Dynamics of nonlinear wave-packet: Soliton, Vortex, Optical Patterns

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Outline

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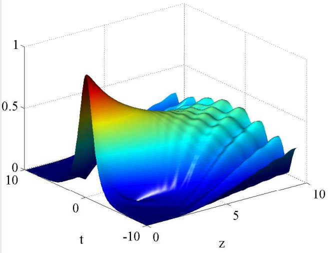
   - Linear/Nonlinear Waves
   - About Solitons
2. Current Solitons Research
   - Nonlocal Solitons
   - Nonlinear Surface Plasmonics
   - Soliton and Vortex in elliptical geometry
   - Optical Pattern Formation: Fromt MI to TI
   - Ultrashort Solitons
3. Conclusion

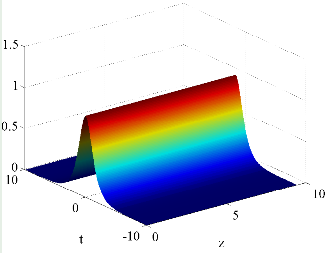
Linear/Nonlinear Wave-Packet

Linear Wave propagation



- Dispersive (time, frequency)
- Diffractive (space, momentum)

Non-Linear Wave propagation



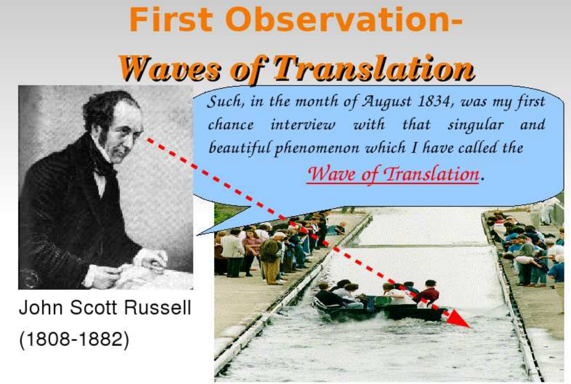
- Temporal localization
- Spatial localization
- Optical Bullet

Fundamentals, instability, transverse instability, oscillatory instability

- Solitons are nonlinear self-guided waves that propagates in a *shape (envelope) preserving* manner.
- Soliton can be bright, dark or gray depending on the form of nonlinearity.
- Modulation instability, transverse modulation instability are important menifesitation to strong nonlinear systems.
- *Modulation instability*: what soliton formation results from.
- When dimension increases, *transverse instability* govern the formation of 2D solitary wave structure. [Quasi-1D soliton stripe]
- When there are multiple components, for example, counter propagating fields, *oscillatory instability* from nonlinear interferences. [Bragg soliton]

Story: First observation of (water surface) soliton

Scottish engineer, John Scott Russell (1808-1882), "Report on Waves": **The wave of translation**, *(Report of the fourteenth meeting of the British Association for the Advancement of Science, York, September 1844 (London 1845), pp 311-390, Plates XLVII-LVII).*



Universal signature of solitons

A Universal phenomenon of self-trapped wave packets

- EM waves in nonlinear optical materials;
- shallow- and deep-water waves;
- charge-density waves in plasmas;
- matter waves in Bose-Einstein condensates;
- excitations on DNA chains;
- domain walls in supergravity, and ”branes” at the end of open strings in superstring theory; to name only a few.

See:M. Segev, *Optics & Photonics News*, pp. 27 (Feb. 2002).

Current Studies on Solitons

In the study of solitons, what is the question in concern currently?

