The lattice study of many flavor QCD with twisted boundary condition

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based on the collaboration work with

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In this work,
We study the running of coupling constant
in QCD-like theory with Nc=3 Nf=12
using lattice simulations.

Outline

Motivation - Walking Techinicolor

Review Lattice Simulations

Method
- Step Scaling Function
- Twisted Polyakov Loop Scheme

Setting for Simulation

Results

Summary

Motivation

Theoretical Interest
+ Candidate for Beyond Standard Model

Technicolor

Origin of Mass: Higgs VEV → Condensate of Techniquarks

Standard Model Higgs

Additive Renormalization, Hierarchy Problem

Plank → → → Weak
10^{19} GeV 250 GeV

Techinicolor

Only Multiplicative Renorm. , Little Hierarchy Problem

Techinicolor → → → Weak
10 TeV??? 250 GeV

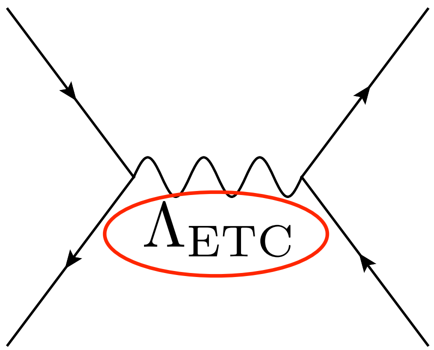
Experiment

Small FCNC & Observed Quark Mass

⇒ ***Walking Technicolor***

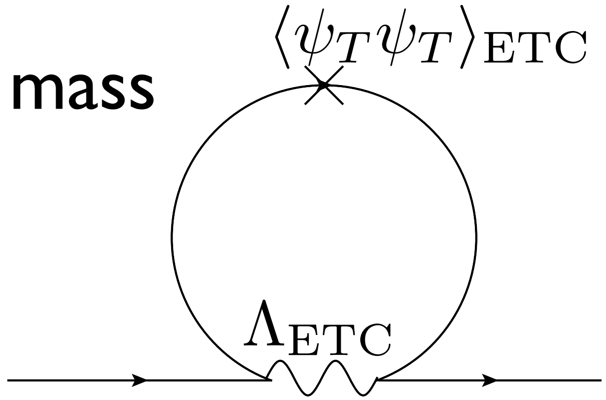
Walking Technicolor

FCNC



$$\frac{\bar\psi\psi\bar\psi\psi}{\Lambda_{\rm ETC}^2}$$

quark mass



$$\frac{\langle\bar\psi_T\,\psi_T\rangle_{\rm ETC}\,\bar\psi\psi}{\Lambda_{\rm ETC}^2}$$

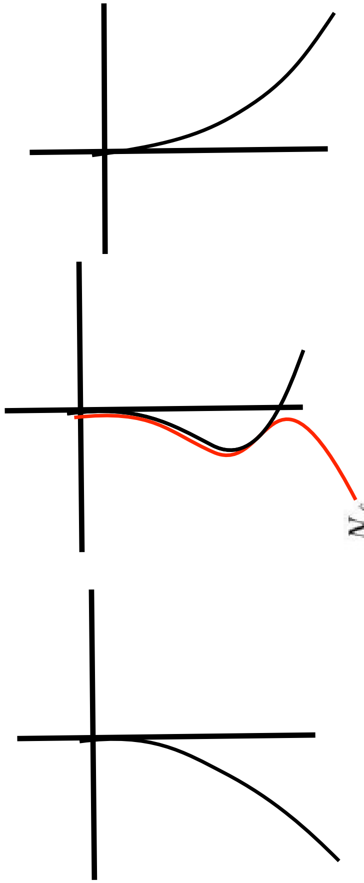
Growth of Technicolor Condensate

$$\langle\psi_T\psi_T\rangle_{\rm ETC} = \langle\psi_T\psi_T\rangle_{\rm TC}\exp\left(\int_{\Lambda_{\rm TC}}^{\Lambda_{\rm ETC}}\gamma(\mu)d\ln\mu\right)$$

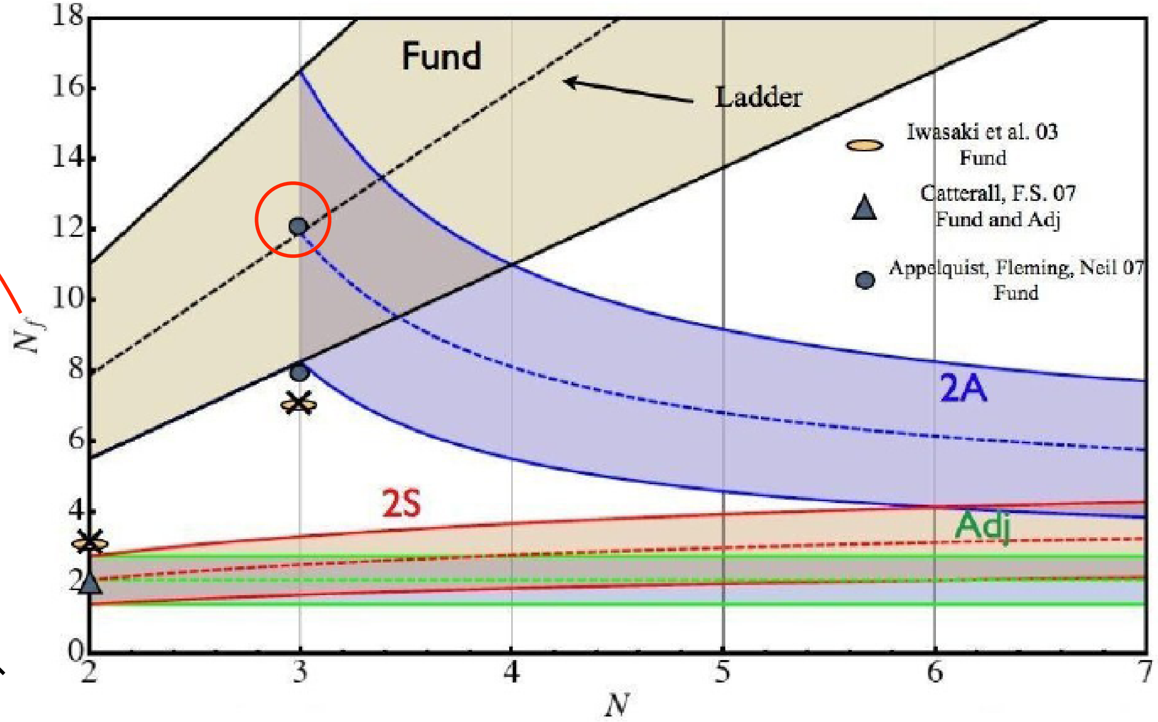
Scale up of QCD $\Rightarrow$ Too Small

Nearly Conformal with $\gamma\sim 1 \Rightarrow$ Large Enhancement of $\langle\psi_T\psi_T\rangle$

Conformal Window



Sannino, Ryottov (2007)



For Nc=3, Nf=12, Controversial Results are given.

