The Topology in QCD

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Therefore, the topological excitations, such as the instantons, plays a central role in understanding the vacuum of QCD.

Since the topological excitations do not occur in the perturbation theory, theoretical calculations starting from the QCD Lagrangian necessarily involves non-perturbative methods, such as lattice QCD.

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(ii) Unquenched simulations with Wilson/staggered fermion do not respect correct chiral or flavor symmetry at finite lattice spacing, and the definition of the topological charge through the Atiyah-Singer index theorem is ambiguous.

(iii) With the HMC algorithm which is based on a continuous evolution of the gauge links, the system is trapped in a fixed topological sector as the continuum limit is approached. Therefore, a proper sampling of different topological sectors cannot be achieved. (Approaching the chiral limit, the suppression of the fermion determinant for $Q \neq 0$ also makes the tunneling a rare event.)

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However, (iii) remains insurmountable, since the correct sampling of topology becomes increasingly more difficult towards realistic simulation with lighter quarks and finer lattices.

A plausible solution is to perform QCD simulations in a fixed topological sector and to extract χ_t from local topological fluctuations. Then any observable measured at a fixed topological charge can be transcribed to its value in the θ vacuum.

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The χ_t is extracted from the constant behavior of the time-correlation of flavor-singlet pseudo-scalar meson two-point function at large distances, which arises from the finite size effect due to the fixed topology.

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In the small m_q regime, our result of χ_t is proportional to m_q as expected from chiral effective theory. Using the formula $\chi_t = m_q \Sigma / N_f$ by Leutwyler-Smilga, we obtain $\Sigma^{\overline{\rm MS}}(2 \text{ GeV}) = [252(5)(10)\text{MeV}]^3$

Outline

- Introduction
- Topology with Overlap Dirac Operator
- Lattice Setup
- Results using $N_f = 2$ Dynamical Overlap Configurations with $Q_t = 0, -2, -4$
- Conclusion and Outlook

Introduction

Theoretically, topological susceptibility is defined as

$$\chi_t = \int d^4x \, \langle \rho(x) \rho(0) \rangle$$

where

$$\rho(x) = \frac{1}{32\pi^2} \epsilon_{\mu\nu\lambda\sigma} \mathrm{tr}[F_{\mu\nu}(x) F_{\lambda\sigma}(x)]$$

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Leutwyler-Smilga relation

$$\chi_t = \frac{m_q \Sigma}{N_f} + \mathcal{O}(m_q^2) \quad \text{(in the chiral limit)}$$

Introduction (cont)

For lattice QCD with fixed topology in a finite volume, χ_t is the most crucial quantity which is used to relate any observable measured in the fixed topology to its physical value.

Brower, Chandrasekaran, Negele, Wiese, PLB 560 (2003) 64

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In other words, the artifacts due to fixed topology can be removed, provided that χ_t has been determined.

Introduction (cont)

Since

$$\chi_t = \int d^4x \, \langle \rho(x)\rho(0) \rangle = \frac{1}{\Omega} \left\langle Q_t^2 \right\rangle , \quad \Omega = \text{volume}$$

where

$$Q_t = \int d^4x \frac{1}{32\pi^2} \epsilon_{\mu\nu\lambda\sigma} \mathrm{tr}[F_{\mu\nu}(x) F_{\lambda\sigma}(x)] = \text{integer}$$

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one can obtain χ_t by counting the number of gauge configurations for each topological sector.

However, for a set of gauge configurations in the topologically-trivial sector, $Q_t = 0$, it gives $\chi_t = 0$

Introduction (cont)

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Thus, one can investigate whether there are topological excitations within any sub-volumes, and to measure the topological susceptibility using the correlation of the topological charges of two sub-volumes.

Introduction (cont)

For any topological sector with Q_t, using χPT, it can be shown that

$$\lim_{|x-y|\to\infty} \langle \rho(x)\rho(y) \rangle = \frac{1}{\Omega} \left(\frac{Q_t^2}{\Omega} - \chi_t - \frac{c_4}{2\chi_t \Omega} \right) + \mathcal{O}(\Omega^{-3})$$

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Thus, in the trivial sector with $Q_t = 0$, for any two widely separated sub-volumes Ω_1 and Ω_2, the correlation of their topological charges would behave as

$$\langle Q_1 Q_2\rangle \simeq -\frac{\Omega_1\Omega_2}{\Omega}\left(\chi_t + \frac{c_4}{2\chi_t\Omega}\right) \qquad Q_i = \int_{\Omega_i} d^4x\, \rho(x)$$

Introduction (cont)

On a finite lattice, consider two spatial sub-volumes at time slices t_1 and t_2, measure the time-correlation functior

$$C(t_1 - t_2) = \langle Q(t_1) Q(t_2) \rangle = \sum_{\vec{x_1}, \vec{x_2}} \langle \rho(x_1) \rho(x_2) \rangle$$

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Then its plateau at large $|t_1 - t_2|$ can be used to extract χ_t provided that

$$|c_4| \ll 2\chi_t^2 \Omega, \qquad c_4 = -\frac{1}{\Omega} \left[\langle Q_t^4 \rangle_{\theta=0} - 3 \langle Q_t^2 \rangle_{\theta=0}^2 \right]$$

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However, on a lattice, it is difficult to extract $\rho(x)$ unambiguously from the link variables !

Topology with Overlap Dirac Operator

It is well known that the topological charge density can be defined via the overlap Dirac operator as

$$\rho(x) = \mathrm{tr}[\gamma_5(1 - rD)_{x,x}], \quad r = \frac{1}{2m_0}$$

where D is the overlap Dirac operator

$$D = m_0(1+V), \quad V = \gamma_5 \frac{H_w}{\sqrt{H_w^2}},$$

$$H_w = \gamma_5(-m_0 + \gamma_\mu t_\mu + W)$$

Topology with Overlap Dirac Operator (cont)

Here $\rho(x) = \mathrm{tr}[\gamma_5(1 - rD)_{x,x}]$ is justified to be a definition of topological charge density since it has been asserted (Kikukawa & Yamada, 1998)

$$\rho(x) \xrightarrow{a \to 0} \frac{1}{32\pi^2} \epsilon_{\mu\nu\lambda\sigma} \mathrm{tr}[F_{\mu\nu}(x) F_{\lambda\sigma}(x)]$$

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Note that the index theorem on the lattice

$$\text{index}(D) = n_+ - n_- = \sum_x \rho(x) = Q_t$$

had been observed by Narayanan and Neuberger in 1995, using the spectral flow of $H_w(m_0)$, before the Ginsparg-Wilson relation was rejuvenated in 1998.

Topology with Overlap Dirac Operator (cont)

It seems natural to use $\rho(x) = \mathrm{tr}[\gamma_5(1 - rD)_{x,x}]$ to compute the topological susceptibility

$$\chi_t = \frac{1}{\Omega}\langle Q_t^2 \rangle = \frac{1}{\Omega}\sum_{x,y}\langle \rho(x)\rho(y) \rangle = \sum_x \langle \rho(x)\rho(0) \rangle$$

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On the other hand, one can derive the relation

$$\text{index}(D) = m \sum_x \mathrm{tr}[\gamma_5(D_c + m)^{-1}_{x,x}] = m\ \mathrm{Tr}[\gamma_5(D_c + m)^{-1}]$$

where

$$D_c = D(1 - rD)^{-1} = 2m_0(1 + V)(1 - V)^{-1}$$

is chirally symmetric but non-local (Chiu & Zenkin, 1998). Note that for the topologically-trivial configurations , D_c is well-defined (without any poles).

