Recent progress on TMD definitions

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presented at CYCU

Dec. 23, 2014

Outlines

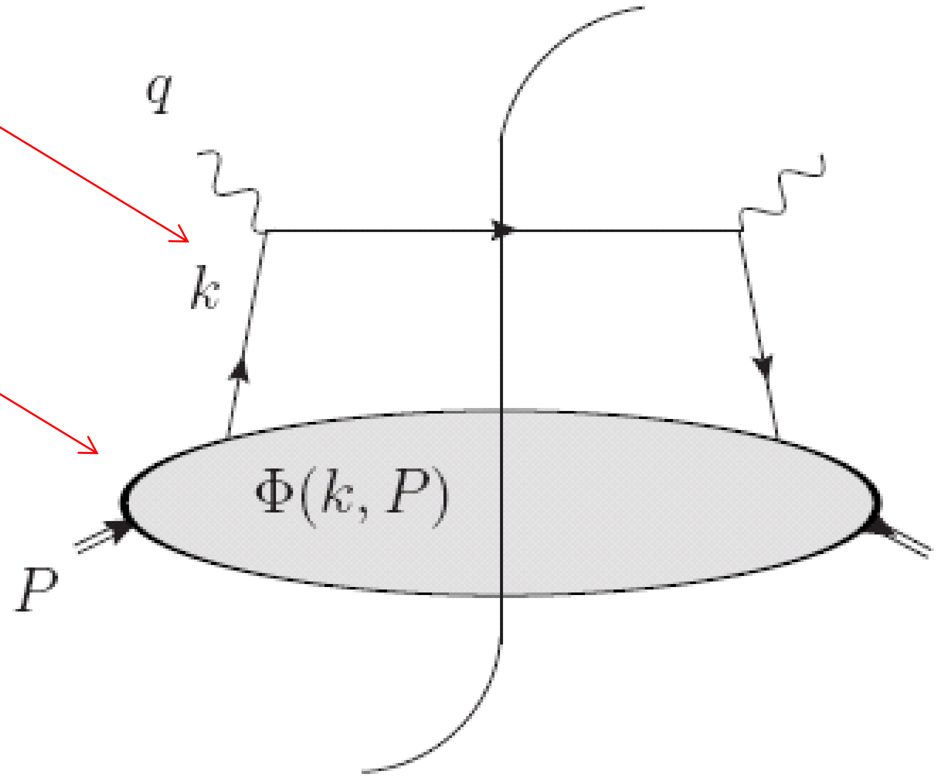
- Introduction
- Naïve definition
- Collins’ definition
- New definition
- Polarized process
- Summary

Introduction

Factorization theorem

- Deeply inelastic scattering (DIS) as an example
- Cross section = Hard (H) * Parton distribution function (PDF)
- H = short distance, LO
  PDF = long distance
- Collinear factorization

$$k = (xP^{+}, 0, 0_{T})$$



Collinear factorization

- Factorization of many processes investigated up to higher twists
- Hard kernels calculated to higher orders
- Parton distribution function (PDF) evolution from low to high scale derived (DGLAP equation)
- PDF database constructed (CTEQ)
- Logs from extreme kinematics resummed
- Soft, jet, fragmentation functions all studied

k_T factorization (less mature)

- k_T factorization applies to small x, or high energy region, especially to LHC physics
- Also to final-state spectra at low q_T, like direct photon and jet production
- Keep k_T in hard kernel, $xP^+ \approx k_T\,, q_T \approx k_T$
- Parton $k = (xP^+, 0, k_T)$ enters hard kernel
- Parton k_T is not integrated out in PDF

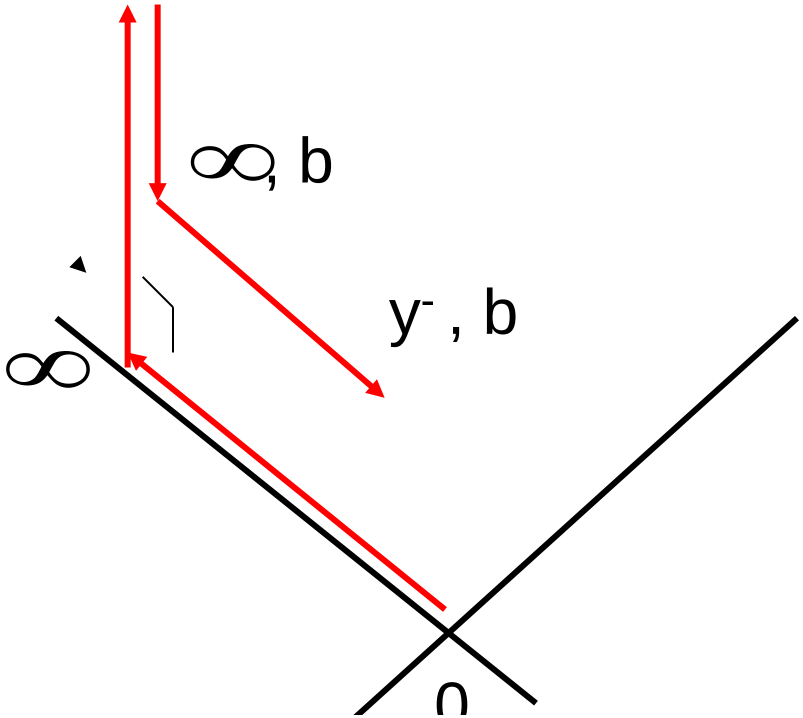
  $\Rightarrow$ k_T dependent (TMD) parton density
- Many aspects of k_T factorization not yet investigated in detail

