Resolutions to $B \to \eta^{(\prime)} K$ branching ratios

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Outlines

- Introduction
- Plausible resolutions
- flavor-singlet contribution
- (gluonic) charming penguin
- chiral mass scale
- axial U(1) anomaly
- Summary and experimental discrimination

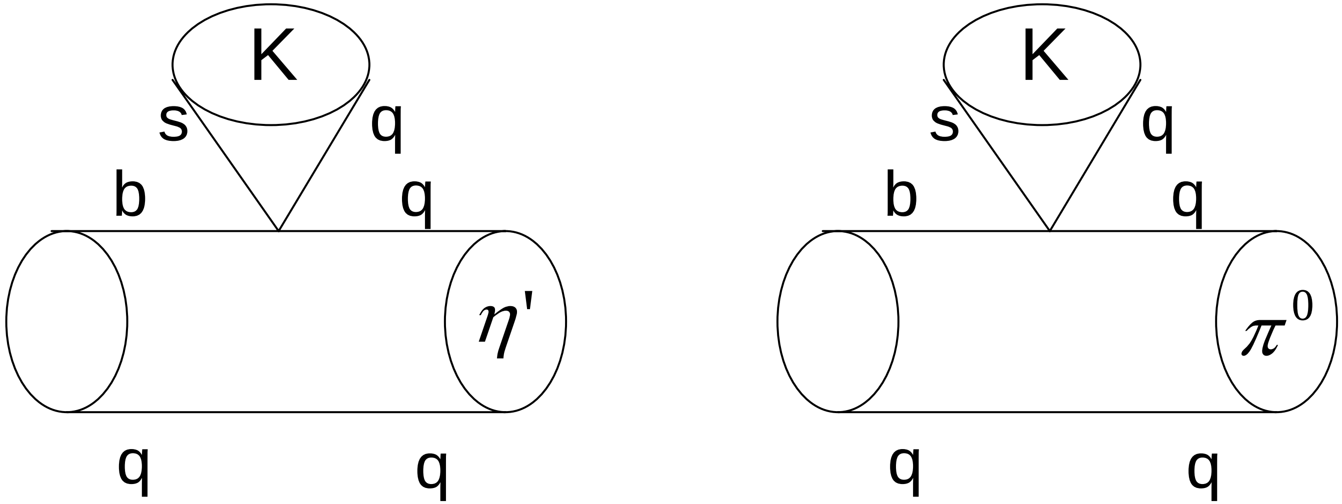
Introduction

A long-standing puzzle

- CLEO (1998,99) measured

  $$Br(B \to \eta' K) \approx 65, 80 \times 10^{-6}$$

- But $Br(B \to \pi^0 K) \approx 10 \times 10^{-6}$
- Why are they so different?



- Why are $Br(B \to \eta K) \approx 2 \times 10^{-6}$ so small?

$\eta - \eta'$ mixing

- Feldmann-Kroll-Stech scheme

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = U(\phi) \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix} \quad U(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

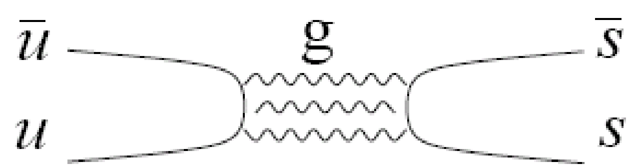
- Flavor states $|\eta_q\rangle = |(u\bar{u} + d\bar{d})/\sqrt{2}\rangle \quad |\eta_s\rangle = |s\bar{s}\rangle$
- Decay constants

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = U(\phi) \begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix}$$

Assumption Of OZI rule: disconnected diagrams are suppressed.