ChPT properties, Z.Fodor et al, '09 ⇒ Similar to QCD

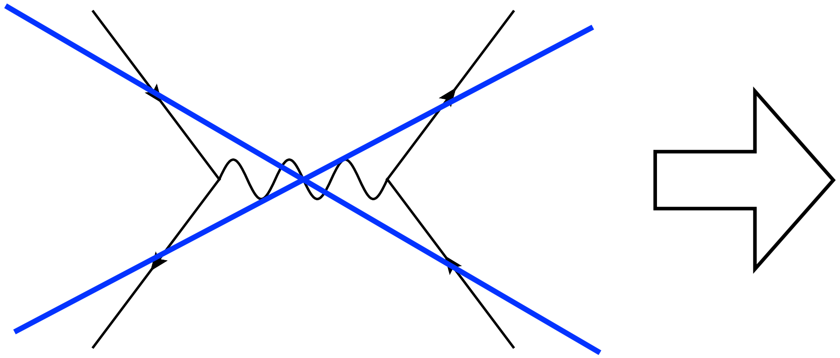
Running of Coupling, T.Appelquist et al., '09 ⇒ Conformal

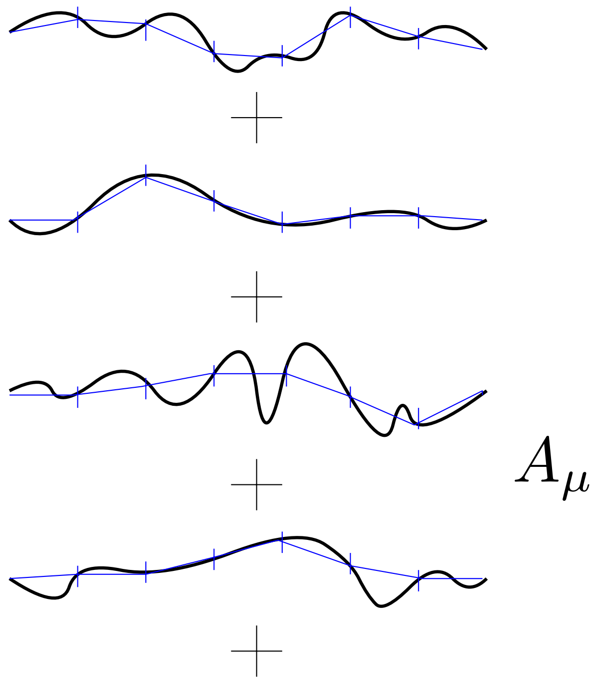
Basics of Lattice Simulation

Purpose of Lattice Simulation:

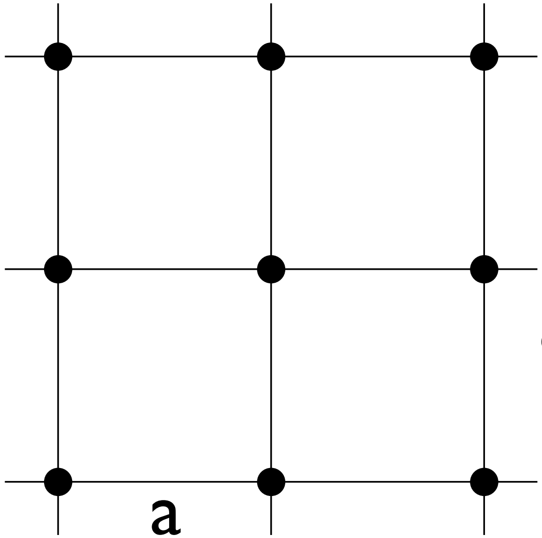
Non-perturbative and direct calculation of Path Integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int dA_\mu d\bar{\Psi} d\Psi e^{-S_g} e^{-S_f} \mathcal{O}$$





Basics of Lattice Simulation



- fermion $\psi(x), \bar{\psi}(x)$

link variable

$$U_\mu(x) = e^{-i \int_x^{x+a_\mu} A_\mu(x')dx'_\mu} \sim e^{-iaA_\mu(x)}$$

a is lattice spacing, a_mu is a vector which goes mu direction with length a

Gauge Symmetry

$$\psi(x)' = \omega(x)\psi(x)$$
$$U_\mu(x)' = \omega(x)U_\mu\omega(x+a_\mu)^\dagger$$

Covariant "Difference"

$$[\partial_\mu^{(+)}\psi](x) \stackrel{\text{def}}{=} U_\mu\psi(x+a_\mu) - \psi(x)$$

Basics of Lattice Simulation

What we measure?
How to set the scale?

What we measure?
How to set the scale?