Topology with Overlap Dirac Operator (cont)

Thus one can regard

$$\rho_1(x) = m \, \mathrm{tr}[\gamma_5 (D_c + m)^{-1}_{x,x}]$$

as a definition of topological charge density, for any valence quark mass m.

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Obviously, the identity index$(D) = m \, \mathrm{Tr}[\gamma_5 (D_c + m)^{-1}]$ can be generalized to

$$\text{index}(D) = m_1 m_2 \cdots m_k \mathrm{Tr}[\gamma_5 (D_c + m_1)^{-1} (D_c + m_2)^{-1} \cdots (D_c + m_k)^{-1}]$$

with the generalized topological charge density

$$\rho_k(x) = m_1 m_2 \cdots m_k \mathrm{tr}[\gamma_5 (D_c + m_1)^{-1} (D_c + m_2)^{-1} \cdots (D_c + m_k)^{-1}]_{x,x}$$

Topology with Overlap Dirac Operator (cont)

Presumably, any ρ_k can be used to compute χ_t.

In general,

$$\chi_t = \frac{m_1 \cdots m_k m_{k+1} \cdots m_l}{\Omega} \langle \mathrm{Tr}[\gamma_5 (D_c + m_1)^{-1} \cdots (D_c + m_k)^{-1}] \times \mathrm{Tr}[\gamma_5 (D_c + m_{k+1})^{-1} \cdots (D_c + m_l)^{-1}] \rangle$$

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It has been pointed out by Lüscher, for $k \geq 2$ and $l \geq 5$, χ_t avoids the short-distance singularities in the continuum limit.

Topology with Overlap Dirac Operator (cont)

However, on a finite lattice,

$$\lim_{|x-y|\gg 1} \langle \rho_1(x)\rho_1(y)\rangle \simeq \frac{1}{\Omega}\left(\frac{Q_t^2}{\Omega} - \chi_t - \frac{c_4}{2\chi_t\Omega}\right) + \mathcal{O}(e^{-m_\pi|x-y|})$$
$$+\mathcal{O}(e^{-m_{\eta'}|x-y|}) + \mathcal{O}(\Omega^{-3}) + \cdots$$

is contaminated by m_π, $m_{\eta'}$, $\cdots$, which can couple to $\langle \rho_1(x)\rho_1(y)\rangle$.

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is contaminated by m_π, $m_{\eta'}$, $\cdots$, which can couple to $\langle \rho_1(x)\rho_1(y)\rangle$.

A better alternative is to compute the correlator of flavor-singlet η', which behaves as

$$\lim_{|x-y|\gg 1} m_q^2 \langle \eta'(x)\eta'(y)\rangle \simeq \frac{1}{\Omega}\left(\frac{Q_t^2}{\Omega} - \chi_t - \frac{c_4}{2\chi_t\Omega}\right) + \mathcal{O}(e^{-m_{\eta'}|x-y|}) + \mathcal{O}(\Omega^{-3}) + \cdots$$

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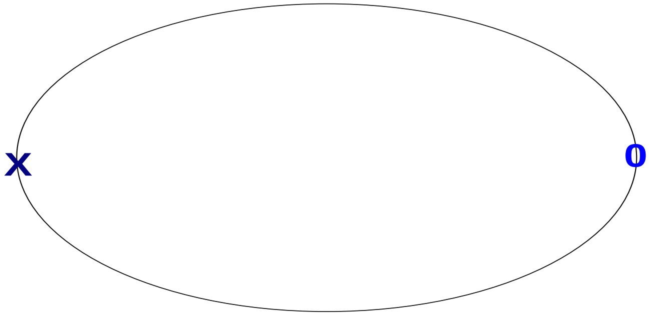
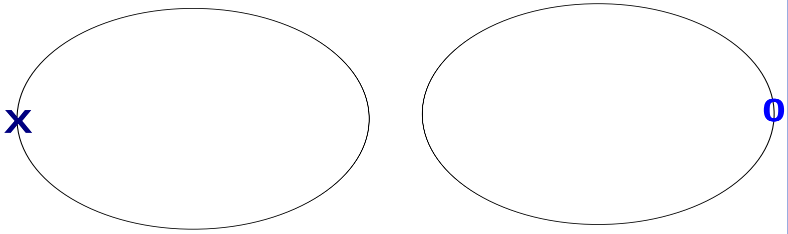
Time-Correlation Function of η'