- What is the Applications?
  soliton based communication system, optical switching, and the like. [Boardman A D and Sukhorukov A P (ed) 2001 *Soliton-Driven Photonics* (Dordrecht: KluwerAcademic)]
- What is the material?
  nonlocal response: thermal-optical (lead glass), photorefractive(SBN), liquid crystal, resonance absorbing media. [A. W. Synder and D. J. Mitchell, "Accessible Solitons," Science **276**, 1538-1541 (1997).]
- What is the boundary effect?
  gap soliton in (subwavelength)lattice, surface waves. [de Sterke C M and Sipe J E 1994 Gap solitons Progress in Optics vol XXXIII ed E Wolf (Amsterdam: North-Holland) pp 20360; Mingaleev S and Kivshar Yu S 2001 Phys. Rev. Lett. 86 5474; Slusher R and Eggleton B (ed) 2003 *Nonlinear Photonic Crystals* (Berlin: Springer);A. V. Zayats, I. I. Smolyaninov, and A. A. Maradudin, Phys. Rep. **408**, 131 (2005).]
- Dissipative or Conservative?
  cavity soliton, laser soliton(autosoliton) [N. Akhmediev and A. Ankiewicz (ed), *Dissipative Solitons: From Optics to Biology and Medicine,* Lect. Notes Phys. 751 (Springer, Berlin Heidelberg (2008).]
- What is the time scale?
  non-instantaneous, ultrashort pulse [P. Kinsler and G.H.C. New, "Few-cycle soliton propagation," Phys. Rev. A **69**, 013805 (2004).]

Nonlocal Response

Nonlinear response related to the neigbhboring materials

$$i\frac{\partial U}{\partial z} + \frac{1}{2}\nabla_{\perp}^{2} U + nU = 0,\ n = \int_{\infty}^{\infty} R(\eta - \eta')|U(\eta')|^{2} d\eta'$$

Diffusive response

$$n - d\nabla_{\perp}^{2} n = |U|^{2}$$

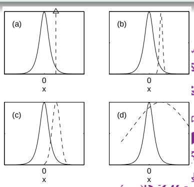
$$\Updownarrow$$

$$R(\eta) = \frac{1}{\sqrt{2d}} e^{-|\eta|/\sqrt{d}}$$

photorefractive material
thermal-optical material
nematic liquid crystal
plsama

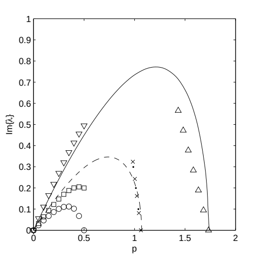
Dipole-dipole force in dipolar BEC

$$R(\vec{\eta}, \vec{\eta}') = \frac{1 - 3\vec{p}\cdot\vec{e}_{\eta-\eta'}}{|\vec{\eta} - \vec{\eta}'|^{3}}$$

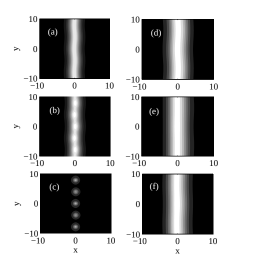


Transverse instability supression in nonlocal nonlinear medium

TI spectrum



nonlinear wave evolution



Nonlocal nonlinear response can suppress soliton transverse instabilities, shown by numerical calculation and asymptotic expansions.

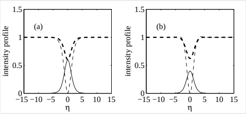
See:Y.Y. Lin et al., *J. Opt. Soc. Am. B* **25**, 576-581 (2008)

Dark-bright soliton pair in nonlocal nonlinear media

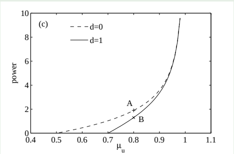
typical nonlocal response $\Longleftrightarrow$ formation power increase !

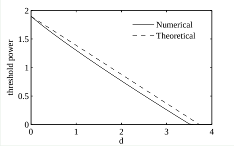
$$i\partial_\eta \psi - \frac{1}{2}\partial_{\xi\xi} + n\psi = 0, \quad \psi = U(\xi;\eta),\ V(\xi;\eta)$$

$$n = \int_{-\infty}^{\infty} R(\xi' - \xi)\left(|U|^2 + |V|^2\right), \quad R(\xi) = \frac{1}{2\sqrt{d}} e^{-|\xi|/\sqrt{d}}$$



(a): d=0 and (b):d=1.





when bright pulse is coupled to dark pulse in nonlocal response $\Longleftrightarrow$ formation power decrease !