Example: QCD

Input Parameters   Bare Coupling, Bare Quark Mass

as g , am

Output   Proton mass am_p

$am_p = 1 \Rightarrow a = 1/m_p$

$am_p = 0.5 \Rightarrow a = 0.5/m_p$

$am_p = 0.2 \Rightarrow a = 0.2/m_p$

$am_p = 0.01 \Rightarrow a = 0.01/m_p$

How to study the running of coupling constant without setting scale explicitly

Lattice We can't set the scale by hand

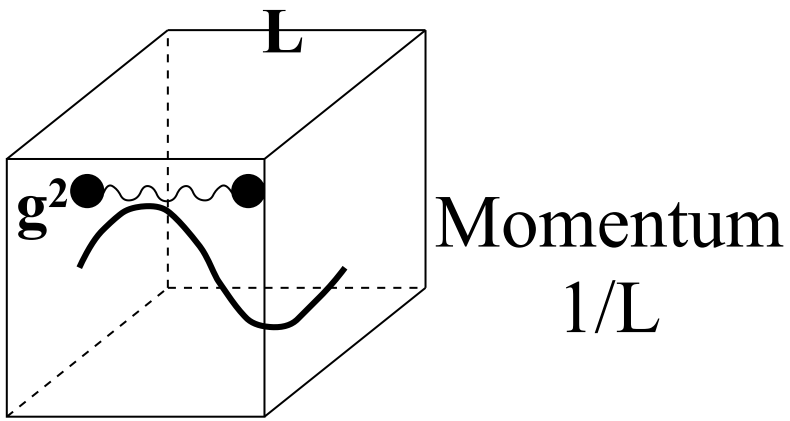
⇒ Extract scale from observables

I. Study in a Finite Box
(Infinite volume limit is not taken)

boundary conditions

II. Mechanism to Suppress Zero Momentum Modes

III. Choose a quantity which is equal to g^2
in the leading order of perturbation



Step Scaling function

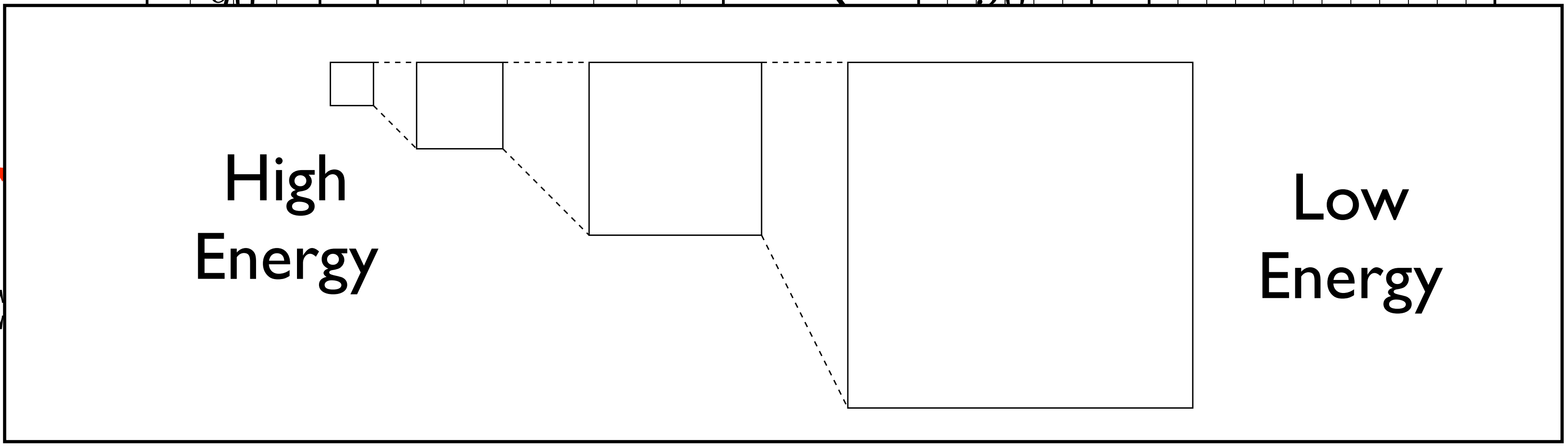
$$\sigma(u) \stackrel{\text{def}}{=} \bar{g}^2(2L)\big|_{u=\bar{g}^2(L)}$$

$$\int_{\sigma(u)}^{u} \frac{dg^2}{\beta(g^2)} = 2$$

relation with beta function

Consider that two system S_A and S_B.
The size of S_B is twice larger than S_A.
SSF is given by a observable in a system S_B.
The argument of SSF is given by
the same observable in S_A.

L



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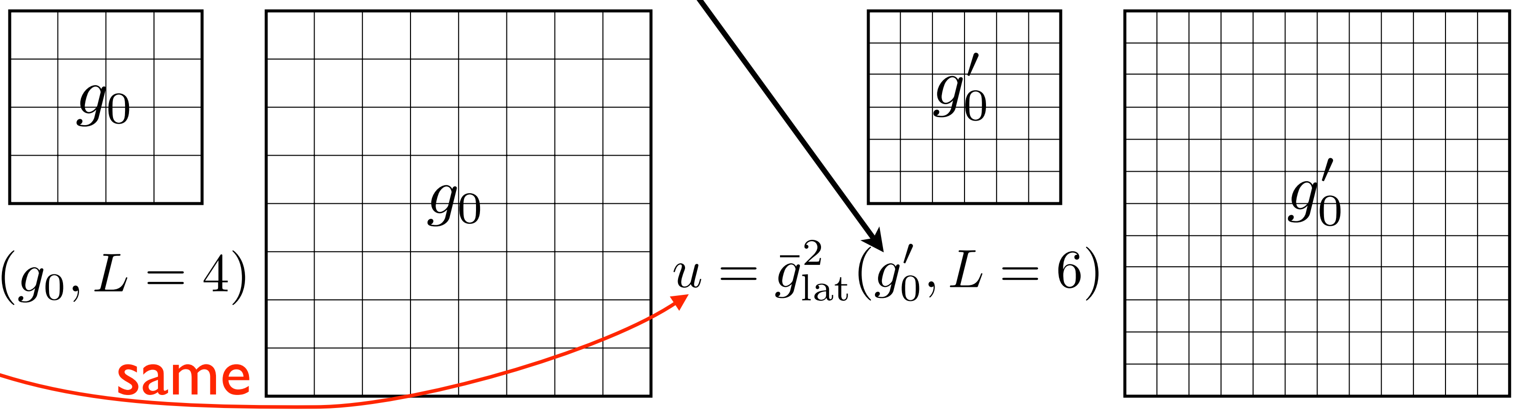
relation with beta function

Lattice SSF → Continuum Limit

$$\sigma(u) = \lim_{L\to\infty} \Sigma(u, L)$$

Tune g'_0 to give the same value of u.

$L = 4$ $\quad$ $L = 6$



$u = \bar{g}^2_{\rm lat}(g_0, L = 4)$ $\quad$ $u = \bar{g}^2_{\rm lat}(g'_0, L = 6)$

same

$$\Sigma(u, L = 4) = \bar{g}^2_{\rm lat}(g_0, L = 8) \qquad \Sigma(u, L = 6) = \bar{g}^2_{\rm lat}(g'_0, L = 12)$$

In this work, $\bar{g}^2_{\rm lat}(g_0,L)$ is given by interpolation

Conformal / Not

Growth Ratio

$$\frac{\sigma(u)}{u} \stackrel{\text{def}}{=} \left.\frac{g(2L,\beta)}{g(L,\beta)}\right|_{u=g(L,\beta)}$$

~ 1 at Infra red => Conformal

1> at Infra red => QCD Like

Polyakov Loop Scheme

G.M.deDivitiis et al., ,Nucl.Phys.B**422**(1994)

Twisted Boundary Condition

$$U_\mu(x+\hat{\nu}L) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger \text{ for } \nu = 1,2$$

with $\Omega_1\Omega_2 = e^{i2\pi/3}\Omega_2\Omega_1, \ \Omega_\mu\Omega_\mu^\dagger = 1,$ single valuedness

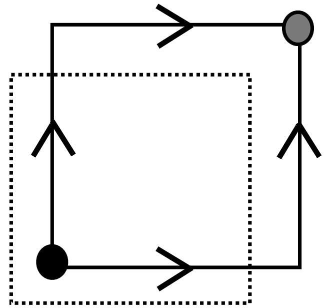
$(\Omega_\mu)^3 = 1, \ \mathrm{Tr}\,[\Omega_\mu] = 0$ SU(3)

e.g.,

$$\Omega_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \Omega_2 = \begin{pmatrix} e^{-i2\pi/3} & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Fermion - “smell” degrees of freedom

$$\psi_\alpha^a(x+\hat{\nu}L) = e^{i\pi/3}\Omega_\nu^{ab}\psi_\beta^b(\Omega_\nu)_{\beta\alpha}^\dagger$$



Polyakov Loop Scheme

Twisted Direction

$$P_1(y,z,t) = \mathrm{Tr}\left(\left[\prod_j U_1\left(x=j,y,z,t\right)\right]\Omega_1 e^{\underline{i2\pi y/3L}}\right)$$

transl. inv.