$$C_{\eta'}(t) = \frac{1}{N_f} \sum_{\vec{x}}$$



$$- \sum_{\vec{x}}$$



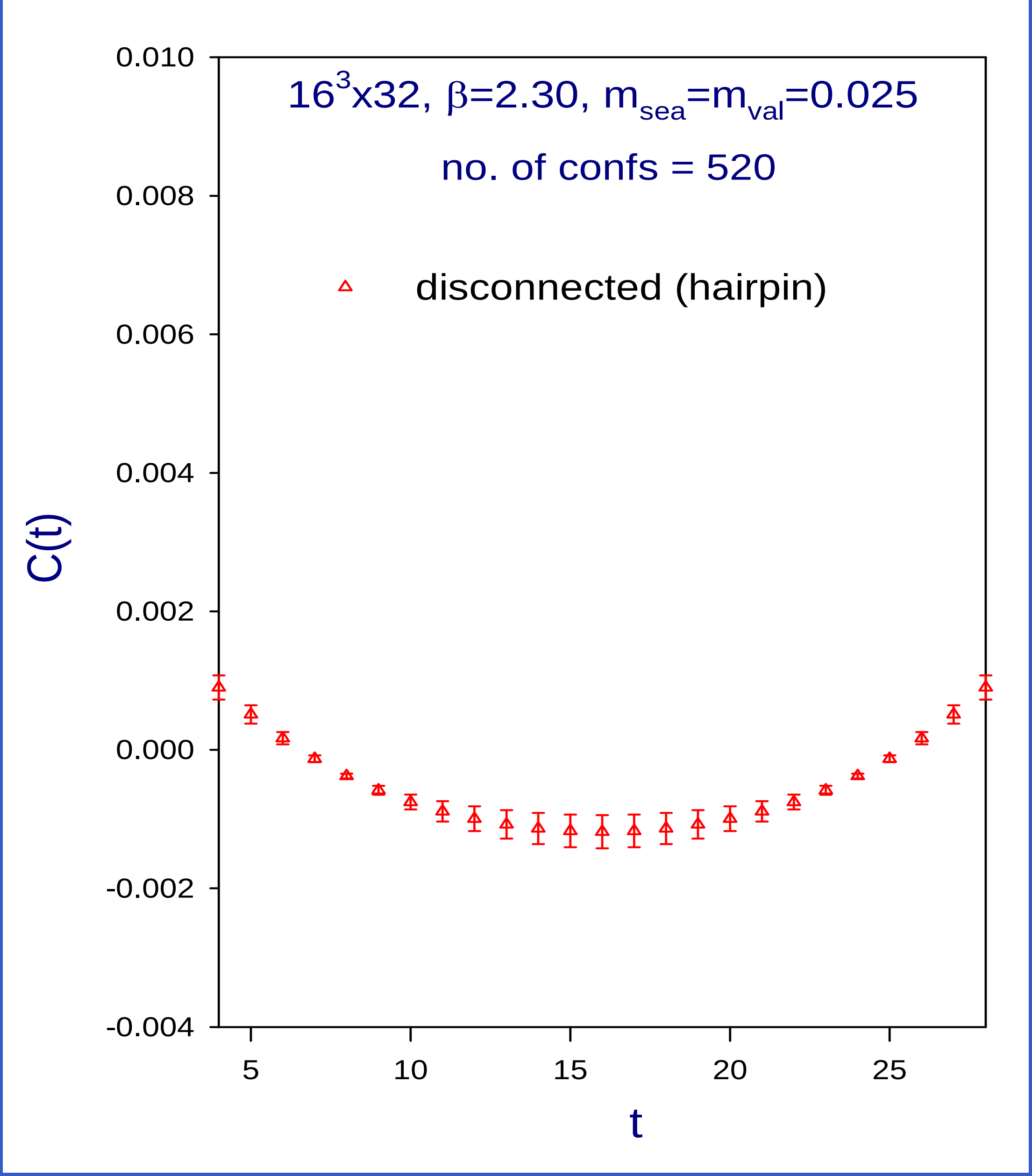
Lattice Setup

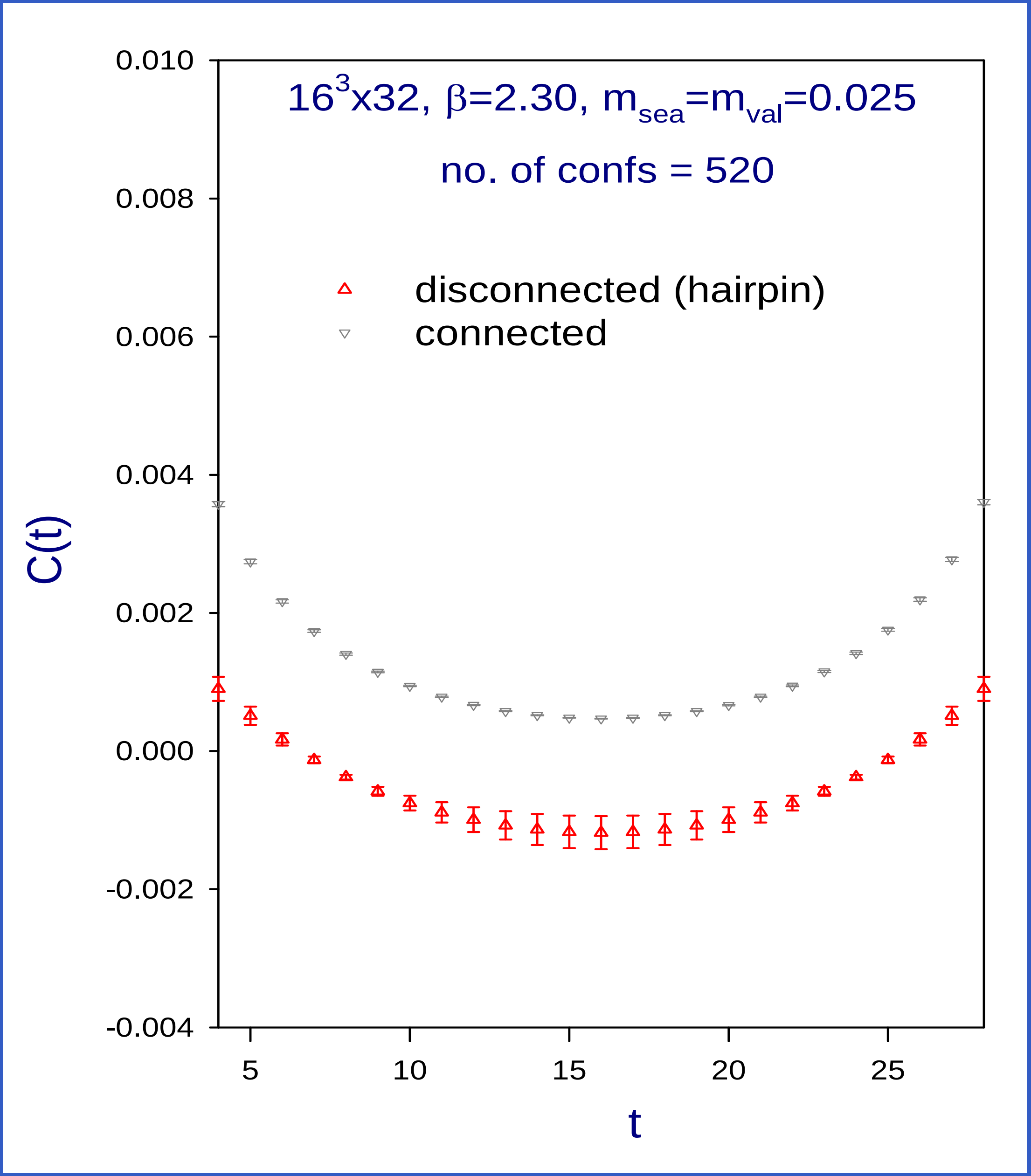
- Lattice size: $16^3 \times 32$
- Gluons: Iwasaki gauge action at $\beta = 2.30$
- Quarks ($N_f = 2$): overlap Dirac operator with $m_0 = 1.6$
- Add extra Wilson fermions and pseudofermions

$$\det(H_{ov}^2) \longrightarrow \det(H_{ov}^2) \frac{\det(H_w^2)}{\det(H_w^2 + \mu^2)}, \quad \mu = 0.2$$

to forbid $\lambda(H_w)$ crossing zero, thus Q_t is invariant.

- Quark masses: m_{sea} = 0.015, 0.025, 0.035, 0.050, 0.070, 0.100, each of 500 confs with $Q_t = 0$. For m_{sea} = 0.05, 250 confs with $Q_t = -2, -4$ respectively.
- For each configuration, 50 conjugate pairs of low-lying eigenmodes of overlap Dirac operator are projected.


16^3x32, β=2.30, m_{sea}=m_{val}=0.025
no. of confs = 520
disconnected (hairpin)
C(t)
t
0.010
0.008
0.006
0.004
0.002
0.000
-0.002
-0.004
5
10
15
20
25