- Global fit

$$\phi = 39.3^\circ \pm 1.0^\circ$$



$$f_q = (1.07 \pm 0.02) f_\pi \ , \qquad f_s = (1.34 \pm 0.06) f_\pi \ ,$$

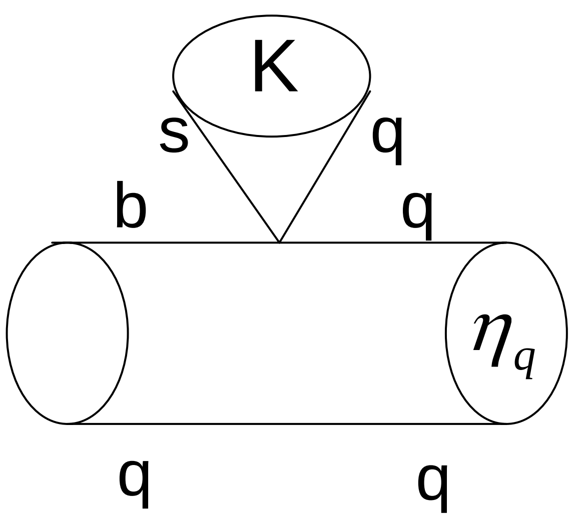
Interference

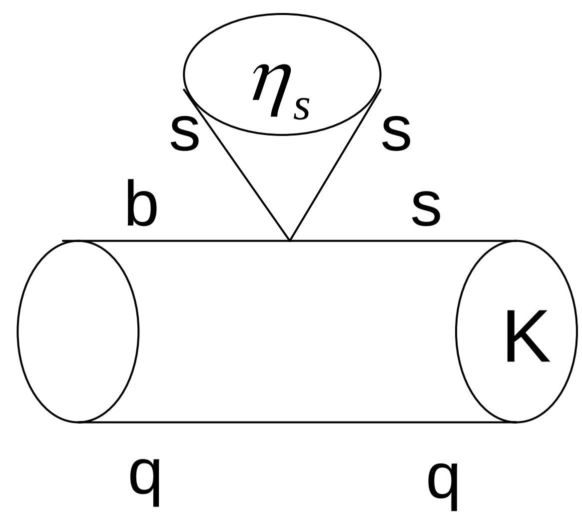
- Decay amplitudes

$$A(B \to \eta' K) = A(\eta_q K)\sin\phi + A(\eta_s K)\cos\phi$$

$$A(B \to \eta K) = A(\eta_q K)\cos\phi - A(\eta_s K)\sin\phi$$

- Feynman diagrams with penguin operators





Naive estimate

- Try to understand the data of branching ratios

$$A(\eta_q K) \approx \left(f_q / f_\pi \right) A(\pi^0 K) = 1.07 A(\pi^0 K)$$

$$A(\eta_s K) \approx \left(f_s / f_\pi \right) \sqrt{2} A(\pi^0 K) = 1.56 A(\pi^0 K)$$

$$B(\eta' K) \approx 4 B(\pi^0 K) \approx 40 \times 10^{-6}$$

$$B(\eta K) \approx 0.2 B(\pi^0 K) \approx 2 \times 10^{-6}$$

- Interference has explained very different $B(\eta' K), B(\eta K)$ to some extent
- Need new mechanism compared to $B \to \pi K$

Recent calculations

- $B \to \eta' K$ branching ratios are not yet completely understood after 10 years

	PQCD	QCDF
$B(B^\pm \to \eta' K^\pm) = (70.2 \pm 2.5) \times 10^{-6}$,	35	42
$B(B^0 \to \eta' K^0) = (64.9 \pm 3.1) \times 10^{-6}$,	31	41
$B(B^\pm \to \eta K^\pm) = (2.7 \pm 0.3) \times 10^{-6}$,	5.7	1.7
$B(B^0 \to \eta K^0) < 1.9 \times 10^{-6}$.	3.0	1.0

- PQCD (Kou, Sanda 01; Akeroyd, Chen, Geng 07) and QCDF (Beneke, Neubert 02) give small results, when predictions for $B \to \pi K$ match data.