Untwisted Direction

$$P_3(x,y,t) = \mathrm{Tr}\left[\prod_j U_3\left(x,y,z=j,t\right)\right]$$

Definition of Renormalized Coupling

$$\frac{\langle P_1(t=0)^\dagger P_1(t=L/2)\rangle}{\langle P_3(t=0)^\dagger P_3(t=L/2)\rangle} \propto g^2$$

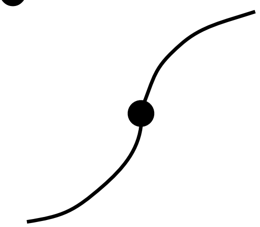
Simulation Setting

Staggered Fermion, m_f=0.0, N_f =12 (4 tastes x 3 smells)

Plaquette Gauge Action, Beta = 4 ~ 20

Beta interpolation is done using bspline,

4 coeffs , 2 uniform break points



Hyper Cubic Box, $N_x = N_y = N_z = N_t$

N_x = 6 , 8 , 10 , 12 , 16 , 20

$\Sigma(L=6)$ $\Sigma(L=8)$ $\Sigma(L=10)$

Three independent $\Sigma(L)$

Size of Step Scaling s = 2

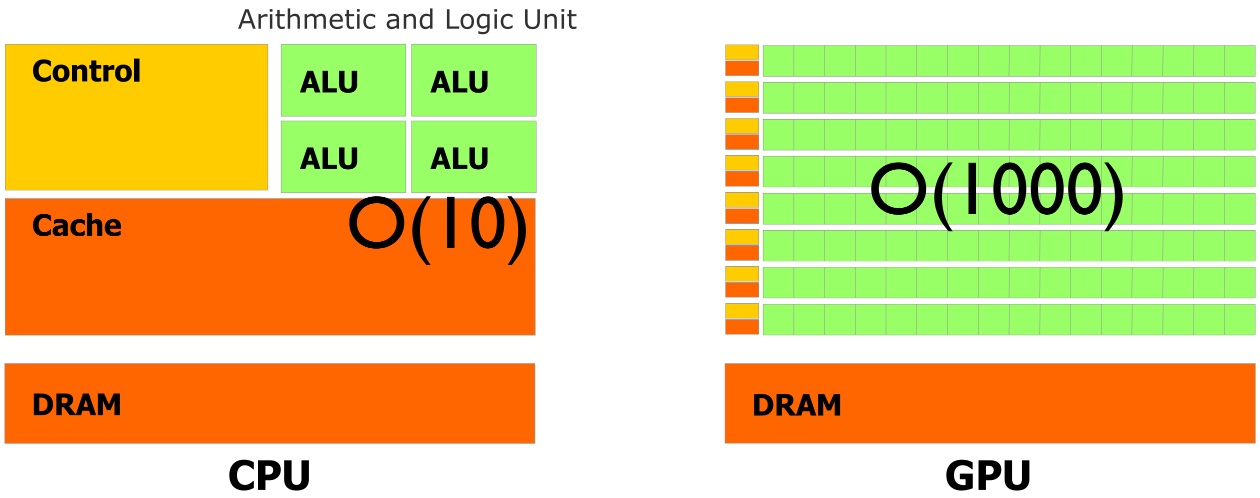
Simulation Done at

NTCU (Hsin-Chu), NCHC (Hsin-Chu), YITP (Kyoto), RCNP (Osaka), Osaka Univ (Osaka), KEK (Tsukuba)

GPU Features

Tesla C1060 Computing Processor

Processor	1 x Tesla T10P
Number of cores	240
Core Clock	1.33 GHz
On-board memory	4.0 GB
Memory bandwidth	102 GB/sec peak
Memory I/O	512-bit, 800MHz GDDR3
Form factor	Full ATX: 4.736" (H) x 10.5" (L) Dual slot wide
System I/O	PCIe x16 Gen2
Typical power	160 W



GPU Simulation

NCHC, NCTU,
Osaka Univ, KEK

Good for SIMD - Single Instruction, Multiple Data

Limitation of Memory ~2GB for Tesla, ~6GB for Fermi

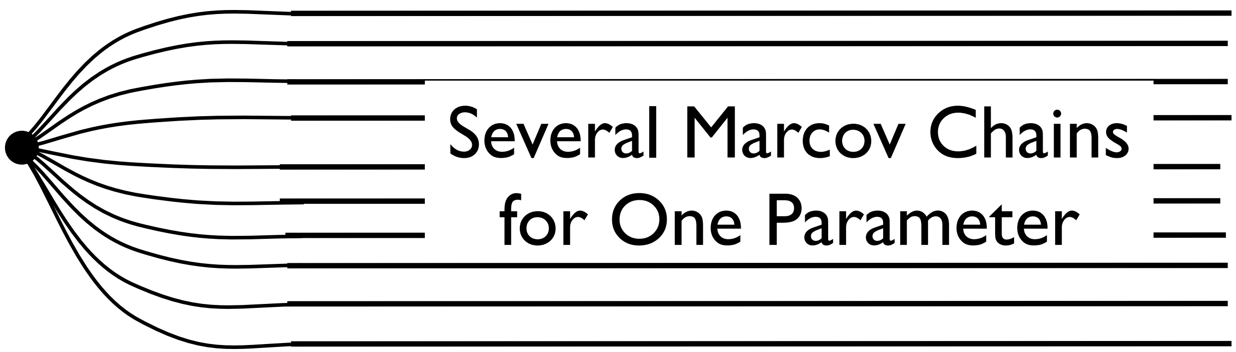
Data Transfer between GPU and GPU make performance lower

Development is Supported by a Large Market (Gamers)