Naïve definition

Transverse Wilson links

- Suppose factorization established. Quark fields nonlocal in transverse directions. Transverse Wilson links introduced
- For PDF, work in axial gauge. Calculate only self-energy corrections.
- Not the case for TMD, because transverse Wilson links contribute in axial gauge.
- Transverse links do not contribute in covariant gauge.

Feynman diagrams



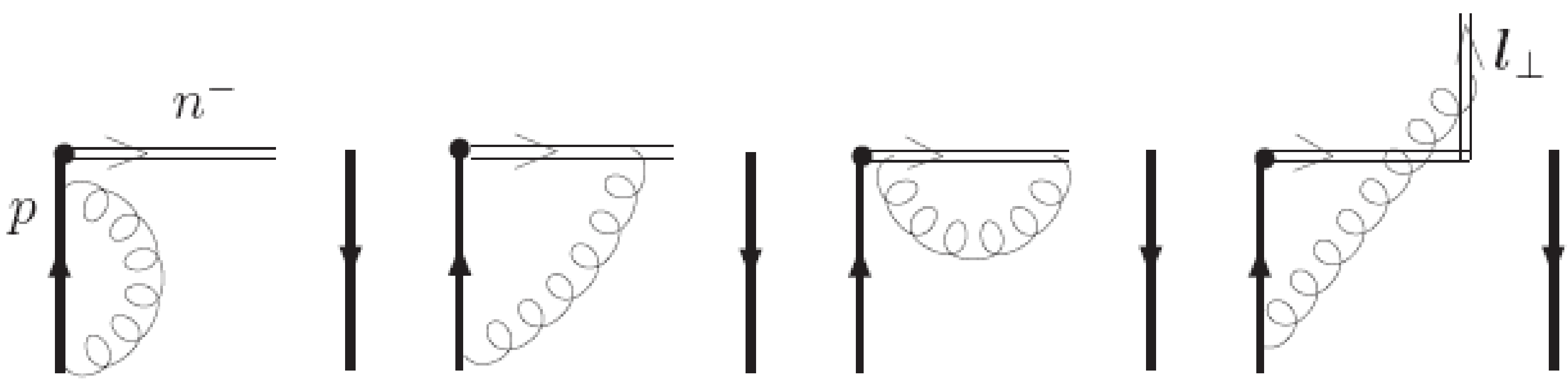
In axial gauge A+=0

$$y^- = \infty$$

$$\downarrow$$

$$\frac{\exp(-il^+\infty)}{l^+} = 2\pi i\delta(l^+)$$

from gluon propagator



Light-cone singularity

- Compute $H^{(1)} = G^{(1)} - \phi^{(1)} \otimes H^{(0)}$
- The pole $1/(n_- \cdot l) = 1/l^+$ from Wilson lines in $\phi^{(1)}$ gives the light-cone singularity.
- They cancel in collinear factorization

$$\phi^{(1)} \otimes H = \int \frac{dl^+}{l^+} [H(x) - H(x + l^+ / P^+)]$$

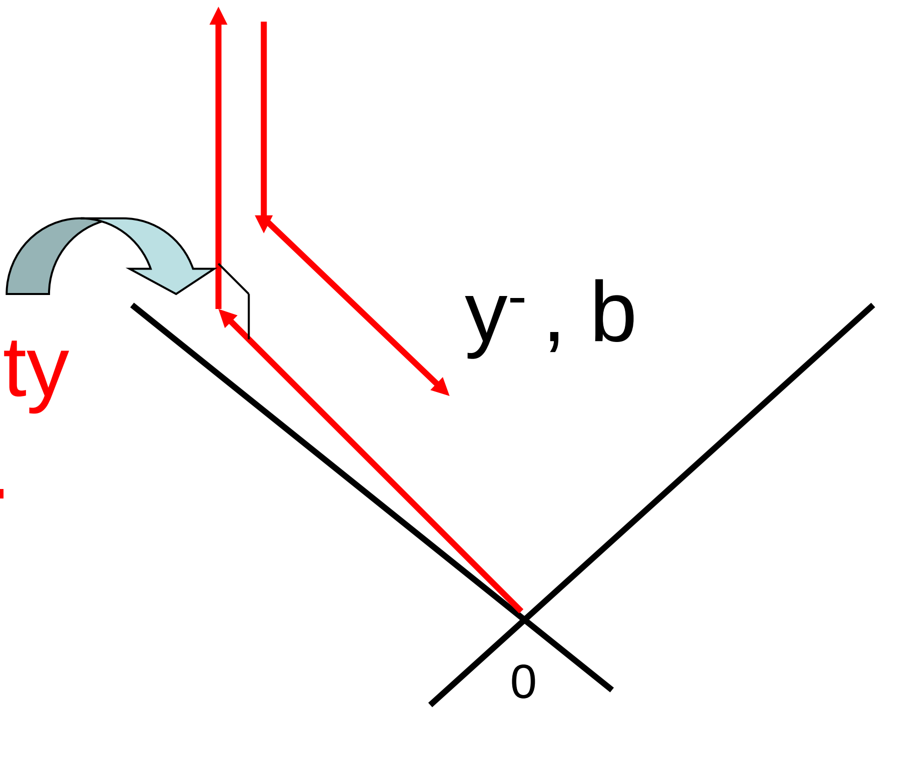
- The difference of $H^{(0)}$ removes singularity.
- They exist in k_T factorization:

$$\int \frac{dl^+}{l^+} [H(x, k_T) - H(x + l^+ / P^+, k_T + l_T)]$$

- because $H(x, k_T) \neq H(x, k_T + l_T)$

Modification

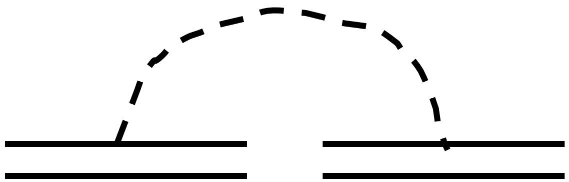
- Naïve TMD is ill-defined
- Modified definition: $n_- \to n, n^2 \neq 0$



- Light-cone singularity is regularized by n^2.

New IR singularity

- Self-energy correction to Wilson links appears



- Proportional to n^2 , vanishes originally as $n_-^2 = 0$
- Its Feynman integrand $$\frac{1}{(n\cdot l + i\varepsilon)(n\cdot l - i\varepsilon)}$$
- 1st pole $n\cdot l = 0$ leads to pinched singularity from 2nd eikonal propagator
- Off-light-cone Wilson links regularize light-cone singularity, but introduce new one

Collins’ definition

Collins' modification

- TMD with light-like Wilson links multiplied by

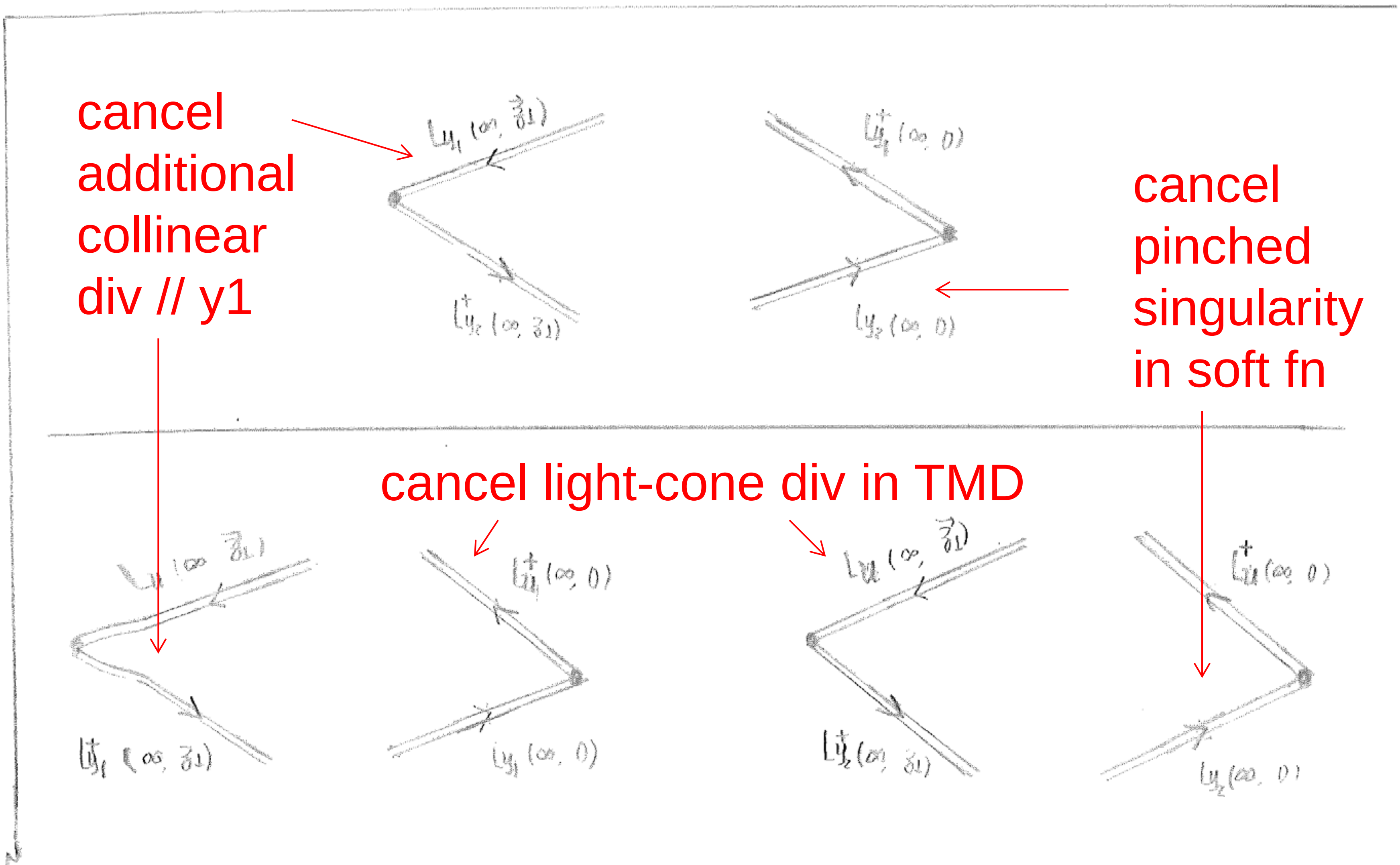
$$\lim_{\substack{y_1 \to +\infty \\ y_u \to -\infty}} \sqrt{\frac{S(z_T; y_1, y_2)}{S(z_T; y_1, y_u)\, S(z_T; y_2, y_u)}}$$