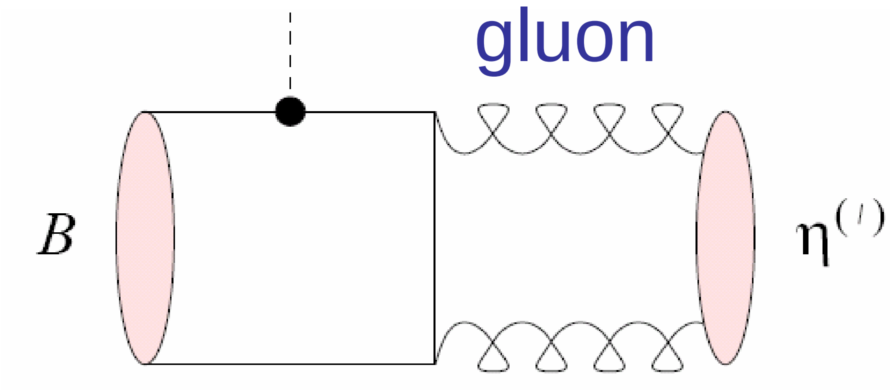
Plausible resolutions

Resolutions

- Large flavor-singlet contribution (BN 02)
- Charming penguins (Williamson, Zupan 06)
- Large chiral scale m_0^q associated with η_q (Akeroyd, Chen, Geng 07)
- Axial U(1) anomaly (Gerard, Kou 06)
- SU(3) (Fu, He, Hsiao 03), including η_1 to form a nonet, additional parameters
- Large $B \to \eta'$ form factor (Pham 07)
- Final-state interaction (Cheng, Chua, Soni 05)
- FSI was fixed by branching ratio data, and then used to predict CP asymmetries

Flavor-singlet contribution

- Absent in $B \to \pi K$, good for QCDF,



Du, Kim, Yang 98
Eeg, Kumericki, Picek 03…

- $B(\eta K)$ are OK, $B(\eta' K)$ too small in QCDF
- Similar for $\eta_q K, \eta_s K$, cancel in ηK, enhance $\eta' K$
- reminded

$$A(B \to \eta' K) = A(\eta_q K)\sin\phi + A(\eta_s K)\cos\phi$$

$$A(B \to \eta K) = A(\eta_q K)\cos\phi - A(\eta_s K)\sin\phi$$

Theo and exp checks

- Parameterized as F_2 in QCDF, increased up to 40% of quark contribution
- Can flavor-singlet contribution be so large?
- Computed by Charng, Kurimoto, Li using PQCD (2005). Inputs for gluonic distribution amplitudes are constrained by other data of $\gamma^*\gamma \to \eta^{(\prime)}$
- found to be few percents at most, negligible!
- No sign from Ds decays (CLEO), consistent with FKS

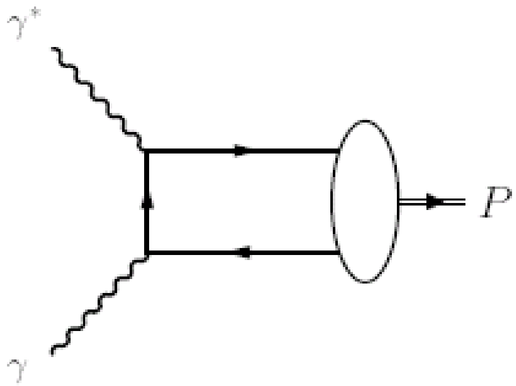
$$\frac{B(D_s \to \eta' \ell \nu)}{B(D_s \to \eta \ell \nu)} = 0.35 \pm 0.09 \pm 0.07$$

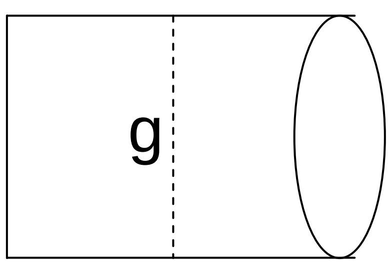
39.3

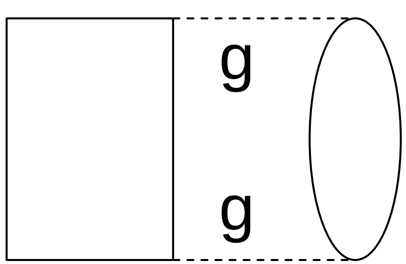
$$\cot\phi = 1.22 \quad \Leftrightarrow \quad \frac{F_+^{D_s\eta'}(0)}{F_+^{D_s\eta}(0)} = 1.14 \pm 0.17 \pm 0.13$$