~ our simulation ~

The action is local. Memory Needed < 500 MB

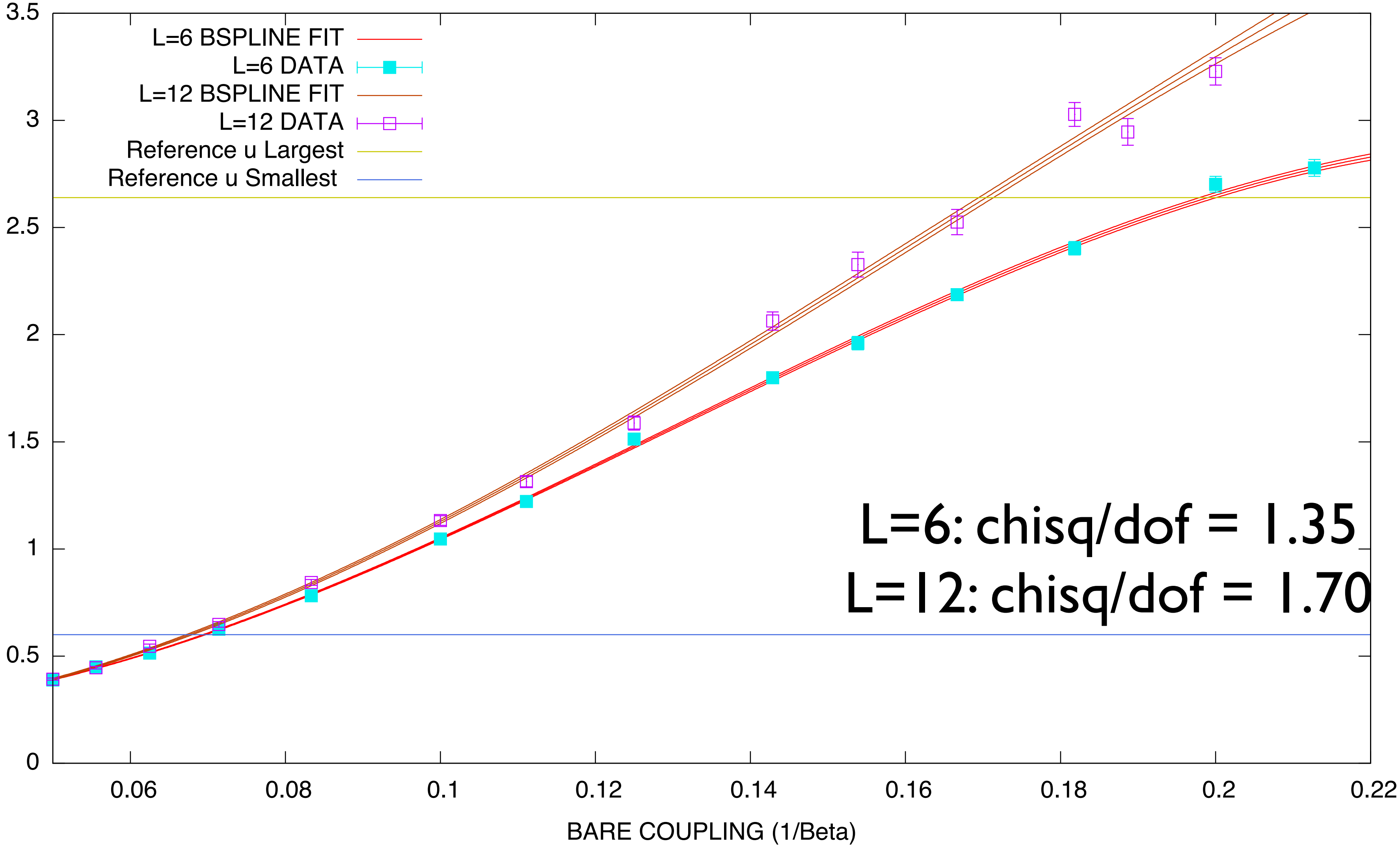
For fixed Beta
"Thermalized "
Configurations



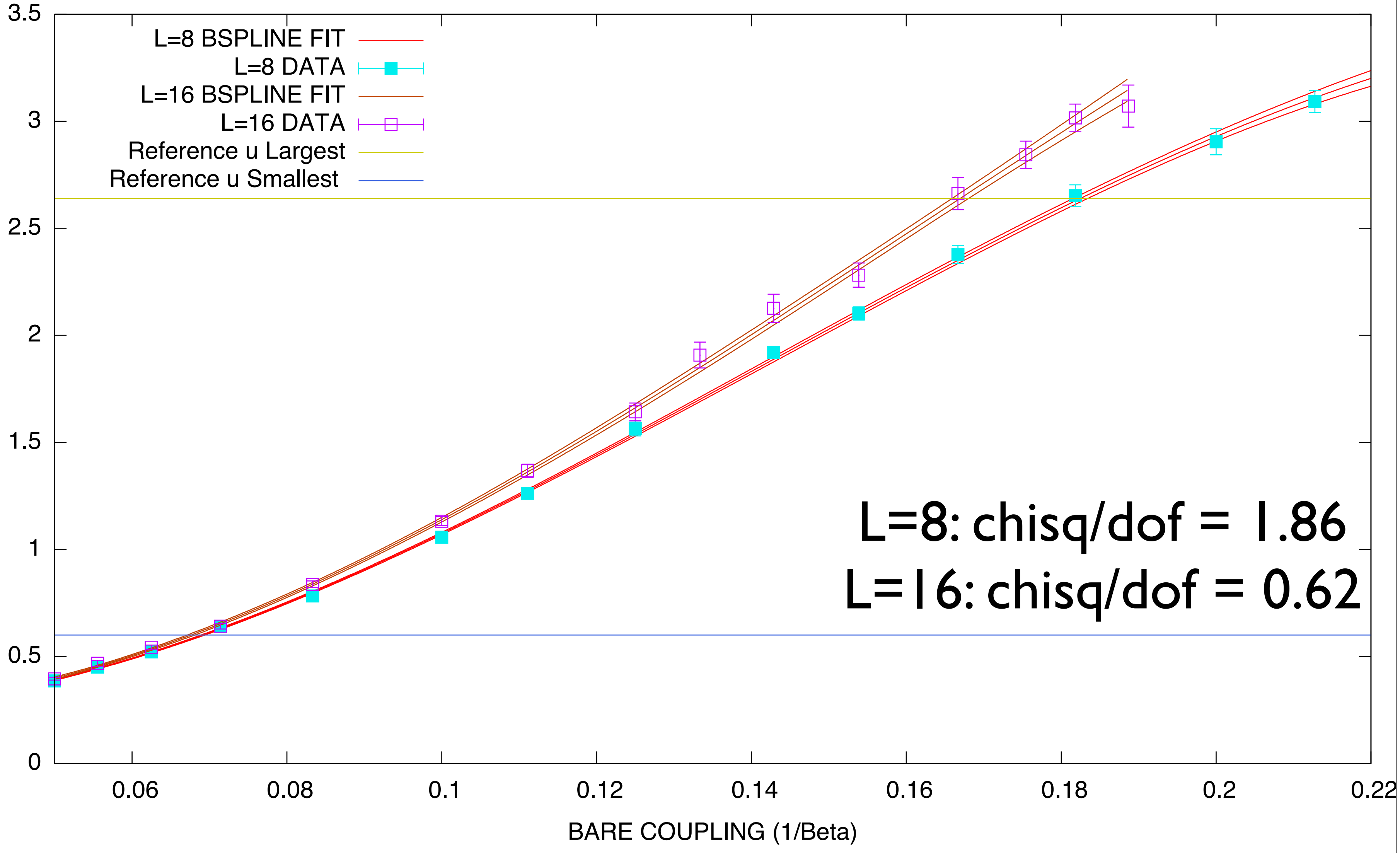
Most of GPU's are Tesla C1060.