Wilson-line rapidity

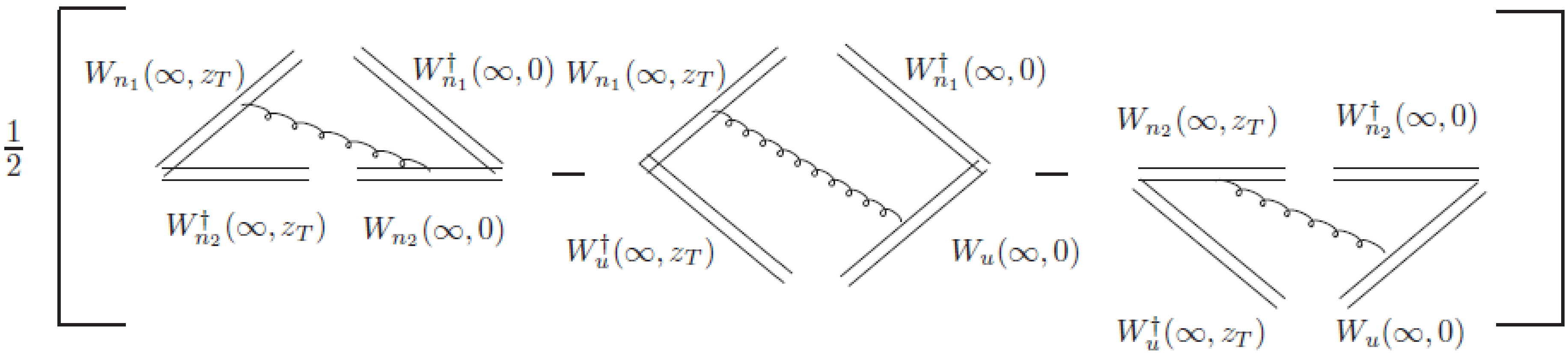
$$n_2 = (e^{y_2}, e^{-y_2}, \mathbf{0}_\mathrm{T})$$

- u and n1 on light cone, n2 off light cone
- Off-light-cone Wilson links move into soft function
- Square root renders calculation difficult

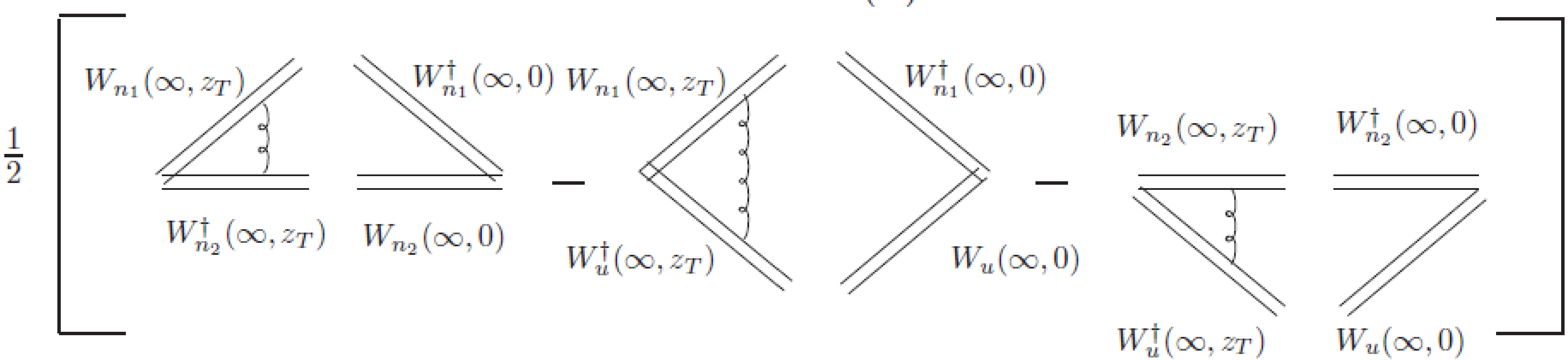
IR cancellations



NLO diagrams



(a)



