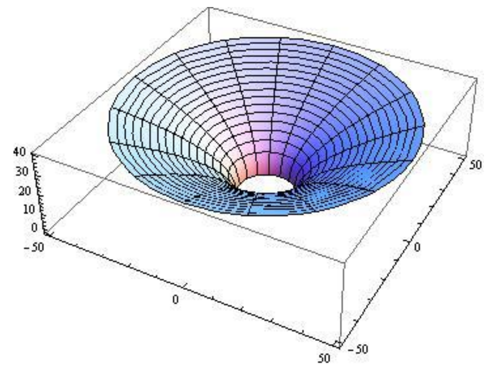
⇕ Arbitrarily large star can be non-singular

Classical regularity condition for pressure $m_0 < \dfrac{1}{3\pi G r_s^2}$
can be violated by arbitrary small m_0

“Embedding” of geometry

Embedding of BH geometry to 3D space

$$ds^2 = \left(1 - \frac{a}{r}\right)^{-1} dr^2 + r^2 d\theta^2$$

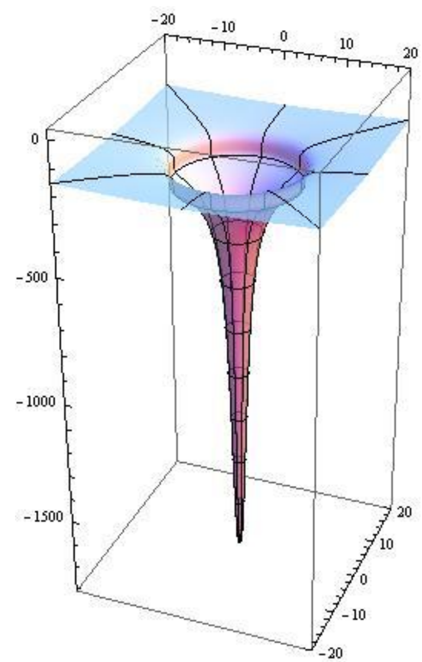


Geometry cannot be embedded into 3D space since proper length in r direction is smaller than r

$$ds^2 = C(r_*) dr_*^2 + r^2(r_*) d\theta^2$$

Embed the following metric, instead

$$ds^2 = dr_*^2 + r^2(r_*) d\theta^2$$



Conclusion

- We have considered back reaction to geometry from quantum effects in energy-momentum tensor.
- For simplest scalar model, anomaly gives negative energy.

  ⇨ wormhole metric
- In stationary case, the quantum energy-momentum tensor (Boulware vacuum) has divergence at the horizon.
- Quantum effects are very small except for near horizon region.
- Back reaction becomes large very near the horizon, and geometry near the horizon must be modified.
- In general, as far as the energy-momentum tensor is non-zero at the horizon, the horizon is killed by the back reaction.

Conclusion (2)

- For interior geometry of "black hole," we considered a star which consists of incompressible fluid.
- If density m_0 is almost appropriate, the geometry continues to $r \sim \mathcal{O}(\ell_p)$, at least.
- The geometry does not have horizon or singularity.
- Density becomes very large if surface of the star is around the wormhole.
- If density of incompressible fluid is sufficiently small, surface of the star cannot be inside the Schwarzschild radius, in contrast to the classical solution.

Thank you