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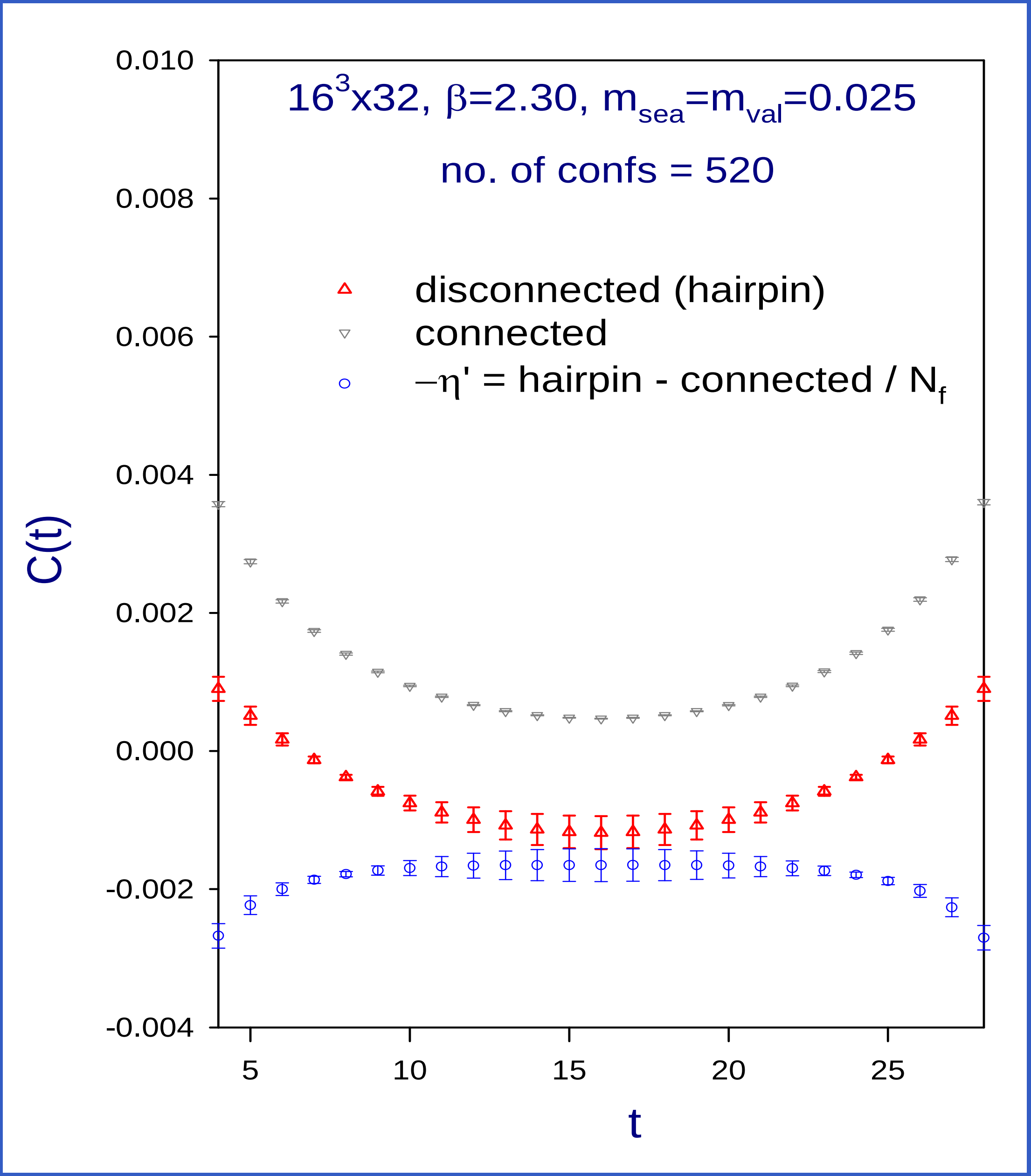

16^3x32, β=2.30, m_{sea}=m_{val}=0.025
no. of confs = 520
disconnected (hairpin)
connected
$-\eta'$ = hairpin - connected / N_f
C(t)
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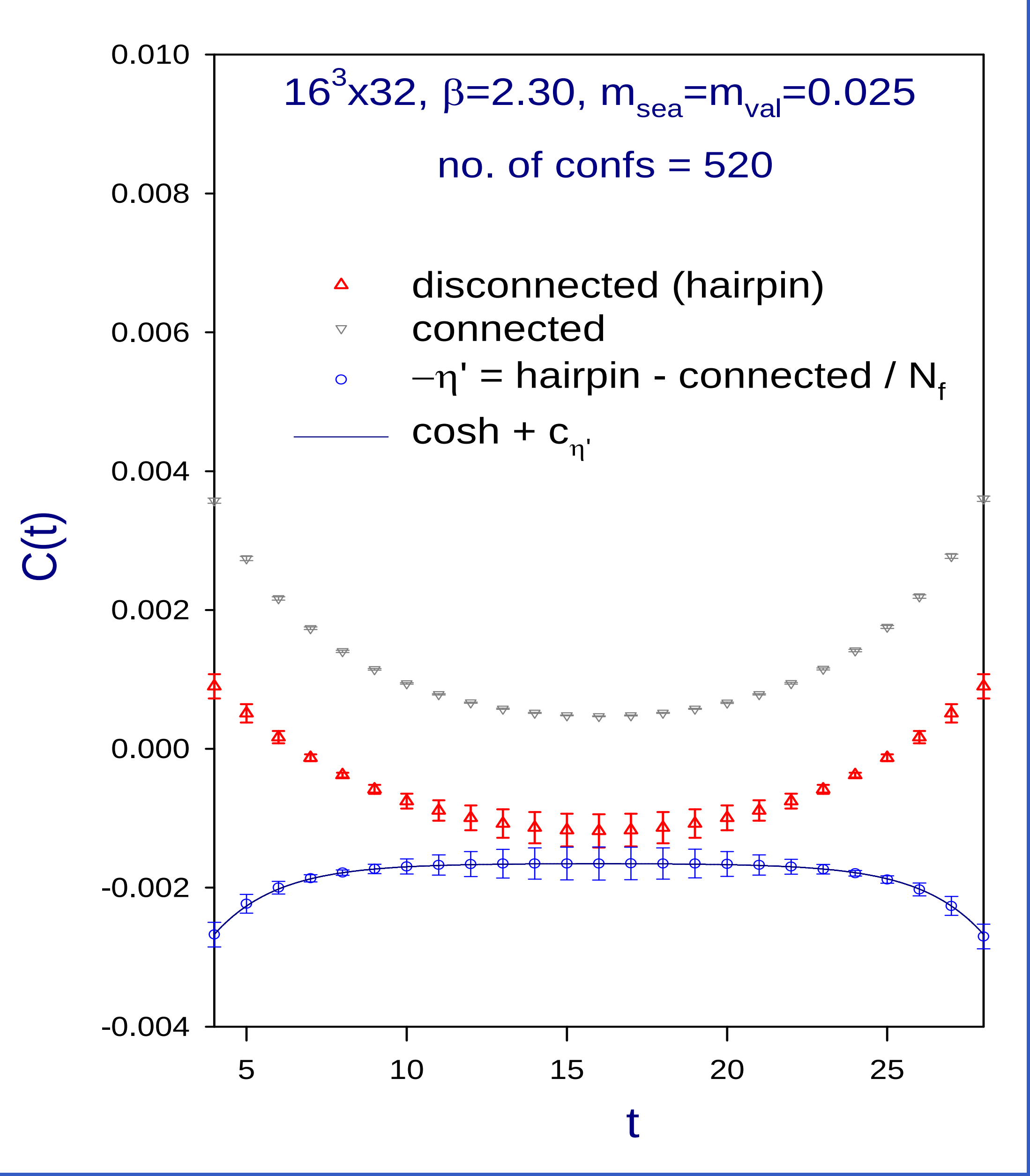