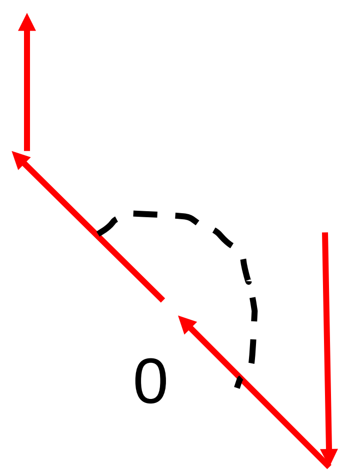
New definition

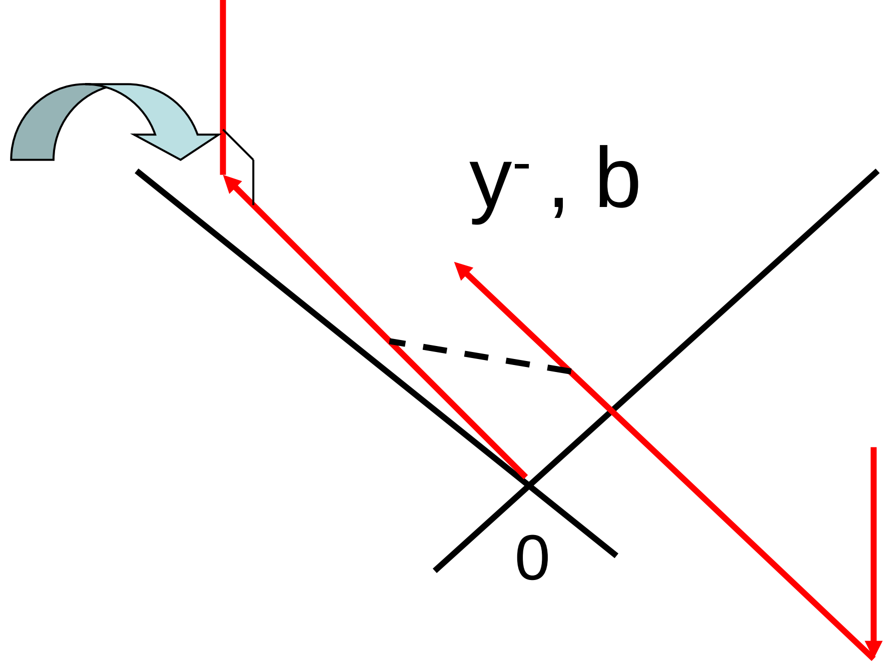
Our modification

- Reverse one of off-light-cone Wilson links
- Pinched singularity reduces to log singularity

$$\frac{1}{(n\cdot l + i\varepsilon)^2}$$

- Introduce soft subtraction to remove this log singularity

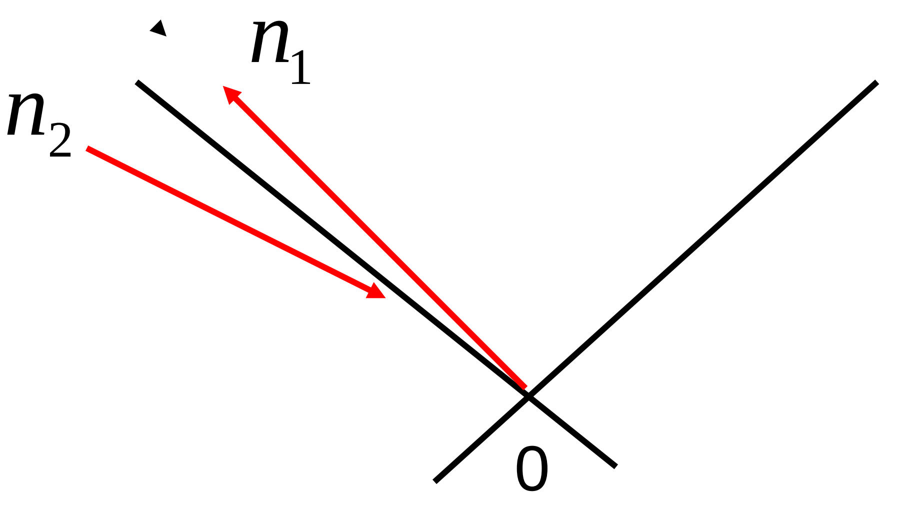




Another modification

- Choose orthogonal gauge vectors for off-light-cone Wilson links
- Pinched singularity disappears
- soft subtraction is not needed

$$\frac{n_1 \cdot n_2 = 0}{(n_1 \cdot l + i\varepsilon)(n_2 \cdot l - i\varepsilon)}$$