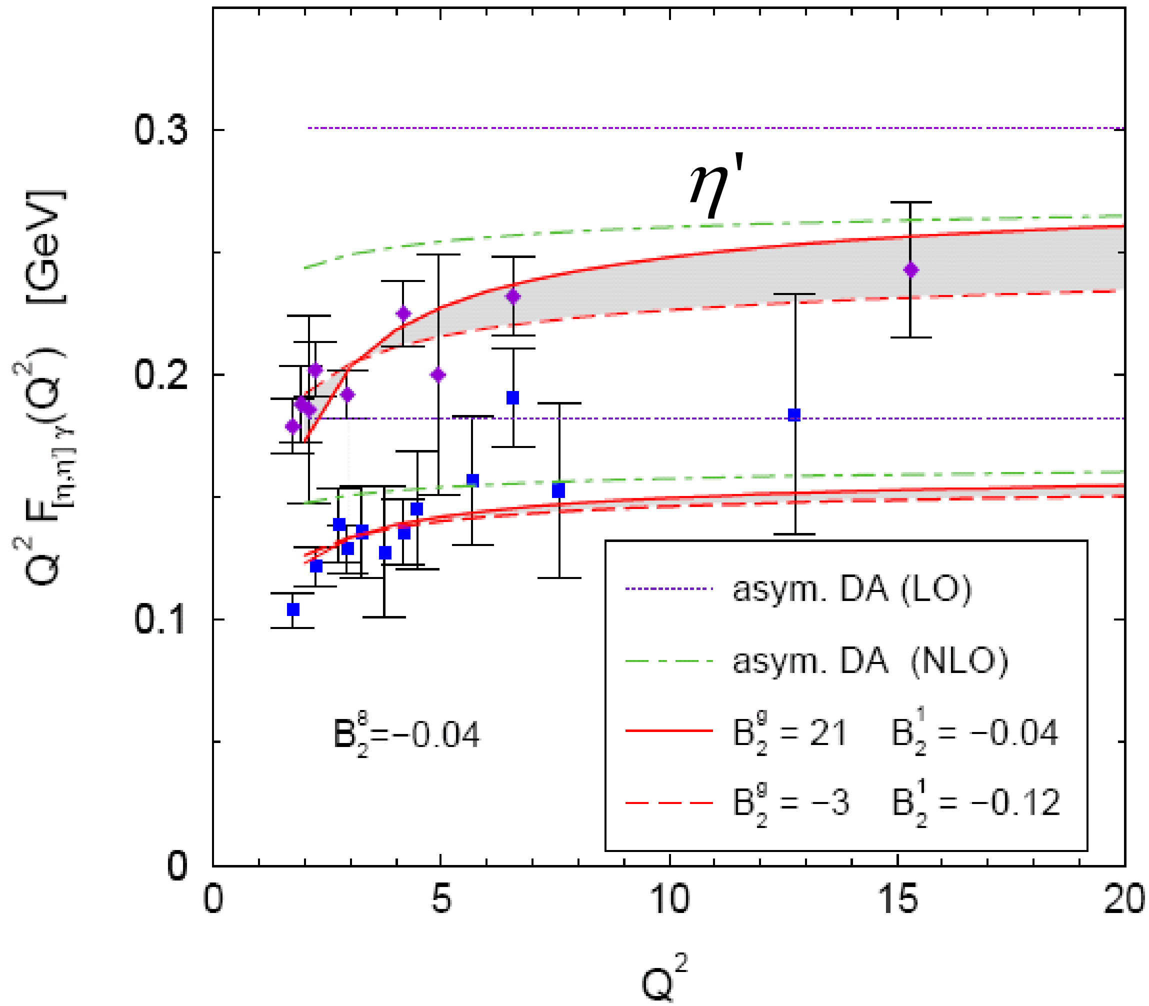
Transition form factor data

- Form factors computed up to NLO









Hint of B decay data

- Use data of semileptonic B decays (Kim, Oh, Yu 04), which have conflict between BaBar