See: Y.Y. Lin et al., *Opt. Express* **15**, 8781-8786 (2007)

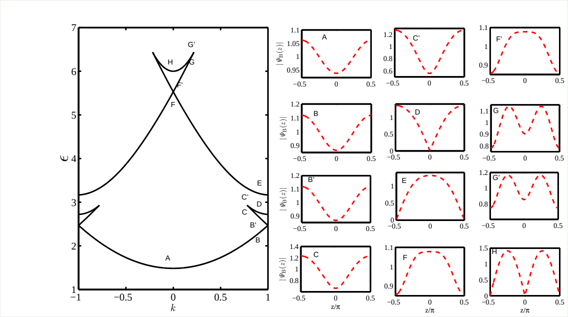
Nonliner bloch wave and swallow-tailed band structure

Nonlinear band diagram with dipolar nonlocal response

$$\mu\Psi(\vec{r}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{trap} + Ng_s|\Psi(\vec{r})|^2 + Ng_d\int d^3r' V_d(\vec{r},\vec{r'})\Psi(\vec{r'})|^2\right]\Psi(\vec{r}), \quad \frac{1-3\vec{p}\cdot\vec{e}_{r-r'}}{|\vec{r}-\vec{r'}|^3}$$

- Cigar shaped dipolar BEC in optical lattice.
- Effective 1D nonlocal model.
- Sufficient Bloch wave expansion up to 3-modes.

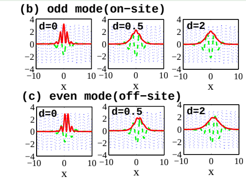
Band structure manipulation by nonlocality control, which is implemented with dipole orientation angle.

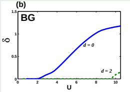


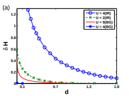
See: Y.Y. Lin et al., Phys. Rev. A **78**, 023629 (2008).

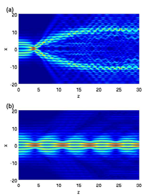
Gap solitons and Bragg Solitons

nonlocal gap soliton









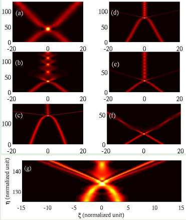
See: Y.Y. Lin et al., J. Opt. A: Pure Appl. Opt. **10** 044017 (2008).

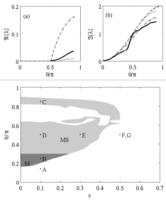
nonlocal bragg solitons

$$i\frac{\partial U}{\partial \eta} + i\frac{\partial U}{\partial \xi} - \frac{1}{2}\left[n_0 + \frac{|V|^2}{1+Dg^2}\right] U + V = 0$$

$$i\frac{\partial U}{\partial \eta} - i\frac{\partial V}{\partial \xi} - \frac{1}{2}\left[n_0 + \frac{|U|^2}{1+Dg^2}\right] V + U = 0$$

$$n_0 - D\frac{\partial^2 n_0}{\partial \xi^2} = |U|^2 + |V|^2$$





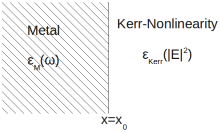
See: Y.Y. Lin et al., Phys. Rev. A, **80**, 013838 (2009).

Nonlinear Surface Plasmonics

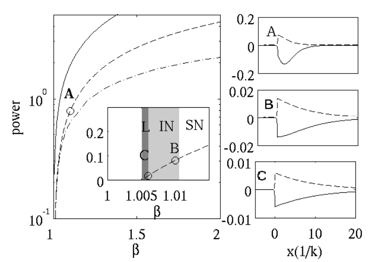
Assume the divergence of displacement vanishes,

$$\left[\nabla^2 + k^2 n^2\right] E = -\nabla \left[E \cdot \nabla \ln n^2\right]$$

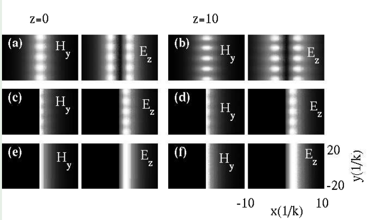
E: vector electric field, k: wave vector in vacuum, n: scalar refractive index($n = \vec{E} \cdot \vec{E}$).



Nonlinear Surface mode



Stable evolution of nonlinear surface plasmons



See: Y.Y. Lin et al., Opt. Lett. **34**, 2982-2984 (2009)

Singular waves: Linear/Nonlinear Vortex

- A singular optical waves with "topological charges", i.e. phase change in the azmuthal direction;
- Possessing Phase singularities;
- Carrying angular momentum;
- Linear or Nonlinear.