Check IR behavior

- Take pion transition form factor as an example, whose hard kernel is simple
- All three definitions give the same collinear logarithm, the same as in QCD diagrams
- They all realize kT factorization at small x

$$\int_{-\infty}^{+\infty} dk'_+ \int_{-\infty}^{+\infty} d^{2-2\epsilon} k'_T \; \phi^{C(1)}(k'_+, k'_T, y_2)\, H^{(0)}(k'_+, k'_T)$$

$$= -\frac{\alpha_s C_F}{4\pi} \left[\ln\left(\frac{k_+}{p_+}\right) + 2\right] \ln k_T^2 \, H^{(0)}(k_+, k_T) + \cdots \, ,$$

x

Check evolution

- Evolution equation in Wilson-line rapidity from Collins' definition

$$\frac{d}{dy_2}\ln\phi^C(k'_+,k'_T,y_2)=\frac{\alpha_s(\mu)\,C_F}{\pi}\left[\ln\left(\frac{2\,k'_+\bar{k}'_+}{\mu^2}\right)+2\,y_2\right]$$

factorization scale

- From our definitions

$$\mu = p_+$$

$$\frac{d}{dy_2}\ln\phi^L(k'_+,k'_T,y_2)=\frac{\alpha_s(\mu)\,C_F}{\pi}\left(\ln\frac{k'_+}{p_+}+1\right)$$

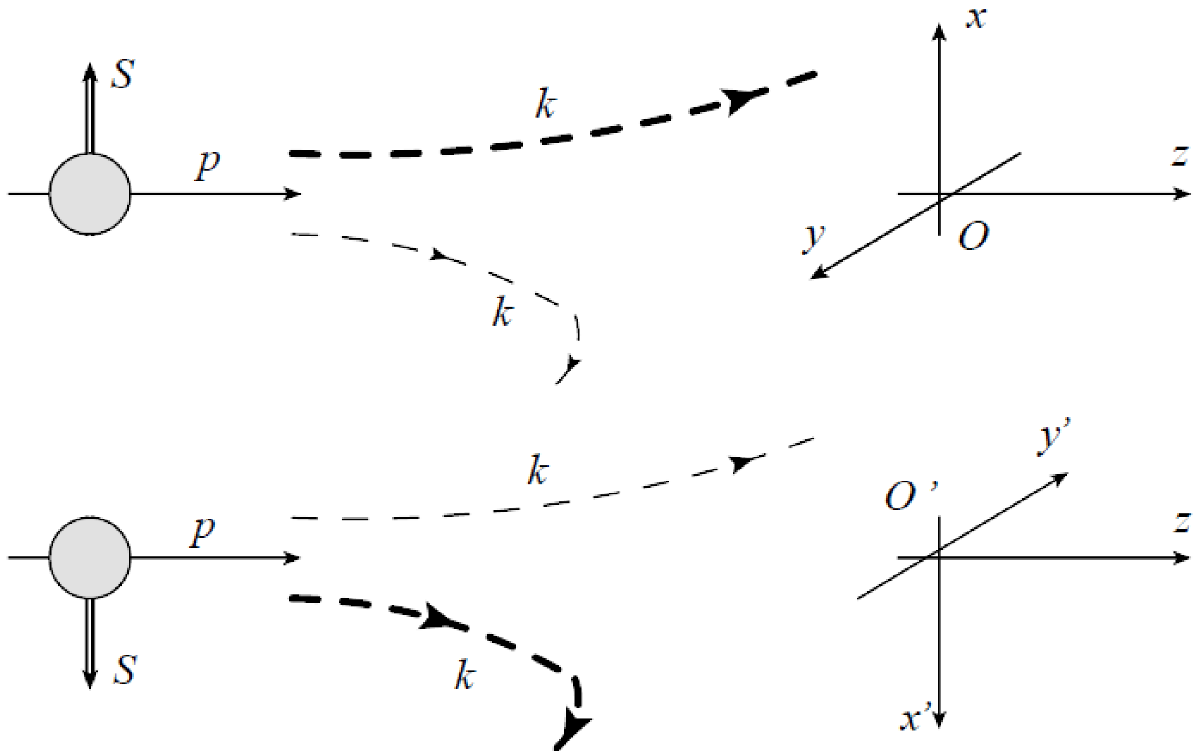
- They are equivalent at small x

Polarized process

Single spin asymmetry (SSA)

$$A_N \equiv \frac{d\sigma^{\uparrow}(\underline{k}) - d\sigma^{\downarrow}(\underline{k})}{2\, d\sigma_{unp}} \propto (\vec{S} \times \vec{k}) \cdot \vec{p} \sim S_x k_y p_z$$