16^3x32, β=2.30, m_{sea}=m_{val}=0.025
no. of confs = 520
disconnected (hairpin)
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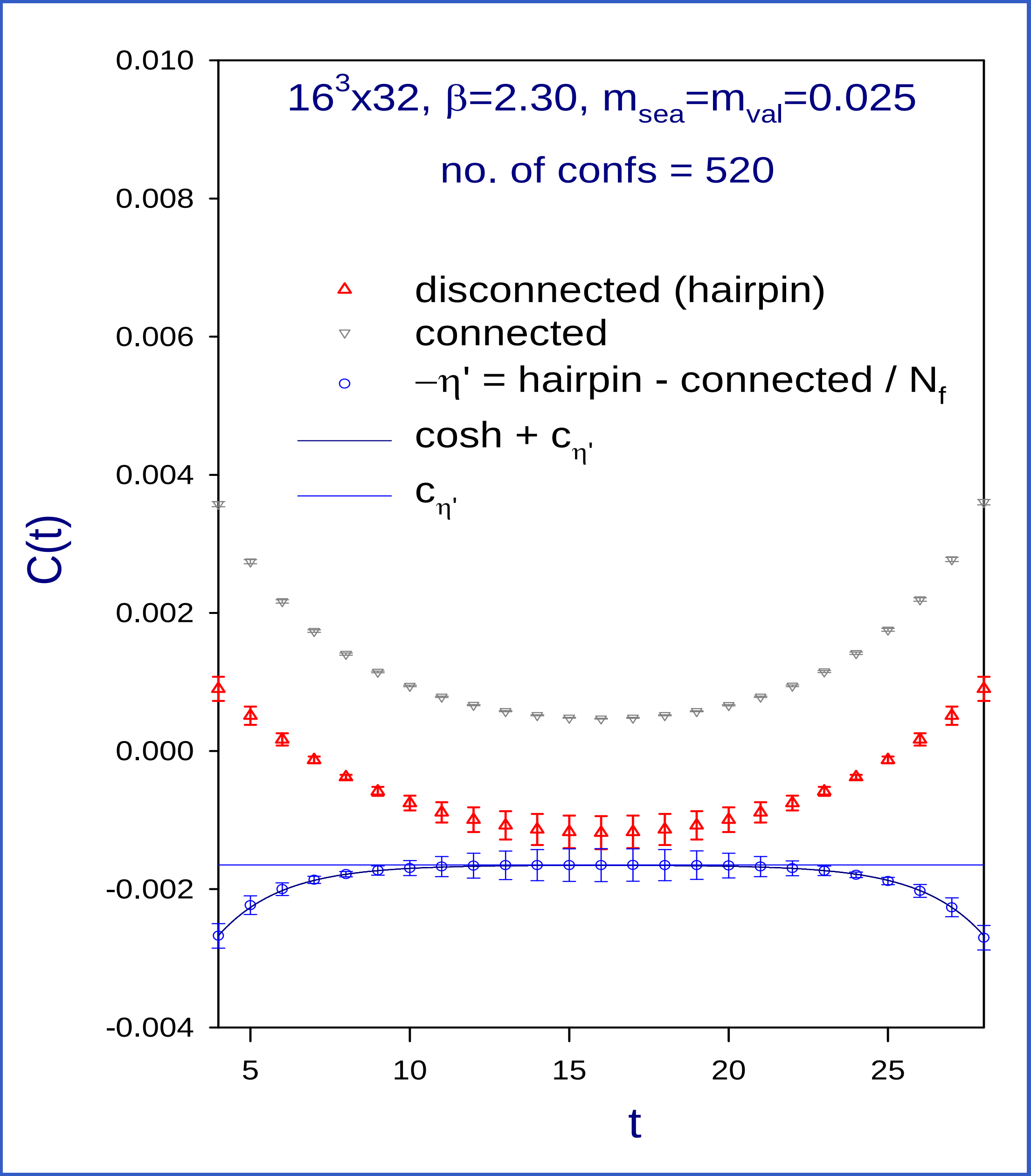
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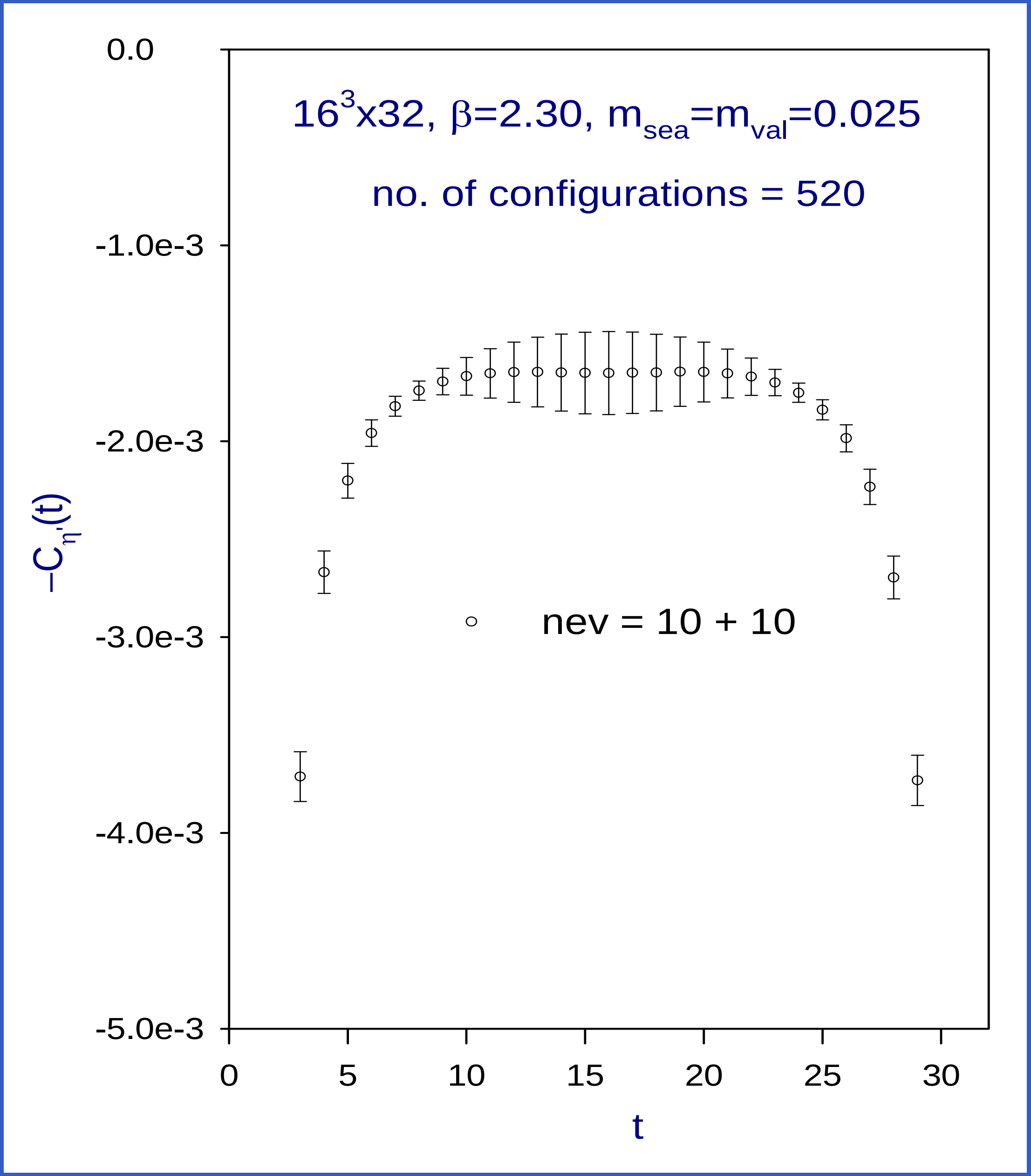
Saturation of η' by low-lying eigenmodes