$$B(B^+ \to \eta \ell^+ \nu) = (0.84 \pm 0.27 \pm 0.21) \times 10^{-4} < 1.4 \times 10^{-4}$$
$$B(B^+ \to \eta' \ell^+ \nu) = (0.33 \pm 0.60 \pm 0.30) \times 10^{-4} < 1.3 \times 10^{-4}$$

- and CLEO

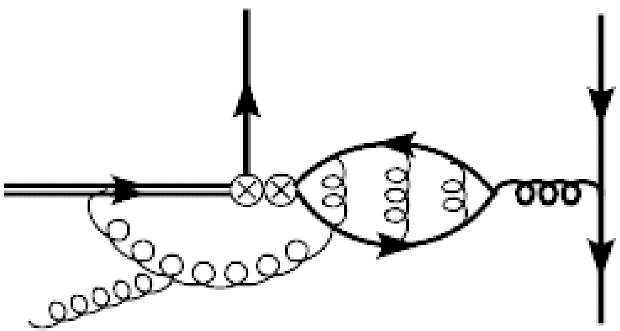
$$< 1.01 \text{ at } 90\% \text{ CL}$$
$$2.66 \pm 0.80 \pm 0.57 \pm 0.04$$

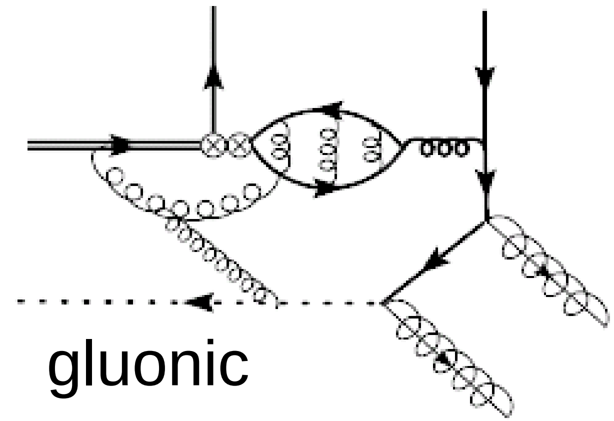
$$R_{\ell\nu} \equiv \frac{B(B \to \eta' \ell \nu)}{B(B \to \eta \ell \nu)} > 2.5 \text{ but } \tan^2 \phi = 0.67$$

- CLEO data imply large flavor-singlet contribution or other new mechanism

(gluonic) charming penguins

- Nonperturbative charming penguins introduced in SCET parameterization





- Due to SU(3) symmetry, charming penguin, contributing to other PP modes, can not be large.
- Gluonic charming penguin responsible for $B(\eta' K) >> B(\pi K)$ in data fitting

Small form factors

- Due to dominance of charming penguins and destructive F_2, form factors are small

$$f_+^{B\eta_q}(0) = \begin{cases} (-2.3 \pm 4.8) \times 10^{-2}, \\ (4.5 \pm 8.6) \times 10^{-2}, \end{cases}$$

Solution I

Solution II

$$f_+^{B\eta_s}(0) = \begin{cases} (-9.9 \pm 2.4) \times 10^{-2}, \\ (-6.6 \pm 4.3) \times 10^{-2}, \end{cases}$$