25 GFlops(sustained) x 100 GPU's x 1 year

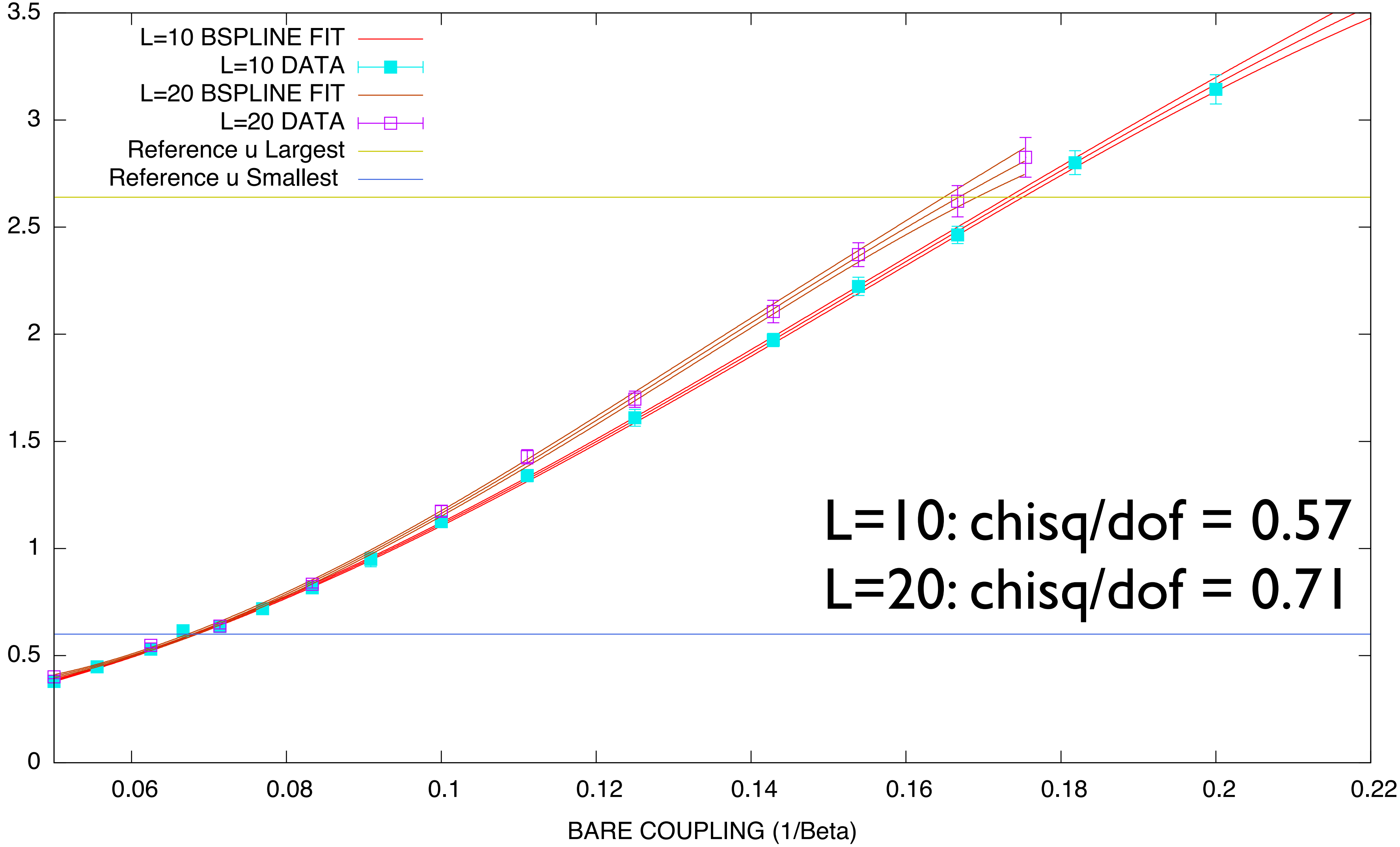
Renormalized Coupling L=6, L=12 ⇒ Σ(L=6)



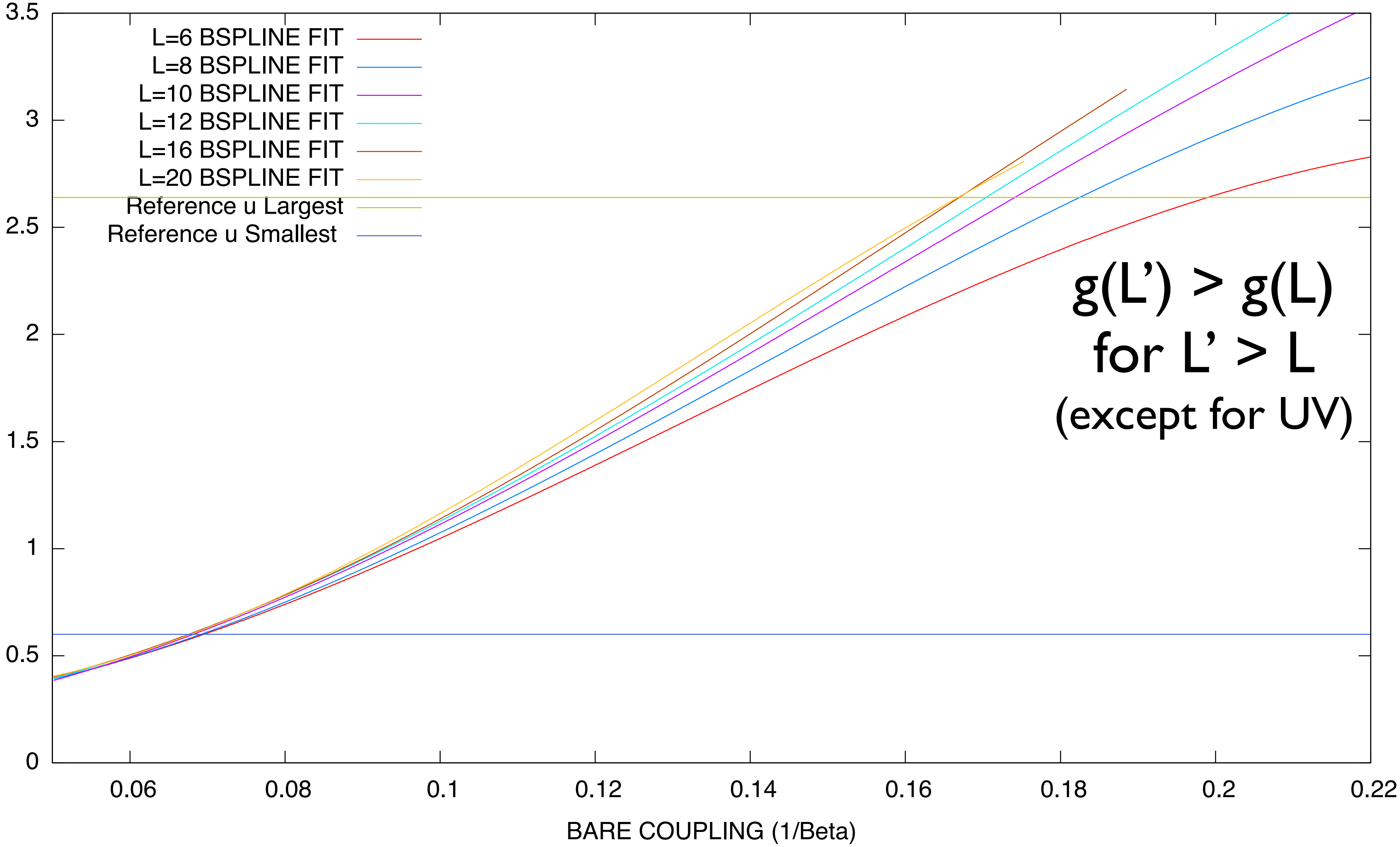
Renormalized Coupling L=8, L=16 ⇒ Σ(L=8)