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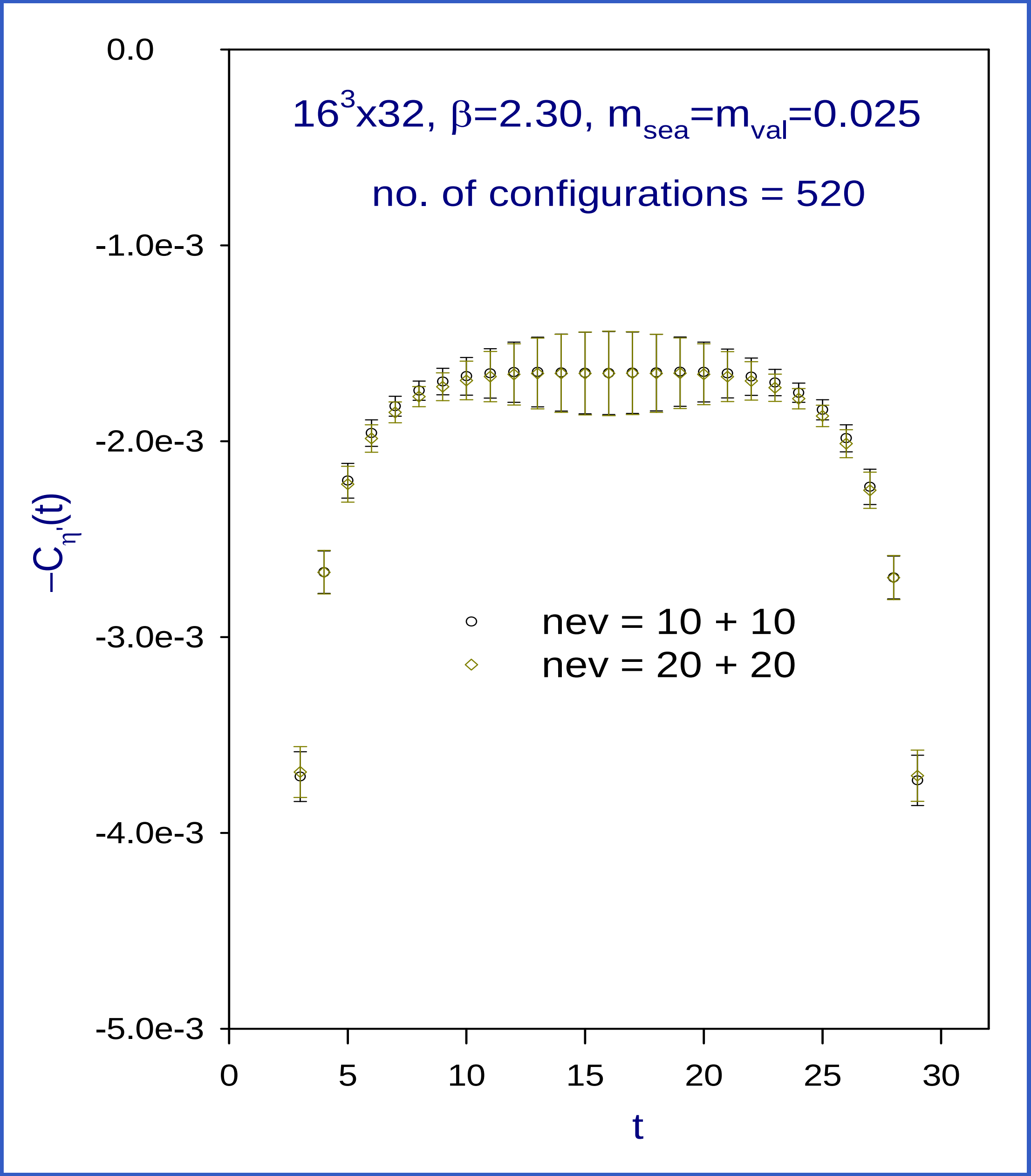

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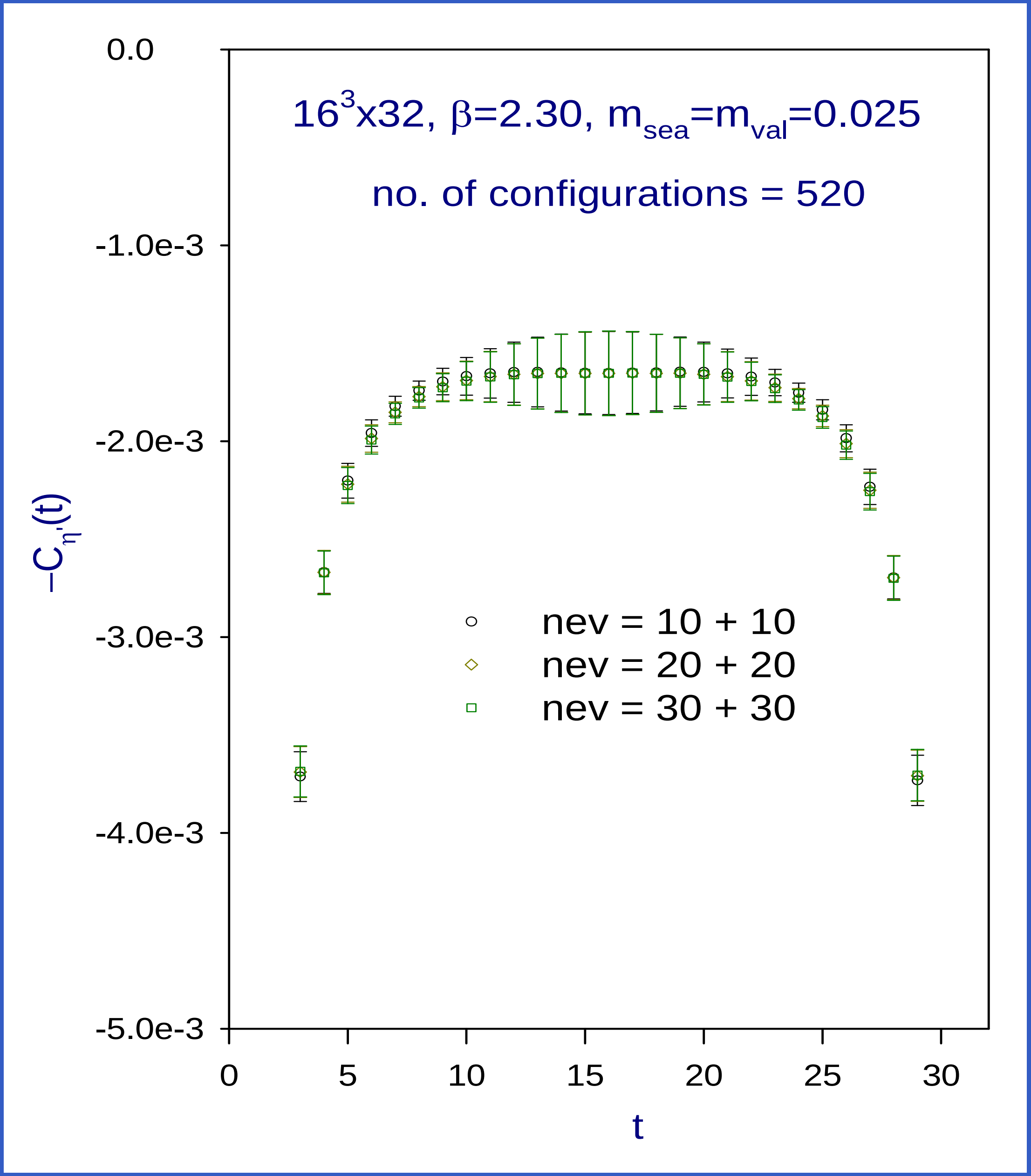