huge uncertainty due to flavor-singlet contribution

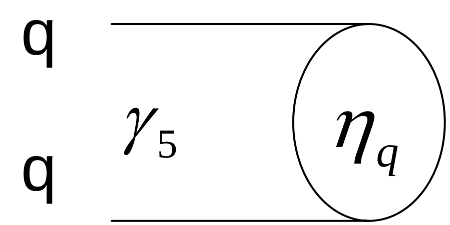
$$F_+^{B\pi}(0) = 0.24 \pm 0.05$$

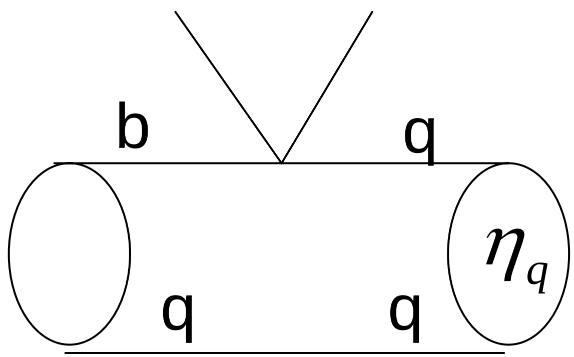
- Violate FKS relation $R_{\ell\nu} \equiv \frac{B(B \to \eta' \ell\nu)}{B(B \to \eta\ell\nu)} \approx \tan^2\phi$

- Lead to small $B(B \to \eta^{(\prime)} l\nu) \sim O(10^{-5})$

Chiral mass scale

- Good for PQCD (ACG 07)
- $B(\eta K)$ are larger, $B(\eta' K)$ smaller in PQCD
- Large chiral scale $m_0^q \equiv m_{qq}^2/(2m_q)$ enhances form factors through twist-3 DA





pseudoscalar density

$$m_{qq}^2 = \frac{\sqrt{2}}{f_q}\langle 0|m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d|\eta_q\rangle$$

$$m_{qq} = 0.14\, GeV \approx m_\pi \rightarrow 0.22\, GeV$$

- Increase $A(\eta_q K)$, more constructive (destructive) interference with $A(\eta_s K)$ leads to larger (smaller) $B(\eta' K)\,[B(\eta K)]$

Check semileptonic data

- Large chiral scale then increases semileptonic branching ratios
- For $m_{qq} = 0.22\,GeV$, PQCD predictions (ACG 07) barely consistent with data

$$B(B^+ \to \eta \ell^+ \nu) \;=\; 1.27 \times 10^{-4}$$ <1.4

$$B(B^+ \to \eta' \ell^+ \nu) \;=\; 0.62 \times 10^{-4}$$ <1.3

- FKS scheme is roughly respected: 0.62/1.27=0.5

$$R_{\ell\nu} \equiv \frac{B(B \to \eta' \ell \nu)}{B(B \to \eta \ell \nu)} \approx \tan^2 \phi$$ =0.67

OZI violating effects

- Can the chiral scale be so large?
- Yes, if OZI violating effects exist at percent level (Hsu, Charng, Li 07)

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = U(\phi) \begin{pmatrix} f_q & f_{sq} \\ f_{qs} & f_s \end{pmatrix}$$

- Equation of motion

gives the chiral scale

$$\partial_\mu (\bar{q}\gamma^\mu \gamma_5 q) = 2 i m_q \bar{q}\gamma_5 q + \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

gives axial U(1) anomaly

$$f_{qs}/f_q\,,\ f_{sq}/f_s < 0.05 \Rightarrow m_q = 0.2\,GeV$$

- 5% OZI violation is common in experiments

Axial U(1) anomaly

- Pseudoscalar density with SU(3) breaking and axial U(1) anomaly

$$\langle 0|\bar{s}\gamma_5 s|\eta\rangle = -i\frac{f_K}{\sqrt{3}}\frac{M_K^2}{m_s+m_q}\left(\sqrt{2}c\theta + s\theta\right) \qquad (17)$$

$$\times\left[1+\frac{M_\eta^2-M_K^2}{\Lambda_0^2}+2(M_K^2-M_\pi^2)\left(\frac{1}{\Lambda_1^2}-\frac{1}{4\Lambda_2^2}\right)\right]$$

$$\langle 0|\bar{s}\gamma_5 s|\eta'\rangle = i\frac{f_K}{\sqrt{3}}\frac{M_K^2}{m_s+m_q}\left(c\theta - \sqrt{2}s\theta\right) \qquad (18)$$

$$\times\left[1+\frac{M_{\eta'}^2-M_K^2}{\Lambda_0^2}+2(M_K^2-M_\pi^2)\left(\frac{1}{\Lambda_1^2}-\frac{1}{4\Lambda_2^2}\right)\right]$$