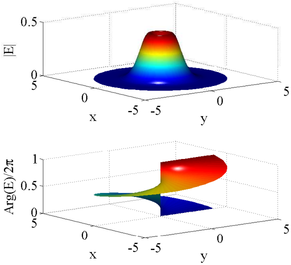
Paraxial *linear* vortex beam

$$E^{(m)}(\rho, \phi, z) = E_S \frac{\rho_S}{w_S} \left(\frac{\rho}{w_S}\right)^{|m|} \exp\left(-\frac{\rho^2}{w_S^2}\right) \exp\left[i\Phi_S(\rho, \phi, z)\right]$$

$$\Phi_S(\rho, \phi, z) = -(|m| + 1)\tan^{-1}\left(\frac{2z}{k\rho_S^2}\right) + \frac{k\rho^2}{2R_S(z)} + m\phi + kz,$$

$$w_S = \sqrt{\rho_S^2 + \left(\frac{2z}{k\rho_S}\right)^2},\ R_S(z) = z + \frac{k^2\rho_S^4}{4z}.$$

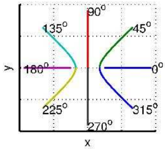
See:L. Allen et al., *Optical Angular Momentum*, Institute of physics Publishing, (2003)

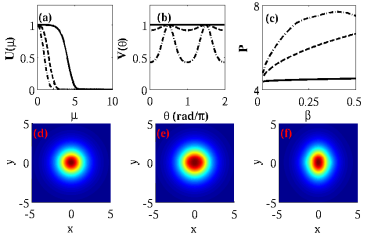


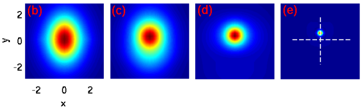
Elliptical Soliton

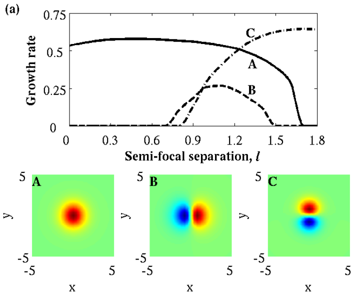
Conformal Mapping

Apply to nonlinear Schrödinger equation, we let $x = l \cdot cosh\mu cos\theta$ and $y = l \cdot sinh\mu sin\theta$, leading to $\nabla_{\perp}^{2} = \frac{1}{l^2(cosh^2\mu - cos^2\theta)}\left[\frac{\partial^2}{\partial\mu^2} + \frac{\partial^2}{\partial\theta^2}\right]$ where l is the semi-focal separation, $\mu > 0$ and $\theta \subset [0, 2\pi]$.



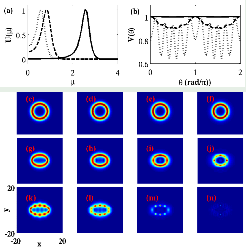




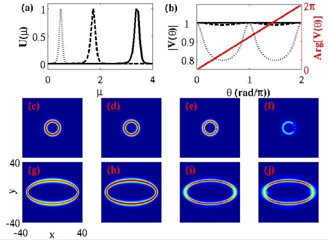


Elliptical Vortex

Ring Soliton (0π)



Vortex (2π)



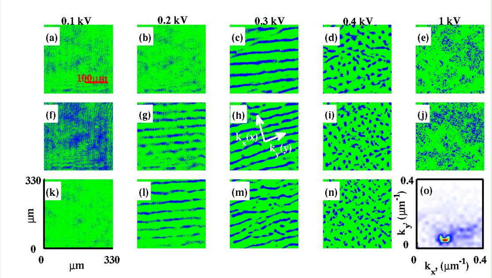
- Elliptical soliton rings evolves to clusters in a specific geometric arrangement ascribable to elliptical symmetry and the corresponding transverse instabilities.
- Elliptical vortex breaks at the major axis, turning into crescents and end up with clusters.

See:YuanYao Lin et al., Opt. Lett. **33**, 1377 (2008).