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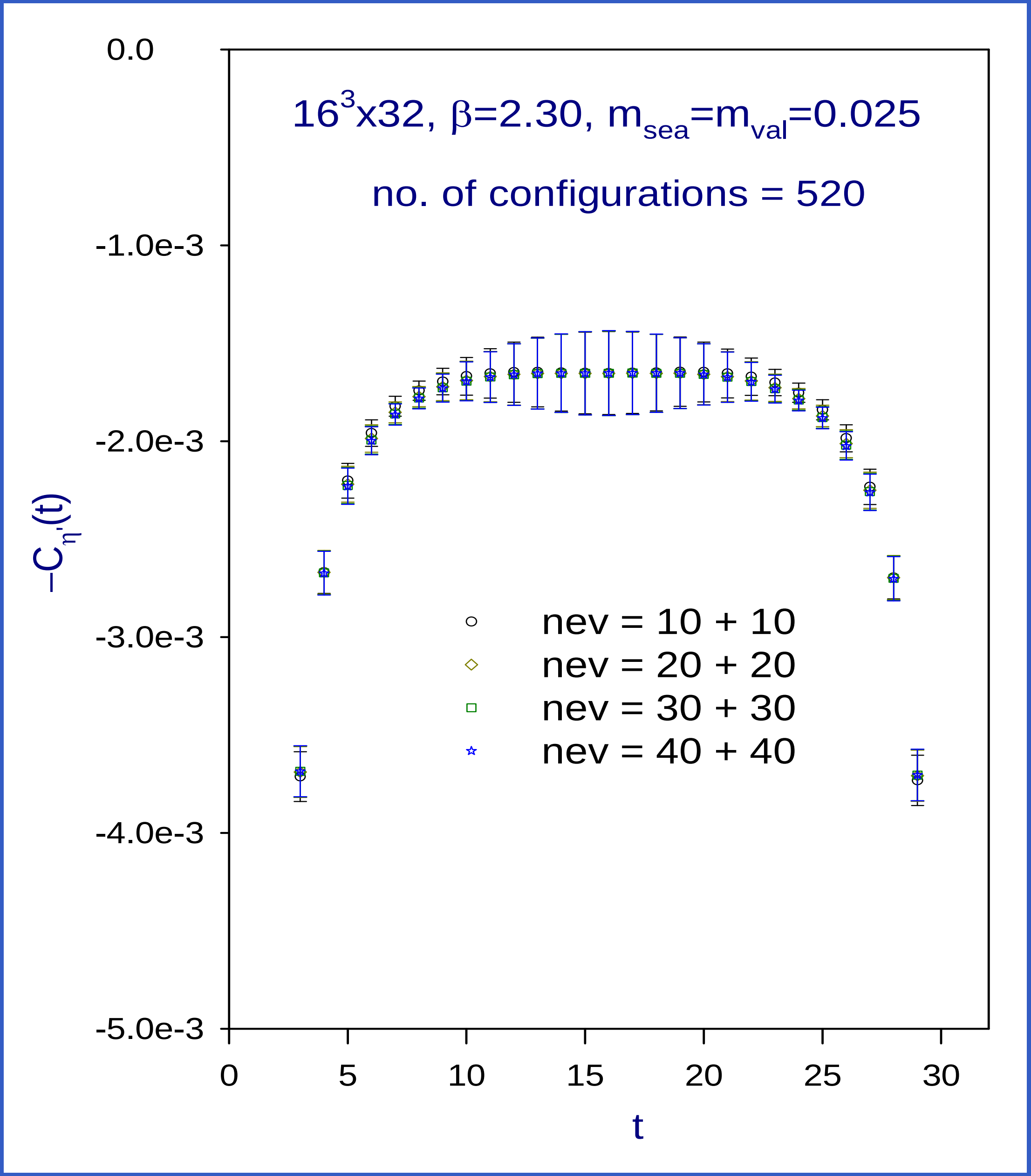

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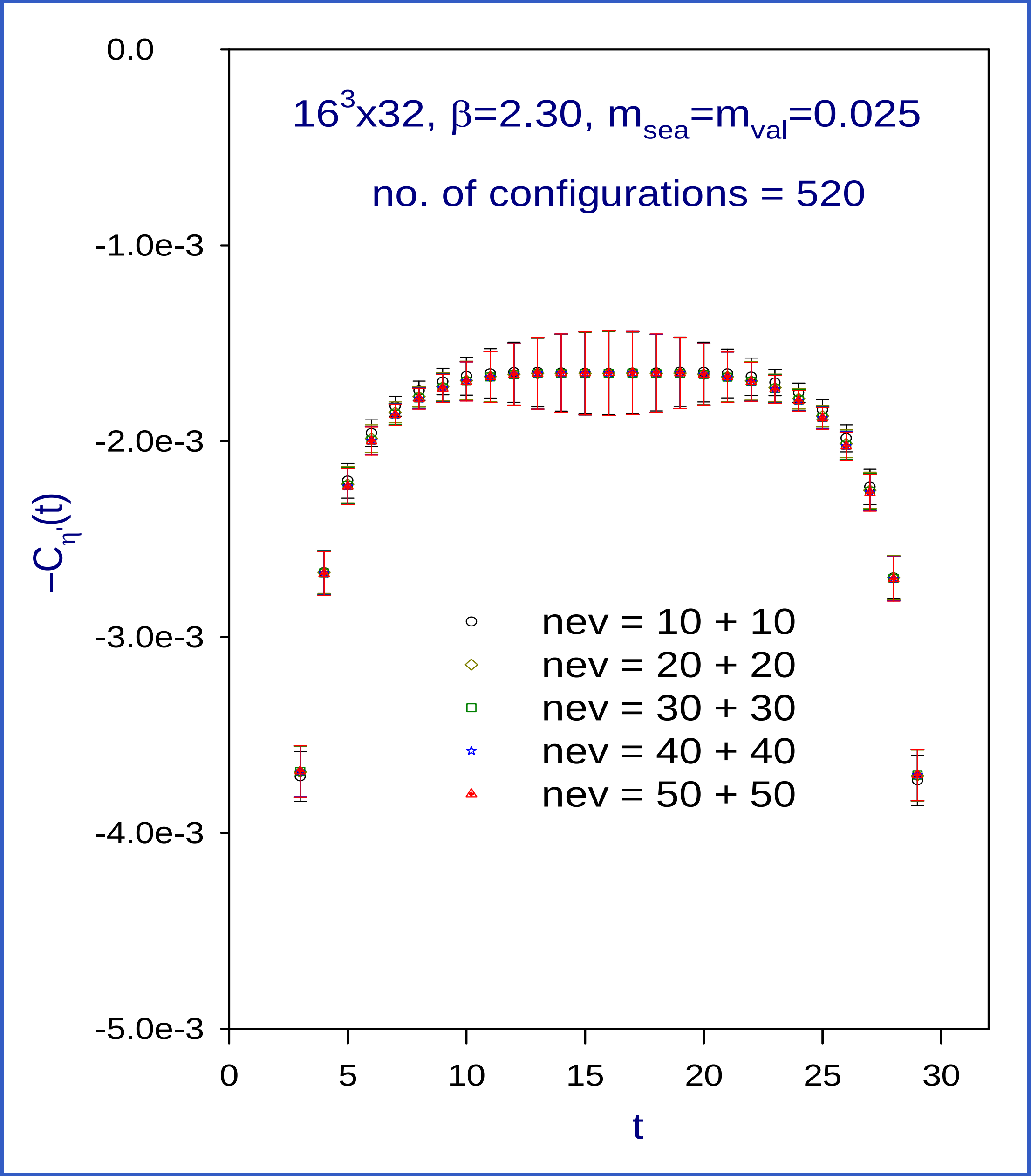
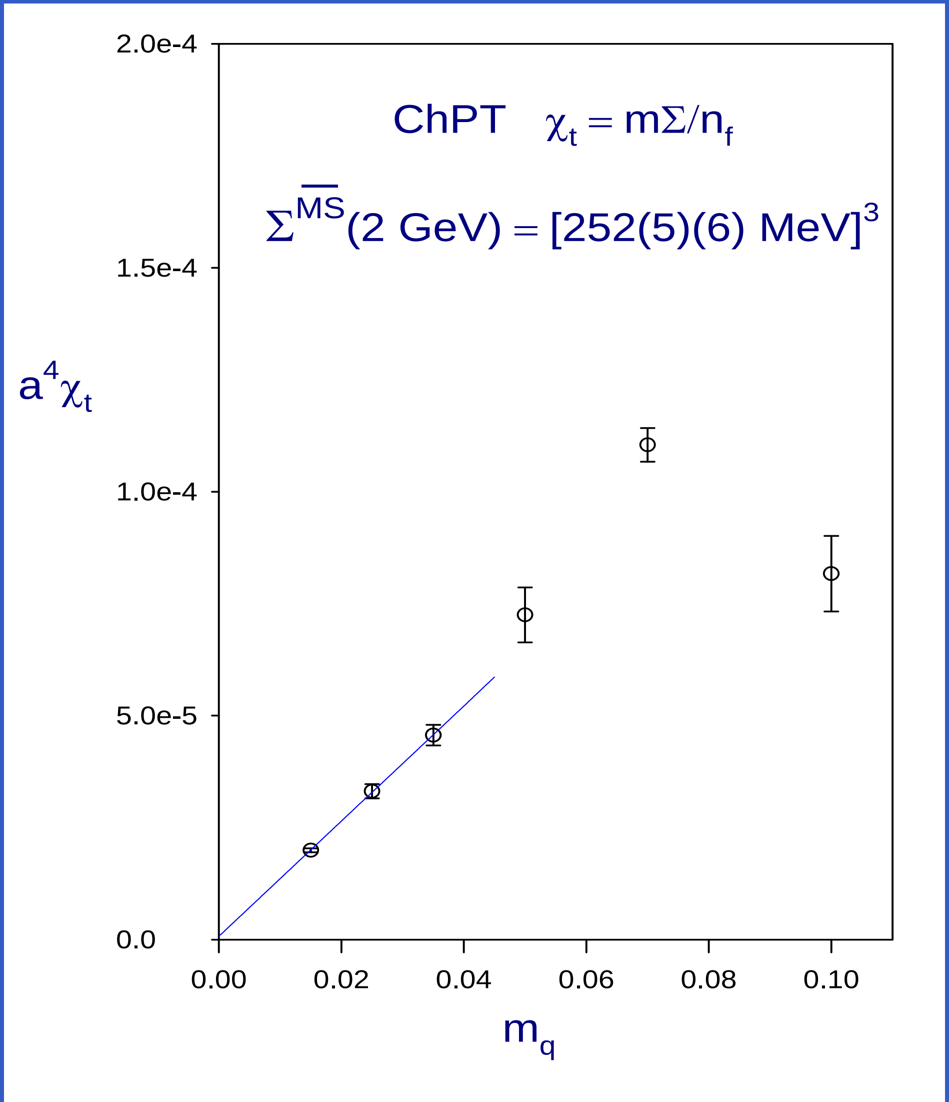