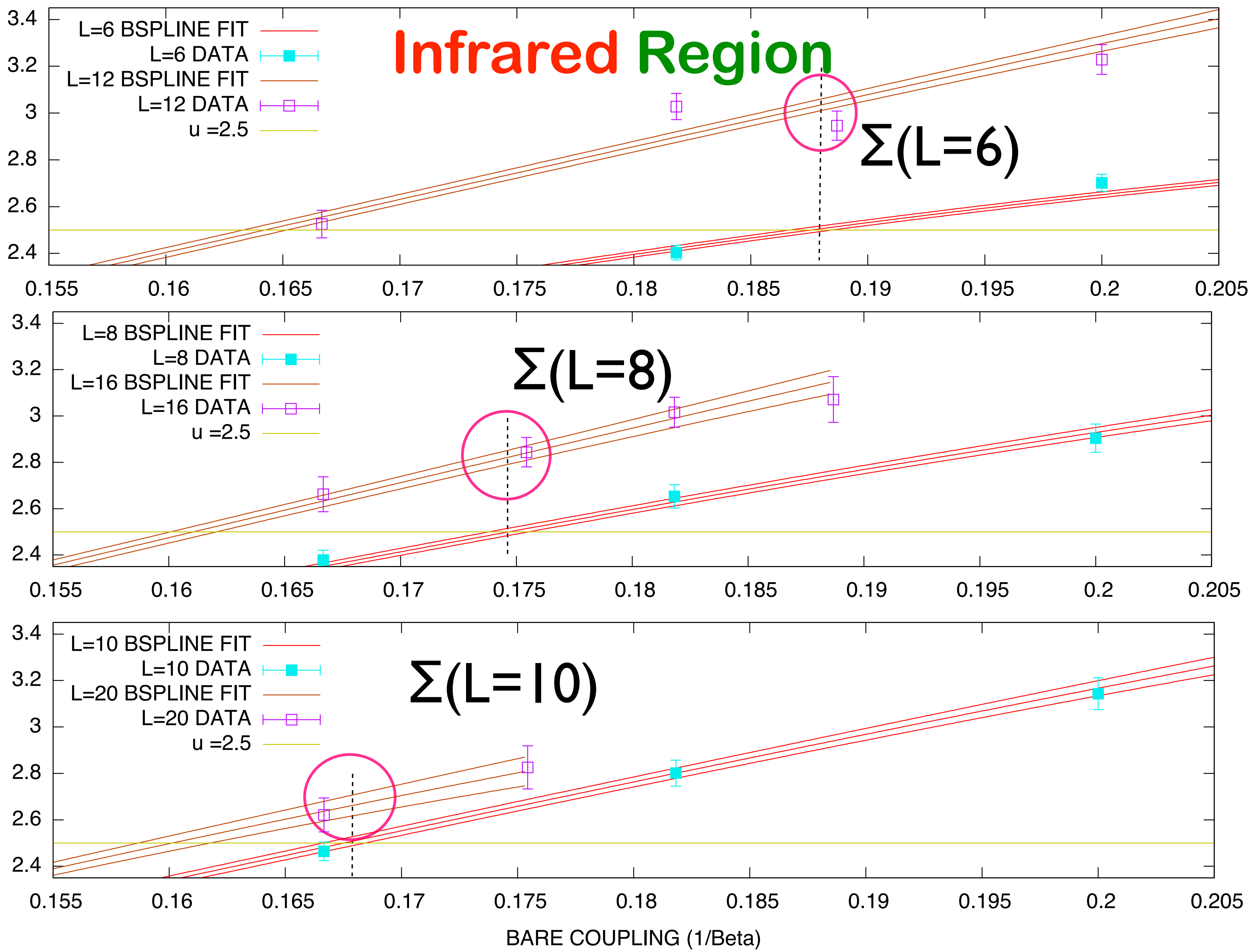


Renormalized Coupling L=10, L=20 ⇒ Σ(L=10)