Optical Pattern in Photorefractive materials

The optical intensity patterns of a coherent beam at the output plane through a nonlinear crystal at different bias voltagesand different input intensities

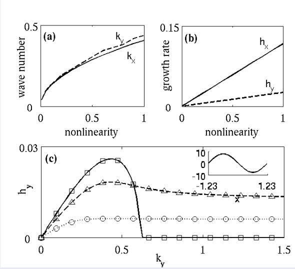
Experimental Result



(a-e):65 $\frac{mW}{cm^2}$; (f-j):96 $\frac{mW}{cm^2}$; (k-n):96 $\frac{mW}{cm^2}$; (o): Fourier spectrum of (h).

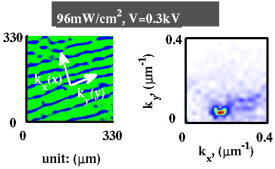
See: C.-C. Jeng, Y.Y. Lin, R.-C. Hong, and R.-K. Lee, Phys. Rev. Lett. **102** 153905 (2009)

MI/TI analysis



- Characteristic spatial vectors for MI and TI are close;
- MI growth rate $\gg$ TI growth rate;
- numerics and analytics match well;
- spatial vector with a maximal growth rate is independent of $\Omega\tau$

Optical Pattern in Photorefractive materials



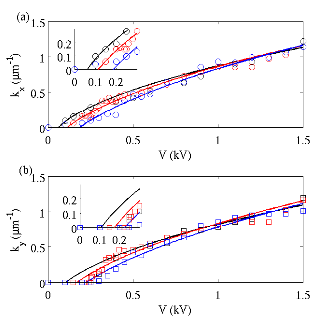
Spatial vectors: k_x (a) and k_y (b)

Spectra of transverse patterns obtained from experimental data for (a) kx and (b) ky . Circles and squares markers accompanied by the fitted curves in black, red and blue refer to optical intensities I = 190, 96, and 65 mW/cm2 , respectively.

Supporting Facts

- second threshold voltage: $V^y_{th} > V^x_{th} \Leftrightarrow h_y < h_x$
- intensity dependent threshold voltage: $I \nearrow \Rightarrow V^y_{th}, V^x_{th} \searrow \Leftrightarrow I \nearrow \Rightarrow \tau \searrow$

Experimental, Theoretical and TI threshold



I	65mW/cm^2	96mW/cm^2	190mW/cm^2
V^x_{th}	0.19kV	0.117kV	0.067kV
V^y_{th}	0.23kV	0.17kV	0.108kV

Ultrashort Solitons: Corrections non-instantaneous(nonlocal) Polarization

For solitons, we study wave packet of special envelope character within the scope of slowly varying envelope apporximation (SVEA).

Under this scope inverse scattering transformation (IST), a well developed method to analyze the interaction between solitons, can be successfully applied of system integrability

When SVEA fails, IST thus fails. It requires new techniques:

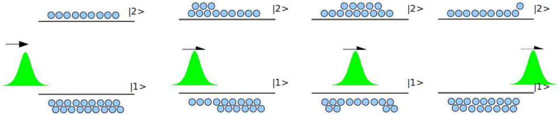
- FDTD [A. Taflove, *Computational Electrodynamics: The Finite Difference Time-Domain Method*, Boston: Artech House, 1995];
- slowly evolving wave approxmimation(SEWA) [M.A. Porras, Phys. Rev. A, **60**,5059 (1999)];
- generalized few-cycle envelope approximation (GFEA)[P. Kinsler, Phys. Rev. A **81**, 013819 (2010)].

See: *arXiv:physic/0212014v4* for details

Self-induced-transparency(SIT) solitons in Two-level-atom(TLA)

- A coherent optical pulse propagate in a resonant absorbing medium without any experiences of loss and distortion when exceed certain power.
- In particular, optical solitons in such a resonance are referred to as self-induced-transparency **SIT** soliton

Ref: S.L. McCall and E.L. Hahn, "Self-Incuced Transparency," Phys. Rev. **183**, 457-489 (1969).