- Exist, if different Nos of particles in +y, -y



Sivers function

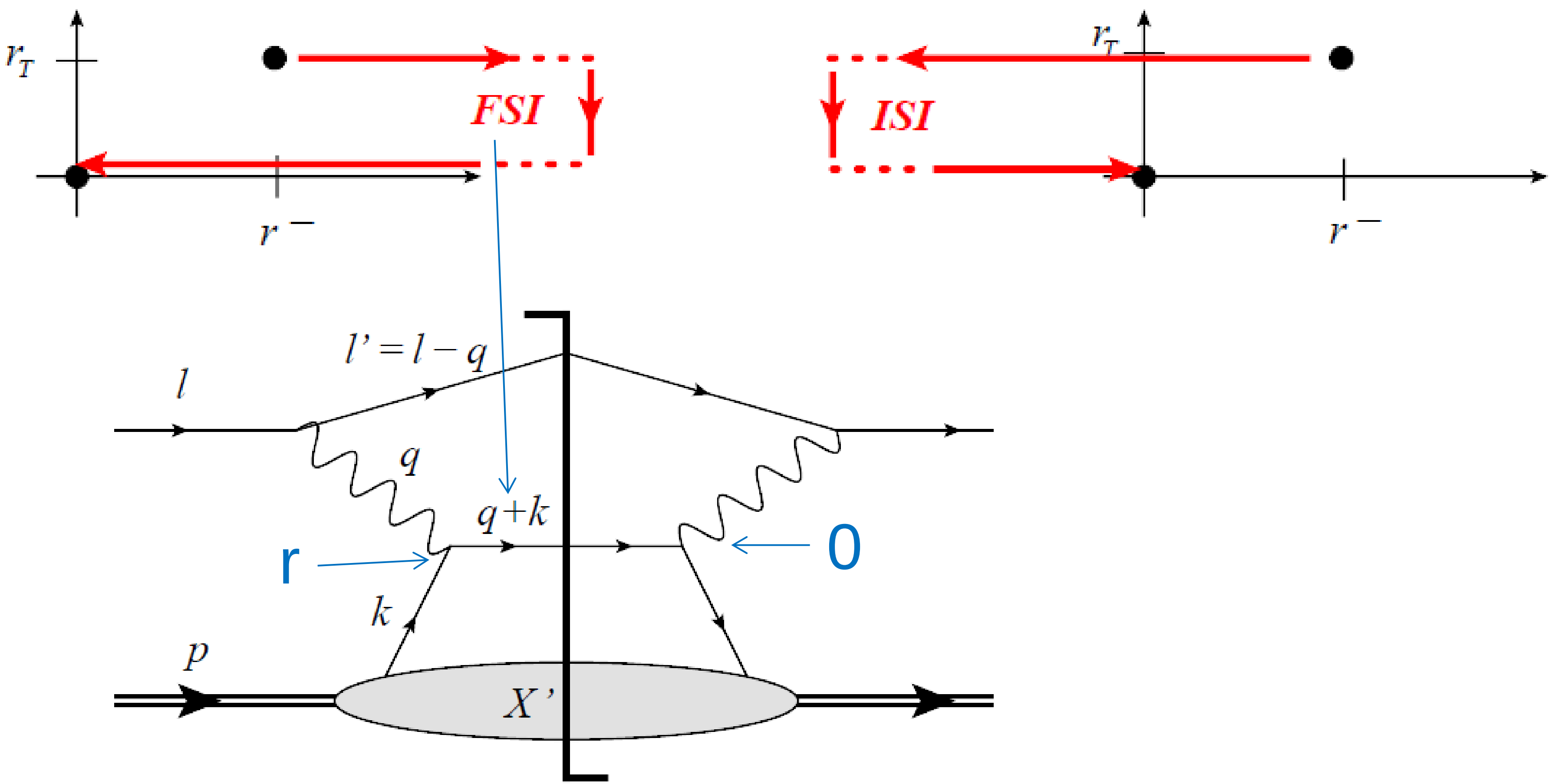
- TMD parton density factorized from polarized collision processes, called Svers function

$$\frac{(\underline{S}\times\underline{k})}{m_N} f_{1T}^{\perp q}(x, k_T)$$

Wilson links

$$= \frac{1}{4}\int \frac{d^{2-}r}{(2\pi)^3} e^{ik\cdot r} \langle p\underline{S}|\, \overline{\psi}(0)\, \gamma^+ U_C[0,r]\, \psi(r)\, |p\underline{S}\rangle - (\underline{S} \to -\underline{S})$$

SIDIS vs DY



PT transformation

$$(PT)\,\psi(x^\mu)\,(PT)^\dagger = \gamma^0\gamma^1\gamma^3\,\psi(-x^\mu)$$

$$(PT)\,\overline{\psi}(x^\mu)\,(PT)^\dagger = -\overline{\psi}(-x^\mu)\,\gamma^0\gamma^1\gamma^3$$

$$\Rightarrow \quad (PT)\;|p,\underline{S}\rangle = e^{i\phi}\,|p,-\underline{S}\rangle$$