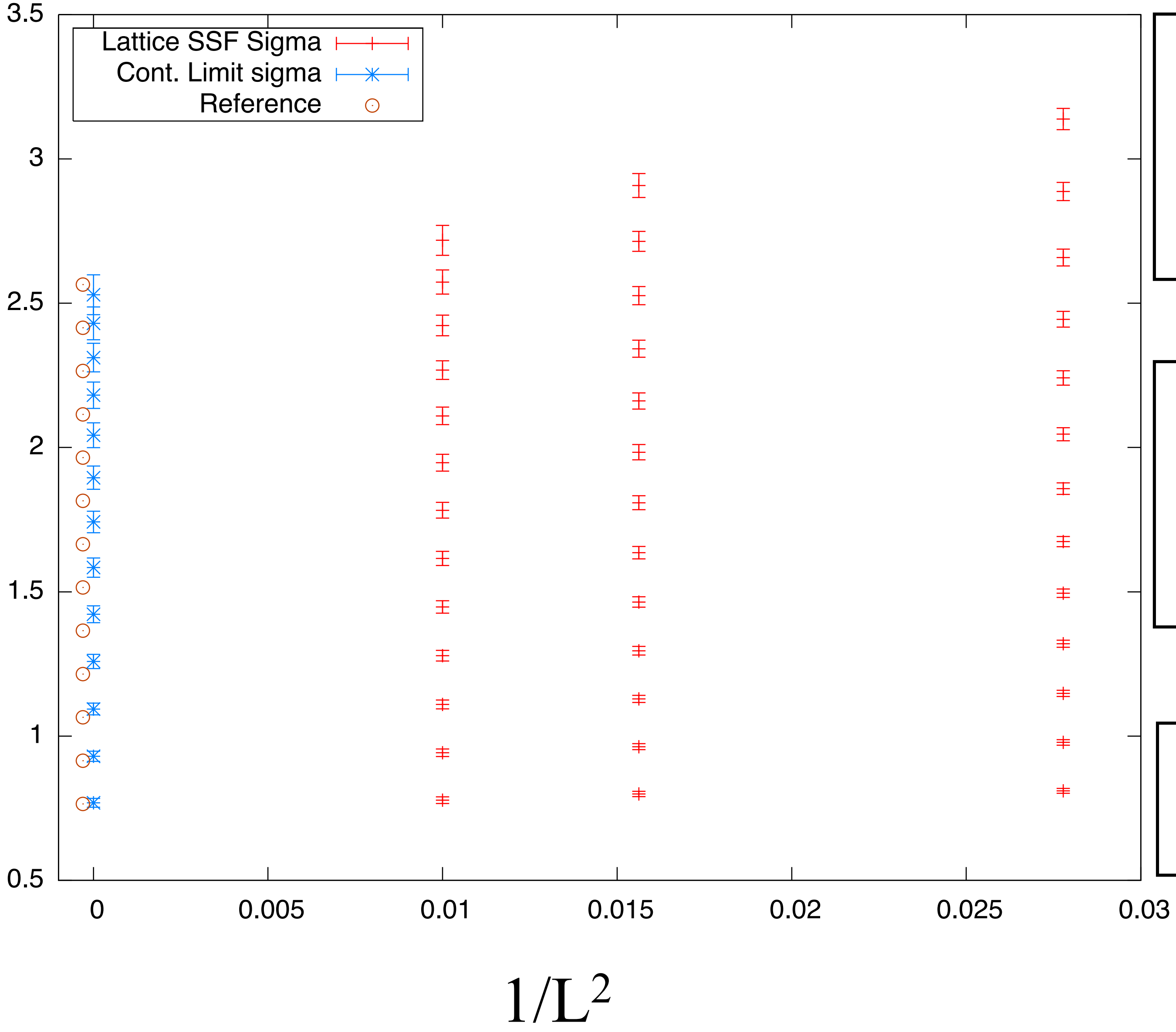
Central Values of Renormalized Coupling





Larger L, Smaller Σ

Continuum Limit

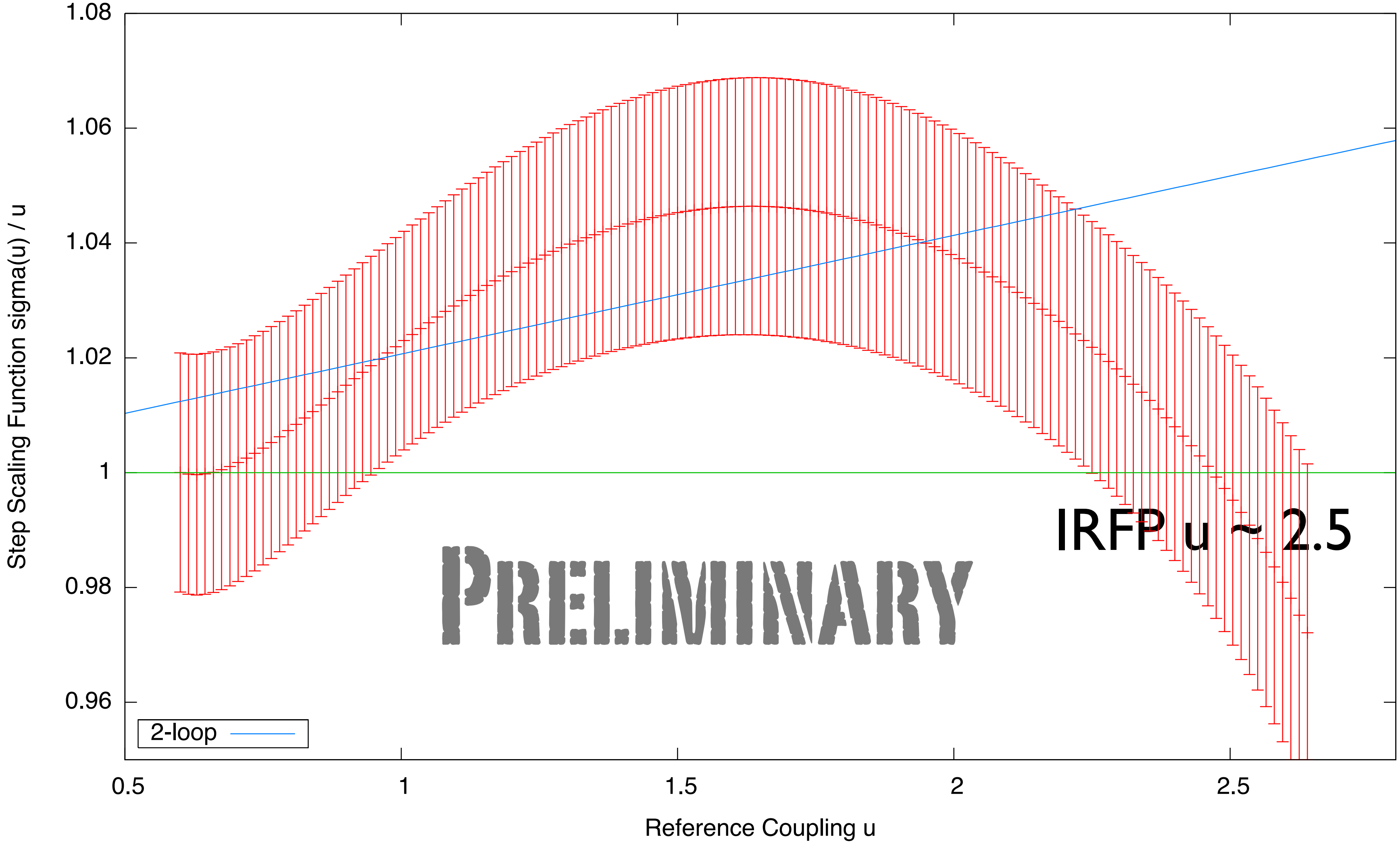


IR
Sigma(u) > u
Continuum Limit
sigma(u) ~ u

Intermediate
Sigma(u) > u
Continuum Limit
sigma(u) > u

UV
Sigma(u) ~ u

Step Scaling Function sigma(u)/ u



Summary

Step Scaling Function
Polyakov Loop Scheme with twisted boundary conditions

GPU's are used for L = 20

Infrared Region
Lattice SSF Sigma(u) > u , Continuum SSF sigma(u) ~ u
u* ~ 2.5 Preliminary