E: Electric field envelope of EM field;
$N \equiv N_{|2\rangle} - N_{|1\rangle}$: population difference;
$P \propto \rho_{12}$: transition dipole moment.

$$k_j = \frac{\partial^j k}{\partial \omega^j}, j = 1, 2, 3 \cdots$$

Maxwell-Bloch Equation

$$\frac{\partial}{\partial t} P = \frac{1}{2} \frac{\mu^2}{\hbar} NE \quad ; \frac{\partial}{\partial t} N = -\frac{1}{\hbar} \left(PE^* + P^* E\right);$$
$$\left[\frac{\partial}{\partial t} + c \frac{\partial}{\partial z}\right] E = \frac{\omega_0}{\epsilon_0} P + \frac{\omega_0}{\epsilon_0} (2 - \frac{\omega_0 k_1}{k_0}) \frac{i}{\omega_0} \frac{\partial}{\partial t} P;$$

Ultrashort SIT solitons

Taking all the arguments and constraints, we can derive a single **cubic-quintic** nonlinear odinary differential equation(**ODE**) from the corrected **MB** equation that governs the solitary wave solution.

cubic-quintic equation for $q = |E|$

$$\frac{\partial^2 q}{\partial \xi^2} - \left(\frac{\Gamma n_c}{4\alpha} - \beta^2\right) q + \left(\frac{1}{2} - \frac{\bar{\sigma}\beta}{2}\right) q^3 + \frac{3\bar{\sigma}^2}{64} q^5 = 0$$

in which $\beta \equiv -\frac{\Delta}{2\alpha} + \frac{\Gamma \bar{\sigma} n_c}{8\alpha}$

temporal phse $\phi(\xi) = \angle E$

$$\frac{\partial \phi}{\partial \xi} = -\frac{3}{8}\bar{\sigma} q^2 + \beta$$

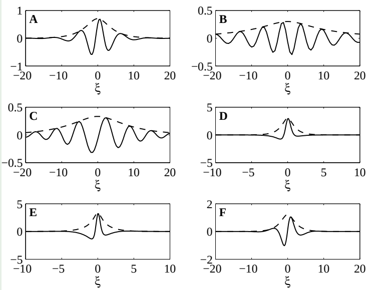
atomic polarizations n, $u = \Re\{P\}$, and $u = \Im\{P\}$

$$n = n_c - \frac{\alpha}{\Gamma} q^2$$

$$u = \frac{2\alpha}{\Gamma} \frac{\partial q}{\partial \xi}$$

$$v = \frac{2\alpha}{\Gamma} \left(\frac{\bar{\sigma}}{8} q^3 - \beta q\right)$$

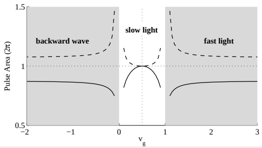
Soliton solutions to $\Re\{E\}$(dashed) and $|E|$:(solid) at $\Delta = 0$



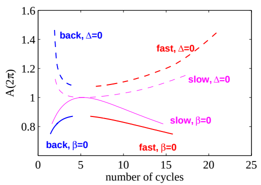
A: $n_c = 1, v_g = -1$; **B**: $n_c = 1, v_g = -0.1$; **C**: $n_c = -1, v_g = 0.1$; **D**: $n_c = -1, v_g = 0.9$; **E**: $n_c = 1, v_g = 1.1$; **F**: $n_c = 1, v_g = 3$.

Stable Backward wave, slow light, superluminal and Area theory breakdown

Area-velocity



Area-Pulse duration



General Soliton family and area theory

- Soliton solution exists only when $n_c\alpha > 0$.
- *slow light* soliton: $n_c < 0$, which means atoms are more populated in the ground state $|1\rangle$, the group velocity of the soliton $v_g < 1$ (normalized to the velocity of light, c).
- *fast light/superluminal* soliton: $n_c > 0$, which means more atoms are excited to $|2\rangle$, the group velocity of the soliton can be $v_g > 1$(group velocity exceeds the velocity of light, c) Phys. Rev. A **75**, 043806(2007).
- *backward wave* soliton: $n_c > 0$, which means more atoms are excited to $|2\rangle$, the group velocity of the soliton can be $v_g < 0$(propagating in the reverse direction with respect to phase velocity).
- stability confirmed by linear stability analysis
- **dark solitons** are not possible due to violation of conserved quantity for bloch vectors.
- Area theory breakdown is illustrated in **real ultrashort SIT solitons**.

conclusion

- Briefly, concepts to solitons is introduced.
- Swiftly, we browse some of the current study associated with solitons: nonlocal effect, special geometry, conserved/dissipative, non-instantaneous effect, and ultrashort soliton.
- Hopefully, interesting physics can be explored from these studies.

Acknowledgement

Colaborators

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