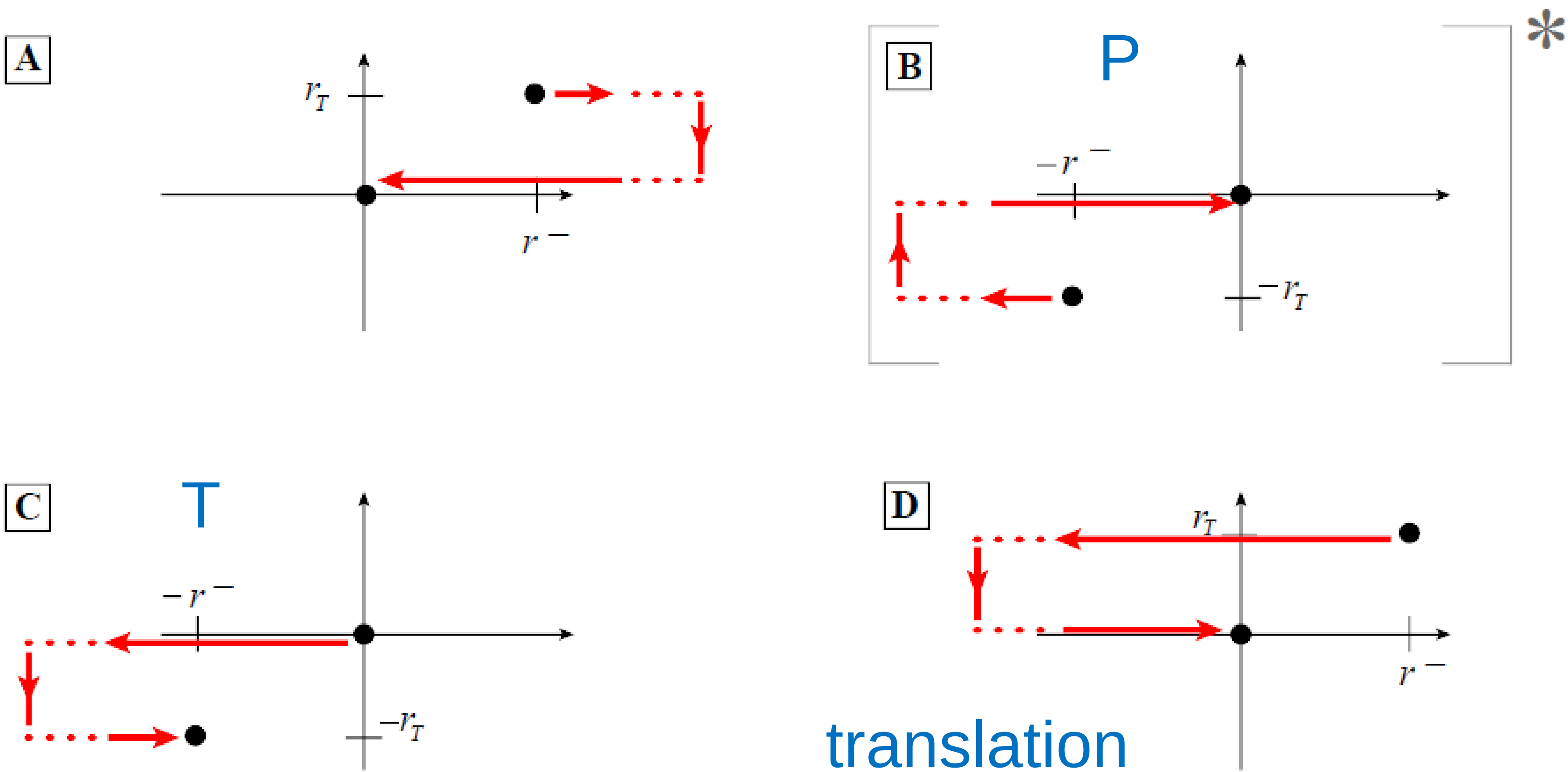
$$(PT)\,const\,(PT)^\dagger = (const)^*$$

$$\langle f|\,(PT)^\dagger\,\hat{O}\,(PT)\,|i\rangle = \langle (PT)f|\,\hat{O}\,|(PT)i\rangle^*$$

$$\langle p\underline{S}|\,\overline{\psi}(0)\gamma^+\psi(r)\,|p\underline{S}\rangle = +\,\langle p,-\underline{S}|\,\overline{\psi}(0)\,\gamma^+\,\psi(r)\,|p,-\underline{S}\rangle$$

- Asymmetry appears at higher orders

PT of Wilson links



$$(PT)^\dagger \, U_{FSI}[0,r]\,(PT) = U^*_{ISI}[0,-r]$$

Relation between SIDIS and DY

$$\langle p\underline{S}|\,\overline{\psi}(0)\gamma^{+}U_{FSI}[0,r]\psi(r)\,|p\underline{S}\rangle = +\,\langle p,-\underline{S}|\,\overline{\psi}(0)\,\gamma^{+}U_{ISI}[0,r]\,\psi(r)\,|p,-\underline{S}\rangle$$

$$\frac{(\underline{S}\times\underline{k})}{m_N}\left[f_{1T}^{\perp q}(x,k_T)\right]_{FSI} = -\frac{(\underline{S}\times\underline{k})}{m_N}\left[f_{1T}^{\perp q}(x,k_T)\right]_{ISI}$$

- famous sign flip of Sivers function in SDS and DY
- 1st new Wilson links leads to vanishing Sivers function
- No definite relation between 2nd new Wilson links for SIDIS and DY
- Anything wrong?

Physical observable

- TMD is not physical, but SSA is
- Employing 1^{st} new Wilson links, SSA is not factorizable
- Hard kernel for SSA starts at NLO, which involves subtraction of TMD effective diagrams from QCD diagrams
- Employing 2^{nd} Wilson links, their arbitrariness under PT will be cancelled by that in hard kernel

Summary

- k_T factorization is more complicated than collinear factorization, and has many difficulties
- TMD definition is one of difficulties, and it took a long time for people to understand the difficulty.
- Simpler definitions facilitate proof of kT factorization, derivation of evolution eqs
- Next goals: fragmentation, resummation, Glauber gluons, applications,…