Realization of Leutwyler-Smilga relation



In the limit $m \to 0$, $\chi_t \to m\Sigma/N_f$, in agreement with ChPT.

Determination of Σ

From the slope of the linear fit of χ_t vs. m_q for $m_q a = 0.015, 0.025$, and 0.035, it gives

$$a^3\Sigma = 0.00257(10)$$

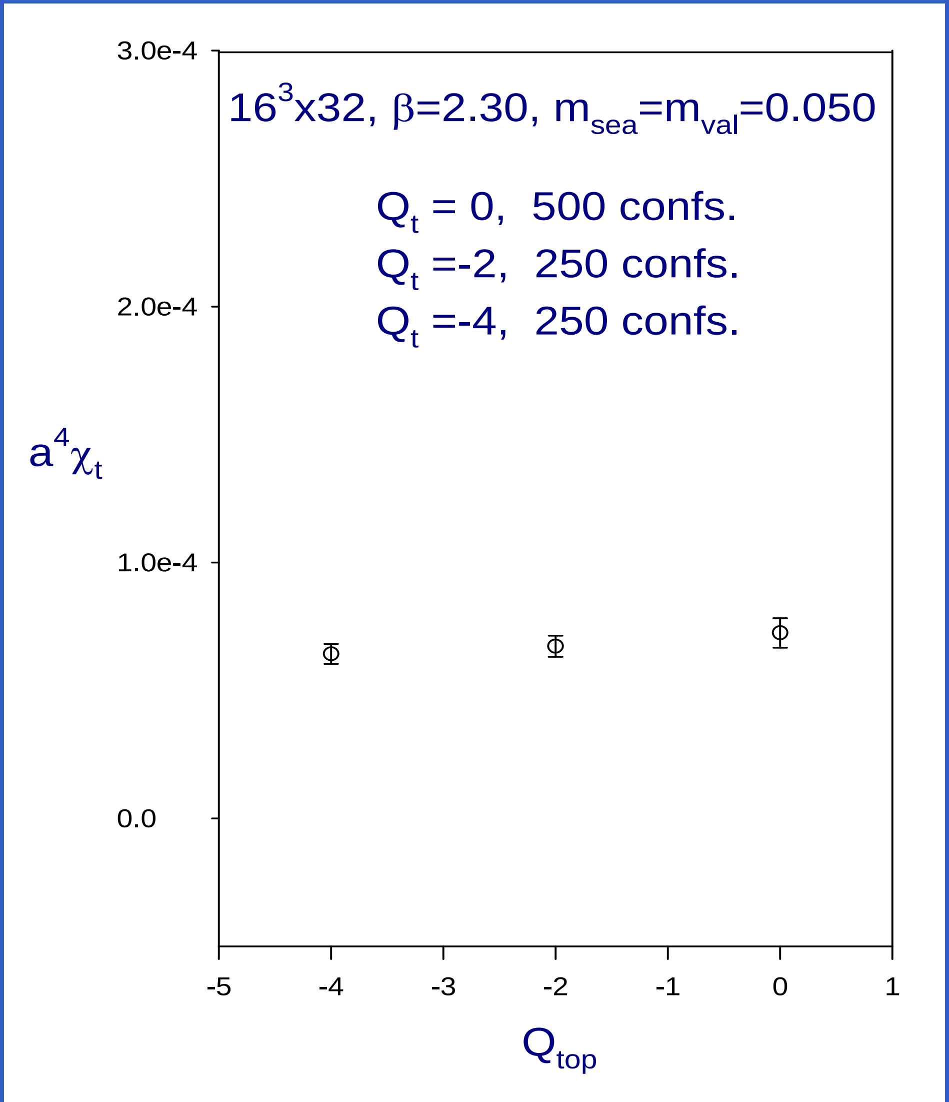
With $a^{-1} = 1670(20)(20)$ MeV, and $Z_m^{\overline{MS}}(2\text{ GeV}) = 0.742(12)$, the value of $a^3\Sigma$ is transcribed to

$$\Sigma^{\overline{MS}}(2\text{ GeV}) = (252 \pm 5 \pm 10\text{ MeV})^3$$

in good agreement with our previous result $251(7)(11)$ MeV obtained in the ϵ-regime.

H. Fukaya et al. (JLQCD-TWQCD) PRL 98 (2007) 172001; PRD 76 (2007) 054503

Universality of χ_t for different Topological Sectors



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- It remains to obtain an upper bound of c_4 (from 2-pt and 4-pt correl. fn.) to see whether $|c_4| \ll 2\chi_t^2\Omega$ is satisfied.