- SU(3) breaking alone

$$(M_\eta^2)_{ideal} = 2M_K^2 - M_\pi^2, \quad (M_{\eta'}^2)_{ideal} = M_\pi^2$$

from chiral pert theory

$$\Lambda_0 \simeq 1.2\ \mathrm{GeV}, \quad \Lambda_1 \simeq 1.2\ \mathrm{GeV}, \quad \Lambda_2 \simeq 1.3\ \mathrm{GeV}$$

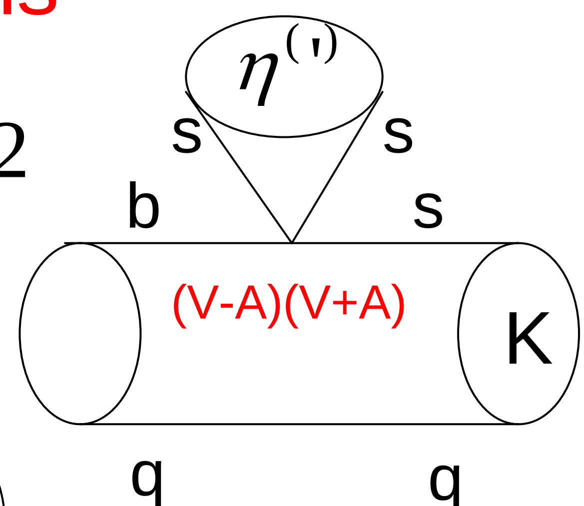
Large pseudoscalar density

- Numerical results

$\theta = -22^\circ$	$U(1)_A \times SU(3)_V$	$SU(3)_V$
$2im_s\langle 0\|\bar{s}\gamma_5 s\|\eta\rangle$	$+0.053 \pm 0.008$	$+0.058$
$2im_s\langle 0\|\bar{s}\gamma_5 s\|\eta'\rangle$	-0.109 ± 0.016	-0.069

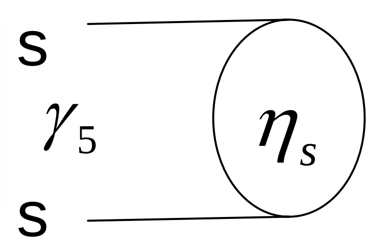
- Enhance (V-A)(V+A) penguins

$$\left|\frac{\langle 0|\bar{s}\gamma_5 s|\eta'\rangle}{\langle 0|\bar{s}\gamma_5 s|\eta\rangle}\right| \approx 2.1 > \cot\phi = 1.22$$



- No sign from Ds decays

$$\frac{F_+^{D_s\eta'}(0)}{F_+^{D_s\eta}(0)} = 1.14 \pm 0.17 \pm 0.13$$



Summary

- Understanding of $B(B \to \eta^{(\prime)} K)$ requires new mechanism compared to $B \to \pi K$
- Flavor-singlet contribution may be too small
- Gluonic charming penguin is a free parameter
- Large axial U(1) anomaly is not seen in other decays
- Large chiral scale is likely due to OZI violating effects
- $B(B \to \eta^{(\prime)} K)$ are still an unsettled issue 10 years after their observation!
- Further exp discrimination is necessary:

Experimental discrimination

- By means of $B \to \eta^{(\prime)} l\nu, \quad B_s \to \eta^{(\prime)} l\nu$
- Ratios

Ratios	FS	CP	CS	AA
$\frac{B(B \to \eta' \ell\nu)}{B(B \to \eta\ell\nu)} \approx \tan^2\phi$	X	X	V	V
		10E(-5) BR		
$\frac{B(B_s \to \eta' \ell\ell)}{B(B_s \to \eta\ell\ell)} \approx \cot^2\phi$	X	--	V	X